PHYSICAL REVIEW D VOLUME 12, NUMBER 7 1 OCTOBER 1975

Masses and other parameters of the light hadrons*

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(Received 2 June 1975)

The masses and static parameters of the light hadrons (the pseudoscalar and vector meson nonets and the baryon octet and decuplet) are calculated using the bag model. The effects of quark kinetic energy, bag energy, strange-quark mass, colored-gluon exchange in lowest order, and energy associated with certain quantum fluctuations are included. These are parameterized by four constants which have fundamental significance and do not change from multiplet to multiplet. The fit to the spectrum is good. The ordering of all the states is given correctly. The intramultiplet splittings are very accurate for the baryon decuplet and reasonably good for the other multiplets. The isosinglet pseudoscalar mesons η and η' are treated separately in light of the special role that gluon intermediate states play in these states. Magnetic moments, weak decay constants, and charge radii are calculated. Where comparison with experiment is possible, we generally obtain improvement over the naive quark model. It is shown that exotic baryons do not exist as narrow resonances in this approximation to the bag theory.

I. INTRODUCTION

During the past decade a quark theory of hadron structure has been developed which is successful in interpreting vast amounts of experimental data in terms of a few extraordinarily simple ideas. The ingredients of this theory are as follows: Hadrons are composed of quarks—three per baryon, two (quark and antiquark) per meson. The quarks come in several "flavors", the three of Gell-Mann and Zweig,[1] augmented perhaps by new quarks for new hadronic degrees of freedom such as charm,[2] and in three colors. The quarks interact among themselves relatively weakly by the exchange of an octet of massless, colored gluons coupled in the manner of Yang-Mills[3] to their color indices. The interaction must be weak at short distances to explain scaling in lepton scattering experiments; it must be weak near zero momentum transfer to account for the lack of large renormalizations of naive quark-model estimates of transitions among light baryons. The SU(3) symmetry generated by the permutation of color indices is unbroken. Quarks of different flavors may have different masses to account for the observed breakdown of Gell-Mann's SU(3) and for the high masses of states composed of charmed quarks—if that is indeed what the $J(3100)$ and $\psi(3700)$ are.

Finally, and essentially, colored quarks and colored gluons are not themselves part of the physical spectrum. To accomplish this we assume that the phenomenological fields which describe the dynamics of quarks and gluons do not permeate all space, but rather are confined to the interior of hadrons. This can be understood in much the same way as the phonon field of a crystal is defined only over the region of space occupied by the crystal. The quark-gluon degrees of freedom may similarly characterize collective variables describing the "low excitation" of hadronic matter. The only way we know of to provide a description of this consistent with Lorentz invariance (at least at the classical level) is by introducing a new term, $-g_{\mu\nu}\theta_s B$, into the energy-momentum tensor of the theory. θ_s is a function which is unity where the quark and gluon fields are defined, zero where they are not. B is a universal constant with the dimensions of pressure. It is then an exact consequence of the unbroken SU(3) color symmetry that all states have conventional quantum numbers. It is tempting to speculate upon an origin for this unconventional term somewhere in the more conventional part of the theory; however, this is irrelevant to our present purposes. Having chosen this mechanism of confinement we have progressed from a general quark model to the MIT bag model.[4]

The object of this paper is to study the spectrum of the lightest baryons (the pseudoscalar and vector-meson nonets, the baryon octet and decuplet) in an approximation to the bag model which takes into account all of the quark model features mentioned above. Previously[5] a "zeroth-order" treatment of the baryon octet and decuplet was given. In Ref. 5 the gluons were ignored entirely as were quark masses and certain quantum fluctuations which we now believe to be important and which will be included here. Some features of that work should be recalled for comparison with the present: the one parameter, B, was used to set the scale of baryon masses; the calculated magnetic moment and charge radius of the proton were found to be in good agreement with the observed values, which are roughly three times the Bohr magneton and Compton wavelength, respectively; the masslessness of the quarks reduced the nucleon axial-vector charge (g_a) from the "nonrelativistic" quark model value of $\frac{5}{3}$ to a value of 1.09. However, that model possessed great degeneracies: The octet and decuplet were degenerate as were the two meson nonets; furthermore, the meson and baryon masses were in the ratio $(\frac{2}{3})^{3/4}=0.74$ which is roughly adequate for the vector mesons, but fails badly for the pseudoscalars.

In the present work these degeneracies will be lifted. It is important to keep track of the number of free parameters. Here there will be four: the bag constant, B; the quark-gluon coupling constant, $\alpha_c \equiv g^2/4\pi$; the strange quark mass, m_s; and a constant which parameterizes the zero-point energy associated with the quantum modes in the bag, Z_0. Z_0 is, in fact, calculable, but that work is in progress and we shall treat it as a parameter here. We will also explore the effects of giving the nonstrange quarks a small mass, m_0, but the spectrum is rather insensitive to the inclusion of such a mass and we do not fit it.

In the remainder of the Introduction we shall review briefly the way in which the various effects discussed above are handled in our model. Each is considered in detail in Sec. II. We then return to compare our approach with other quark models.

The bag model is defined by equations of motion and boundary conditions.[4] For each field degree of freedom there is an equation of motion inside the bag and a homogeneous boundary condtion at the bag's surface. In addition, there is another "quadratic" boundary condition which assures that the pressure of the constituent fields balances the pressure B locally on the surface. These equations determine both the field motion and the motion of the surface, and are difficult to solve in general. Solutions with static, spherical boundaries appropriate to particles at rest were found in Ref. 5. This "cavity" may be populated with quark (or gluon) modes satisfying the appropriate homogeneous boundary condition. However, the quadratic boundary condition allows only quark modes with total angular momentum $\frac{1}{2}$. We shall not relax that condition here, and shall consider only bags with static, spherical boundaries. In Sec. II A it is shown how the equation of motion and homogeneous boundary condition fix the form of quark wave functions in the cavity, and how the boundary condition determines the frequency of a quark mode in terms of the quark mass and the cavity radius, R. This is very important because it expresses the kinetic energy of a quark mode as a function of its mass and R. Also in Sec. II A the contribution of individual quark modes to certain operators (i.e., magnetic moment, axial-vector charge) is computed.

In addition to the energy associated with occupied modes in the cavity there must be an energy $\pm\frac{1}{2}\hbar\omega$ associated with each mode (unoccupied and occupied) of frequency ω (+ for bosons, − for fermions). This gives rise to an infinite term proportional to the cavity volume, which is absorbed into a renormalization of B, and a finite term in the Hamiltonain proportional to $1/R$ ($E_0=-Z_0/R$), which is observable because R changes from hadron to hadron. This effect is discussed in some detail in Sec. II B and will be discussed further in a forthcoming publication.[6]

The gluon interaction is considered in Sec. II C. It is treated to lowest order in α_c. The relevant diagrams are shown in Fig. 1. Its principal effect is to produce a non-SU(3)-invariant spin-spin interaction between quarks. Since the gluons are also confined to the bag, the gluon propagators shown in Fig. 1 include terms produced by the bag boundary conditions. The interaction energy, Fig. 1(a), can be computed exactly (analytically). The treatment of the self-energy, Fig. 1(b), is more delicate. A substantial fraction of the self-energy is absorbed in renormalization of the quark mass m_0 or m_s. Including all of Fig. 1(b) would therefore be double counting. However, the gluon field generated by the interaction, Fig. 1(a), does not satisfy the required boundary conditions. We have therefore adopted the somewhat arbitrary prescription of including only that part of Fig. 1(b) which must be added to Fig. 1(a) to meet the gluon boundary conditions. A more consistent procedure would be to calculate the whole of Fig. 1(b) and perform the required renormalization, much in the fashion that Chodos and Thorn[7] have computed the electrodynamic self-energy of massless quarks confined to a cavity.

The result of Sec. II is an effective Hamiltonian for hadrons including quark, gluon-interaction, zero-point, and bag energies, in terms of the parameters B, α_c, Z_0, m_s (m_0), and the cavity radius R. We next implement the quadratic boundary condition. For angular momentum $\frac{1}{2}$ quarks in a spherical cavity achieving the local pressure balance is equivalent to minimizing the Hamiltonian

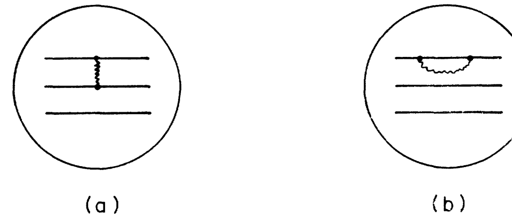

FIG. 1. Lowest order (in α_c) gluon interaction diagrams for a baryon. Mesons are similar. (a) Gluon exchange; (b) gluon self-energy.

with respect to R. This fixes R for a given hadron in terms of the other parameters and its quark content. In Sec. III this apparatus is used to compute the masses of the light hadrons. The masses of the nucleon, Ω^- baryon, Δ, and ω resonances are taken as input.

In Sec. IV the baryon magnetic moments, the axial-vector couplings of octet β-decays, and various charge radii are computed and compared with experiment. Section V is devoted to a summary of our results and a discussion of some outstanding problems.

