PHYSICAL REVIEW D VOLUME 12, NUMBER 7 1 OCTOBER 1975

Shadowing in inclusive reactions on composite systems and large-$p_\perp$ inclusive reactions in nuclei*

Paul M. Fishbane and J. S. Trefil
Department of Physics, University of Virginia, Charlottesville, Virginia 22901
(Received 5 March 1975)

We show that there are no shadowing corrections to certain types of inclusive reactions in nuclei, in the sense that the results one finds when shadowing corrections are applied are the same as if no corrections are applied. Among these processes is production of particles at fixed angle. We suggest that this process should have a linear A dependence at large transverse momentum, in contradiction with experimental data. We discuss possible resolutions of this difference, and find we cannot explain it in a natural way, suggesting this process may harbor even more interesting new physics than it has already revealed. We also mention a possible linear A dependence for the production of high-mass dileptons on nuclei.

I. INTRODUCTION

The question of whether or not there are rescattering corrections of final-state hadronic interactions in inclusive reactions has been of interest recently.[1] Since these arguments are customarily couched in terms of the parton model, some light can be shed on the subject by examining other composite models and trying to see, in detail, how this problem is resolved. In this note, we investigate inclusive reactions in nuclei as a paradigm case of the problem of inclusive reactions in composite systems. We show that when *all* final-state interactions are taken into account in an optical approximation, the inclusive cross section becomes identical to the cross section in which no final-state interactions occur; this result holds for a wide class of inclusive processes. This conclusion is natural in this essentially classical model.

As an example of an application of the formalism, we discuss the recent Fermilab data on large-$p_\perp$ inclusive reactions from nuclear targets.[2] We discuss the difficulty of explaining these data by conventional ideas of nuclear or elementary-particle physics, so that these data constitute an important problem whose solution is not obvious at the present time.

We also comment on the production of large mass dilepton states in nuclear targets.

II. SHADOWING IN INCLUSIVE REACTIONS ON COMPOSITE SYSTEMS

The extension of the Glauber theory to the case of inclusive reaction on nuclei has been carried out in Ref. 3. A reaction is visualized as taking place as follows (see Fig. 1): The projectile enters from the left, and is allowed to scatter elastically any number of times (including zero) before it makes its first inelastic collisions at the position z_1 (the z axis is taken to be along the beam direction). At z_1, an inelastic reaction occurs which excites either the target nucleon or the projectile (or both) to some state. This excited state then propagates to the point z_2, scattering elastically any number of times. At z_2, another inelastic event occurs, and the process repeats to the point z_3. Clearly, both "shadowing" and/or "final-state interactions" are present in this way of considering the process.

In Ref. 3 it was shown that the cross section for a projectile to produce a given final-state particle in an inclusive reaction with nucleus A when there are exactly N inelastic reactions is

$$\sigma_N^A = \sum_{\text{perms}} \int d^2B \left[\frac{\Gamma_{N,N+1}}{\sigma_{Np}^{\text{inel}}} \sigma_{Np}^{\text{inel}} A \int_{z_N}^{\infty} \rho(B,z)dz \exp\left(-A\sigma_{N+1,p}^{\text{inel}} \int_{z_n}^{\infty} \rho(B,z)dz \right) \right] \times \cdots$$

$$\times \left[\frac{\Gamma_{12}}{\sigma_{1p}^{\text{inel}}} A\sigma_{1p}^{\text{inel}} \int_{z_1}^{z_2} \rho(B,z)dz \exp\left(-A\sigma_{1p}^{\text{inel}} \int_{z_1}^{z_2} \rho(B,z)dz \right) \right] \left[\exp\left(-A\sigma_{pp}^{\text{inel}} \int_{-\infty}^{z_1} \rho(B,z)dz \right) \right]. \qquad (2.1)$$

