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Inclusive trident electroproduction as a test of parton model and bilocal algebra

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We analyze deep-inelastic trident production, $e+p\to e+\mu^+\mu^-+$ anything, in the approprate scaling region, assuming the validity of the parton model and the bilocal algebra. It is shown that the measurement of the difference between the scaling inclusive trident cross section of the electron and that of the positron will allow the determination of a proton structure function $V(x)$, which involves the cube of the parton charges, in unexplored kinematic regions. This structure function obeys an exact sum rule, independent of the momentum distribution of partons; the sum rule therefore holds for nuclear targets as well. Since $V(x)$ is related to odd-charge-conjugation exchanges in the t channel, the Pomeranchukon and other C-even contributions are not present, so that $V(x)$ should have a readily integrable quasielastic peak. In the light-cone approach the same difference of cross sections measures the symmetric function $S_{Q^3}(\eta)$, which cannot be obtained from electroproduction and neutrino-induced-production data. So far as scaling is concerned, the two approaches seem to give similar results.

I. INTRODUCTION

The scaling observed in SLAC-MIT electron scattering experiments[1] has generated considerable interest in various models of hadron structure. The parton model,[2] which is based on a pointlike structure of hadrons, had a notable success in explaining various deep-inelastic phenomena.[3] An alternative approach to the problem of hadron structure is the algebra of bilocal operators on the light cone.[4] It is suggested that this algebra has the same structure as that implied by free or interacting quark field theories.[4] The bilocal operators which arise in commutators of currents on the light cone[4,5] provide a succinct description of Bjorken scaling[6] in deep-inelastic scattering. In the case of deep-inelastic electron-proton scattering, where only local electromagnetic currents are involved, the implications of the bilocal algebra were found to be in agreement with all the results of the parton model,[2] which do not depend on the explicit assumptions about the momentum distribution of partons.

In spite of all these successes, the recent experiments at electron-positron colliding-beam accelerators[7] create considerable doubt as to the meaning of the success of the light-cone algebra and the parton model in describing deep-inelastic scattering. These experiments indicate that the ratio $R=\sigma(e^+e^-\to\gamma\to\text{hadrons})/\sigma(e^+e^-\to\mu^+\mu^-)$ increases linearly with the center-of-mass energy squared, at least up to 25 GeV^2, rather than being constant according to naive scaling expectation. Such a nonscaling behavior is difficult to understand using parton models, which, because of precocious scaling in inelastic electron-proton scattering, predict early scaling in e^+e^- annihilation also. Although there seems to be a change in the interpretation of nonscaling behavior of R because of the discovery of new resonances,[8] no firm conclusions can be drawn at this stage. A possible theoretical understanding of the behavior of R can be that there seems to be a threshold near 4 GeV and that new hadronic degrees of freedom are being excited.[9] In any case, the naive parton model has got to be modified before it can accommodate these experimental results.

In view of this situation it is, therefore, important to obtain and test all the predictions of the parton model as well as of bilocal algebra, particularly in unexplored kinematic regions and in cases where the number of new assumptions is minimal. With this motivation we study in this paper the trident process

$$e+p\to e+\mu^++\mu^-+\text{anything} \tag{1}$$

in an appropriate "scaling" region as a test of the parton model as well as of bilocal algebra. Unlike earlier tests[10] using this reaction, which were plagued by background and interference terms, our aim is to provide a test which is free of these difficulties. More precisely, measurement of the difference between the scaling inclusive trident cross sections of the electron and positron will allow determination of a structure function which depends upon the cube of the charge of various partons.[11] Such a difference in cross sections, which could be measured in future colliding-beam facilities,[12] depends only on the odd-charge-conjugation piece of the parton distribution functions

$$V(x) = \sum_a \lambda_a{}^3 U_a(x)$$

$$= \sum_a \lambda_a{}^3 U_a^{\text{odd}}(x) ,$$

$$U_a^{\text{odd}}(x) = \tfrac{1}{2}[U_a(x) - U_{\bar{a}}(x)] ,$$

where $U_a(x)$ is the probability of finding a parton of type a with charge λ_a and fraction x of the proton's momentum in the infinite momentum reference frame. Unlike $\nu W_2^{ep}(x)$, which is sensitive to the squared charges of partons and which obtains contributions from even-charge-conjugation t-channel exchange terms, the new structure function should show a quasielastic peak; sum rules involving the integral of $V(x)$ can be expected to converge in an experimentally accessible region. Moreover, integrals over $V(x)$ are determined by quantum-number conservation rules (e.g., charge, baryon number, hypercharge) and thus provide a test for fractional charge without the need for making additional assumptions about the parton distributions,[13] as is the case with $\nu W_2^{ep}(x)$. Further, we note that since $U_a^{\text{odd}}(x)$ are related in parton models to the structure functions for highly inelastic neutrino scattering, $V(x)$ should be completely determined by the results of the neutrino experiments.

In the light-cone approach[14] the difference in cross sections of the electron and positron, which involves a commutator of a bilocal operator with a local operator, will measure the symmetric scaling function $S_{Q^3}(\eta)$. This function, which cannot be obtained from the analysis of the electroproduction and neutrino-induced production processes, can thus be measured from this experiment. Further, the particular form of scaling obtained for the difference cross section provides a severe test of the bilocal algebra. The form of scaling obtained in bilocal algebra is similar to that of the parton model. Although we use ordered limits in our application of bilocal algebra, the results obtained are similar to those obtained with a more general approach.[14] Note that since both the photons in the trident experiments are far off the mass shell the singularities due to leading Regge behavior are avoided.[15]

In Sec. II we state the limits in which the difference cross section can be evaluated using the parton model. Sum rules for the structure function $V(x)$ are discussed briefly in Sec. III. The evaluation of the cross section using bilocal algebra is given in Sec. IV.

