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Nonstrange quark mass in the bag model*

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The MIT bag model of the nucleon is extended to include the effect of finite nonstrange quark mass. If the nucleon axial-vector coupling constant is used to fix the quark mass, then using $g_A = 1.25$ we find $m = 122$ MeV for the mass of the degenerate up and down quarks. Other attributes of the nucleon are evaluated in this model and the limiting case of infinite quark mass is studied as a check on the calculation.

I. INTRODUCTION

Recently, a field-theoretic framework for describing hadronic states of finite spatial extent was presented.[1] Basic properties of this "bag" model were enumerated, and the transition from classical to quantum fields was studied in some detail. In a second paper, the model was used to describe the structure of nonstrange baryons.[2] Simplifying assumptions were made. The quarks were assumed to be massless, and the bag boundary was taken as a fixed sphere. The sole parameter of the model, B (the positive, constant energy density within the bag), was fixed in terms of the (degenerate) N-Δ mass. The spectrum of low-lying excited states was thereupon determined, and various attributes of the nucleon, such as its magnetic moment, were calculated. Results were encouraging, especially in view of the simplifying assumptions made, and tended to lend credence to the over-all approach.

As an example of the potential utility of such a model, a calculation of various single-particle intermediate-state contributions to the chiral charge algebra was performed.[3] The bag model was used to estimate matrix elements of the axial charge density evaluated between single baryon states. Although, the data are inadequate at present to judge the phenomenological merit of this description, we can state that the predicted matrix elements at least do not lead to oversaturation.

It is our purpose in this paper to extend the work of Ref. 2 by allowing the quark masses to be nonzero. In a sense, this serves as a check on the work done there by probing the response of various "good" results to the presence of quark mass. A more important motivation is that we can employ the model as a way of phenomenologically estimating quark mass. While it is true that the simplicity of the model forces us to interpret the resulting estimate with due caution, our calculation points out one of the more interesting ways in which refined models of the future can be applied. Since several distinct methods exist for estimating the mass of nonstrange quarks, it is of interest to compare them with one based on bag phenomenology. In fact, it is our feeling that the bag estimate should not be disregarded in comparison to any of the others, because so many properties of low-mass baryons appear to be described so well by the bag model.

By allowing the quarks to have nonzero mass, we introduce a second free parameter into the calculation. We have arbitrarily decided to fix this second parameter by fitting the model to the nucleon axial-vector coupling constant, g_A. In the massless-quark limit,[2] it was found that $g_A = 1.09$. As pointed out in Ref. 1, even though the quarks are massless, axial current is not generally conserved because the bag surface can act as a source of axial charge. On the other hand, the nonrelativistic quark prediction for the axial-vector coupling constant is known to be $g_A = \frac{5}{3}$. Presumably, this corresponds to the limit of infinite quark mass. Therefore, the experimental value, $g_A \cong 1.25$, should be obtainable in bag model where the quark mass is finite. Later in this paper, we shall study the infinite-quark-mass limit as well.

In order to make this paper self-contained, and also to facilitate interpretation of some formulas in Sec. II, we reproduce here certain portions of the formalism adopted in Ref. 2. The fields associated with massless quarks are assumed to obey

$$\gamma \cdot \partial \psi_\alpha(x) = 0 \tag{1}$$

within the bag, and

$$i n \cdot \gamma \psi_\alpha(x) = \psi_\alpha(x), \tag{2a}$$

$$\sum_\alpha n \cdot \partial [\bar{\psi}_\alpha(x) \psi_\alpha(x)] = 2B \tag{2b}$$

on its surface. The index α labels color and isospin, and the interior 4-normal to the bags surface is taken here to be $n_\mu = (0, -\hat{r})$. As a consequence of this form for n_μ, only modes of total angular momentum $j = \frac{1}{2}$ can exist within the bag.

The field $\psi_\alpha(x)$ may be expressed as a linear combination of $j=\frac{1}{2}$ solutions to the free Dirac equation,

$$\psi_\alpha(\vec{x},t)=\sum_{n\kappa m} N(\omega_{n\kappa})a_\alpha(n\kappa m)\psi_{n\kappa m}(\vec{x},t), \tag{3}$$

where $N(\omega_{n\kappa})$ is a normalization factor, $\omega_{n\kappa}$ is the frequency of a given mode and is determined by Eq. (2a), κ distinguishes between states of opposite parity, and m labels the z component of the angular momentum. An important constraint which follows from Eq. (2b) is