Here the ith inelastic collision is one in which the state i goes to the state $i+1$; $\sigma_{ip}^{\text{inel}}$ is the total inelastic cross section for this state on a nucleon, B is the impact parameter of the scattering state, and $\rho(B,Z)$ is the nuclear ground-state density. Thus, the term on the far right in the integrand represents the elastic scattering of the projectile to the point z_1, the next term to the left represents the creation of the state "1" and its propagation to the point z_2, and so forth. The quantity $\Gamma_{i,i+1}$ is an appropriately integrated inclusive cross section. For example, we could integrate over transverse momentum and longitudinal momentum, leaving $\Gamma_{i,i+1}$ dependent in principle only on the energy:

$$\Gamma_{i,i+1}=\int dx_{i+1}d^2k_{i+1}\frac{d^3\sigma_{i,i+1}}{d^2k_{i+1}dx_i}. \qquad (2.2)$$

One may not wish to integrate over either k or x in the last step. Insofar as the intermediate states are different from asymptotic hadronic states, $\Gamma_{i,i+1}$ is then simply an integrated inelastic cross section for the transition $i\to i+1$. Depending on the application, it may label certain quantum numbers carried by these states. For example, in Ref. 3, promotion to a state of high strangeness was of interest, so $\Gamma_{i,i+1}$ represents an inelastic cross section with $\Delta S\neq 0$. It should be noted that this expression depends on σ^{inel} rather than σ^{tot}, as one might naively expect. This is due to the fact that one is considering an incoherent rather than a coherent sum over final nuclear states, as is discussed in Ref. 3.

Equation (2.1) takes on a more transparent form if we assume all inelastic cross sections $\sigma_{ip}^{\text{inel}}$ have a common value,

$$\sigma_N^A=\prod_{i=1}^{N}(\Gamma_{i,i+1}/\sigma^{\text{inel}})\frac{1}{N!}\int d^2B[t(B)]^Ne^{-t(B)}, \qquad (2.3)$$

where $t(B)$ is an optical thickness defined by

$$t(B)=A\sigma^{\text{inel}}\int_{-\infty}^{\infty}\rho(B,z)dz. \qquad (2.4)$$

If we are interested in one particular final state (for example, one produced at large $p_\perp$, and if we have reason to believe that it is produced in a single collision, then we would interpret the chain of inelastic collisions in the following way: We would say that at some particular location, z_j, the state of the projectile was changed. For example, z_j could be the position at which the large momentum transfer was imparted to the projectile. The other inelastic scatterings at z_1, z_2, etc., are then "normal" forward going inelastic collisions, of the type that have been discussed elsewhere[4] and occur in small momentum transfer collisions. We shall discuss below the possibility of several "special" (e.g., large-$p_\perp$) collisions within the nucleus.

At every location except z_j, then, we want to sum over all possible final states $i+1$, since we are not interested in any particular intermediate step in the chain. In that case, by definition,

$$\Gamma_{i,i+1}=\sigma_{pp}^{\text{inel}}, \qquad (2.5)$$

so that Eq. (2.3) becomes linear in Γ_h, where Γ_h is now the inclusive cross section for the single hard collision in the chain.

For the case of an N step chain, the hard collision can occur on any of the N inelastic sites, so the actual measured cross section for producing the final state of interest is

$$\sigma_1^A=\Gamma_h\sum_{N=1}^{A}N\cdot\frac{1}{N!}\int d^2B[t(B)]^Ne^{-t(B)}. \qquad (2.6)$$

To carry out the sum, we note that

$$S=\sum_N\frac{1}{(N-1)!}\int d^2B\,t(B)^Ne^{-t(B)},$$

and, interchanging the summation and integration,

$$S=\int d^2B\,t(B)\sum_{l=0}^{\infty}\frac{1}{l!}[t(B)]^le^{-t(B)}$$

$$=\int d^2B\,t(B). \qquad (2.7)$$

Using the definition of $t(b)$ in Eq. (2.4), and the fact that the density is normalized, we finally arrive at

$$\sigma_1^A=\Gamma_hA \qquad (2.8)$$

as our final result.

Equation (2.8) is precisely the result we would have obtained had we assumed that the incoming particle made only one hard collision and no soft ones (i.e., if we had assumed that there was no shadowing). However, the physical picture behind the result in this case is very different. We have allowed an arbitrary number of elastic and inelastic collisions in addition to the single hard collision, but have found a result which is formally equivalent to the single hard collision alone.