II. THE INTERFERENCE CROSS SECTION

The diagrams contributing to process (1) are shown in Fig. 1. The Compton diagram has been evaluated by using bilocal algebra[16] as well as the parton model.[17] However, as is obvious, the Compton contribution is difficult to isolate experimentally because of background diagrams and interference terms. On the other hand, the difference of inclusive cross sections

$$d\sigma(e^- + p \to e^- + \mu^+ + \mu^- + X) - d\sigma(e^+ + p \to e^+ + \mu^+ + \mu^- + X)$$

integrated over the relative phase space of the muon pair is due to the interference of the amplitude of Fig. 1(a) with that of Fig. 1(b) and Fig. 1(b′) (see Fig. 2). The difference cross section can be written as[18]

$$\frac{d\sigma(e^-)}{(d^3p_+/E_+)(d^3p_-/E_-)(d^3p'/E')} - \frac{d\sigma(e^+)}{(d^3p_+/E_+)(d^3p_-/E_-)(d^3p'/E')} = \frac{16\alpha^4}{(2\pi)^4}\frac{M}{P\cdot p}\frac{1}{q^2(q-q')^2q'^4} L_{\alpha\beta\mu}(p_{+\beta}p_{-\nu} + p_{-\beta}p_{+\nu} - p_+\cdot p_- g_{\beta\nu})W_{\nu\mu\alpha} , \quad (2.1a)$$

where

$$L_{\alpha\beta\mu} = \tfrac{1}{2}\,\mathrm{Tr}[(q'+p')^{-2}(\not{p}\gamma_\alpha\not{p}'\gamma_\beta\not{p}'\gamma_\mu + \not{p}\gamma_\alpha\not{q}'\gamma_\beta\not{p}'\gamma_\mu) + (q'-p)^{-2}(\not{p}\gamma_\beta\not{p}\gamma_\alpha\not{p}'\gamma_\mu - \not{p}\gamma_\beta\not{q}'\gamma_\alpha\not{p}'\gamma_\mu)] , \quad (2.1b)$$

and the hadronic tensor $W_{\nu\mu\alpha}$ is a particular discontinuity of the three-photon Compton amplitude

$$W_{\nu\mu\alpha}(P,q,q') = \frac{4\pi^2 E_p}{M}\int d^4y\, d^4z\, e^{iq'\cdot y}e^{i(q-q')\cdot z}\langle P|J_\alpha(z)T^*[J_\nu(y)J_\mu(0)]|P\rangle . \quad (2.1c)$$

The same amplitude, but with $q'^2 = 0$, also contributes to the inclusive bremsstrahlung experiment discussed by Brodsky *et al.*[11] We now proceed to evaluate this amplitude in the parton model. In this model the leading contribution to $W_{\nu\mu\alpha}$ arises when all three photons interact with an individual parton and the contribution is given by the kinematical factor multiplied by the scale-invariant function $V(x)$ (see Fig. 3). This result is derived from the following considerations.

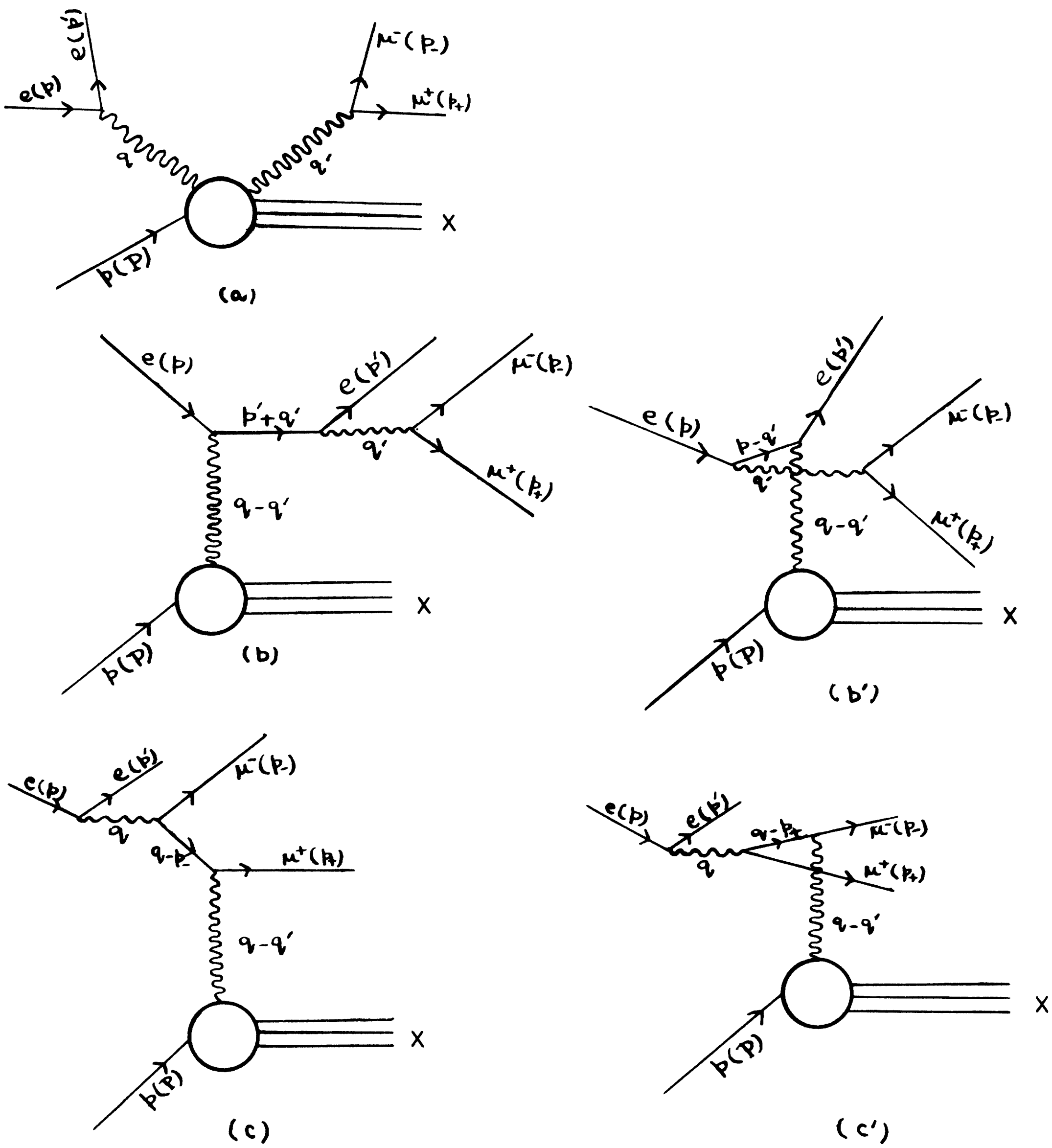

FIG. 1. Feynman diagrams contributing to trident electroproduction of massive muon pairs. By integrating over the relative phase space of the muon pair, the interference between diagrams (c), (c′) and diagrams (a), (b) (b′) can be eliminated. The interference contribution requires that all amplitudes have the same final state.