Although the whole of this paper is presented in the framework of the bag model, we believe the analysis to be quite general. To see this consider the terms in the bag Hamiltonian: First, consider the quark kinetic energy, determined by its mass and a boundary condition—which amounts to the recognition that the quark is confined to a region approximately the size of a proton. We expect that any theory which is consistent with the confinement picture (quasifree quarks moving in a finite domain) will behave similarly. If the quarks are turned back at the edge of the hadron by a strong confining interaction, the wave function will be strongly damped in that region. The penetration into the domain of the strong interaction will therefore be minimal. The linear boundary condition provides a realization of this property for a Dirac field. The quark momentum is large. It ranges from $2.04/R$ for a massless quark to π/R for an infinitely heavy quark. For $R \sim 1$ F and a quark mass of 300 MeV the momentum is approximately 500 MeV. It is therefore inconsistent with the uncertainty principle to consider a nonrelativistic quark model, recent resurrections not withstanding[8]: Some measure of the quark kinetic energy is essential. Second, let us consider the bag energy (BV). A similar term is necessary in any confinement scheme in recognition of the fact that the confining "potential" must itself be a dynamical object which, in particular, will carry energy. Models which ignore such contributions to the mass are manifestly at odds with relativity. Furthermore, a prescription for reconciling the confining potential locally with relativity is necessary.[9] In the bag theory this is provided naturally by the nonlinear boundary condition which locally determines the domain of the confining potential. The third input is the gluon interaction energy. The form and significance of this term is shared by most models.[8] Finally we have the zero-point energy. This is present in any extended system when the dynamics is characterized by phenomenological collective variables describing excitations with a scale related to the size of the object. Again, phonons are an excellent physical example. Zero-point energies are also encountered in the only other relativistic extended model studied in great detail, namely, the dual resonance model.[10]

Before passing on to the body of the paper we wish to note that certain parts of our analysis overlap recent work by others. Golowich[11] has pointed out the effect of a nonstrange quark mass on the calculation of g_a for neutron β decay. Barnes[12] has included a strange quark mass in the "zeroth"-order model of Ref. 5 and computed its effect on octet baryon magnetic moments[13] and axial-vector changes. His results do not differ substantially from ours.

II. TERMS IN THE (PHENOMENOLOGICAL) QUARK HAMILTONIAN

A. Wave-functions, quark frequencies, and single-particle operators

Here we shall discuss the wave functions which will be used for the quarks. In doing so, we are generalizing the discussion of Ref. 5 to allow for nonzero quark mass. In the hadron states considered, the quarks will all occupy the lowest mode for the free Dirac field in a spherical cavity with radius R. Of course, as we will see, the actual quark energy will change from state to state because the radius R will vary from hadron to hadron in order to satisfy the nonlinear boundary condition of the bag model. This was mentioned in the Introduction and will be discussed further in Sec. III.

Here R will stand for a general value of the radius. The equation for the mode wave functions is[14]

$$(-i\vec{\gamma}\cdot\vec{\nabla}+\gamma^0\omega+m)q=0,$$

which is solved with the linear boundary condition

$$-i\vec{\gamma}\cdot\hat{r}q=q, \quad r=R. \tag{2.1}$$

The lowest-mode solutions take the simple form,

$$q(\vec{x},t)=\frac{N(x)}{\sqrt{4\pi}}\begin{pmatrix}\left(\dfrac{\omega+m}{\omega}\right)^{1/2} ij_0(xr/R)U \\ -\left(\dfrac{\omega-m}{\omega}\right)^{1/2} j_1(xr/R)\vec{\sigma}\cdot\hat{r}U\end{pmatrix}e^{-i\omega t/R}. \tag{2.2}$$

N is taken so that $\int_{\text{bag}} d^3x q^\dagger q=1$:

$$N^{-2}(x)=R^3 j_0^{\,2}(x)\frac{2\omega(\omega-1/R)+m/R}{\omega(\omega-m)}. \tag{2.3}$$

j_i are spherical Bessel functions and U are two-component Pauli spinors. We choose to express the frequency of the lowest mode in the form

$$\omega(m,R)\equiv\frac{1}{R}[x^2+(mR)^2]^{1/2}, \tag{2.4}$$

where $x=x(mR)$, and obeys the eigenvalue equation obtained from Eqs. (2.1) and (2.2),

$$\tan x = \frac{x}{1 - mR - [x^2 + (mR)^2]^{1/2}}. \qquad (2.5)$$

A graph of the smallest positive root of Eq. (2.5), $x(mR)$, is presented in Fig. 2. Each occupied quark mode of mass m in a cavity of radius R contributes a term $\omega(m, R)$ to the energy of the system.

Using the wave function Eq. (2.2) we may compute various parameters associated with the quark occupying this mode, such as its magnetic moment, mean square charge radius, g_a, etc. All of these quantities will depend parametrically on $x(mR)$ and R, where x is given by Eq. (2.5). We list these quantities, their expression in terms of the quark wave functions, and the values computed using the wave function Eq. (2.3) in Table I. It is interesting to study the extreme relativistic and nonrelativistic limits of certain of these. In the case of the magnetic moment note that when m is zero, the quark moment takes on the value used previously.[5] For very large m it approaches the value $1/2m$, which is the magnetic moment of a free particle with mass m or, equivalently, the magnetic moment of a massive Dirac particle moving in a nonrelativistic S-state. Consider, for example, the case of a particle with mass $m = 330$ MeV, moving in a hadron with the radius $1/R = 200$ MeV (1 F), for which $mR = 1.15$, giving $\mu = 0.54/2m$. This differs significantly from the naive (nonrelativistic) value. We take this as a further example of the inadequacy of nonrelativistic quark-model calculations. Similarly, consider the axial-vector coupling for neutron β decay. For massless quarks this has the value 1.09. In the limit $m \to \infty$ we find $g_A = \frac{5}{3}$—the result of the nonrelativistic quark model. As noted by Golowich[11] a value of $mR \sim 1$ reproduces the experimental value $g_A = 1.25$.

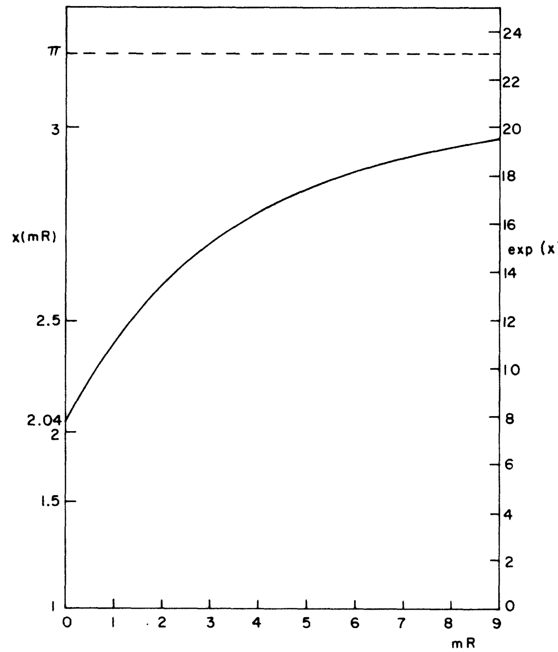

FIG. 2. Eigenfrequency $x(mR)$ of the lowest quark mode with mass in a spherical cavity of radius R.

B. Energy of zero-point fluctuations

We turn now to the field fluctuation energy. Previously[5] we considered only the field energy associated with the occupied modes in the cavity. It is well known that in any quantum theory there is another source of field energy, namely, the zero-point energy of all the modes. In conventional field theory, the "zero-point" energy which is extensive is discarded since the volume filled by the fields does not change in any process. In the bag model, since the volume depends upon the

TABLE I. Contributions of a single quark in the lowest cavity eigenmode to certain static parameters of hadrons. The magnetic moment and charge radius are diagonal. The axial-vector charge connects either nonstrange quarks ($i = j$; $\Delta S = 0$) or one strange and one nonstrange quark ($i = s$, $j = u$, or d; $\Delta S = 1$). Notation: $\lambda \equiv mR$, $x = x(mR)$ is the root of Eq. (2.5), and $\alpha = R\omega(mR)$, where $\omega(mR)$ is defined by Eq. (2.4). Note $\alpha^2 = \lambda^2 + x^2$.

Single-particle operator	Definition	Value in lowest quark eigenmode
Magnetic moment $\mu(m_i, R)$	$\left(\int_{\text{bag}} d^3x \frac{1}{2}\vec{r} \times :q^+(x)\vec{\alpha}q(x):\right)_z$	$\mu = \frac{R}{6}\frac{4\alpha + 2\lambda - 3}{2\alpha(\alpha - 1) + \lambda}$
Axial-vector coupling constant $(g_a)_{ij}$	$\left(\int_{\text{bag}} d^3x :q_i^\dagger(x)\vec{\sigma}q_j(x):\right)_z$	$(g_a)_{ij} = \frac{2x_i x_j}{3(x_i^2 - x_j^2)} \frac{2(\alpha_i - \alpha_j) + \lambda_i - \lambda_j}{[2\alpha_i(\alpha_i - 1) + \lambda_i]^{1/2}[2\alpha_j(\alpha_j - 1) + \lambda_j]^{1/2}}$
Mean square Charge radius $\langle r^2 \rangle$	$\int_{\text{bag}} d^3x \, \lvert \vec{x} \rvert^2 :q^+(x)q(x):$	$\langle r^2 \rangle = R^2 \frac{\alpha[2x^2(\alpha - 1) + 4\alpha + 2\lambda - 3] - \frac{3}{2}\lambda(4\alpha + 2\lambda - 2x^2 - 3)}{3x^2[2\alpha(\alpha - 1) + \lambda]}$

mass of the state, the question of the zero-point energy is less straightforward. Conventional divergences are handled in relativistic field theory by introducing a cutoff. Physical quantities which are cutoff sensitive are isolated and taken to be uncomputable parameters associated with the theory (so-called "renormalized" quantities). Quantities which are insensitive to the cutoff when it is large in comparison to all relevant scales are the predictive elements in the theory. We shall employ the same procedure with the zero-point energy (and all other short-wavelength sensitive quantities) of the confined fields in the bag theory. We may hope that short-wave sensitive quantities will not be affected by the confinement of the fields which is associated with a long scale, namely, R. However, we are aware that the classical scale which separates the inside of the hadron from the outside is zero; that is, the surface is "sharp." Hence, short-wavelength quantum effects could lead to difficulties near the surface. We have found that these potential difficulties do *not* arise in a study of the zero-point energy of massless fields.