The crucial fact which leads to this result is the

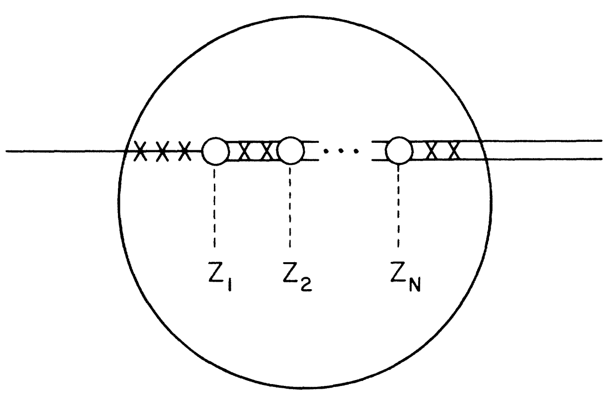

FIG. 1. A typical production process in a nucleus. The "X" represent elastic scatterings, and the inelastic scatterings occur at the vertices labeled "z_1", "z_2", etc. Further discussion is given in the text.

cancellation in Eq. (2.7) between those terms representing multiple elastic scattering and explicit integrated inelastic scattering. This cancellation is in fact local in z, so that Eq. (2.1) also exhibits it, as we shall now show in abbreviated form by generalizing our result to the case where the elementary cross sections after the hard collision are different from those before it. In this case, the expression for σ_N^A would be

$$\sigma_N^A = \sum_{\substack{n,j \\ n+j=N-1}} \Gamma_h \int d^2B\, dz_1 \rho(B,z_1) \frac{1}{j!}(\Gamma_1 T_L)^j \times \frac{1}{n!}(\Gamma_2 T_R)^n e^{-(\sigma_1^{\text{inel}} T_L + \sigma_2^{\text{inel}} T_R)}, \tag{2.9}$$

where the incoming particle and its excitations previous to the "hard" collision, characterized by inelastic cross section σ_1^{inel}, scatter j times to the left of z_1, the position of the hard collision, and the outgoing states, characterized by σ_2^{inel}, scatter n times to the right. We have defined

$$T_L = A\int_{-\infty}^{z_1} \rho(B,z)dz \tag{2.10a}$$

and

$$T_R = A\int_{z_1}^{\infty} \rho(B,z)dz\,. \tag{2.10b}$$

Carrying out the sum over n and j subject to the constraint that $n+j+1=N$, we can retrace the steps leading to Eq. (2.6) to find

$$\sigma_1^A = \Gamma_h \sum_{N=1}^{A} N \int d^2B \int dz_1 \rho(B,z_1) \times (\sigma_1^{\text{inel}} T_L + \sigma_2^{\text{inel}} T_R)^{N-1} \times \exp[-(\sigma_1^{\text{inel}} T_L + \sigma_2^{\text{inel}} T_R)]\,, \tag{2.11}$$

which, when the sum over N is performed, will lead directly back to Eq. (2.8).

The only way that a result which is not proportional to A can be obtained in this approach is to allow more than one hard collision. For future reference, we present here the derivation of the double "hard" scattering term, from which the generalization to any number of "hard" scatterings can easily be made. As before, we are not required to make assumptions on the excitation cross sections.

The analogous expression to Eq. (2.6) for two hard collisions, $N-2$ normal inelastic collisions, and any number of elastic scatterings is just

$$\sigma_2^A = \frac{\Gamma_h^{(1)}}{\sigma^{\text{inel}}} \frac{\Gamma_h^{(2)}}{\sigma^{\text{inel}}} \times \sum_{N=2}^{\infty} \frac{N(N-1)}{N!} \int d^2B\, t(B)^N e^{-t(B)}, \tag{2.12}$$

where Γ^1 ($\Gamma^{(2)}$) refers to the first (second) hard collision. Proceeding as before, we find

$$\sigma_2^A = \frac{\Gamma_h^{(1)}}{\sigma^{\text{inel}}} \frac{\Gamma_h^{(2)}}{\sigma^{\text{inel}}} (A\sigma^{\text{inel}})^2 \times \int d^2B \int dz \int dz' \rho(B,z)\rho(B,z')\,. \tag{2.13}$$