(a) In an infinite momentum frame if both spacelike photons $(q-q')$ and q have large transverse momenta and are such that the outgoing timelike photon q' also has large transverse momentum, then only those diagrams in which all the photons interact with the same parton need to be considered. All those diagrams in which photons interact with more than one parton are strongly suppressed. This assumption is, in general, not satisfied for inelastic Compton processes. In case of small transverse momentum of the outgoing photon, multiple parton processes can be important even in the scaling region.[19]

(b) In order to neglect the interparton interactions during the time period of photon-parton interaction (impulse approximation) and the final-state interactions (incoherence approximation), we require that $-q^2$, $-(q-q')^2$, $2P\cdot q$, and $2P\cdot(q-q')$ all be large.

The standard assumptions of the parton model

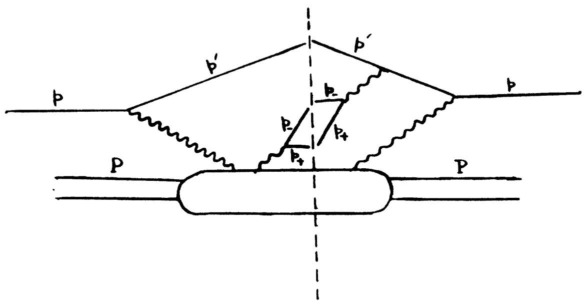

FIG. 2. A typical absorptive amplitude contributing to the $e^{\mp} p \to e^{\mp}\mu^{+}\mu^{-}X$ cross-section difference, from the interference of the diagrams of Fig. 1(a) and 1(b).

imply that we work in the following kinematic region:

$$-q^2,\, q'^2,\, -(q-q')^2,\, q\cdot q' \gg M^2\,,$$
$$2P\cdot q,\, 2P\cdot(q-q') \gg M^2\,, \qquad (2.2)$$

with

$$x = -(q-q')^2/2P\cdot(q-q') \ \text{ fixed}.$$

The difference of electron and positron inclusive trident cross sections scales and is proportional to the cube of the charge of the partons. We now proceed along usual lines to obtain the parton-model expression for the tensor $W_{\nu\mu\alpha}$. Denoting the fraction of the proton's momentum in an infinite momentum frame carried by parton i as η_i we can write

$$W_{\nu\mu\alpha} = \frac{1}{2P\cdot(q-q')} \times \frac{1}{2Mx}\sum_{n,i} |a_n|^2 \langle n|\delta(\eta_i - x)\lambda_i{}^3|n\rangle M^i_{\nu\mu\alpha}\,, \qquad (2.3)$$

with

$$|a_n|^2 = 1\,.$$

The expressions for the kinematic factor $M^i_{\nu\mu\alpha}$ for spin-$\frac{1}{2}$ and spin-0 cases are

$$M^i_{\nu\mu\alpha} = \tfrac{1}{2}\,\mathrm{Tr}\,\not{P}_i\gamma_\alpha(\not{P}_i + \not{q} - \not{q}')[(P_i+q)^{-2}\gamma_\nu(\not{P}_i + \not{q})\gamma_\mu + (P_i - q')^{-2}\gamma_\mu(\not{P}_i - \not{q}')\gamma_\nu]\,, \qquad (2.4\mathrm{a})$$

$$M^i_{\nu\mu\alpha} = 2(P_i+q)^{-2}[(2P_i+q-q')_\alpha(P_i+q)_\nu(2P_i+q)_\mu] + 2(P_i-q')^{-2}[(2P_i+q-q')_\alpha(2P_i+q-2q')_\mu 2P_{i\nu}] - [4g_{\mu\nu}(2P_i+q-q')_\alpha]\,, \qquad (2.4\mathrm{b})$$

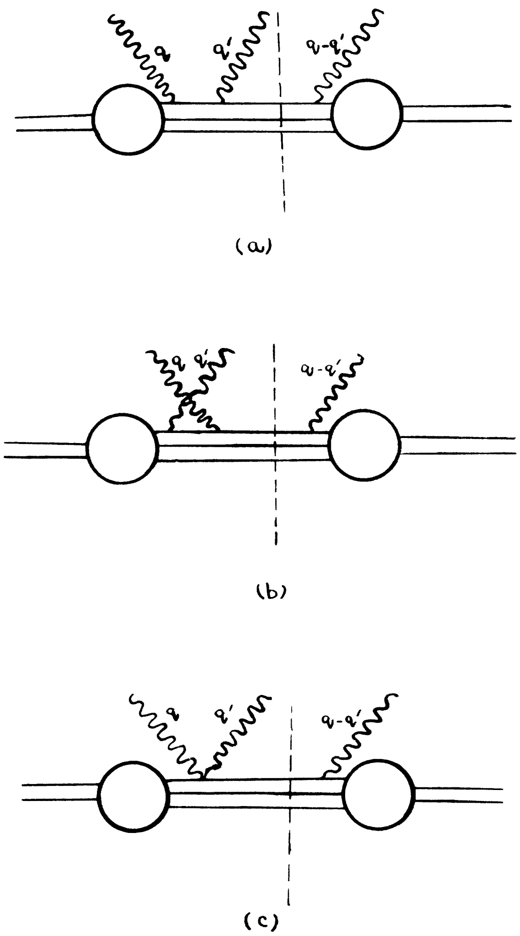

FIG. 3. Dominant parton-model contribution to the interference amplitude in the scaling limit. The kinematical restrictions require that all three photons interact with the same parton. Diagram c contributes to spin-$\frac{1}{2}$ partons only.

respectively. From $W_{\nu\mu\alpha}$ we can extract the structure function $V(x)$

$$V(x) = \sum_{n,i} |a_n|^2\langle n|\delta(\eta_i - x)\lambda_i{}^3|n\rangle = \sum_a \lambda_a{}^3 U_a(x)\,, \qquad (2.5)$$

the sum being over all partons and their antipartons of different types denoted by α.

