Some meson coupling constants in broken SU(4) and the radiative decays of ψ and η_c†

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We calculate vector-pseudoscalar-pseudoscalar and vector-vector-pseudoscalar coupling constants in broken SU(4) using current algebra and partial conservation of the axial-vector current. As applications of our results, estimates of the decay widths $\Gamma(\psi \to X^0\gamma)$, $\Gamma(\psi \to \eta_c\gamma)$, and $\Gamma(\eta_c \to 2\gamma)$ are presented.

Following the experimental discovery of the narrow resonances $\psi(3105)$ and $\psi'(3700)$ at SLAC, Brookhaven, and elsewhere,[1] a major effort is afoot to understand the nature of these particles. Among the host of proposals put forth so far, the scheme in which ψ and ψ' are considered as bound states of charmed and anticharmed quarks is most promising.[2,3] An integral part of such considerations is the search for a group-theoretic classification of the newly discovered particles among the hadrons. Actually even before the discovery of ψ and ψ', Gaillard, Lee, and Rosner,[4] in a remarkable paper, considered the consequences of the existence of charmed objects which would fall into suitable representations of SU(4). In analogy to the ϕ meson (which is almost entirely built from a strange quark and its antiparticle), one would expect the existence of a vector meson ϕ_c which is built from a $c\bar{c}$ pair, c denoting a charmed quark. Such a vector meson would belong to the representation $1\oplus 15$ of SU(4). Recently Borchardt *et al.*[2] have independently suggested the same representation for ψ and worked out the masses of various particles which are encountered in an SU(4) classification scheme. The symmetry-breaking Hamiltonian is assumed to be a sum of scalar densities. One of these transforms as the eighth component of the regular representation of SU(4) and the other as the fifteenth component. In this paper we present a calculation of the vector-pseudoscalar-pseudoscalar (VPP) and vector-vector-pseudoscalar (VVP) coupling constants by making use of the techniques of current algebra, partial conservation of the axial-vector current (PCAC), and the aforementioned form of the SU(4) symmetry-breaking Hamiltonian. Since the SU(4) symmetry must be rather badly broken in nature, it is all the more important that one obtain a relatively more realistic estimate of these coupling constants which one expects to encounter frequently, for example, in the calculation of decay rates. As expected, we find large deviations from the SU(4) symmetric values. As examples of the use of our results, we estimate the $\eta_c \to 2\gamma$, $\psi \to X^0\gamma$, and $\psi \to \eta_c\gamma$ decay widths.

We write the total Hamiltonian as $\mathcal{H}=\mathcal{H}_0+\mathcal{H}'$, where $\mathcal{H}_0=a_0u^0$ and $\mathcal{H}'=a_8u^8+a_{15}u^{15}$. Here $\mathcal{H}_0$ is invariant under SU(4), but $\mathcal{H}'$ breaks the symmetry. Consider the process $V_i(p)\to P_j(q)+P_k(q')$, where i,j,k are the SU(4) indices and p, q, and q' the momenta. In the standard fashion

$$\lim_{q\to 0}\,[2q_0(2\pi)^3]^{1/2}\langle P_j(q), P_k(q')|\mathcal{H}|V_i(p)\rangle = if_j^{-1}\left(\frac{a_0}{\sqrt{2}}f_{jki}+a_8d_{j8l}f_{lki}+a_{15}d_{j15l}f_{lki}\right)G\ , \tag{1}$$

where f_j is the decay constant for the pseudoscalar P_j and G is the SU(4) reduced matrix element (which, however, contains the Lorentz structure of the matrix element). It follows that the VPP interaction Lagrangian in the case of broken SU(4) is given by

$$\mathcal{L}_{\rm int}(VPP)=iG_0\,{\rm Tr}(P\overleftrightarrow{\partial}_\mu PV^\mu)+iG_8\,{\rm Tr}(\{P,\lambda_l\}\lambda_8)\overleftrightarrow{\partial}_\mu{\rm Tr}([P,\lambda_l]V^\mu)+iG_{15}\,{\rm Tr}(\{P,\lambda_l\}\lambda_{15})\overleftrightarrow{\partial}_\mu{\rm Tr}([P,\lambda_l]V^\mu)\ . \tag{2}$$

Here P and V are the 4×4 pseudoscalar- and vector-meson matrices of SU(4). In writing (2), we have set

$$f_\pi=f_K=f_\eta=f_D=f_F=f_{\eta_c'}\ . \tag{3}$$

This result follows if one extends the Gell-Mann–Oakes–Renner[5] treatment to the breaking of SU(4); it therefore fits in naturally in the present discussion. In other words, if the SU(4) breaking is not spontaneous, the above result should hold to a good approximation. The particle notation used here follows that used in the paper of Gaillard, Lee and Rosner. The λ_l are the 4×4 SU(4) matrices.