This is as far as we can take the general problem, but more insight can be gained by choosing particular forms for the density. For example, if we choose

$$\rho(B,z) = \frac{1}{(\sqrt{\pi}R)^3} e^{-(z^2+B^2)/R^2} \tag{2.14}$$

then the integrals can be evaluated explicitly to yield

$$\sigma_2^A = \Gamma_h^{(1)} \Gamma_h^{(2)} \frac{(A)^2}{4\pi R^2}\,. \tag{2.15}$$

If, in addition, we assume that $R = R_0 A^{1/3}$, then we see that the double hard scattering term will increase with A as $A^{4/3}$, in comparison to $A^{1.0}$ for the single hard scattering. Clearly, this argument can be extended to any number of hard scatterings. For M hard collisions,

$$\sigma_M^A \sim A^{(2+M)/3}\,. \tag{2.16}$$

III. LARGE-$p_\perp$ REACTIONS ON NUCLEI

Having studied the general problem, we turn to the case which first sparked our interest. Recent experiments, including the experiment of Cronin *et al.*[2] on nuclear targets, show that inclusive cross sections for $p_\perp \gtrsim 1$ GeV/c depart dramatically from the usual exponential fall in p^2 seen at small $p_\perp$ and instead behave more like $p_\perp^{-m}$, where m depends on the detected particle but is always $\gg 2$. As we shall discuss below, this behavior is compatible only with nuclear processes in which the large p state is produced in a single hard collision. We would then expect this situation to be adequately described by our normal extension of the Glauber theory. This, in turn, means that for $p_\perp \lesssim 1$ GeV/c, we expect a cross section of the usual geometrical size $\sim A^{2/3}$ passing over into an inclusive cross section $\sim A^{1.0}$ as $p_\perp$ increases into the region where the power-law falloff becomes dominant.

Cronin *et al.*[2] find a strikingly different result. We can summarize their findings in the following way: In the reaction

$$p+A \to H+X,$$

where A is the nuclear target and H is the measured hadron at large $p_\perp$, the invariant cross section is of the form

$$E\frac{d^3\sigma}{d^3p_H}=A^{n(p_\perp)}f(p_H, E). \tag{3.1}$$

Figure 2 shows the dependence of n on $p_\perp$ for detected pions. n is significantly greater than 1.0 in the region where power falloff is important. This qualitative effect persists for all detected particles, with the "asymptotic" value of n (see Fig. 2) being 1.1 for pions, 1.3 for protons, and 1.4 for antiprotons.

Moreover, the *form* of Eq. (3.1) seems well established. Figure 3 shows a log-log plot of the cross section vs A. The three available points are extremely well fitted by a straight line. If this experimental result persists, then a power series in A is of course ruled out.

The discrepancy between the experimental result and the result predicted by a straightforward extension of current ideas leads us to suppose that one of two things is happening. Either (1) there is something wrong with the basic formalism leading to Eq. (2.8), or (2) the inputs into the formalism are wrong at large $p_\perp$. The latter alternative would imply that some interesting new physics is manifesting itself in the anomalous A dependence.

Clearly, alternative (2) is the more interesting one. In order to be able to conclude that there is new physics, however, it will be necessary to rule out alternative (1). Unfortunately, the only way to do this is by explicitly checking as many "uninteresting" alternatives as we can think of. It is to this task that we now turn, along with some preliminary discussion of possible new physics.

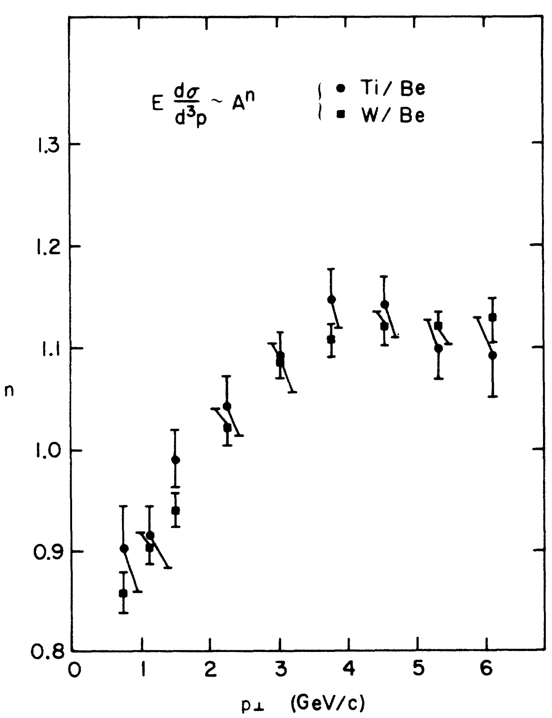

FIG. 2. Exponent of A in the process $p+A \to \pi+X$ at $\theta^*=90°$ and a bombarding momentum of 300 GeV/c. Data are from Ref. 2.