We have computed the zero-point energy with a cutoff,

$$E_0 \equiv \tfrac{1}{2}\sum \omega e^{-\omega/\Omega} \tag{2.6}$$

in a slab of thickness L (which is the only shape cavity allowing an analytic calculation).[15] When the constituents are vector fields and Dirac fields (but not if they are scalar fields) in the limit $\Omega \gg L$, we find

$$E_0(\text{vector}) = \Omega^4 \frac{3V}{\pi^2} - \frac{\pi^2}{720}\frac{V}{L^4}, \tag{2.7}$$

$$E_0(\text{dirac}) = -\Omega^4 \frac{6V}{\pi^2} - \frac{7}{4}\frac{\pi^2}{720}\frac{V}{L^4}. \tag{2.8}$$

The finite term is independent of the form of the cutoff function $f_c(\omega/\Omega)$ which in Eq. (2.6) is exponential. It should be noted that $\pi^2/720 \simeq 0.0137$ is numerically small. In the case of scalar fields there is also a cutoff-dependent term proportional to $A\Omega^3$. ($A = V/L$ is the cross-sectional area of the slab.) It can be shown for arbitrary smooth surfaces that there is no term proportional to $A\Omega^3$ for vector and Dirac fields. Fortunately, our relativistic model contains only quarks and gluons (Dirac and vector fields), so the problem of the scalar field does not concern us. Therefore, the sharp boundary does not introduce any new quantum divergences in this approximation to the quark-gluon bag. We also note that in the case of the slab there are no terms proportional to $\Omega^2 V/L^2$, $\Omega V/L^3$, or $V/L^4 \log\Omega$. In calculations done with spherical geometry these also seem to be absent, but we have no general demonstration of this. We shall assume that they are absent. The divergent terms in Eqs. (2.7) and (2.8) are proportional to V and independent of the shape of the cavity. This is the "traditional" extensive divergent zero-point energy. In its covariant form it corresponds to an infinity in the stress tensor proportional to $g^{\mu\nu}$ which is put equal to zero in conventional theory by the introduction of a cutoff-dependent counter term in the Lagrange density which is taken to cancel it *exactly*. In our theory we merely choose the counter term to renormalize the divergence. The counter term is therefore the "bare" version of the bag constant B. If we wish, we may associate B with the quantum fluctuations of the quark and gluon fields: The fields are confined by their zero-point energy.

In the absence of a numeral calculation of E_0 in the spherical case (which is currently in progress),[6] we shall simply parameterize the finite part of the zero-point energy by including in our mass a term

$$E_0 = -Z_0/R. \tag{2.9}$$

Since not all constituents are massless Z_0 can depend upon (mR). We shall neglect this dependence since (1) most constituents are massless, and (2) the form of Z_0 is the same for all hadrons. It changes only with variation in R, which is generally not too large from one hadron to another, except in the case of the lightest mesons. From the slab calculation we may get a crude estimate of the magnitude and sign of Z_0. If in Eqs. (2.7) and (2.8) we substitute $L \cong R$ and $V \cong \frac{4}{3}\pi R^3$, and take eight colors of gluons and nine (= three flavors × three colors) for the number of quarks we find

$$Z_0(\text{slab}) = \frac{\pi^3}{540}(8 + \tfrac{7}{4}9) = 1.36.$$

Below it will appear that the value of Z_0 needed in the phenomenology is

$$Z_0 \sim 2.0.$$

This is consistent with the crude estimate, but of course a real calculation of Z_0 is needed, and will eventually be provided.

C. Quark-gluon interaction (two-particle operators)

Here we calculate the quark interaction energy due to their coupling to colored gluons. The interaction will be calculated only to lowest order in $\alpha_c = g^2/4\pi$, which shall be the only parameter of the interaction: The appropriate diagrams may be evaluated exactly for a spherical cavity as a function of its radius. The gluon interaction will lift the spin degeneracies of our model, splitting the nucleon from the Δ resonance, and the ρ from the π. Such an interaction has been discussed often in the past,[16] most recently by De Rújula, Georgi, and Glashow.[8] The differences between previous treatments and the one given here are, first, that ours is entirely relativistic and, second, that we are able to calculate the reduced matrix elements of the interaction in all quark configurations (mesons and baryons), rather than fit them as parameters. We use no group theory since the radius of the cavity changes from state to state: Higher-order representation-mixing effects are implicitly involved.

The lowest-order gluon-exchange graphs are shown in Fig. 1. Since the quarks remain in the lowest cavity mode, the current at the vertices in Fig. 1(a) is time-independent. Consequently, only the static part of the gluon propagator contributes in Fig. 1(a). This is not true for the self-energy diagrams: In Fig. 1(b) the intermediate quark may be in any cavity mode. For reasons which will be explained below we shall truncate this sum and consider only the term in which the quark remains in the lowest mode. The gluon propagator in Fig. 1(b) will therefore also be the static propagator.

To lowest order in α_c the non-Abelian gluon self-coupling does not contribute—the gluons act as eight independent Abelian fields. The problem reduces to ordinary electro- (or magneto-) statics with the boundary conditions on the surface,[4]

$$\hat{r}\cdot\vec{E}^a=0\ , \tag{2.10}$$

$$\hat{r}\times\vec{B}^a=0\ . \tag{2.11}$$

The index "a" denotes color and runs from 1 to 8. $\vec{E}^a$ and $\vec{B}^a$ are gluon electric and magnetic field vectors. These boundary conditions are necessary in order to confine the gluons to the bag and correspond for confined vector fields to the boundary condition Eq. (2.2) obeyed by the quark fields.

The electrostatic interaction energy of a static charge distribution is

$$\Delta E_E=\tfrac{1}{2}g^2\sum_a\int_{\text{bag}}d^3x\,\vec{E}^a(x)\cdot\vec{E}^a(x)\ . \tag{2.12}$$

Similarly, the magnetostatic interaction energy is

$$\Delta E_M=-\tfrac{1}{2}g^2\sum_a\int_{\text{bag}}d^3x\,\vec{B}^a(x)\cdot\vec{B}^a(x)\ , \tag{2.13}$$

where $g^2=4\pi\alpha_c$. $\vec{E}^a$ and $\vec{B}^a$ are determined from the quark charge and current distributions by Maxwell's equation and the boundary conditions Eqs. (2.10) and (2.11).

Before evaluating the gluon field energy it is necessary to discuss further the treatment of the self-energy diagrams [Fig. 1(b)] which are included in Eqs. (2.12) and (2.13). These diagrams contribute to a renormalization of the quark masses. A proper treatment would separate this contribution from the rest of Fig. 1(b) since its effect is already included in the phenomological quark mass we use. We have not attempted this analysis, but realize that including some parts of Fig. 1(b) risks recounting effects already included in the quarks' masses. We have adopted the following minimal prescription: Only those parts of Fig. 1(b) necessary to enable $\vec{E}^a$ and $\vec{B}^a$ to satisfy the boundary conditions, Eqs. (2.10) and (2.11) will be included in our calculation. We have checked that including more of Fig. 1(b) does not alter our spectrum substantially though it does (as expected) require rather large alterations of the quark masses and of Z_0.[17]

The consequences of our prescription for Fig. 1(b) are simply stated: It can be ignored for calculation of magnetic energies since the $\vec{B}^a$ field of Fig. 1(a) satisfied Eq. (2.11) by itself; the static term (quark remaining in the same energy state) must be included in the calculation of electric energy. With these provisos Eqs. (2.12) and (2.13) may be rewritten:

$$\Delta E_E=\tfrac{1}{2}g^2\sum_a\sum_{i,j}\int_{\text{bag}}d^3x\,\vec{E}_i^a(x)\cdot\vec{E}_j^a(x)\ , \tag{2.14}$$

$$\Delta E_M=-g^2\sum_a\sum_{i>j}\int_{\text{bag}}d^3x\,\vec{B}_i^a(x)\cdot\vec{B}_j^a(x)\ . \tag{2.15}$$

$\vec{B}_i^a$ ($\vec{E}_i^a$) is the color magnetic (electric) field generated by the ith quark in the hadron.