After integrating over the relative phase space of the muon pair the difference in cross sections, Eq. (2.1a), can be written as

$$\frac{d\sigma(e^-)}{(d^3p'/E')d^4q'}-\frac{d\sigma(e^+)}{(d^3p'/E')d^4q'}=\frac{8\alpha^4}{(2\pi)^4}\frac{1}{P\cdot p\,q'^4}\sum_a |T^a_{\rm int}|^2\lambda_a{}^3U_a(x)\,, \tag{2.6a}$$

where

$$|T^a_{\rm int}|^2=\int d^4q''\delta(q'^2+q''^2-4m_\mu{}^2)\delta(q'\cdot q'')(p_{+\beta}p_{-\nu}+p_{+\nu}p_{-\beta}-p_+\cdot p_-g_{\beta\nu})\frac{L_{\alpha\beta\mu}M^a_{\alpha\nu\mu}}{(-q^2)(q-q')^4}\,,$$
$$q''=p_+-p_-\,. \tag{2.6b}$$

The evaluation of $|T_{\rm int}|^2$ is straightforward, although cumbersome. Evaluate the leptonic trace (2.1b), the hadronic traces (2.4), multiply the two, and then integrate over the relative phase space to get $|T_{\rm int}|^2$. The expressions are given in the Appendix for both spin-0 and spin-$\frac{1}{2}$ partons. Here we concentrate on a particularly simple kinematic region, namely,

$$2q\cdot q',\, q'^2 \ll -(q-q')^2 \ll 2P\cdot p,\, 2P\cdot p',\, 2P\cdot q',\, 2p\cdot q',\, 2p'\cdot q'\,, \tag{2.7}$$

in which the formulas simplify considerably. We choose this region for illustrative purposes only—in general, the full formulas of the Appendix must be used. In this region, we have

$$|T_{\rm int}|^2=\frac{\pi}{3(-q^2)}[x^2(5ut+2vt-2t^2)+x(-t+4u+8v-2tv-ty+10uy-7uz-12vy+4uz)$$
$$+(-2tz+2uz-2y+3z-2tz)],\quad \text{for spin-}\tfrac{1}{2}\text{ partons}\,, \tag{2.8a}$$

$$=\frac{\pi}{3(-q^2)}[-12tx^2(u+v)+x(14t-9ty-8uy+2uz-4vy+8uz-4u)$$
$$+(7y+8z-6yz+8z^2)],\quad \text{for spin-0 partons}\,, \tag{2.8b}$$

where

$$2P\cdot p=K^2u\,,\quad 2P\cdot p'=K^2v\,,\quad 2P\cdot q'=K^2t\,,\quad 2p\cdot q'=K^2y\,,\quad 2p'\cdot q'=K^2z\,,$$
$$2q\cdot q'=K^2(y-z)\,,\quad q'^2=K^2l\,,\quad -q^2=K^2(l-y+z+1)\,,\quad K^2=-(q-q')^2\,. \tag{2.8c}$$

The different dependences of spin-0 and spin-$\frac{1}{2}$ terms on the invariants allow one, in principle, to distinguish the parton's spin. We note that besides simplifying the formulas this kinematic region may satisfy the important experimental requirement that the interference be a substantial fraction of the signal,[11] i.e., the interference-to-signal ratio $[d\sigma(e^-)-d\sigma(e^+)]/[d\sigma(e^-)+d\sigma(e^+)]$ be of the order of unity. Therefore, it should be quite feasible to measure the odd-charge-conjugation structure function $V(x)=\sum_a\lambda_a{}^3U_a(x)$ from the $e^- - e^+$ cross section difference by the relation

$$V(x)=\sum_a\lambda_a{}^3U_a(x)=\frac{[d\sigma(e^-)/(d^3p'/E')d^4q'-d\sigma(e^+)/(d^3p'/E')d^4q']}{[8\alpha^4/(2\pi)^4](1/P\cdot p\,q'^4)|T_{\rm int}|^2}\,. \tag{2.9}$$

Equation (2.9) severely tests the parton model by requiring that the right-hand side be a function of x alone.

In concluding this section, we note that it is possible to relate the structure function $V(x)$ to the structure functions measured in deep-inelastic neutrino and electron scattering off protons and neutrons.[3] For example, in the quark model, one obtains a general relation

$$V(x)=\frac{1}{108}\left(\frac{9}{x}[F_2^{\nu n}(x)-F_2^{\nu p}(x)]-7[F_3^{\nu n}(x)+F_3^{\nu p}(x)]-\frac{1}{27}[U_\lambda(x)-U_{\bar\lambda}(x)]\right)\,. \tag{2.10}$$

The last term drops for nonstrange baryons if one assumes $U_\lambda(x)=U_{\bar\lambda}(x)$ for such targets. Alternatively, the last term can be expressed in terms of $\Delta S=1$ deep-inelastic neutrino scattering. Note that the integral $\int_0^1 dx[U_\lambda(x)-U_{\bar\lambda}(x)|=S$ vanishes for nonstrange baryons.

III. SUM RULES

We now write down sum rules satisfied by the structure function $V(x)$. All these sum rules follow from quantum-number conservation. Specifically we have

$$Q=\int_0^1 dx \sum_a \lambda_a U_a(x),$$
$$Y=\int_0^1 dx \sum_a y_a U_a(x), \qquad (3.1)$$
$$B=\int_0^1 dx \sum_a b_a U_a(x),$$

where Q, Y, and B (λ_a, y_a, and b_a) are the charge, hypercharge, and baryon numbers of the target hadron (parton) of interest. These sum rules depend only on the odd–charge-conjugation part of $U_a(x)$:

$$U_a^{\text{odd}}=\tfrac{1}{2}[U_a(x)-U_{\bar{a}}(x)].$$

In general, it is possible to reduce λ_a^3 to a linear combination of λ_a, y_a, and b_a so that the integral

$$\int_0^1 V(x)\,dx=\int_0^1 dx\sum \lambda_a^3 U_a(x) \qquad (3.2)$$

is determined by quantum-number conservation alone.

This is in striking contrast to the sum rules involving the electroproduction structure functions $\nu W_2^{ep}(x)$ and $\nu W_2^{en}(x)$, defined by

$$W_2^{ep}(x)=x\sum_a \lambda_a^2 U_a(x),$$
$$W_2^{en}(x)=x\sum_a \lambda_a^2 U_{\hat{a}}(x), \qquad (3.3)$$

where $\hat{a}$ is the isospin reflection of the parton a. Recall that the usual sum rules for sums over the squares of the parton's charges involving $\nu W_2(x)$ depend on a variety of assumptions.[13] Since νW_2^{ep} and νW_2^{en} depend on the combination $U_a^{\text{even}}=\frac{1}{2}[U_a(x)+U_{\bar{a}}(x)]$, they are unrelated to the conserved quantum numbers. Sum rules involving $\nu W_2(x)$ are valid only with specific assumptions about the distribution of partons in the nucleon.[11]

Sum rules for the odd-charge-conjugation structure function do not suffer from such difficulties:

(a) In all models with partons of charge 0 or ±1 (e.g., Drell-Levy-Yan; Han-Nambu; σ model; etc.) $\lambda_a^3=\lambda_a$, so that

$$\int_0^1 dx\,V(x)=Q=\begin{cases}1 \text{ for protons}\\ 0 \text{ for neutrons.}\end{cases} \qquad (3.4)$$

