Six-quark model for the suppression of $\Delta S = 1$ neutral currents*

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(Received 9 June 1975)

We propose an alternative arising naturally in a six-quark model to the Glashow-Iliopoulos-Maiani mechanism for the suppression of $\Delta S = 1$ neutral currents. While violating the Cabibbo form of universality, it maintains the $\tan^2\theta_C$ suppression of strange particle decays and introduces an additional $\tan\theta_C$ suppression of charmed-particle decays.

It has been suggested recently that the color interactions which are presumed to be responsible for the confinement of quarks may have an algebraic origin. In this view, the SU(3) of color is considered to be the maximal subgroup of the automorphism group G_2 of the octonion algebra. Octonions, like quaternions, are generalizations of complex numbers. The connection between quark structure and octonions provides a way in which the number of quark colors can be predicted correctly. Therefore one might also rely on it to predict the number of quark flavors. This is done through a unification of colors and flavors at a fundamental level resting, as remarked before,[1] on the identification of charge space with exceptional observables of Jordan, von Neumann, and Wigner,[2] which are 3×3 Hermitean octonionic matrices.

It is well known that the orthogonal, unitary, and symplectic Lie groups can be generated by anti-Hermitean matrices whose matrix elements are respectively real, complex, and quaternionic numbers. To the extent that octonions are the remaining generalization of complex numbers, we may expect to find a similar way to generate Lie groups with octonions. However, octonions have a nonassociative algebra, so that such a simple construction will not in general yield the required Jacobi identities. However, owing to a remarkable property of the octonions called "triality," it is still possible to build a selected set of Lie groups, the so-called exceptional Lie groups. For example, G_2 and F_4 are constructed as the automorphism groups of (respectively) the octonions and the Jordan exceptional observables. There are five exceptional Lie groups, listed here with their relevant maximal subgroups:

G_2 $SU^c(3)$

F_4 $SU(3)\otimes SU^c(3)$

E_6 $SU(3)\otimes SU(3)\otimes SU^c(3)$

E_7 $SU(6)\otimes SU^c(3)$

E_8 $E_6\otimes SU^c(3)$.

In each maximal subgroup, we interpret one SU(3) to be the unbroken SU(3) of color. The remainder is interpreted to be the symmetry group of quark flavors. It is thus inferred that if there are more than three flavors—as indicated by the recent discovery of new particles[3] and the rise in $R=\sigma(e^+e^-\to \text{hadrons})/\sigma(e^+e^-\to\mu^+\mu^-)$ (see Ref. 4)—there ought to be six.[5] Let us call them $\mathcal{P}, \mathfrak{N}, \lambda, \mathcal{P}', \mathfrak{N}', \lambda'$. We assign to them the charges $\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}$. As a consequence, the electric charge is among the SU(6) of flavor, and the R ratio is predicted to equal 4.

We now wish to present an alternative to the Glashow-Iliopoulos-Maiani mechanism[6]—operating within the four-quark model based on SU(4) (see Ref. 7)—for the suppression of the weak $\Delta S = 1$ neutral currents. Our alternative fits in harmoniously with the doubling of the quarks. We propose that the charged weak current be[8]

$$j_\mu^+(x) = \bar{\mathcal{P}}(\alpha)\gamma_\mu \frac{1+\gamma_5}{2}\mathfrak{N}(\alpha) + \bar{\mathcal{P}}'(\alpha)\gamma_\mu\frac{1+\gamma_5}{2}\lambda'(\alpha), \quad (1)$$

where

$$\mathcal{P}(\alpha) = \mathcal{P}\cos\tfrac{1}{2}\alpha + \mathcal{P}'\sin\tfrac{1}{2}\alpha,$$
$$\mathcal{P}'(\alpha) = -\mathcal{P}\sin\tfrac{1}{2}\alpha + \mathcal{P}'\cos\tfrac{1}{2}\alpha \quad (2)$$

and similarly for $\mathfrak{N}(\alpha)$, $\mathfrak{N}'(\alpha)$, $\lambda(\alpha)$, and $\lambda'(\alpha)$. Explicitly,

$$j_\mu^+(x) = \cos^2\tfrac{1}{2}\alpha\,\bar{\mathcal{P}}\gamma_\mu\frac{1+\gamma_5}{2}\mathfrak{N} + \sin^2\tfrac{1}{2}\alpha\,\bar{\mathcal{P}}\gamma_\mu\frac{1+\gamma_5}{2}\lambda + \sin^2\tfrac{1}{2}\alpha\,\bar{\mathcal{P}}'\gamma_\mu\frac{1+\gamma_5}{2}\mathfrak{N}' + \cos^2\tfrac{1}{2}\alpha\,\bar{\mathcal{P}}'\gamma_\mu\frac{1+\gamma_5}{2}\lambda'$$
$$+\sin\tfrac{1}{2}\alpha\cos\tfrac{1}{2}\alpha\left(\bar{\mathcal{P}}\gamma_\mu\frac{1+\gamma_5}{2}\mathfrak{N} + \bar{\mathcal{P}}\gamma_\mu\frac{1+\gamma_5}{2}\mathfrak{N}' - \bar{\mathcal{P}}'\gamma_\mu\frac{1+\gamma_5}{2}\lambda - \bar{\mathcal{P}}\gamma_\mu\frac{1+\gamma_5}{2}\lambda'\right). \quad (3)$$