The color magnetic field must satisfy

$$\vec{\nabla}\times\vec{B}_i^a=\vec{j}_i^a,\quad r<R \tag{2.16a}$$

$$\vec{\nabla}\cdot\vec{B}_i^a=0,\quad r<R \tag{2.16b}$$

$$\hat{r}\times\sum_i\vec{B}_i^a=0,\quad r=R \tag{2.16c}$$

where $\vec{j}_i^a$ is the color current of the ith quark:

$$\vec{j}_i^a=q_i^\dagger\vec{\alpha}\lambda^a q_i \tag{2.17}$$

$$=-\frac{3}{4\pi}\hat{r}\times\vec{\sigma}_i\lambda_i^a\frac{\mu_i'(r)}{r^3}\ . \tag{2.18}$$

Here $\mu_i'(r)$ is the scalar magnetization density of a quark of mass m_i in the lowest cavity eigenstate. The integral of $\mu_i'(r)$ yields the quark modes' magnetic moment:

$$\mu(m_i,R)=\int_0^R dr'\,\mu_i'(r')\ , \tag{2.19}$$

which is listed in Table I. Equations (2.16) may be integrated to determine $\vec{B}_i^a(x)$:

$$\vec{B}_i^a(x) = \frac{\lambda_i^a \vec{\sigma}_i}{4\pi}\left(2M_i(r) + \frac{\mu(m_i,R)}{R^3} - \frac{\mu(m_i,r)}{r^3}\right) + \frac{3\lambda_i^a}{4\pi}\hat{r}(\vec{\sigma}_i\cdot\hat{r})\,\mu(m_i,r)/r^3 \quad , \tag{2.20}$$

where $\mu(m_i, r)$ is the integral of $\mu_i'(r')$ to a radius r, and

$$M_i(r) \equiv \int_r^R \frac{dr'}{r'^3}\,\mu_i'(r') \ . \tag{2.21}$$

This expression for $\vec{B}_i^a$ may be substituted into Eq. (2.15):

$$\Delta E_M = -3\alpha_c \sum_a \sum_{i>j} (\vec{\sigma}_i\lambda_i^a)\cdot(\vec{\sigma}_j\lambda_j^a)\frac{\mu(m_i,R)\,\mu(m_j,R)}{R^3}\,I(m_iR,m_jR) \ , \tag{2.22}$$

where

$$I(m_iR,m_jR) = 1 + 2\int_0^R \frac{dr}{r^4}\,\mu(m_i,r)\,\mu(m_j,r) \tag{2.23}$$

$$= 1 + (x_i\sin^2 x_i - \tfrac{3}{2}y_i)^{-1}(x_j\sin^2 x_j - \tfrac{3}{2}y_j)^{-1} \times\{-\tfrac{3}{2}y_i\,y_j - 2x_ix_j\sin^2 x_i\sin^2 x_j + \tfrac{1}{2}x_i\,x_j[2x_i\,\mathrm{Si}(2x_i) + 2x_j\,\mathrm{Si}(2x_j) - (x_i+x_j)\,\mathrm{Si}(2(x_i+x_j)) - (x_i-x_j)\,\mathrm{Si}(2(x_i-x_j))]\} \ , \tag{2.24}$$

where $y_i = x_i - \sin x_i\cos x_i$, x_i is the root of Eq. (2.5) for a given $m_i R$ and

$$\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}\,dt \ .$$

The color and spin dependence of Eq. (2.22) may be simplified considerably. For a color-singlet meson

$$(\lambda_1^a + \lambda_2^a)\,|M\rangle = 0 \ . \tag{2.25}$$

Squaring this and using

$$\sum_a (\lambda_i^a)^2 = \tfrac{16}{3} \ , \tag{2.26}$$

we find

$$\sum_a \lambda_1^a\lambda_2^a = -\tfrac{16}{3} \ . \tag{2.27}$$

Likewise for baryons:

$$\sum_{i=1}^{3} \lambda_i^a\,|B\rangle = 0 \ , \tag{2.28}$$

whence

$$\sum_a \lambda_i^a\lambda_j^a = -\tfrac{8}{3}, \quad i\neq j. \tag{2.29}$$

The final expression for the magnetic interaction energy is

$$\Delta E_M = 8\alpha_c\lambda\sum_{i>j}(\vec{\sigma}_i\cdot\vec{\sigma}_j)\frac{\mu(m_i,R)\mu(m_j,R)}{R^3}I(m_iR,m_jR) \tag{2.30}$$

$$\equiv \sum_{i>j}\lambda M_{ij}(\vec{\sigma}_i\cdot\vec{\sigma}_j). \tag{2.31}$$

Here $\lambda = 1$ for a baryon, 2 for a meson. The only parameter in ΔE which is not dimensionless is R. M_{ij} is linear in α_c and calculable from Eqs. (2.22) and (2.24) and from the values of $\mu(m_i,R)$ in Table I. Equation (2.31) has the interesting property that the sign of the energy shift is the opposite of that found in atoms. This is because the quark colors are "opposite" since the hadrons are color singlets.

The magnetostatic energy shifts are easily calculated for different hadrons. They will be discussed in Sec. III. To illustrate the effect of such an interaction consider the splitting of the Λ and Σ hyperons. This mass splitting is due entirely to the dependence of the magnetic interaction on the quark mass: In the Λ particle the u and d quarks are in a spin-zero state so that $\vec{\sigma}_u\cdot\vec{\sigma}_d = -3$, while $\vec{\sigma}_u + \vec{\sigma}_d = 0$ so $\vec{\sigma}_s\cdot(\vec{\sigma}_u+\vec{\sigma}_d) = 0$. The Σ, on the other hand, contains a u and d quark in a spin-one state so $\vec{\sigma}_u\cdot\vec{\sigma}_d = 1$, while $\vec{\sigma}_s\cdot(\vec{\sigma}_u+\vec{\sigma}_d) = -4$.

We turn now to the gluon electrostatic energy. The color electric field of a single quark must satisfy

$$\vec{\nabla}\cdot\vec{E}_i^a = j_i^{0a}, \quad r<R \tag{2.32a}$$

$$\vec{\nabla}\times\vec{E}_i^a = 0, \quad r<R \tag{2.32b}$$

$$\sum_i \hat{r}\cdot\vec{E}_i^a = 0, \quad r=R \tag{2.32c}$$

where $j_{ai}^0(r)$ is a single quark's color charge density

$$j_i^{0a} = q_i^\dagger\lambda^a q_i \tag{2.33}$$

$$= \frac{\lambda_i^a}{4\pi r^2}\rho_i'(r). \tag{2.34}$$

Here $\rho_i'(r)$ is the charge density of a quark of mass m_i in the lowest cavity eigenmode and satisfies

$$\int_0^R dr' \rho_i'(r') = 1. \tag{2.35}$$

The color electric field is obtained from Gauss's law:

$$\vec{E}_i^a = \frac{\lambda_i^a}{4\pi r^2} \hat{r} \rho_i(r), \tag{2.36}$$

where $\rho_i(r)$ is the integral of $\rho_i'(r)$ out to a radius r.

Now if all of the quarks in a given hadron have the same mass, then $\rho_i(r)$ is independent of the index i and the total color electric field is given by

$$\vec{E}^a = \frac{\hat{r}}{4\pi r^2} \rho(r) \sum_i \lambda_i^a. \tag{2.37}$$

For a color-singlet hadron $\sum \lambda^a |H\rangle = 0$. Therefore, $\vec{E}^a = 0$. Simply stated the color charge density vanishes locally. From Eq. (2.14) we see $\Delta E_E = 0$. Notice that it was essential to include the static self-interaction $(\vec{E}_i^a \cdot \vec{E}_i^a)$ to obtain this cancellation. In the same manner note that the gluon electric field of a single quark will not satisfy the boundary condition, Eq. (2.32c), while the sum over all quarks does. It is this which has forced us to keep the static self-interaction terms.

Even when the quarks have different masses, so long as the masses are not too different, ΔE_E is very small. For massless u and d quarks and for a strange quark with $m_s < 300$ MeV, the gluon electrostatic interaction energy is always less than 5 MeV. We will therefore present the details of the calculation of electrostatic energy in Appendix A. We do include this term in our fits to the spectra. If two quarks have greatly different masses, as is possibly the case for charmed mesons and baryons, this term may be more important.

III. HADRON MASSES

We can now write down the expression for the Hamiltonian which determines the mass of each hadron. Each term was discussed in detail in Sec. II. We summarize them briefly:

(a) The quantum fluctuations contribute two terms which depend only on the radius of the hadron. The volume term is

$$E_v \equiv \tfrac{4}{3}\pi B R^3 . \tag{3.1}$$

The remainder of the zero-point energy is

$$E_0 \equiv -Z_0/R . \tag{3.2}$$

As the discussion of Sec. II has shown, we can expect Z_0 to be positive and of order unity.