(b) In the standard quark parton model $\lambda_a^3=\frac{1}{3}\lambda_a+\frac{2}{9}b_a$, so that

$$\int_0^1 dx\,V(x)=\tfrac{1}{3}Q+\tfrac{2}{9}B=\begin{cases}\frac{5}{9} \text{ for protons}\\ \frac{2}{9} \text{ for neutrons.}\end{cases} \qquad (3.5)$$

Since the sum rule is independent of parton distribution, similar results hold for nuclear targets as well:

$$\int_0^1 dx\,V(x)=(3Z+2A)/9 \quad \text{(quark model)}. \qquad (3.6)$$

For nuclei with $A=2Z$, the quark-model sum rule gives $\frac{7}{9}$ of the corresponding result for integrally charged constituents. The sum rules (3.4), (3.5), and (3.6) thus provide a stringent test of fractionally charged versus integrally charged quark models. These sum rules have the further advantage that the tests can be performed on nuclear targets as well, with the additional benefit of large cross sections.

IV. BILOCAL ALGEBRA

An alternative approach to the question of scaling in deep-inelastic processes is the algebra of bilocal currents on the light cone. This algebra postulates that when all possible separations are lightlike

$$(x-y)^2=(u-v)^2=(x-u)^2=(y-u)^2=(x-v)^2=(y-v)^2=0, \qquad (4.1)$$

the commutation relations of the bilocal operators are those as suggested by the free-quark model[4]:

$$[J_\mu^i(x,u),J_\nu^j(y,v)]\hat{=}\partial_\rho\{\epsilon(x_0-v_0)\delta((x-v)^2)\}(if_{ijk}-d_{ijk})[S_{\mu\nu\rho\sigma}J_\sigma^k(y,u)+i\epsilon_{\mu\nu\rho\sigma}J_\sigma^{k,5}(y,u)]$$
$$+\partial_\rho\{\epsilon(u_0-y_0)\delta((u-y)^2)\}(if_{ijk}+d_{ijk})[S_{\mu\nu\rho\sigma}J_\sigma^k(x,v)-i\epsilon_{\mu\nu\rho\sigma}J_\sigma^{k,5}(x,v)], \qquad (4.2)$$

with similar relations for axial currents. The limited validity of the commutator is implied by the symbol $\hat{=}$, denoting equality on the light cone.

Relations of this type have already been tested successfully in SLAC-MIT experiments. In these cases, however, the currents $J_\mu^i(x,u)$ and $J_\nu^j(y,v)$ are ordinary local electromagnetic currents, with $x=u$ and $y=v$. These tests involve single commutators in which the initial densities are local operators. Recently, several experiments[20] have been proposed to test the algebra of bilocal operators. Unfortunately, all these tests are difficult to perform because of background contributions. We now show that the interference cross section can be used as a test of bilocal algebra. Although the cross section involves a retarded commutator of a bilocal operator with a local operator, the test is free of background complications.

We begin by ignoring possible singular terms in the T^* product which destroy the covariance of T product. The T^* is then replaced by T, and $W_{\nu\mu\alpha}$ becomes

$$W_{\nu\mu\alpha}=\frac{4\pi^2E_p}{M}\int d^4y\,d^4z\,e^{i(q'\cdot y+q\cdot z-q'\cdot z)}\theta(y_0)\times\langle P|[J_\alpha(z),[J_\nu(y),J_\mu(0)]]|P\rangle. \quad (4.3)$$

In arriving at Eq. (4.3), we have used the spectrum conditions, $q_0'>0$ and $(q_0-q_0')>0$, to replace the T product by a retarded commutator, and the resulting product with another commutator. We now use the usual stationary-phase argument in order to conjecture that the integral in (4.3) is dominated by the light cone with respect to all pairs of space-time points. To achieve this, we first take $-q^2$, q'^2, and $2P\cdot q$ large with fixed ratios, keeping $-(q-q')^2$ and $2P\cdot(q-q')$ finite. In the second step we take the limit $-(q-q')^2$, $2P\cdot(q-q')\to\infty$, keeping the ratio $\eta=x=-(q-q')^2/2P\cdot(q-q')$ finite. The ordering of the two limiting procedures is important in this argument.[21] Then causality implies that all the three space-time differences in the integral (4.3) are lightlike. Therefore, we can use the commutators of Eq. (4.2), and obtain

$$W_{\nu\mu\alpha}=\frac{1}{2M}\frac{S_{Q^3}(\eta)}{2P\cdot(q-q')}N_{\nu\mu\alpha}, \quad (4.4a)$$

where

$$N_{\nu\mu\alpha}=2P_kR_\beta[L_\rho L^{-2}(S_{\nu\mu\rho\sigma}S_{\alpha\sigma\beta k}-\epsilon_{\nu\mu\rho\sigma}\epsilon_{\alpha\sigma\beta k}) - N_\rho N^{-2}(S_{\nu\mu\rho\sigma}S_{\alpha\sigma\beta k}+\epsilon_{\nu\mu\rho\sigma}\epsilon_{\alpha\sigma\beta k})], \quad (4.4b)$$

$$N=q'-\eta P,\quad L=q+\eta P,\quad R=q-q'+\eta P,$$

$$S_{\alpha\sigma\beta k}=g_{\alpha\beta}g_{\sigma k}+g_{\alpha k}g_{\sigma\beta}-g_{\alpha\sigma}g_{\beta k},$$

$$\langle P|S_\sigma(y,z)|P\rangle=\frac{P_\sigma}{2E_p}\frac{1}{(2\pi)^3}\tilde{S}_{Q^3}(P\cdot(y-z)) =\frac{P_\sigma}{2E_p}\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}d\eta\, e^{-i\eta P\cdot(u-z)}S_{Q^3}(\eta), \quad (4.5)$$

and

$$S_\sigma(y,z)=J_\sigma(y,z)+J_\sigma(z,y).$$

$S_\sigma(y,z)$ contains the matrix Q^3, due to three electromagnetic currents. The SU(3) content of the currents $J_\sigma(y,z)$ and the scaling function $S_{Q^3}(\eta)$ is given below:

$$J_\sigma(y,z)=\tfrac{1}{2}:\bar\psi(y)\gamma_\sigma Q^3\psi(z):,$$
$$S_{Q^3}(\eta)=\frac{4}{27}S^{(0)}(\eta)+\frac{1}{3}S^{(3)}(\eta)+\frac{1}{3\sqrt3}S^{(8)}(\eta). \quad (4.6)$$