The corresponding neutral current,

$$j_\mu^3(x) = \tfrac{1}{2}\int d^3y [j_0^+(y), j_\mu^-(x)]$$

$$= \tfrac{1}{2}\left(\bar{\mathcal{P}}(\alpha)\gamma_\mu \frac{1+\gamma_5}{2}\mathcal{P}(\alpha) - \bar{\mathcal{N}}(\alpha)\gamma_\mu \frac{1+\gamma_5}{2}\mathcal{N}(\alpha)\right) + \tfrac{1}{2}\left(\bar{\mathcal{P}}'(\alpha)\gamma_\mu \frac{1+\gamma_5}{2}\mathcal{P}'(\alpha) - \bar{\lambda}'(\alpha)\gamma_\mu \frac{1+\gamma_5}{2}\lambda'(\alpha)\right), \quad (4)$$

has no strangeness-changing piece.

Because in spontaneously broken gauge theories, the lightest vector bosons are coupled to the generators of the least-broken symmetries, we feel that it is somewhat more natural that the W boson couple to $\mathcal{P}^\dagger\eta$ and $\mathcal{P}'^\dagger\lambda'$ (before the α rotation) than to $\mathcal{P}^\dagger\eta$ and $\mathcal{P}'^\dagger\lambda$ as it does in the Glashow-Iliopoulos-Maiani model (before the Cabibbo rotation) since $\mathcal{P}'$ and λ' have presumably comparable masses while $\mathcal{P}'$ and λ do not.

Equation (3) shows that we must have[9]

$$\tan^2 \tfrac{1}{2}\alpha = \tan\theta_C = 0.242 \quad \text{or} \quad \alpha = 52^\circ. \quad (5)$$

In order to preserve the universality of the weak interactions, the α rotation must also be applied to the leptons. However, this is automatic in gauge theories based on exceptional groups, because in the latter the leptons and the quarks are placed in the same multiplets. For instance, in the E_7 scheme, the leptons and the quarks are united in the $\underline{56}$ multiplet of E_7, which has the $SU(6)\otimes SU^c(3)$ decomposition

$$\underline{56} = (\underline{20}, \underline{1}) + (\underline{6}, \underline{3}) + (\underline{\bar{6}}, \underline{\bar{3}}). \quad (6)$$

This multiplet contains six quarks and their antiparticles plus ten leptons and their antiparticles. α is the relative angle in SU(6) charge space between the direction in which the vector-boson mass matrix is diagonal and the direction in which the $\underline{56}$-plet mass matrix is diagonal, and is thus the same angle for the leptons as for the quarks. Consequently, we find for the total weak charged current

$$j_\mu^+ = \cos\tfrac{1}{2}\alpha\left(\bar{\nu}\gamma_\mu \frac{1+\gamma_5}{2} e + \bar{\nu}'\gamma_\mu \frac{1+\gamma_5}{2}\mu + \bar{\mathcal{P}}\gamma_\mu \frac{1+\gamma_5}{2}\mathcal{N} + \tan^2\tfrac{1}{2}\alpha\,\bar{\mathcal{P}}\gamma_\mu \frac{1+\gamma_5}{2}\lambda + \dots\right), \quad (7)$$

where the dots represent pieces containing charmed-quarks or heavy leptons.

Besides the over-all $\cos^2\frac{1}{2}\alpha$ factor, Eq. (7) still differs from Cabibbo theory by the absence of a $\cos\theta_C$ factor for the hadronic terms. The bare coupling constants for β decay and muon decay are predicted to be equal. However, the radiative corrections to these coupling constants are of the same order of magnitude as the correction due to the $\cos\theta_C$ factor, and thus need to be calculated[10] before comparison with experiment is possible. This calculation is quite difficult in our model[10,11] owing to the presence in it of positively charged heavy leptons.[11] In a gauge theory, the over-all $\cos^2\frac{1}{2}\alpha$ factor merely reduces the mass of the intermediate W-boson.

Finally, we see from Eq. (3) that, while our current duplicates the effect of the Cabibbo suppression by means of a factor $\tan^2\frac{1}{2}\alpha$, it also predicts a somewhat lesser suppression of the weak decay of charmed particles by a factor of $\tan\frac{1}{2}\alpha$. Furthermore, here charmed particles do not decay preferentially into strange particles. These features may help explain the absence of charmed-particle production and decays in recent experiments.[12]

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*Research supported by the U. S. Energy Research and Development Administration under Contract No. AT(11-1)3075.

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