$$\sum_\alpha a^*_\alpha(n\kappa m)a_\alpha(n'\kappa' m')=0, \tag{4}$$

unless $n=n'$, $\kappa=\kappa'$ or $n=-n'$, $\kappa=-\kappa'$. In the quantum formulation of this model, the coefficients a_α are interpreted as creation and annihilation operators,

$$\begin{aligned} a_\alpha(n\kappa m)&=b_\alpha(n\kappa m), \quad n>0\\ &=d^\dagger_\alpha(-n-\kappa m), \quad n<0 \end{aligned} \tag{5}$$

which obey the usual anticommutation relations, e.g.,

$$[b_\alpha(n\kappa m), b^\dagger_\alpha(n\kappa m)]_+=1. \tag{6}$$

The occupation-number interpretation of field quanta, with a number operator

$$N_\alpha=\int_{\text{bag}} d^3x\psi^\dagger_\alpha(x)\psi_\alpha(x) \tag{7}$$

and zero-particle state (empty bag)

$$\begin{aligned} b_\alpha(n\kappa m)|0\rangle&=0,\\ d_\alpha(n\kappa m)|0\rangle&=0 \end{aligned} \tag{8}$$

also exist here in the usual way.

Upon constructing an energy operator for the model and using a weighted average for the N, Δ mass (equaling 1180 MeV), a value of $B^{1/4}=120$ MeV is found.[2] Various static properties of the nucleon can then be calculated, among them the axial-vector coupling constant ($g_A=1.09$), the proton gyromagnetic ratio ($g_p=2.6$), and the proton charge radius ($\langle r^2\rangle_p^{1/2}=1.04$ fm). The bag radius R has the numerical value $1/R=144$ MeV. The two contributions to the nucleon mass come from the quark field energy $E_Q=885$ MeV and the bag volume energy $E_B=295$ MeV. The first excited state has energy 1421 MeV with $E_Q=1066$ MeV and $E_B=355$ MeV.

II. EFFECT OF QUARK MASS

The model just described is remarkable for its simplicity. The bag boundary is a static sphere, and the quarks enclosed within are massless and do not interact with each other. Several extensions of this model, each relaxing one of the simplifying assumptions, are clearly possible. For example, the quarks may be allowed to interact by means of either a vector gluon field or any other reasonable candidate for quark dynamics. In this way, the spin degeneracy of the "zeroth-order" model can be lifted. A more difficult extension would be an attempt to let the bag boundary have a more realistic dynamical behavior. In the following, we explore the consequences of dealing with massive quarks, while at the same time retaining the assumptions that their interaction may be neglected and that the bag boundary is a static sphere of radius R. As will be seen, the good results of the previous section are relatively unchanged, and the fit to the nucleon axial-vector coupling constant can be made arbitrarily precise.

The equation of motion obeyed by the quark fields within the bag is now

$$(-i\gamma\cdot\partial+m)\psi_\alpha(\vec{x},t)=0. \tag{9}$$

The boundary conditions (2a)–(2b) remain as before. The $j=\frac{1}{2}$ solutions to Eq. (9), to be used as expansion functions in Eq. (3), exhibit the spatial dependence[4]

$$\psi_{n-1\,m}(\vec{x})=[4\pi(E+m)]^{-1/2}\begin{pmatrix} i(E+m)^{1/2}j_0((E^2-m^2)^{1/2}r)u_m\\ -(E-m)^{1/2}j_1((E^2-m^2)^{1/2}r)\vec{\sigma}\cdot\hat{r}u_m\end{pmatrix}, \tag{10a}$$

for $\kappa=-1$, and

$$\psi_{n1m}(\vec{x})=[4\pi(E-m)]^{-1/2}\begin{pmatrix} i(E+m)^{1/2}j_1((E^2-m^2)^{1/2}r)\vec{\sigma}\cdot\hat{r}u_m\\ (E-m)^{1/2}j_0((E^2-m^2)^{1/2}r)u_m\end{pmatrix}, \tag{10b}$$

for $\kappa=+1$. The j_i are spherical Bessel functions and u_m is a two-component Pauli spinor. Although our normalization for the $\kappa=+1$ solution is not the one conventionally used to study the $m\to\infty$ limit, we find it convenient to employ in the following analysis. We have suppressed the dependence of the energy E in (10a)–(10b) on the quantum numbers n, κ. A natural dimensionless variable for the mode frequency is[5]

$$\omega\equiv ER. \tag{11}$$

The linear boundary condition Eq. (2a) can then be written as

$$\tan[(\omega^2 - m^2R^2)^{1/2}] = \frac{\kappa(\omega^2 - m^2R^2)^{1/2}}{(\omega - \kappa mR) + \kappa}. \quad (12)$$