Q is the charge matrix

$$Q=\begin{pmatrix}\frac23 & 0 & 0\\ 0 & -\frac13 & 0\\ 0 & 0 & -\frac13\end{pmatrix}.$$

From neutrino data we know $S^{(3)}(\eta)$ because it is proportional to $F_2^{\nu p}-F_2^{\nu n}$, and the combination $(\frac23)S^{(0)}(\eta)+(1/\sqrt3)S^{(8)}(\eta)$ because it is proportional to $F_3^{\nu p}+F_3^{\nu n}$. But we do not know $S^{(0)}(\eta)$ and $S^{(8)}(\eta)$ separately. Therefore, $S_{Q^3}(\eta)$ cannot be evaluated using electron scattering and neutrino scattering data. This theoretically interesting function can, however, be measured from the difference of cross sections for incident electrons and positrons.

At this point, we should emphasize that although the ordered limiting procedure is an important part of the above argument, similar results can be obtained in a different manner.[22] We further mention that since both photons are off mass shell the complications due to Regge behavior do not arise here. Note also that the result (4.4a), although not exactly equal to the parton model result (2.3), is nevertheless similar in structure to it. We thus see that, so far as scaling is concerned, light-cone analysis and the parton model give equivalent results. This point will be further discussed in the following.

The calculation of the difference cross section is now straightforward. We substitute the expression (4.4) for $W_{\nu\mu\alpha}$ in Eq. (2.1) and, after integrating over the relative phase space of the muon pair, obtain

$$\frac{d\sigma(e^-)}{(d^3p'/E')d^4q'}-\frac{d\sigma(e^+)}{(d^3p'/E')d^4q'}=\frac{8\alpha^4}{(2\pi)^4}\frac{1}{P\cdot pq'^4}S_{Q^3}(\eta)|L_{\rm int}|^2, \quad (4.7)$$

where

$$|L_{\rm int}|^2=\int d^4q''\delta(q'^2+q''^2-4m_\mu^2)\delta(q'\cdot q'')(p_{+\beta}p_{-\nu}+p_{-\beta}p_{+\nu}-p_+\cdot p_-g_{\beta\nu})\frac{L_{\alpha\beta\mu}W_{\nu\mu\alpha}}{q^2(q-q')^2 2P\cdot(q-q')}.$$

The evaluation of $|L_{\rm int}|^2$ is similar to that of $|T_{\rm int}|^2$ in the parton model and is equally cumbersome. The complete expressions are, therefore, given in the Appendix. Here we concentrate on the simple kinematic region (2.7)

$$2q\cdot q',\ q'^2 \ll -(q-q')^2 \ll 2P\cdot p,\ 2P\cdot p',\ 2P\cdot q',\ 2p\cdot q',\ 2p'\cdot q',$$

in which the expression for $|L_{\rm int}|^2$ simplifies, and becomes

$$|L_{\rm int}|^2 = \frac{\pi}{3(-q^2)}\,(2\eta^2 ut)\,. \tag{4.8}$$

This result, although not same as the parton-model result (for $|T_{\rm int}|^2$), should be of the same order of magnitude. The scaling function $S_{Q^3}(\eta)$ is obtained from the experiment by the relation

$$S_{Q^3}(\eta) = \left[\frac{d\sigma(e^-)}{(d^3p'/E')d^4q'} - \frac{d\sigma(e^+)}{(d^3p'/E')d^4q'}\right] \Big/ \left[\frac{8\alpha^4}{(2\pi)^4}\frac{|L_{\rm int}|^2}{P\cdot p\, q'^4}\right]. \tag{4.9}$$

The requirement that the right-hand side of Eq. (4.9) be a function of η alone thus serves as a test of bilocal algebra.

In conclusion, we note that the result (4.7) obtained from bilocal algebra is similar in structure to the parton-model result (2.6a). However, for exact equality of the two approaches, we should have $(x\equiv\eta)$

$$S_{Q^3}(\eta)|L_{\rm int}|^2 = V(x)|T_{\rm int}|^2\,, \tag{4.10}$$

where complete expressions, given in the Appendix, should be used for L and T. Since the scaling functions $S_{Q^3}(\eta)$ and $V(x)$ are experimentally observable functions, Eq. (4.10) could provide a possible test for the equivalence of the parton model and the bilocal algebra.

V. CONCLUDING REMARKS

We have shown that the parton model predicts a very specific scaling form for the interference cross section for trident production and that the right-hand side of Eq. (2.9) depends on the scaling variable x alone and not on any other dimensionless ratio of invariants. This provides a strong test of the validity of the parton model. A similar test of the bilocal algebra is provided by Eq. (4.9), which can also be used to obtain the theoretically interesting structure function $S_{Q^3}(\eta)$. Further, since the structure function $V(x)$ is proportional to the cube of the parton charge, it is possible to obtain exact sum rules which would provide a test of whether the constituents of proton have fractional or integral charges. Since $V(x)$ is odd with respect to charge conjugation, these sum rules are expected to converge in an experimentally accessible region. We further note that the results of the parton model for the structure function $W_{\nu\mu\alpha}$ are similar to those of bilocal algebra. We obtain the relation (4.10), between $V(x)$ and $S_{Q^3}(\eta)$, for the exact equality of the two approaches. The nominal order of the magnitude of this process is down by a factor of α^2 as compared to the electroproduction experiments. All these observations coupled with the fact that there exists a simple kinematic region in which the interference-to-signal ratio is maximal should make inclusive trident production a feasible experiment for proton and nuclear targets.