(b) The quarks contribute their rest and kinetic energies to the hadron's mass. If N_0, N_s, m_0, and m_s are the respective numbers and masses of the nonstrange and strange quarks, and if $\omega(y)$ is the frequency defined by Eq. (2.4), then this term is

$$E_Q \equiv N_0 \omega(m_0, R) + N_s \omega(m_s, R). \tag{3.3}$$

(c) The gluon interaction has color magnetic exchange and color electric parts as discussed in Sec. II. The color magnetic exchange term will be written in the form

$$E_M = a_{00} M_{00} + a_{0s} M_{0s} + a_{ss} M_{ss} \tag{3.4}$$

by evaluating $\sum_{ij} (\vec{\sigma}_i \cdot \vec{\sigma}_j) M_{ij}$ in each state. In Eq. (3.4) M_{00} is the color magnetic interaction between two nonstrange quarks, M_{0s} is that between a nonstrange and strange quark and M_{ss}, the interaction energy between two strange quarks. The values of M_{00}, M_{0s}, and M_{ss} can be read off of Fig. 3. The coefficients a_{ij} depend on the state and are listed in Table II. The color electric energy is given by

$$E_E = b\epsilon , \tag{3.5}$$

where ϵ is the color electric interaction energy of a strange and a nonstrange quark including both

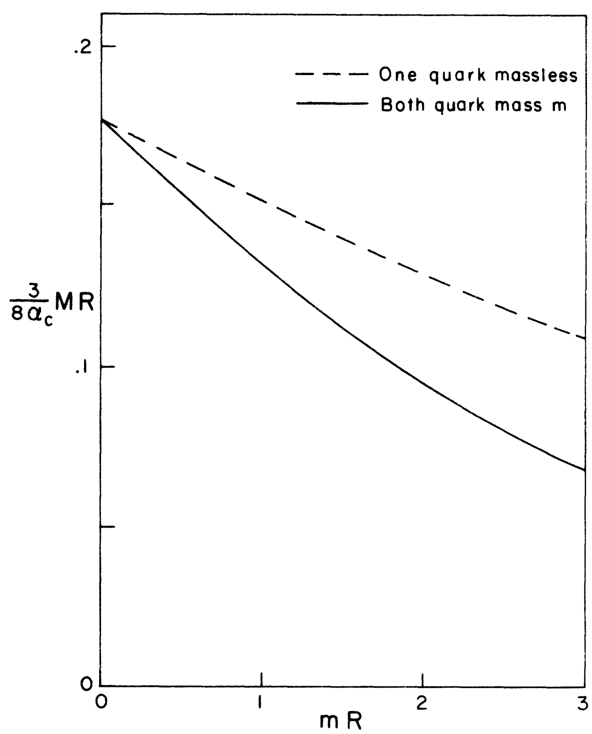

FIG. 3. Magnetic gluon exchange energy of two quarks as a function of mR. M is the quantity referred to in Eq. (3.4). The solid line gives the interaction energy between equal-mass quarks (M_{00} or M_{ss}), the dashed line gives the interaction energy between a massless quark and a quark of mass m (M_{0s}).

self-interaction and exchange graphs. The coefficient b is one or zero depending upon whether the quark content of the hadron is mixed or not. ϵ is given in terms of the dimensionless quark frequencies and α_c in Eqs. (A3)–(A5).

The mass of hadron of radius R is then given by

$$M(R)=E_V+E_0+E_Q+E_M+E_E\,, \tag{3.6}$$

where the individual terms are given by Eqs. (3.1)–(3.5).

So far we have not used the quadratic boundary condition of the bag model.[4,5] This requires that the quark and gluon field pressures balance the "external" pressure B locally on the bag's surface. It can be shown that for static spherical bags containing quarks with angular momentum $\frac{1}{2}$, the quadratic boundary condition is equivalent to minimizing $M(R)$ with respect to R. Thus the true radius of the hadron, R_0, is determined by $\partial M/\partial R=0$ and its mass is given by $M(R_0)$.

In order to become more familiar with these equations, it is useful to consider the nonstrange particles (Δ, p, ω, π) in the case that the nonstrange quark mass is zero. Considerable simplification results. All expressions are evaluated with $mR=0$, and $x(mR)=x(0)=2.0428$. Furthermore, $E_E=0$ in all these states. Accordingly, we have

$$M_i(R)=\tfrac{4}{3}\pi BR^3+A_i/R\,, \tag{3.7}$$

where

$$A_i=-Z_0+N_0x(0)+a_iM_{00}\big|_{m_0=0}\,. \tag{3.8}$$

A_i is independent of R, so we can easily carry out the minimization. The observed masses of the Δ, p, and ω can then be used to fix the parameters B, Z_0, and α_c.

The various mass splittings arise in the following way: If α_c were zero, baryons would be split from mesons since baryons have three quarks and mesons two. The Δ and the proton would be degenerate as would be the ρ and π. However, when we turn on the color magnetic interaction the Δ and the proton are split apart since $a=3$ for the Δ and -3 for the proton. The ρ and ω remain degenerate, but are split from the π. The ratio of the magnetic interaction $2:-6=1:-3$ for the (ρ,ω) and π follows simply from the fact that the quarks are in a triplet state $(\vec{\sigma}_1\cdot\vec{\sigma}_2=1)$ in the (ρ,ω) and are in a singlet state $(\vec{\sigma}_1\cdot\vec{\sigma}_2=-3)$ in the π.

In order to break SU(3), we give the strange quark a larger mass than the nonstrange quarks. We have chosen this mass to fit the Ω^-. If the only effect of this SU(3) breaking were in the quark mass-kinetic term E_Q, the Σ and Λ would remain degenerate. However, the presence of the strange-quark mass modifies the strange-quark wave function and therefore causes a secondary SU(3) breaking through the gluon interaction. This splits the Σ and Λ in the right direction, and in our case with a magnitude about $\frac{1}{2}$ of the observed difference.

Various SU(3) and SU(6) mass formulas may be obtained using Table II if one were to assume that R does not change from state to state. For instance, the "linear" Gell-Mann-Okubo formula for octets and decuplets is equivalent to

$$\tfrac{1}{2}(M_{00}+M_{ss})=M_{0s}\,.$$

This is well-satisfied for the baryon and vector-meson octets, where $m_sR\sim1.5$, but fails badly for the pseudoscalar mesons, where $m_sR\sim1.2$. The SU(6) relation[18]

$$M(\Sigma^*)-M(\Sigma)=M(\Xi^*)-M(\Xi)$$

is immediate.

It should be pointed out that these mass formulas do not apply linearly in the baryon masses and quadratically in the meson masses. All hadrons are treated on the same footing.

Because the eigenvalue equation for $x(mR)$ is transcendental and because R and M_{ij} change in a complicated (although smooth) way from particle to particle, we cannot quote any universal mass formulas based on group properties. Our results are given for two cases in Tables III and IV, and for case A in Fig. 4.

Case A. Here the nonstrange-quark mass is held at zero. B, Z, and α_c are adjusted to fit the Δ, p, and ω. The strange-quark mass is adjusted to fit the Ω^-.

Case B. Here the nonstrange quark mass is set at 108 MeV. The other parameters are determined in the same way as in case A.

The masses of the η and η' have not been included

TABLE II. Parameters which specify the gluon magnetostatic and electrostatic energy of light hadrons. a_{00}, a_{0s}, and a_{ss} are defined by Eq. (3.4); b by Eq. (3.5).

Hadron	a_{00}	a_{0s}	a_{ss}	b
p	−3	0	0	0
Λ	−3	0	0	1
Σ	1	−4	0	1
Ξ	0	−4	1	1
Δ	3	0	0	0
Σ^*	1	2	0	1
Ξ^*	0	2	1	1
Ω^-	0	0	3	0
ρ	2	0	0	0
K^*	0	2	0	1
ω	2	0	0	0
ϕ	0	0	2	0
π	−6	0	0	0
K	0	−6	0	1

TABLE III. Masses of light hadrons for the case $m_0=0$. All masses are quoted in GeV, R_0 in GeV^{-1}. The five contributions to the hadron mass, E_V, E_0, E_Q, E_M, and E_E, defined by Eq. (3.6) and preceding equations, are listed and compared with experiment.

Particle	M_{exp}	M_{bag}	R_0	E_0	E_V	E_Q	E_M	E_E
p	0.938	0.938	5.00	−0.367	0.234	1.226	−0.155	0
Λ	1.116	1.105	4.95	−0.371	0.227	1.400	−0.156	0.005
Σ^+	1.189	1.144	4.95	−0.371	0.227	1.400	−0.116	0.005
Ξ^0	1.321	1.289	4.91	−0.374	0.222	1.572	−0.136	0.005
Δ	1.236	1.233	5.48	−0.336	0.308	1.119	0.141	0
Σ^*	1.385	1.382	5.43	−0.338	0.301	1.292	0.122	0.005
Ξ^*	1.533	1.529	5.39	−0.341	0.293	1.465	0.106	0.005
Ω^-	1.672	1.672	5.35	−0.343	0.287	1.636	0.092	0
ρ	0.77 ± 0.01	0.783	4.71	−0.390	0.196	0.868	0.110	0
K^*	0.892	0.928	4.65	−0.395	0.189	1.039	0.091	0.004
ω	0.783	0.783	4.71	−0.390	0.196	0.868	0.110	0
ϕ	1.019	1.068	4.61	−0.399	0.183	1.207	0.076	0
K	0.495	0.497	3.26	−0.564	0.065	1.407	−0.415	0.003
π	0.139	0.280	3.34	−0.549	0.070	1.222	−0.462	0

$B^{1/4}=0.145$ GeV, $Z_0=1.84$, $\alpha_c=0.55$, $m_s=0.279$ GeV

in these calculations. Usual considerations would require the η' and π to be degenerate as are the ω and ρ (see below). However, the η and η' masses are affected by an additional interaction, namely, quark-antiquark annihilation into two gluons as shown in Fig. 5. This occurs only between spin-0, SU(3) - singlet states. We have not yet calculated this interaction energy. It affects only the $\eta-\eta'$ system. In Appendix B we discuss the expected size of this interaction in our model and relate it to the observed η and η' masses.

From Fig. 4 we see that in the baryon decuplet the fit is very good. The equal spacing rule is well verified. In the baryon octet the Gell-Mann–Okubo

TABLE IV. Masses of the light hadrons for the case $m_0=0.108$ GeV. All masses are quoted in GeV, R_0 in GeV^{-1}. The five contributions to the hadron mass, E_V, E_0, E_Q, E_M, and E_E, defined by Eq. (3.6) and preceding equations, are listed and compared with experiment.