Notice that ω depends explicitly upon R in Eq. (12). Thus, unlike the massless case, ω for a given mode will depend upon the particle state, e.g., ω for an $n=1$, $\kappa=-1$ quark will change slightly as we go from the nucleon to any of its excited states. Before discussing the nonlinear boundary condition (2b), let us first consider the number operator N_α as defined in Eq. (7). Given the standard anticommutation relations for the creation and annihilation operators, the number operator will have integer eigenvalues within the bag if it can be written as

$$N_\alpha = \sum_{n\kappa m} [b_\alpha^\dagger(n\kappa m)b_\alpha(n\kappa m) - d_\alpha^\dagger(n\kappa m)d_\alpha(n\kappa m)]. \quad (13)$$

This implies for the normalization factor of Eq. (3) the form

$$N(\omega_{n\kappa}) = \left(\frac{(\omega^2 - m^2R^2)^2}{R^3(2\omega^2 + 2\kappa\omega + mR)\sin^2[(\omega^2 - m^2R^2)^{1/2}]}\right)^{1/2}. \quad (14)$$

We are now in a position to write an expression for the nonlinear boundary condition (2b). In this paper, we shall deal only with baryon states, that is, states containing no antiquarks. Thus, we write relation (2b) in terms of quark creation and annihilation operators only,

$$2\pi BR^4 = \sum_{n\alpha\kappa m} \frac{(\omega+\kappa)(\omega^2 - m^2R^2)}{2\omega^2 + 2\kappa\omega + mR} b_\alpha^\dagger(n\kappa m)b_\alpha(n\kappa m). \quad (15)$$

This completes the initial phase of our presentation. Next, we go on to discuss the calculation of various observables associated with the nucleon and its excited states. To summarize, Eq. (12) provides a relation between ω and the dimensionless quantity mR for a given value of κ. Below, we shall employ a relation between ω and mR of another type which will allow us to solve for these unknowns separately. The nonlinear boundary condition (2b) written in the form (15) can be used in conjunction with the energy operator (see below) to provide equations to be used in calculating B and R.

The energy contained within the baryon bag has contributions from the quark field energy and from the volume energy of the quantity B.[6] For a baryon state within a spherical bag, the energy operator has the form[7]

$$H = \tfrac{4}{3}\pi BR^3 + \sum_{n\alpha\kappa m} \frac{\omega}{R} b_\alpha^\dagger(n\kappa m)b_\alpha(n\kappa m). \quad (16)$$

Because the nucleon and $\Delta(1236)$ are degenerate in this model, we take as input to calculation the weighted average for the ground-state mass $(4M_N + 16M_\Delta)/20 = 1180$ MeV. This implies the following relation between B and R:

$$1180 = \tfrac{4}{3}\pi BR^3 + 3\omega_{1,-1}/R. \quad (17)$$

The nucleon axial-vector coupling constant g_A can be calculated from the formula[2]

$$g_A = \left\langle P(s_z=\tfrac{1}{2}) \right| \int_{\text{bag}} d^3x \psi^\dagger(x)\tau_3\sigma_z\psi(x) \left| P(s_z=\tfrac{1}{2}) \right\rangle. \quad (18)$$

Evaluation of the matrix element in (18) is straightforward, and we find

$$g_A = \frac{5}{9}\left(\frac{2\omega_{1,-1}{}^2 + 4mR\omega_{1,-1} - 3mR}{2\omega_{1,-1}{}^2 - 2\omega_{1,-1} + mR}\right). \quad (19)$$

At this point, we have a battery of equations sufficient to allow calculation of the four unknowns $\omega_{1,-1}$, B, R, and the quark mass m. For definiteness, we shall employ the numerical value $g_A = 1.25$, although the sensitivity of our determination of quark mass to changes in g_A will be given shortly. Equations (12) and (19) were solved numerically on a computer, and gave $\omega_{1,-1} = 2.61$, $mR = 1.01$. Some care must be taken in solving these equations. Accuracy well beyond 1% is needed to pin down the value of the unknowns to better than one per cent. The energy equation (16) and nonlinear boundary condition (15) then imply $B^{1/4} = 99.9$ MeV and $1/R = 120.6$ MeV. Finally, we obtain for the mass of nonstrange quarks the value $m = 122.3$ MeV.

Several comments on the nature of this solution are in order. In Table I, we exhibit the dependence of quark mass m and bag pressure B upon the value of g_A. Hereafter, we shall refer to the particular solution to $g_A = 1.25$. Of the 1180 MeV in mass of the baryon ground state, 238 MeV is the volume energy of B, and 942 MeV is the quark field energy. The quarks are definitely relativistic. The ratio of momentum to rest mass is $|\vec{p}|/m = 2.37$ for each quark.