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APPENDIX

Here we give the complete expressions for $|T_{\rm int}|^2$ and $|L_{\rm int}|^2$ which are involved in parton-model and light-cone calculations, respectively. Some of the variables in the following have been defined in (2.8c) and (4.5), and the rest will be defined presently. We ignore the lepton and proton masses. For $|T_{\rm int}|^2$ we write

$$|T_{\rm int}|^2 = \frac{\pi}{3(-q^2)}\,(P_{11}T_1 + P_{12}T_2 + P_{21}T_3 + P_{22}T_4 - 2d_1T_5 - 2d_2T_6) \tag{A1}$$

where for spin-$\frac{1}{2}$ partons

$$\begin{aligned}T_1 = {}&[xu' + (2l+z)(y-z) + 2l(l+1)][x^2(6uv+2tu) + 2xu(1-z) - 2ul'] \\ &+x^2(l't'v') - 2x(xt'-l')[2lv(-l+y-z-1) + uz(3l+z) + vyz] + xl't'[z(y-z) - 2ll''] \\ &+x(2ll''-yz)[xt^2 - 2l(u-v) + t(y-z)] + xt'z[xu'' + y(y-z) + ll''] - xu''(-xv+zl') + x^2t^2(2ll''-yz) \\ &+xl'z[xt^2 + t(y-z) - 2l(u-v)] + xty[xu' + z(y-z) - 2ll''] \\ &+2xuz\{x[t(y-z) - 2l(u-v)] + (y-z)^2 + 4ll''\} + 6l[x^2(tvy - l''t^2) + l''z] + 2xz(u-v-t)[y(y-z) + 2ll''],\end{aligned} \tag{A2}$$

$$T_2 = x(yz - 2ll'')[x(u-v)(4t-u+3v)+t(y-z)+(u-v)(-6l-2y+2)+2lt]$$
$$-6xl[x(tvy+l''t^2-uz)(u-v)+l''(2lt+ty-tz+uz-vz)+vy(y-z)-z^2(u-v)]$$
$$+xv'\{x^2[u(3t+8v)-2v(t+2v)]$$
$$+x[(l+1)(4t+2u-6v)+(y-z)(-6t+2u)-(u-v)(2l+z)-2u(2l+y+2z)-4ty]+l'(2l+y+z)\}$$
$$+xz^2(u-v)(xu-l'')+xl't[z(y-z)-2ll'']-xu''z(xt'+l'z)-2xtz[y(y-z)+2ll'']$$
$$-xl'(u-v)[xv'+z(y-z)-2ll'']+x^2t^2z[-2xu+2(l+z+1)]+xz(-2xu+y)[xt^2+t(y-z)-2l(u-v)], \quad \text{(A3)}$$

$$T_3 = x[xu''+y(y-z)+2ll''][4xv+3y(u-v)-2l''t'-2v(y-z)+2lv+tz-4uy]$$
$$+x(yz-2ll'')\{x[(u-v)(2t-4v)+t(t+2u)]-2l''(t-2u)+2u(y-z)\}$$
$$+xy^2\{x[2vt-4u(u-v)]+2l'v\}+2x^2v'(-lt+ty-uy)+x(2xvy-l''y)[xt^2+t(y-z)-2l(u-v)]$$
$$+2xuy\{x^2t^2+x[2t(y-z)-2l(u-v)]+(y-z)^2+4ll''\}$$
$$-6xl\{x[tuz+tvy-l''t^2-2vy(u-v)]-l''v''+vy(y-z)\}, \quad \text{(A4)}$$

$$T_4 = xu''\{2x^2v(3u-2t)+2x[3v(l-y)+2l''(u-v)-v(y-z)+z(t-3u)]+z(-2l+3y+z+2)\}$$
$$+xl't[y(y-z)+2ll']+x^2t^2y(z-2l)$$
$$+x(yz-2ll'')\{x[(u-v)(2u-t)-t^2]+l'(u-v)-t(y-z)\}$$
$$+2xy^2(u-v)(xt+l')-xl'(u-v)[u''z+y(y-z)+2ll'']$$
$$+x[y(u-v)-2lu][xv'+z(y-z)-2ll'']$$
$$+xty[z(y-z)-2ll'']+x[xt^2+t(y-z)-2l(u-v)](2xvy+2ll''-2yz)+x^2t^2(2xvy+2l''y)$$
$$-6xl\{x[(u-v)(ty-uz)+l''[3t(u-v)-t^2-(u-v)^2]+utz]$$
$$+y(u-v)(y-2l)+l''(2lt-4lu+uy-uz-vy+vz)-tyz\}, \quad \text{(A5)}$$

$$T_5 = T_6 = 0 .$$

For spin-0 partons we have

$$T_1 = [xv'+z(y-z)+2ll''][4x^2(tu+2uv)-4xu(l+y)-l''(l+z)]$$
$$+[xu''+y(y-z)+2ll'']\{x[z(u-v)-lv]-l''(l+y)\}$$
$$+[-2xuz+l''(l+z)]\{2x^2t^2+2x[2t(y-z)-3l(u-v)]+(y-z)^2+2ll''\}, \quad \text{(A6)}$$

$$T_2 = xu''[4x(lv-tz+uz)+2l''(l-z)+4lz-4z(y-z)]$$
$$+x[t(2xt+y-z)-2l(u-v)](-4xuz+4yz-2ll'')$$
$$+xv'\{8x^2u(t+2v)+2x[2l''(u-v-t)+2u(-3l+y-4z)-3y(t+2u)]+2ll''+4y(l+2z)\}, \quad \text{(A7)}$$

$$T_3 = [xu''+y(y-z)+2ll''][4x^2(2uv-tv)-4xvy+l''(l-y-z)-z(y-z)]$$
$$+\{2x^2t^2+3x[t(y-z)-2l(u-v)]+(y-z)^2+4ll''\}[2xvy-l''(l-y)]$$
$$+[xv''+z(y-z)-2ll'']\{2x[lu-y(u-v)]-l''(l-y)\}, \quad \text{(A8)}$$

$$T_4 = xu''\{8x^2v(2u-t)+x[2l''(5t-2u)-4v(3l+4y)+2z(t+2u)]+2l''(y-l)\}$$
$$+xv'\{4x(2ty-y(u-v)-2lu]+2l''(l+y)-4ly+4y(y-z)\}$$
$$+x[2xt^2+t(y-z)-2l(u-v)][4xvy+2l''(l+y)-4yz], \quad \text{(A9)}$$

$$T_5 = z^2[4xu-2(l+z+1)]+(yz-ll'')[4x(v+t)-(l+z-1)]$$
$$-6l[2x(-tl''-uz+vy)+l''(l+2z)]-2l''[2xv'+z(y-z)-2ll''], \quad \text{(A10)}$$

$$T_6 = z^2(4xv+2l'')+(yz-2ll'')[4x(u-y+z)+2(l-y+1)]$$
$$-6l[2x(tl''-uz+vy)+2l''(y-l)]-2l''[2xu''+y(y-z)-2ll'']. \quad \text{(A11)}$$