Particle	M_{exp}	M_{bag}	R_0	E_0	E_V	E_Q	E_M	E_E
p	0.938	0.938	5.59	−0.348	0.181	1.266	−0.160	0
Λ	1.116	1.103	5.51	−0.354	0.173	1.443	−0.163	0.004
Σ^+	1.189	1.145	5.53	−0.353	0.174	1.439	−0.120	0.004
Ξ^0	1.321	1.286	5.42	−0.360	0.165	1.620	−0.143	0.004
Δ	1.236	1.233	6.38	−0.305	0.269	1.132	0.137	0
Σ^*	1.385	1.381	6.29	−0.310	0.257	1.312	0.119	0.004
Ξ^*	1.533	1.528	6.20	−0.314	0.247	1.489	0.103	0.004
Ω^-	1.672	1.672	6.11	−0.319	0.236	1.666	0.089	0
ρ	0.77 ± 0.01	0.783	5.47	−0.356	0.170	0.860	0.110	0
K^*	0.892	0.925	5.37	−0.363	0.160	1.033	0.091	0.004
ω	0.783	0.783	5.47	−0.356	0.170	0.859	0.110	0
ϕ	1.019	1.063	5.26	−0.370	0.151	1.206	0.076	0
K	0.495	0.371	0.73	−2.688	0	5.865	−2.807	0.001
π	0.139	0.175	1.13	−0.796	0.022	3.717	−1.822	0

$B^{1/4}=0.125$ GeV, $Z_0=1.95$, $\alpha_c=0.75$, $m_s=0.353$ GeV

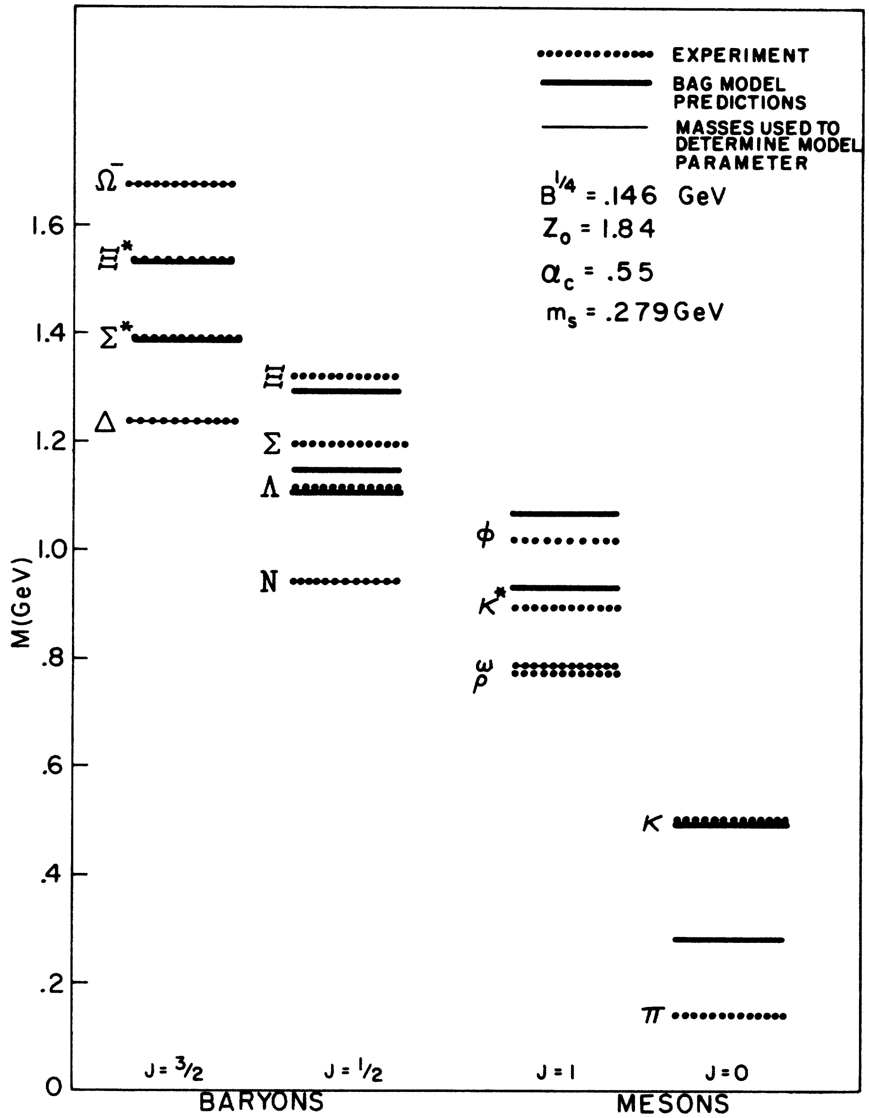

FIG. 4. Fit to the hadron masses with $m_0=0$, with B, α_c, Z_0, and m_s as shown. The actual masses are given by dotted lines for comparison. The masses of the N, Δ, Ω, and ω were used to determine the parameters, all other masses are predicted.

formula is satisfied very well. The mass of the Ξ is about 30 MeV too low and the $\Sigma-\Lambda$ splitting is about $\frac{1}{2}$ the observed value.

At this level of approximation, the ω and ρ remain degenerate. The observed splitting is about 13 MeV. Our ϕ mass is about 50 MeV too high. [Our Hamiltonian's eigenstates are pure strange (ϕ) and pure nonstrange (ω).] As usual, the pseudoscalar mesons are the source of some difficulty. The $K-\pi$ mass splitting we get is about

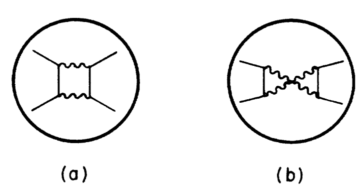

FIG. 5. Lowest-order gluon annihilation diagrams present only for spin-0 flavor-SU(3) singlet mesons.

210 MeV versus an experimental value of about 350 MeV. With zero mass for the nonstrange quarks, the K comes out about 2 MeV too high and the π is found at 280 MeV. As the nonstrange-quark mass is increased (and the other parameters are readjusted to keep the Ω, Δ, p, and ω correct), the π and K begin to drop together. In fact, this is the only substantial change in the fit, as can be seen from Tables III and IV. At a nonstrange quark mass of 108 MeV the π mass is 175 MeV. By the time the quark mass reaches 109.5 MeV the equation $\partial M/\partial R = 0$ no longer has a root for $R > 0$ for the pion. The pressure is not sufficient to balance B. Thus the π meson is no longer a state in our model. Actually this is encouraging: The pion has always been difficult to reconcile with a quark model. A pion made of two quarks but nearly degenerate with the vacuum seems like a step in the right direction.[19] Finally, it is also possible to compute the masses of "exotic" hadrons. In Appendix C it is shown that all exotic baryons are unstable in our model.

IV. MAGNETIC MOMENTS, AXIAL-VECTOR COUPLING CONSTANTS, AND CHARGE RADII

The parameters of our model, a_c, Z_0, B, and m_s have been determined by fitting certain hadron masses. The quark wave functions of any hadron are determined by these parameters and by its radius, as described in Sec. III. It is now possible to put this information together with the formulas listed in Table I and calculate various static (or near static) parameters of the hadrons. We shall evaluate matrix elements treating the bag as a nailed-down cavity containing free quark fields. The reader should be warned that we are not addressing the deep question of whether totally satisfactory local current operators can be defined for an extended object such as the bag. We do not include order a_c corrections to the parameters calculated in this section. The effect of recoil on matrix elements measured away from zero momentum transfer (for example, the charge radius) is ignored, as is the slight difference in radius between initial and final states in hyperon β decay. It is possible that some of these questions may be addressed before a full theory of local currents is developed.

A. Magnetic moments

Here we shall be brief since the subject has been discussed by Barnes[12] and by Allen.[13] The magnetic moments are computed from the radii of our baryons (Table III or IV) and the first equation of Table I. The results are listed in Table V. We list the ratio of the magnetic moments to that of the proton to allow comparison with SU(3). Experimental data, where available, are also listed in Table V. The nonrelativistic quark model also makes predictions for magnetic moments. Its predictions are obtained by sending the quark masses to infinity then adjusting the strange to nonstrange-quark mass ratio to fit the magnetic moment of the Λ hyperon. The magnetic moments of other hyperons in units of the proton's are determined by this ratio. For a given μ_Λ/μ_p the predictions of the bag model and the nonrelativistic quark model for other hyperons are identical. The advantage of the bag calculation is, first, that μ_Λ/μ_p is predicted not fit, and, second, that it makes sense to calculate magnetic moments with less than infinite quark mass (cf. the discussion of Sec. II A).

The bag model does less well on the over-all normalization of the octet magnetic moments. Previously[5] we found $2M_p\mu_p = 2.6$ (experiment is 2.79). The present model leads us to a larger value of $B^{1/4}$ than in Ref. 5 and a consequently smaller proton radius. Because of this μ_p is somewhat too small: $2M_p\mu_p = 1.9$. We regard this as the most serious discrepancy among the predictions we are able to make. In comparison, the nonrelativistic quark model allows for *no* consistent prediction for the magnitude of μ_p since quarks light enough to give a reasonable value for μ_p certainly *cannot* be considered nonrelativistic (see Sec. II A).