After a somewhat tedious calculation, one obtains the expressions of the various coupling constants in terms of the three parameters G_0, G_8, and G_{15}. To determine these parameters we use the experimental information on the widths of ρ, K^*, and ϕ. Thus

$$\frac{1}{\sqrt{2}} G_0' - 4(\tfrac{2}{3})^{1/2} G_8' - \frac{4}{\sqrt{3}} G_{15}' = 0.849\,, \tag{4}$$

$$\frac{1}{\sqrt{2}} G_0' - (\tfrac{2}{3})^{1/2} G_8' - \frac{4}{\sqrt{3}} G_{15}' = 0.910\,, \tag{5}$$

$$G_0' + \frac{4}{\sqrt{3}} G_8' - 4(\tfrac{2}{3})^{1/2} G_{15}' = 1.251\,, \tag{6}$$

where $G_i' = G_i/\sqrt{4\pi}$. Unfortunately Eqs. (4) through (6) do not determine G_0' and G_{15}' separately but only a certain combination of them. From Eqs. (4) and (5), we find that $G_8' = 0.022$ and $G_0' - 4(\frac{2}{3})^{1/2} G_{15}' = 1.304$. To disentangle G_0' and G_{15}', we use the result of Borchardt *et al.*,[4] according to which the mass formula determines the ratio of G_{15}' to G_8':

$$G_{15}' = 21.61 G_8'\,. \tag{7}$$

Using this, $G_0' = 2.878$ and $G_{15}' = 0.482$. Actually, we may obtain two more solutions for the G_i': one by solving Eqs. (5), (6), and (7), and the other by solving (4), (6), and (7). The first one gives comparable values for G_0' and G_{15}' and is unlikely on physical grounds. The second one gives $G_0' = 1.779$, $G_8' = 0.007$, and $G_{15}' = 0.151$. For both sets of solutions G_8' is much smaller than G_0' and G_{15}' and may be dropped without introducing too much inaccuracy. There result the relations

$$\begin{aligned}
&g_{\rho\pi\pi} = g_{\rho KK} = g_{K^*K\pi} = \frac{1}{\sqrt{3}} g_{K^*K\eta} = g_{\omega KK} = -\frac{1}{\sqrt{2}} g_{\phi KK} = -\frac{1}{\sqrt{2}} G_0 + \frac{4}{\sqrt{3}} G_{15}\,,\\
&g_{K^*DF} = -\sqrt{2} g_{\rho DD} = -\sqrt{2} g_{\omega DD} = g_{\phi_c DD} = -g_{\phi FF} = g_{\phi_c FF} = G_0 + 4(\tfrac{2}{3})^{1/2} G_{15}\,,\\
&g_{D^*D\pi} = \sqrt{3} g_{D^*D\eta} = \frac{-1}{\sqrt{2}} g_{D^*KF} = \frac{-1}{\sqrt{2}} g_{F^*DK} = -\frac{\sqrt{3}}{2} g_{F^*F\eta} = -\frac{1}{\sqrt{2}} G_0\,,\\
&g_{D^*D\eta_c'} = -g_{F^*F\eta_c'} = \frac{2}{\sqrt{3}} G_0 + 4\sqrt{2} G_{15}\,,\\
&g_{D^*D\eta'} = g_{F^*F\eta'} = 4(\tfrac{2}{3})^{1/2} G_{15}\,.
\end{aligned} \tag{8}$$

Following a procedure similar to the one discussed earlier, the interaction Lagrangian for the VVP interactions is

$$\mathcal{L}_{\text{int}}(VVP) = \epsilon^{\mu\nu\lambda\sigma}[g_0 \mathrm{Tr}(P\partial_\mu V_\nu \partial_\lambda V_\sigma) + g_8 \mathrm{Tr}(\{P,\lambda_l\}\lambda_8)\mathrm{Tr}(\partial_\mu V_\nu \partial_\lambda V_\sigma \lambda_l) + g_{15}\mathrm{Tr}(\{P,\lambda_l\}\lambda_{15})\mathrm{Tr}(\partial_\mu V_\nu \partial_\lambda V_\sigma \lambda_l)]\,. \tag{9}$$