The propagators P_{ij} $(i, j=1,2)$ and other quantities are

$$P_{ij}=d_i D_j\ ,$$
$$d_1=\frac{K^2}{(q'+p')^2}\ ,\quad d_2=\frac{K^2}{(q'-p)^2}\ , \tag{A12}$$
$$D_1=\frac{K^2}{(xP+q)^2}=\frac{K^2}{L^2}\ ,\quad D_2=\frac{K^2}{(xP-q')^2}=\frac{K^2}{N^2}\ ,$$

$$\begin{aligned} &tz-2lu=u',\quad ty-2lu=u'',\quad tz-2lv=v',\quad ty-2lv=v'',\\ &-t+2u-2v=t',\quad 2l-y+2z+2=l',\quad l-y+z+1=l''. \end{aligned} \tag{A13}$$

The light-cone interference function is

$$|L_{\rm int}|^2=(2\pi/3)\frac{8\eta}{(-q^2)(q-q')^4}(L_1+L_2+L_3+L_4)\ , \tag{A14}$$

where

$$\begin{aligned} L_1=\left[\frac{1}{L^2(q'+p')^2}\right]\{&(L\cdot q'p'\cdot q'-L\cdot p'q'^2)[R\cdot pP\cdot(2p'+q')+P\cdot pR\cdot(2p'+q')-R\cdot Pp\cdot q']\\ &-(L\cdot q'P\cdot q'-L\cdot Pq'^2)[2R\cdot pp'\cdot q']-(p'\cdot q')^2(2R\cdot pL\cdot P)\\ &+(L\cdot q'R\cdot q'-L\cdot Rq'^2)(P\cdot p'p\cdot q'-P\cdot pp'\cdot q'+P\cdot q'q^2/2)\\ &+(P\cdot q'p'\cdot q'-P\cdot p'q'^2)[R\cdot pL\cdot(2p'+q')+L\cdot pR\cdot(2p'+q')+L\cdot Rp\cdot q']\\ &+(R\cdot q'p\cdot q'-R\cdot pq'^2)(L\cdot Pp'\cdot q'+L\cdot p'P\cdot q'-L\cdot q'P\cdot p')\\ &+(R\cdot q'P\cdot q'-R\cdot Pq'^2)(-L\cdot q'q^2/2-L\cdot pp'\cdot q'-L\cdot p'p\cdot q')\\ &+(p\cdot q'p'\cdot q'+q^2q'^2/2)[R\cdot PL\cdot q'-P\cdot q'L\cdot R-L\cdot PR\cdot(2p'+q')]\\ &-3q'^2(L\cdot PR\cdot pp'\cdot q'+L\cdot q'R\cdot Pq^2/2+R\cdot PL\cdot p'p\cdot q'-L\cdot RP\cdot q'q^2/2-L\cdot RP\cdot p'p\cdot q')\}\ , \end{aligned}$$
$$\begin{aligned} &L_2=-L_1(L\leftrightarrow N, R\leftrightarrow P),\\ &L_3=L_1(p\leftrightarrow p', q'\leftrightarrow -q')\ ,\\ &L_4=-L_1(L\leftrightarrow N,\ R\leftrightarrow P,\ p\leftrightarrow p', q'\leftrightarrow -q'). \end{aligned} \tag{A15}$$

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[1]For a recent review, see E. D. Bloom, in *Proceedings of the Sixth International Symposium on Electron and Photon Interactions at High Energies, Bonn, 1973*, edited by H. Rollnik and W. Pfeil (North-Holland, Amsterdam, 1974), p. 227.

[2]R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, Mass., 1972); J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).

[3]C. H. Llewellyn Smith, Phys. Rep. 3C, 261 (1972).

[4]H. Fritzsch and M. Gell-Mann, in *Broken Scale Invariance and the Light Cone*, edited by M. Dal Cin *et al.* (Gordon and Breach, New York, 1971); J. M. Cornwall and R. Jackiw, Phys. Rev. D 4, 367 (1971); D. J. Gross and S. B. Treiman, *ibid.*, 1059 (1971).

[5]Y. Frishman, Ann. Phys. (N. Y.) 66, 373 (1971); Phys. Rev. Lett. 25, 966 (1970).

[6]J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[7]A. Litke *et al.*, Phys. Rev. Lett. 30, 1189 (1973); B. Ritcher, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974*, edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. IV-37.

[8]J.-E. Augustin *et al.*, Phys. Rev. Lett. 33, 1406 (1974); J. J. Aubert *et al.*, *ibid.* 33, 1404 (1974); C. Bacci *et al.*, *ibid.* 33, 1408 (1974); G. S. Abrams *et al.*, *ibid.* 33, 1453 (1974).

[9]For a review, see F. J. Gilman, invited talk presented at the Orbis Scientiae II, University of Miami, Coral Gables, 1975, SLAC Report No. SLAC-PUB-1537 (unpublished).

[10]J. Iliopoulos and E. A. Paschos, Phys. Rev. D 6, 1340 (1972); H. Fritzsch, Nuovo Cimento 12A, 1916 (1972); B. L. Koul, P. N. Pandita, and Y. Singh, Phys. Rev. D 10, 298 (1974).

[11]S. J. Brodsky, J. F. Gunion, and R. L. Jaffe, Phys. Rev. D 6, 2487 (1972) and references therein. The structure function $V(x)$ introduced in the present paper reduces to that of Brodsky *et al.* in the limit $q'^2=0$ (i.e., the outgoing photon q' being real). In this sense the present analysis is in a more general kinematic

domain.

[12]SLAC Report No. 146 (unpublished).

[13]J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969); Phys. Rev. D 1, 1450 (1970).

[14]Joe Kiskis, Phys. Rev. D 10, 1219 (1974); J. Iliopoulos and E. A. Paschos, Ref. 10.

[15]Y. Frishman, *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill. 1972*, edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 4, p. 119.

[16]J. Iliopoulos and E. A. Paschos, Ref. 10; H. Fritzsch, Ref. 10.

[17]B. L. Koul *et al.*, Ref. 10.

[18]We use the metric and other conventions of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[19]S. J. Brodsky and P. Roy, Phys. Rev. D 3, 2914 (1971); R. L. Jaffe, *ibid.* 4, 1507 (1971); J. F. Gunion, S. J. Brodsky, and R. Blankenbecler, *ibid.* 6, 2652 (1972).

[20]D. J. Gross and S. B. Treiman, Phys. Rev. D 4, 2105 (1971); H. Terazawa, *ibid.* 5, 2259 (1972); J. Iliopoulos and E. A. Paschos, *ibid.* 6, 1340 (1972); H. Fritzsch, Nuovo Cimento 12A, 1016 (1972).

[21]J. Iliopoulos and E. A. Paschos, Ref. 10.

[22]Joe Kiskis, Ref. 14.