A. Geometrical effects

The Glauber theory is derived under the eikonal assumption. The apparatus of Cronin *et al.*,[2] however, measured particles at 77 mrad in the laboratory frame (which is the rest frame of the nucleus). This means that the emerging large-$p_\perp$ state travels a nonforward path through nuclear matter on its way out of the nucleus. Could this introduce a spurious A dependence?

To investigate this question, we considered a single hard collision process is a uniform spherical nucleus. In this case, for radius R and density ρ, we have

$$T(b)=A\rho\sigma^{\text{inel}}(R^2-b^2)^{1/2}. \tag{3.2}$$

The geometrical effect is taken into account by replacing the exponential absorption factor in Eq. (2.3) by a term

$$e^{-T(b)}e^{A\rho\sigma^{\text{inel}}\Delta r}, \tag{3.3}$$

where Δr is the difference in the actual path length

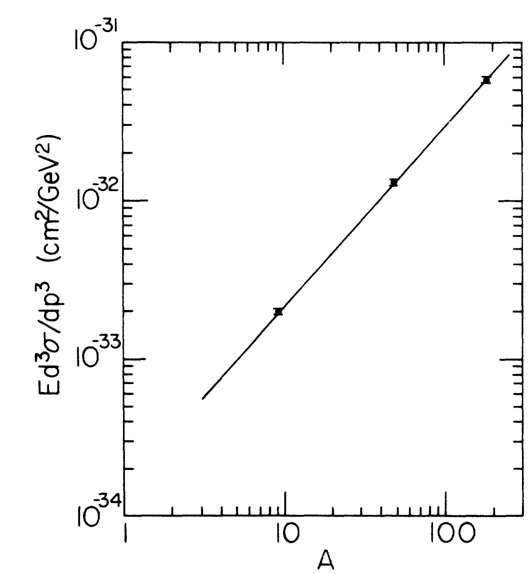

FIG. 3. Illustration of power dependence of inclusive cross section on A. The data, taken from Ref. 2, are for π^- production at $p_\perp=4.58$ GeV/c, bombarding energy 300 GeV/c.

traveled by the large $p_\perp$ state and one that would have gone straight forward.

It is straightforward geometry to show that for a single hard collision at z,

$$\Delta r = B_x \alpha \left[-1 + \frac{z}{(b^2 - R^2)^{1/2}}\right] + B_y \phi \left[-1 + \frac{z}{(b^2 - R^2)^{1/2}}\right], \qquad (3.4)$$

where α and ϕ are the direction cosines of the new particle direction and B_x and B_y are the x and y components of the impact parameter, respectively. We have dropped higher-order terms in α and ϕ because we are dealing with small angles.

Substituting back into Eq. (2.5) we now find for the modified nuclear weight

$$\int d^2B\, T(b) \exp\{-[\sigma^{\text{inel}}\, T(B)(1 + B_x\alpha + B_y\phi)]\}$$
$$\approx \int d^2B\, T(B) \exp\{-[\sigma^{\text{inel}}\, T(B)][1 + O(\alpha^2, \phi^2, \alpha\phi)]\}. \qquad (3.5)$$

Thus, we see that geometrical effects will come in only at the level of the scattering angle squared. This means that it would be a 1% effect in the experiment we are considering, and is not responsible for the A dependence anomaly. Although we have proved this result only for a uniform density, we expect that it will hold for any spherically symmetrical density.

B. Cross-section difference between pre-hard-collision and post-hard-collision states

It is certainly conceivable that the state emitted at large $p_\perp$ differs in its properties from the normal hadronic state, and specifically in its cross section. Nonetheless, the expectation of a value of the index n of unity is independent of the choices of the parameters which govern the interactions of the final state, as is clear from Eq. (2.9) and the argument that follows. Therefore, this effect cannot give rise to a spurious A dependence.