Our main objective was to obtain an estimate of quark mass by means of a fit to the axial-vector coupling g_A. However, it is also of interest to observe the effect of quark mass upon the proton gyromagnetic ratio and charge radius, as well as the energy of the first excited state. We define the magnetic moment operator as

$$\mu_p = \left\langle P(s_z = \tfrac{1}{2}) \left| \int_{\text{bag}} d^3x \tfrac{1}{2}(\vec{r} \times \psi^\dagger \vec{\alpha}\, Q\psi)_3 \right| P(s_z = \tfrac{1}{2}) \right\rangle, \tag{20}$$

where Q is the usual quark charge matrix. Upon evaluating the matrix element, we have

$$\mu_p = \frac{R}{6}\, \frac{4\omega_{1,-1} + 2mR - 3}{2\omega_{1,-1}{}^2 - 2\omega_{1,-1} + mR}\,. \tag{21}$$

The proton gyromagnetic ratio $g_p = 2m_p \mu_p$ is found to be $g_p = 2.61$, practically unchanged from the numerical value of Ref. 2. However, the relation between μ_p and R has changed: For $m = 0$, if was found in Ref. 2 that $\mu_p = 1.2R/6$, whereas in our $m \neq 0$ calculation the corresponding relation is $\mu_p = R/6$. Of course, the fact that R has changed accounts for the agreement between the two calculations of g_p. The proton charge radius is defined by

$$\langle r^2 \rangle_p = \left\langle P(s_z) \left| \int_{\text{bag}} d^3x \psi^\dagger(x) Q\psi(x) |\vec{x}|^2 \right| P(s_z) \right\rangle, \tag{22}$$

and implies the following lengthy expression for $\langle r^2 \rangle_p$:

$$\langle r^2 \rangle_p = \frac{R^2}{6}\, \frac{A}{(\omega_{1,-1}{}^2 - m^2R^2)(2\omega_{1,-1}{}^2 - 2\omega_{1,-1} + mR)}, \tag{23}$$

where A is a quartic in $\omega_{1,-1}$,

$$\begin{aligned} A = {} & 4\omega_{1,-1}{}^4 - 4\omega_{1,-1}{}^3 + \omega_{1,-1}{}^2 (8 + 6mR - 4m^2R^2) \\ & + \omega_{1,-1}(-6 - 8mR + 4m^2R^2) \\ & + (9mR - 6m^2R^2 - 6m^3R^3). \end{aligned}$$

Numerically, we find $(\langle r^2 \rangle_p)^{1/2} = 1.12$ fm, in somewhat poorer agreement with experiment than the $m = 0$ value, 1.04 fm.

Finally, we discuss our calculation of the lowest-lying excited state. We felt that it was of no special interest to proceed to any higher excitations because the static bag model under consideration does not do a particularly good job of describing the $S = 0$ spectrum of baryons taken as a whole. A more general bag boundary must be employed before improvement can be expected. At any rate, we calculated the energy of the $1S_{1/2}{}^2\, 1P_{1/2}$ quark configuration (two quarks with $n = 1$, $\kappa = -1$ and one quark with $n = 1$, $\kappa = 1$). The unknowns are now $\omega_{1,-1}$, $\omega_{1,1}$, R, and the excited-state energy E. The quantities m and B have been determined by the ground-state calculation. It was found that $\omega_{1,-1} = 2.647$, $\omega_{1,1} = 4.065$, $1/R = 113.2$ MeV, and $E = 1346$. The quark field energy is $E_Q = 1058$ MeV, and the volume energy of B is $E_B = 288$ MeV. Notice the slight shift in $\omega_{1,-1}$ relative to its value in the ground state.

III. CONCLUSION

For the most part, we have endeavored to discuss this calculation as a modified version of the work done in Ref. 2. However, there is an instructive aspect to our model not possible in the $m = 0$ calculation, namely, passage to the limit of nonrelativistic quarks. We shall interpret this limit as $m \to \infty$ with R remaining finite. Of course, because the quarks are assumed to be noninteracting, the mass of a nucleon cannot remain constant in this limit, but must itself become infinite. Consider the linear boundary-value equation for $n = 1$, $\kappa = -1$ quarks,

$$\tan[(\omega_{1,-1}{}^2 - m^2R^2)^{1/2}] = -\frac{(\omega_{1,-1}{}^2 - m^2R^2)^{1/2}}{\omega_{1,-1} + mR - 1}. \tag{24}$$