Thus all the VVP-type coupling constants are linear combinations of the three parameters g_0, g_8, and g_{15}. Nevertheless, it is obvious that

$$g_0 : g_8 : g_{15} :: G_0 : G_8 : G_{15}\,.$$

To fix the normalization, we use the value of the $\omega\rho\pi$ coupling constant determined from the $\omega \to 3\pi$ experimental decay width and the use of a Gell-Mann–Sharp–Wagner[6] model. Accordingly, we use the number

$$(g_{\omega\rho\pi}{}^2/4\pi) = 0.455/m_\pi{}^2$$

in the following calculations. Corresponding to the solutions given above for VPP we obtain $g_0' = 0.37/m_\pi$, $g_8' = 0.003/m_\pi$, $g_{15}' = 0.06/m_\pi$ and $g_0' = 0.42/m_\pi$, $g_8' = 0.002/m_\pi$, $g_{15}' = 0.04/m_\pi$, respectively. Not surprisingly, g_8 is quite small compared to g_0 and g_{15}. Even if we did not drop g_8, the number of independent coupling constants would be much smaller here than in the case of VPP. With g_8 ignored, the economy is drastic:

$$\begin{aligned}
g_{\omega\rho\pi} &= 2g_{K^*K^*\pi} = 2\sqrt{3} g_{\rho\rho\eta} = 2\sqrt{3} g_{\omega\omega\eta} = 2\sqrt{3} g_{D^*D^*\eta} = 2\sqrt{2} g_{\rho\rho\eta'} = 2\sqrt{2} g_{\omega\omega\eta'} = 2\sqrt{6} g_{\rho\rho\eta_c'} = 2\sqrt{6} g_{\omega\omega\eta_c'}\\
&= 2g_{D^*D^*\pi} = 2g_{\rho K^*K} = \sqrt{2} g_{K^*K^*\eta'} = \sqrt{6} g_{K^*K^*\eta_c'} = 2g_{\omega K^*K} = \sqrt{2} g_{\phi K^*K} = \sqrt{2} g_{F^*D^*K} = -\sqrt{3} g_{\phi\phi\eta}\\
&= -\sqrt{3} g_{F^*F^*\eta} = 2\sqrt{2} g_{\phi\phi\eta'} = -2\sqrt{3} g_{K^*K^*\eta} = 2\sqrt{6} g_{\phi\phi\eta_c'} = \sqrt{2} g_0 + \frac{4}{\sqrt{3}} g_{15}\,,\\
g_{D^*D^*\eta'} &= \sqrt{2} g_{\rho D^*D} = g_{F^*K^*D} = \sqrt{2} g_{\omega D^*D} = g_{\phi_c D^*D} = g_{F^*F^*\eta'} = g_{D^*K^*F} = g_{\phi F^*F} = g_{\phi_c F^*F} = g_0 - 2(\tfrac{2}{3})^{1/2} g_{15}\,,\\
g_{\phi_c\phi_c\eta'} &= -\frac{1}{\sqrt{3}} g_{\phi_c\phi_c\eta'} = \tfrac{1}{2} g_0 - \sqrt{6} g_{15}\,,\\
g_{D^*D^*\eta_c'} &= g_{F^*F^*\eta_c'} = -\frac{1}{\sqrt{3}} g_0 + \frac{10\sqrt{2}}{3} g_{15}\,.
\end{aligned} \tag{10}$$

It is amusing to note that both in the case of *VPP* and *VVP* many of the ratios between coupling constants are unchanged to first order in the SU(4) symmetry breaking.

As an application of these results, we consider the radiative decays $\phi_c \to X^0\gamma$, $\phi_c \to \eta_c\gamma$, and $\eta_c \to 2\gamma$. Here X^0 and η_c are obtained by diagonalizing the mass matrix for the 0, 8, and 15th components. Thus $m_{\eta_c} = 2.758$ GeV and $m_{X^0} = 0.958$ GeV. We use a vector-dominance model for these decays. If the coupling between the photon and a vector meson V is denoted by em_V^2/g_V, then

$$\Gamma(\phi_c \to P\gamma) = \frac{\alpha}{24}\,(g_{\phi_c\phi_c P}^2/4\pi)(g_{\phi_c}^2/4\pi)^{-1}\left(\frac{m_{\phi_c}^2 - m_P^2}{m_{\phi_c}}\right)^3, \quad P = X^0, \eta_c\,, \tag{11}$$

and

$$\Gamma(\eta_c \to 2\gamma) = \frac{\alpha^2}{16}\, m_{\eta_c}^3 \left| \sum_{V=\rho,\omega,\phi,\phi_c} \frac{g_{VV\eta_c}}{\sqrt{4\pi}} \left(\frac{g_V^2}{4\pi}\right)^{-1} \right|^2 . \tag{12}$$