C. Multiple hard collisions

One of the classical results of the Glauber theory was the explanation of diffraction bumps in particle-nucleus differential cross sections on the basis of the fact that at large momentum transfers (although not nearly as large as those being discussed here), it is easier for the nucleus to transfer momentum in a series of small bits rather than all at once. This, in turn, means that at large t, the multiple scattering terms dominate the cross section. This result depends on an exponential fall in t, and multiple scattering terms retain an exponential behavior.

We must first say that the curve of Fig. 3 is in fact easily fitted by functions of the form

$$E\frac{d^3\sigma}{d^3p} \sim C_1 A + C_2 A^{4/3} + \dots, \qquad (3.6)$$

where the second term represents double hard collisions, the third three hard collisions, etc. In fact, only C_1 and C_2 are necessary.

However, the $p_\perp$ dependence of the double scattering term would in principle be wrong. (This may not be easily observable at present energies.) If we had a double hard scattering, we would have a term proportional to

$$I = \int \frac{d^2k}{(p_\perp - k)^m k^m}, \qquad (3.7)$$

where $p_\perp$ is the momentum transferred to the final state, and k is the momentum transferred in the first collision. We have assumed here that the large $p_\perp$ cross section falls off as $(p_\perp)^{-m}$. The region of integration over k cannot include $k=0$ and $k=p_\perp$, as one might expect at first glance, because it was assumed in the derivation of Eq. (2.8) that the nucleus was partitioned among those nucleons on which elastic, ordinary inelastic, and hard collisions took place. A "hard" collision with zero $p_\perp$ would, by definition, be one of the former types, and we would therefore be double counting if we took this region of the integral into account. Similarly, if $p_\perp = k$, then the second collision has zero $p_\perp$, and the argument repeats itself.

If we exclude the regions $|k| < \epsilon p_\perp$ and $|k - p_\perp| < \epsilon p_\perp$ then the above integrand is perfectly well behaved, and, by dimensional arguments, gives a result

$$I \sim \frac{1}{(p_\perp)^{2m-2}}. \qquad (3.8)$$

This means that the coefficient of the double hard scattering term will fall much more quickly with $p_\perp$ than will the coefficient of the single hard scattering term. Consequently, at sufficiently large $p_\perp$, these terms will become negligible, and we will be back to the prediction of $n=1$.

The reader should note that if the region $|k| < |k_0|$, $|k_0|$ fixed, rather than $|k| < \epsilon p_\perp$, is chosen, then $p_\perp^{-m}$ behavior is recovered. The choice between these can in principle be determined by experiment.

D. Many-body interactions

Another proposal which has been advanced to explain the data is that the large-$p_\perp$ particles are the result of interactions in which two or more nucleons are the effective target, so that the center-of-mass effect from a heavier target will come into play. In this picture, a particle which is seen and interpreted as a large-$p_\perp$ event on a single nucleon is actually the result of a smaller-$p_\perp$ event on a larger effective target.

There are two ways in which this sort of argument can be advanced. It might be thought that the projectile, after suffering the collision through large $p_\perp$, gains mass in subsequent collisions. This mechanism for boosting the effective energy in a collision in a nucleus has been discussed in detail elsewhere.[5,6] We could also postulate the existence in the nucleus of some sort of coherent cluster which behaves as a single particle in large $p_\perp$ collisions. We shall see that the first suggestion will not explain the data of Cronin *et al.*, and present arguments against the second. We study these propositions in turn.

The state which is produced in the hard collision may have properties which differ from normal hadronic matter. We mentioned cross-section differences in part B. Although assumptions of this type take us into the realm of new physical ideas, the data may well warrant such excursions.

As an example, suppose the excitation produced at large $p_\perp$ picks up mass with successive collisions within the nucleus but that it decays into a *fixed* number of pions when it emerges. It is then conceivable that the fastest detected pions would come from the state which has undergone the most collisions within the nucleus. Clearly the larger A is, the more collisions will occur, so this effect could introduce an A dependence. As an exercise of the kind of calculation which might be done, we shall give results for this particular approach.