B. Axial-vector coupling constants

The axial-vector charges of octet baryon β decay can be calculated if their dependence on the

TABLE V. Baryon magnetic moments in units of μ_p. The bag-model predictions are for $m_s = 0.279$. The SU(3) predictions depend on the F/D ratio for the magnetic-moment operator $(c = -\frac{2}{3}[1/(\frac{1}{3} + F/D)])$. The figures in parentheses correspond to a (conventional) choice of $F/D - \frac{2}{3}$.

	Magnetic moment (μ/μ_p)		
Hadron	Experiment	Bag model	SU(3)
n	-0.685	$-\frac{2}{3}$	c $(-\frac{2}{3})$
Λ	-0.240 ± 0.021	-0.255	$\frac{1}{2}c$ $(-\frac{1}{3})$
Σ^+	0.93 ± 0.16	0.97	1 (1)
Σ^0		0.31	$-\frac{1}{2}c$ $(\frac{1}{3})$
Σ^-	-0.53 ± 0.13	-0.36	$-1-c$ $(-\frac{1}{3})$
Ξ^0		-0.56	c $(-\frac{2}{3})$
Ξ^-	-0.69 ± 0.27	-0.23	$-1-c$ $(-\frac{1}{3})$

momentum transfer in the decay can be ignored. This is probably a reasonable approximation phenomenologically since the matrix elements are not pion-pole dominated. The contribution of an individual quark to the weak axial-vector charge is listed on the second line of Table I. For a specific decay it is necessary only to add up the contributions of the various quarks. Since the spin-SU(3) structure of our baryon wave functions is that of SU(6), the quark counting is identical to that well-known model. The difference in the bag model comes from the relativistic nature of the quark wave functions. The lower components of the Dirac wave function reduce the contribution of an individual quark from unity (suitably normalized) to 0.653 for a fully relativistic quark ($m_q = 0$). This result can be read off of Table I for $x = 2.0428$. The factor 0.653 is therefore appropriate to $\Delta S = 0$ transitions, which involve only massless quarks. Thus, for example, the nonrelativistic quark-model prediction for neutron β decay ($n \to pe\nu$) is reduced from $\frac{5}{3}$ to 1.09. Likewise for the decay $\Sigma \to \Lambda e\nu$, g_a is reduced from $\frac{2}{3}$ to 0.53.

In strangeness-changing decays a quark of mass 285 MeV changes into one of zero mass. The appropriate reduction factor, read off of Table I, is 0.707. Thus, for the decay $\Lambda \to pe\nu$ the nonrelativistic prediction of $(\frac{3}{2})^{1/2}$ is reduced to 0.87.

Our results are most easily summarized and compared with experiment by introducing the language of Cabibbo parameterization.[20] There, in an SU(3)-symmetric world, all octet β decays are fit by two reduced matrix elements F_a and D_a. Each measurement of g_a determines a line (actually a band when experimental uncertainties are included) in the F_a, D_a plane. This plot is shown in Fig. 6. The overlap of the various bands determines a best fit to F_a and D_a. The most recent values of these are[21]

$$F_a^{\text{expt}} = 0.41 \pm 0.02\,, \quad D_a^{\text{expt}} = 0.83 \pm 0.02\,.$$

The nonrelativistic quark model predicted[22]

$$F_a^{\text{NR}} = \tfrac{2}{3}\,, \quad D_a^{\text{NR}} = 1\,.$$

The bag model does not predict a unique F_a and D_a because of the SU(3) breaking induced by the strange-quark mass. Instead we find for $\Delta S = 0$ transitions a reduction by a factor of 0.653.

$$F_a^{\Delta S=0} = 0.44\,, \quad D_a^{\Delta S=0} = 0.65\,,$$

and for $S = 1$ transitions a reduction by a factor of 0.707:

$$F_a^{\Delta S=1} = 0.47\,, \quad D_a^{\Delta S=1} = 0.71\,.$$

These values, together with the predictions of the nonrelativistic quark model, are shown in Fig. 6.

The relativistic correction of the bag model improves agreement with experiment substantially (e.g., in the decays $\Sigma \to \Lambda e\nu$; $n \to pe\nu$; $\Lambda \to pe\nu$), though it goes the wrong way slightly for the decay $\Sigma^- \to ne\nu$. The limitation of the bag model in this regime is apparent from Fig. 6. The corrections reduce F_a and D_a in proportion. Thus, the nonrelativistic quark-model prediction may be shifted along a line through the origin. Since the experimental point does not lie quite on this line, no choice of strange- and nonstrange-quark masses will reproduce this point. Golowich[11] has chosen to adjust the nonstrange-quark mass to fit g_a from neutron β decay. While this shifts the bag predictions slightly it does not substantially change the over-all agreement with experiment.

C. Charge radii

Data on hadron charge radii are substantially less numerous than either magnetic moments or axial-vector charges. What data do exist are obtained from measurements of the derivatives of the elastic electric form factor at zero momentum transfer. The proton's charge radius is measured

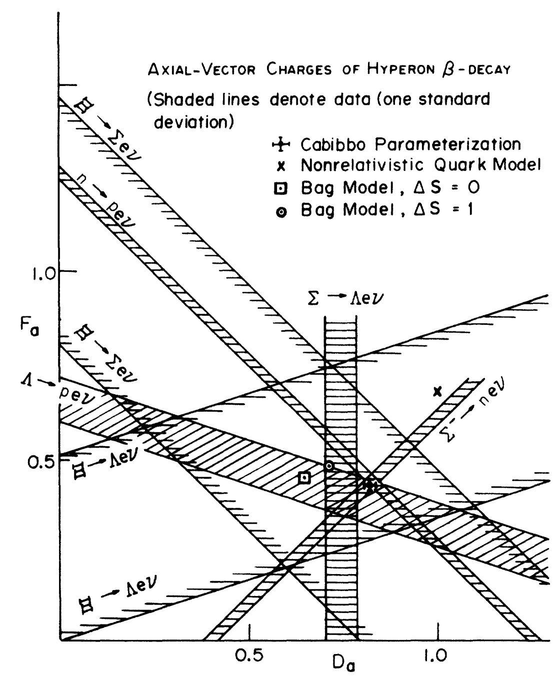

FIG. 6. Axial-vector coupling constants for octet β decays. Each experimental measurement (branching ratio or angular distribution) determines a line in the F_a-D_a plane. Experimental uncertainty broadens the line to a band. The data are summarized by the Cabibbo parameterization. Model predictions are summarized by values of F_a and D_a as shown.

to be[23]

$$\langle r_p{}^2\rangle_{\rm exp}^{1/2} = 0.88 \pm 0.03 \text{ fm},$$

and neutron's is

$$\langle r_n{}^2\rangle_{\rm exp} = -0.12 \pm 0.01 \text{ fm}^2 .$$

There is also a rather controversial measurement of the pion charge radius[24]

$$\langle r_\pi{}^2\rangle_{\rm exp}^{1/2} = 0.78 \pm 0.10 \text{ fm}.$$

The bag-model predictions for these may be calculated from the third line of Table I:

$$\langle r_p{}^2\rangle^{1/2} = 0.73 \text{ fm},$$

$$\langle r_n{}^2\rangle^{1/2} = 0,$$

$$\langle r_\pi{}^2\rangle^{1/2} = 0.49 \text{ fm}.$$

The same effect which reduced the proton's magnetic moment in comparison with Ref. 5 also reduces its charge radius.

V. CONCLUSIONS AND DISCUSSION

We have shown that the structure of the levels of the lightest SU(6) multiplets for baryons and mesons can be well understood in terms of a limited number of parameters, all of which have a fundamental significance and can be used to calculate other hadronic states and processes. This should be contrasted to other fits of the mass spectrum which in general leave the reduced matrix elements of differnece SU(3) multiplets as distinct, unrelated parameters. In our case the parameters are the colored quark-colored gluon coupling constant, the mass of the strange quark, the scale of the confinement potential (B), and the magnitude of the zero-point energy. Among these, the latter will eventually be related by calculation to the structure of the constituents. For the present it is fitted. It is probably important to remark that the energy scale is related to the confinement (B) rather than to the quark masses. The quark masses seem only to play a role in establishing the magnitude of the SU(3)-symmetry violation. The model includes non-SU(3)-symmetric spin-spin interaction which accounts for the breaking of SU(6). Not only does the model yield a reasonable mass spectrum, but the magnitude of the differences between the calculated and actual masses are of the size to be expected when one takes into account higher-order effects using the same parameters. That is, the fit is consistent.

Using the quark wave functions the calculations of static parameters also produces a general improvement over the results obtained in nonrelativistic quark models.

Then, what problems remain? Two important extensions of this work are in progress. First, the spectrum of the low-lying negative-parity baryon resonances (the $L=1$ 70 of the SU(6) quark model) is being computed. The study of these states introduces no new parameters and provide a crucial test of our ideas. Second, with the addition of a charmed quark of mass m_c, the spectrum of charmed hadrons may be computed. We regard this more as a test of the notion of charm—that is, of a new flavor of quark interacting in much the same way as the old—than as a test of the dynamics of quark confinement.

Another problem is to evaluate the effect of the surface fluctuations about the spherical shape. These have been ignored in our calculations. We must provide justification for this by taking them into account. We must also study the $\eta-\eta'$ states to which we have alluded. One must determine whether the resonant effect present in the bag (mentioned in Appendix B) works quantitatively.