As $mR \to \infty$, the frequency $\omega_{1,-1} \to \infty$ as well, such that, in the limit, $\omega_{1,-1}{}^2 - m^2R^2 \to \pi^2$. Equation (24) then reads $0 = 0$. With this knowledge, we can proceed to study the behavior of the various nucleon observables. From Eq. (19), we see that $g_A \to \frac{5}{3}$ as $mR \sim \omega_{1,-1} \to \infty$. This is the standard nonrelativistic result, and serves as a check on our calculation. The proton gyromagnetic ratio demands a somewhat more careful study because the magnetic moment goes as an inverse mass in natural units. Therefore, we must first understand the limiting behavior of the proton mass. The nonlinear boundary condition (15) evaluated between nucleon states implies that $BR^4 \to 0$ as $mR \to \infty$ (i.e., that B vanishes if R remains non-

TABLE I. Dependence of quark mass and bag pressure on g_A. Both m and $B^{1/4}$ are expressed in units of MeV.

m	g_A	$B^{1/4}$
19.9	1.113	117.7
26.2	1.121	116.7
32.34	1.128	115.7
38.29	1.136	114.8
44.11	1.143	113.8
49.79	1.151	112.9
55.33	1.158	112.0
60.76	1.166	111.1
71.27	1.179	109.3
81.35	1.193	107.5
91.09	1.206	105.8
100.45	1.219	104.0
109.52	1.232	102.3
118.32	1.244	100.6
126.81	1.256	99.0

zero). The energy equation (16) then implies that the proton mass approaches $3m$ as $m\to\infty$. This results in a limiting value of the proton gyromagnetic ratio, $g_p\to 3$. Again, this is the standard quark-model result, as is derived, for example, in terms of a nice physical argument in Ref. 8. It is even possible to obtain a value for $\langle r^2\rangle_p$ as $m\to\infty$ from Eq. (23). Here, particular attention must be paid to the constraint $\omega_{1,-1}{}^2 - m^2R^2 \to \pi^2$. We find

$$\langle r^2\rangle_p \to (2\pi^2-3)R^2/(6\pi^2) = 0.283R^2,$$

which at least is finite and positive, if somewhat obscure in interpretation. Thus, our formulas reproduce the results of Ref. 2 as $m\to 0$ and of the nonrelativistic quark model as $m\to\infty$. It is clear, at least in this kind of model, why the predictions of the nonrelativistic quark model must always be limited in their success. The quarks within a nucleon appear to be relativistic,[9] a feature which must be explicitly taken into account.

It turned out that we were able to estimate the mass of the nonstrange quarks because (i) we arbitrarily chose to fit the axial-vector coupling constant g_A, and (ii) the other nucleon observables were less sensitive to the effect of quark mass compared to g_A. In other words, our feeling about the result $m=122$ MeV is that it is clearly not definitive, but rather represents the kind of information a realistic dynamical model can be expected to furnish. If quarks do indeed exist but are permanently confined within hadrons, then such calculations constitute our only means for determining quark mass. Given the above caveat, we feel that it is worth taking our value of m seriously enough to compare it with alternative estimates. These fall into two classes; "heavy," with $m=350$ MeV (see Ref. 9 for example) and "light," with $m\cong 10$ MeV (see Ref. 10, for example). We have no explanation at present for this wide range of estimates. There has been at least one estimate of quark mass which agrees with our "moderate" value. Jaffe and Llewellyn Smith[11] find $m\cong 120$ MeV using the (apparently) very different method of a subtracted sum rule for the σ operator. Perhaps it is noteworthy that they achieve this result by assuming the quark-gluon coupling to be negligible.

We feel that the problem of understanding the reason for the large differences in estimates of quark mass is an important one, as is further effort to elucidate the physics of the bag model. Work is in progress on each of these subjects.

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[3]E. Golowich, Phys. Rev. D 11, 589 (1975).

[4]The time dependence for our stationary states is the usual $\psi(\vec{x},t)=\psi(\vec{x})\exp(-iEt)$.

[5]We shall generally suppress the n,κ dependence of ω except in situations where it is necessary for clarity.

[6]We have no comment to make at this time beyond the discussion in Ref. 2 regarding zero-point energy within the bag.

[7]At this point, it appears that a physically more appealing frequency parameter would be $\omega' = (\omega^2 - m^2R^2)^{1/2}$, so that the field energy per quantum for a given mode would be $(\omega^2+m^2R^2)^{1/2}/R$. However, we have found it generally more useful to work with the variable ω as defined in Eq. (11).

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