We imagine starting with a finite excitation mass m_0 and large available energy s after the hard collision. After M collisions, the resulting excitation has mass m_M and energy E_M. The energy and momentum transfer variables in the Mth collision are $s^{(M)}$ and $t^{(M)}$. Our key dynamical assumption will be that m_M is the following function of s:

$$m_M{}^2 = x_M{}^2 s^{(M)} . \tag{3.9}$$

For simplicity, we shall take $x_M = x$ for all M. x may be s-dependent.

It is then straightforward to compute m_M, $s^{(M)}$, E_M, etc., in terms of x and s. Since we are interested in average transverse momenta $\langle p_\perp^\pi\rangle$ of, say, detected pions, we compute the momentum of the excitation, emerging after M collisions at angle θ, and divide this momentum by α, the decay multiplicity. In general, $\alpha = \alpha(m_M)$ (for example, in the old nova model,[7] $\alpha \propto \log m_M$). Then we would have

$$\langle p_\perp^\pi\rangle = \sin\theta \langle p^\pi\rangle = \sin\theta \frac{1}{\alpha}(E_M{}^2 - m_M{}^2)^{1/2} . \tag{3.10}$$

In this formula,

$$E_M \approx \frac{s}{2m_p} x^M , \tag{3.11a}$$

$$m_M = x[s x^{M-1}(1+x)^{M-3}(1+x+x^2)]^{1/2} , \tag{3.11b}$$

$$m_3 = x\sqrt{s} , \tag{3.11c}$$

$$m_5 = x[sx(1+x)]^{1/2} . \tag{3.11d}$$

Now since $M \sim A^{1/3}$ so long as cross sections do not depend strongly on M, we are interested in a positive slope for $\langle p_\perp^\pi\rangle$ as a function of M; a positive slope will then give an increase of n with $p_\perp$. Therefore, we compute

$$\frac{d\langle p_\perp^\pi\rangle}{dM} = \frac{d\langle p_\perp^\pi\rangle}{dM}\bigg|_{\alpha\ \mathrm{fixed}} - \frac{1}{2\alpha}\langle p_\perp^\pi\rangle \frac{d\alpha}{dm_M} m_M \ln x(1+x) , \tag{3.12}$$

where the slope term for fixed multiplicity is

$$\frac{d\langle p_\perp^\pi\rangle}{dM}\bigg|_{\alpha\ \mathrm{fixed}} = \frac{\sin\theta}{\alpha^2\langle p^\pi\rangle}[E_M{}^2 \ln x - \tfrac{1}{2}m_M{}^2 \ln(x(1+x))] . \tag{3.13}$$

It is instructive to consider first the fixed multiplicity term alone. For any fixed (i.e., s-independent) value of x, the term $\sim E_M{}^2$ dominates the term $\sim m_M{}^2$, since $E_M{}^2 \sim s^2$ while $m_M{}^2 \sim s$. Since $\ln x < 0$ $(x<1)$, this term is negative, and rather than growing with M, $\langle p_\perp\rangle$ in fact decreases with increasing M. On the other hand, if $x = O((m^2/s)^\beta)$, then both E_M and m_M drop with increasing M, so that a high A nucleus would favor production of slow particles.

In order to assess the importance of the second term in Eq. (3.12), we ask the straightforward question: What is the bound on $d\alpha/dm_M$ which makes $d\langle p_\perp^\pi\rangle/dM$ positive? We find

$$\frac{d\alpha^2}{dm_M} \geq \frac{4}{\langle p^\pi\rangle^2 m_M \ln(x(1+x))}[E_M{}^2 \ln x - m_M{}^2 \ln(x(1+x))] . \tag{3.14}$$

For fixed x, we can again drop m_M terms compared to E_M terms. The equality point of Eq. (3.14) gives

$$\alpha = \exp\left(\frac{\ln m_M \ln x^2}{\ln x(1+x)}\right) . \tag{3.15}$$

Since $\ln x^2/\ln x(1+x) > 1$, α must grow faster than $m_M \sim \sqrt{s}$, which is not kinematically allowed. The case $x = O((m^2/s)^\beta)$ similarly does not save the situation. We must conclude that it is not possible to explain the data with this approach.