There exists no reliable estimate of the relative strengths of the photon–vector-meson couplings in the literature. In the Glashow-Iliopoulos-Maiani model[7] where the quarks are fractionally charged, the charge operator is given by

$$Q = \frac{\sqrt{2}}{3} F_0 + F_3 + \frac{1}{\sqrt{3}} F_8 - (\tfrac{2}{3})^{1/2} F_{15}\,.$$

Hence, if exact SU(4) is assumed,

$$\frac{g_\rho^2}{4\pi} : \frac{g_\omega^2}{4\pi} : \frac{g_\phi^2}{4\pi} : \frac{g_{\phi_c}^2}{4\pi} : : \tfrac{1}{9}\quad 1 : \tfrac{1}{2} : \tfrac{1}{8}\,. \tag{13}$$

However, the use of (13) leads to too large a decay width for $\phi_c \to X^0\gamma$. For example, using the first set of solutions we get $\Gamma(\phi_c \to X^0\gamma) \approx 345$ keV, which is much too large considering the experimental number of about 70 keV for $\Gamma(\phi_c \to \text{all})$. From the same numbers we get $\Gamma(\phi_c \to \eta_c\gamma) \approx 33$ keV and $\Gamma(\eta_c \to 2\gamma) \approx 1.4$ keV, which is reasonable. Since SU(4) is badly broken and mass differences are important, we are reluctant to take (13) too seriously. If we retain the masses, (13) is modified to

$$(g_\rho^2/4\pi) : (g_\omega^2/4\pi) : (g_\phi^2/4\pi) : (g_{\phi_c}^2/4\pi) : : \tfrac{1}{9} : (m_\omega^2/m_\rho^2)^2 : \tfrac{1}{2}(m_\phi^2/m_\rho^2)^2 : \tfrac{1}{8}(m_{\phi_c}^2/m_\rho^2)^2\,. \tag{14}$$

Again using the first solution, $\Gamma(\phi_c \to X^0\gamma) \approx 1.3$ keV $\Gamma(\phi_c \to \eta_c\gamma) \approx 0.1$ keV, and $\Gamma(\eta_c \to 2\gamma) \approx 2.2$ keV.

Finally, we mention the relation suggested by Yennie[8] which is equivalent to

$$(m_\rho/9)\left(\frac{g_\rho^2}{4\pi}\right)^{-1} : m_\omega\left(\frac{g_\omega^2}{4\pi}\right)^{-1} : (m_\phi/2)\left(\frac{g_\phi^2}{4\pi}\right)^{-1} : (m_{\phi_c}/8)\left(\frac{g_{\phi_c}^2}{4\pi}\right)^{-1} : : (0.72 \pm 0.10) : (0.76 \pm 0.08) : (0.67 \pm 0.07) : (0.65 \pm 0.07)\,. \tag{15}$$

The use of (15) and the first solution results in the estimates $\Gamma(\phi_c \to X^0\gamma) \approx 77$ keV, $\Gamma(\phi_c \to \eta_c\gamma) \approx 7.5$ keV and $\Gamma(\eta_c \to 2\gamma) \approx 0.9$ keV.

Of the three alternatives for the photon–vector-meson couplings considered above, (14) seems most plausible. Further work on the applications of our results is in progress.

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[1] J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974); J.-E. Augustin *et al.*, *ibid.* 33, 1406 (1974); C. Bacci *et al.*, *ibid.* 33, 1408 (1974); G. S. Abrams *et al.*, *ibid.* 33, 1453 (1974); L. Criegee *et al.*, Phys. Lett. 53B, 489 (1975).

[2] S. Borchardt, V. S. Mathur, and S. Okubo, Phys. Rev. Lett. 34, 38 (1975); S. Okubo, V. S. Mathur, and S. Borchardt, *ibid.* 34, 236 (1975). See also D. H. Boal, R. H. Graham, J. W. Moffat, and P. J. O'Donnell, *ibid.* 34, 541 (1975).

[3] T. Appelquist and H. D. Politzer, Phys. Rev. Lett. 34, 43 (1975); A. De Rújula and S. L. Glashow, *ibid.* 34, 46 (1975); T. Appelquist, A. De Rújula, H. D. Politzer, and S. L. Glashow, *ibid.* 34, 365 (1975).

[4] M. K. Gaillard, B. W. Lee, and J. L. Rosner, Rev. Mod. Phys. 47, 277 (1975).

[5]M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968); see also Ref. 4.
[6]M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Lett. 8, 261 (1962).
[7]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).
[8]D. R. Yennie, Phys. Rev. Lett. 34, 239 (1975).