If we cannot think of collisions with several nucleons in a microscopic sense (that is, if we cannot consider the collisions separately), could not a genuine many-body interaction have occurred? Of course, this possibility cannot categorically be ruled out, but a number of points would have to be made. First, if nuclei cluster into high "effective mass" states, why is there no evidence for such states in normal small t production processes? Second, the physics of an hadronic state which could remain coherent up to $\sqrt{t}=0$ (5 GeV/c) is rather foreign to our present experience. Finally, if this is indeed the correct solution to the large-$p_\perp$ problem, it would be necessary that the probability of hitting a cluster vary with A in just such a way as to produce a line on a plot such as that in Fig. 3. Again, while this cannot be ruled out, the predictive power of such a scheme appears rather limited.

E. Comments and conclusions

The authors[8] who have written on the large-$p_\perp$ problem have proposed some of the solutions we have discussed in this section. We find, upon close examination, that these solutions simply will not work when they are confronted with the precise data[2] of Cronin *et al.* This means that whatever the final solution of the problem turns out to be, it will involve an "interesting" result, as opposed to the "uninteresting" alternatives we have been considering.

IV. A COMMENT ON DILEPTON PRODUCTION IN NUCLEI

Another important application of the theorem on A dependence which was presented above lies in the process[9]

$$p+A\to l^+l^-+X$$

at large invariant dilepton mass. This is a process which can be used to produce the new ψ particles at proton accelerators.

From our results, we expect that the cross section for this process will be

$$\frac{d^3\sigma}{Edp}=A\sigma(p+p\to l^+l^-+X)$$

and not, as has been postulated,[9]

$$\frac{d^3\sigma}{Edp}=A^{2/3}\sigma(p+p\to l^+l^-+X)\,.$$

Thus, in going from nuclear production data to cross sections on protons, the extrapolation will be as A^1 as opposed to $A^{2/3}$.

Of course, as in the case of large $p_\perp$, there might be more surprises in store for us (could we find A^n, $n>1$?), but $A^{1.0}$ represents our best expectation at the moment.

ACKNOWLEDGMENTS

P. M. F. would like to thank A. S. Goldhaber and R. Slansky for helpful conversations. J. S. T. also thanks J. Kuhn for useful conversations.

*Work supported in part by NSF Grant No. GP-39179 and a Cottrell Grant from the Research Foundation.

[1] F. Henyey and R. Savit, Phys. Lett. 52B, 71 (1974); M. Einhorn and F. Henyey, Phys. Rev. D 11, 2009 (1975); C. E. DeTar, S. D. Ellis, and P. V. Landshoff, Nucl. Phys. B87, 176 (1975); J. L. Cardy and G. A. Winbow, Phys. Lett. 52B, 95 (1975).

[2] J. W. Cronin *et al.*, Phys. Rev. Lett. 31, 1426 (1973); paper presented at the XVII International Conference on High Energy Physics, London (1974); Phys. Rev. D 11, 3105 (1975).

[3] J. S. Trefil and F. von Hippel, Phys. Rev. D 7, 2000 (1973).

[4] P. M. Fishbane and J. S. Trefil, Phys. Lett. 51B, 139 (1974); Phys. Rev. Lett. 31, 734 (1973); Phys. Rev. D 9, 168 (1974); K. Gottfried, Phys. Rev. Lett. 32, 957 (1974); G. Calucci, R. Jengo, and A. Pignotti, Phys. Rev. D 10, 1468 (1974); A. Dar and J. Vary, *ibid.* 6, 2412 (1972); B. Andersson and I. Otterlund, Lund report (unpublished).

[5] W. D. Walker, Phys. Rev. Lett. 24, 1143 (1969).

[6] P. M. Fishbane and J. S. Trefil, Phys. Rev. D 9, 168 (1974).

[7] M. Jacob and R. Slansky, Phys. Rev. D 5, 1847 (1972); R. Hwa, Phys. Rev. Lett. 26, 1143 (1971).

[8] J. Pumplin and E. Yen, Phys. Rev. D 11, 1812 (1975); G. Farrar, Phys. Lett. 56B, 185 (1975).

[9] J. H. Christenson *et al.*, Phys. Rev. Lett. 25, 1523 (1970); Phys. Rev. D 8, 2016 (1973).