Finally, as we have remarked, the magnetic moment of the proton is too small (2 as opposed to 2.79). This is a consequence of the fact that the magnetic moment of a quark is associated with the overlap of the small and large components of the Dirac wave function and this is rather small ($\mu = 0.2R$, for massless quarks). Since our radius decreased from our earlier fit[5] (good for the charge radius) our magnetic moment has gotten too small. It will be important to see if higher-order effects will allow us to keep R roughly the same and increase the 0.2.

APPENDIX A: GLUON ELECTROSTATIC INTERACTION ENERGY

Here we complete the discussion, begun in Sec. II C, of the energy due to the "electric" gluon energy of quarks in the bag. If the quarks in a given bag have the same mass, ΔE_B is zero as shown in the text. Here we consider the case where some are massive and others not. Separating Eq. (2.14) into interaction and self-energy parts, we write

$$\Delta E_B = g^2 \sum_a \left(\tfrac{1}{2}\sum_i \int_{\rm bag} d^3x\,|\vec{\rm E}_i^a(x)|^2 + \sum_{i>j}\int_{\rm bag} d^3x\,\vec{\rm E}_i^a(x)\cdot\vec{\rm E}_j^a(x)\right), \quad \text{A1)}$$

where $\vec{\rm E}_i^a$ is given by Eq. (2.36). The sum over the color index, a, may again be performed with the aid of Eqs. (2.26), (2.27), and (2.29), with the following result:

$$\Delta E_B = -\frac{8\alpha_c\lambda}{3R}\left(\sum_{i>j} f(x_i, x_j) - \frac{1}{\lambda}\sum_i f(x_i, x_i)\right), \quad \text{(A2)}$$

where $\lambda = 1$ for a baryon, 2 for a meson. $f(x_i, x_j)$ is analogous to M_{ij} which described the magnetostatic interaction. Although $f(x_i, x_j)$ may be written in terms of elementary functions, the expression is intractibly long. We content ourselves with the integral representation:

$$f(x_i, x_j) = R \int_0^R \frac{dr}{r^2} \rho_i(r)\rho_j(r), \tag{A3}$$

where

$$\rho_i(r) \equiv \frac{\omega[x_i y - (\sin^2 x_i y)/x_i y] - m[\sin x_i y \cos x_i y - (\sin^2 x_i y)/x_i y]}{\omega[x_i - (\sin^2 x_i)/x_i] - m[\sin x_i \cos x_i - (\sin^2 x_i)/x_i]}, \tag{A4}$$

with $x = (mR)$ given by Eq. (2.5), and $\omega = \omega(m, R)$ given by Eq. (2.4). $\rho_i(r)$ is the fraction of the quark charge density within a radius r. $f(x_i, x_j)$ was evaluated by computer.

We note that if the quark masses are the same, then $x_i = x_j$ for any i or j and Eq. (A2) implies $\Delta E_E = 0$ for either meson or baryon. ΔE_E is not zero for strange mesons, nor for baryons with strangeness -1 or -2. In all of these cases, evaluation of Eq. (A2) yields

$$\Delta E_E = -\frac{8\alpha_c}{3R}[2f(x_0, x_s) - f(x_0, x_0) - f(x_s x_s)], \tag{A5}$$

where x_0 is the root of Eq. (2.5) appropriate to a nonstrange quark, and x_s is that appropriate to a strange quark. To write a general expression for the electrostatic gluon energy we need only multiply Eq. (A5) by a factor (b) which is 1 for $|S| = 1$ mesons and $S = -1$ or -2 baryons and zero otherwise. The assignments of b among the light hadrons are listed in Table II.

APPENDIX B: η-η' PROBLEM

It is well known that the η and η' have special problems in the ordinary quark model. The same is true in our case. If we treat the η and η' on the same footing as the ϕ and ω, then one state will be

$$\eta = s\bar{s},$$

and the other

$$\eta' = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}),$$

in which case the η' will be degenerate with the π (as the ω is with the ρ). In fact the η' is very massive (958 MeV). The above prediction for η is also bad (693 MeV), as opposed to the experimental number (549 MeV). These fits are considerably inferior to any of those for the other states.

There is an effect which will raise the η' and lower the η and at the same time not give rise to a large ρ, ω splitting, nor make significant changes in other states. The candidate is the annihilation process, shown in Fig. 5, which admittedly involves diagrams of higher order in α_c. Since the gluons are exactly invariant under SU(3) the above diagrams couple only to the SU(3)-singlet states, and hence will imply a mechanism which acts only on the SU(3)-singlet component of the states η' and η. Further, because the gluons are vectors, the above diagrams vanish in the vector-meson states. Hence, they act only on the η and η'. Below we shall suggest a mechanism may greatly enhance the contribution of these diagrams above "normal" fourth-order effects.

The above diagrams are diagonal in the SU(3) octet, singlet states, which in the exact SU(3) limit correspond to the quark states

$$\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}), \quad \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}).$$

In fact, the diagrams are nonvanishing only in the singlet state. In the present absence of an actual calculation of the diagrams, let us simply postulate a large singlet matrix element which will mix the states $s\bar{s}$ and $u\bar{u} + d\bar{d}$. If we do this, we may obtain a prediction of one mass, given the other.

Thus, let us compute the energy of the quarks, assuming only that the singlet matrix element exists. By dimensional arguments in the limit where we forget about SU(3) breaking, the matrix element in the singlet state will be a/R where a is a number (proportional to α_c^2). Then, using standard methods, the mass matrix will be

$$M = \begin{pmatrix} E_0 + \frac{2}{3}\frac{a}{R} & \frac{\sqrt{2}}{3}\frac{a}{R} \\ \frac{\sqrt{2}}{3}\frac{a}{R} & E_s + \frac{1}{3}\frac{a}{R} \end{pmatrix} + \tfrac{4}{3}\pi B R^3, \tag{B1}$$

where

$$E_0 \equiv \frac{2\omega(m_0, R) - 6M_{00} - Z_0}{R},$$

$$E_s = \frac{2\omega(m_s, R) - 6M_{ss} - Z_0}{R}.$$

We can now diagonalize M and compute M_η and $M_{\eta'}$ by minimizing the eigenvalues with respect to R. We determine a to fit $M_{\eta'} = 958$ MeV. We can then compute M_η. We find $a = 1.54$, $a/R = 330$ MeV for the η', and $M = 463$ MeV, in reasonable agreement with reality ($M_\eta^{\exp} = 549$ MeV).

We finally mention the possible explanation for the size of a. In our approximation the gluon propagators in the diagrams of Fig. 5 are the ones appropriate to a cavity with radius R. Hence, they are represented as a sum over all the modes. The gluon propagator contains two types of modes, TE and TM. The lowest frequency of one of these modes is $4.49/R$. This frequency is close to the frequency of nonstrange-quark-antiquark-meson system, namely, $2\times(2.04)/R = 4.08/R$. Hence, a resonant effect may enhance the diagrams of Fig. 5 above their nominally higher-order value. This point is presently under investigation.

APPENDIX C

We have omitted any reference to "exotic" hadrons. These could be, for example, particles with six quarks, two quarks and two antiquarks, etc. Of course, the coupling to colored glue automatically excludes noncolor singlets among these. Even if we restrict our attention to nonstrange particles, as many as twelve colored nonstrange quarks could occupy the lowest mode. In the approximation considered before,[5] the mass of these multiple quark states was proportional to $n^{3/4}$, where n is the total number of quarks and antiquarks in the lowest mode. Hence, there were a considerable number of exotics, where the most massive were the most stable. Including the effect of the quark-gluon interaction completely changes this. Here we will discuss in detail only the multiple quark hadrons. The color magnetic interaction energy for an n-quark color-singlet nonstrange baryon is

$$E_M = \tfrac{1}{2}[n(n-6) + J(J+1) + 3I(I+1)]M_{00},$$

where J is the total spin and I the total isospin. All quarks occupy the lowest mode, so the allowed values of n are 3, 6, 9, and 12. It is interesting that this is negative only for the nucleon ($n = 3$, $J = \frac{1}{2}$, $I = \frac{1}{2}$). The lowest value of E_M for a possible state of six quarks occurs with $J = 1$, $I = 0$, and hence $E_M = M_{00} = 0.26/R$. Its mass would therefore be

$$M_6 = \tfrac{4}{3}(4\pi B)^{1/4}(6\times 2.04 + 0.26 - 1.85)^{3/4} = 2.16 \text{ GeV}.$$

[The allowed values of (J, I) are (3, 0), (0, 3), (2, 1), (1, 2), (1, 0), and (0, 1).] Thus, this state and all higher six-quark states are unstable. Presumably as a resonance, the width would be large since no quarks need be created for the particle to decay into two nucleons. It is also interesting that the lowest six-quark state has the quantum numbers of the deuteron. This adds considerable weight to the picture of the nuclear force proposed by Fairley and Squires.[25] Since in our previous case, the most stable state was the twelve-quark state, we also will compute its mass. Here $n = 12$, and we have a closed shell so $I = 0$, $J = 0$. Therefore,

$$M_{12} = \tfrac{4}{3}(4\pi B)^{1/4}(12\times 2.04 + 36\times 0.26 - 1.85)^{3/4} = 4\times 1.22 \text{ GeV}.$$

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*This work is supported in part through funds provided by Energy Research and Development Administration under Contract No. AT(11-1)-3069.

†National Science Foundation Graduate Fellow.

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state. The extra third is the energy (BV) necessary to "support" the square-well potential. Note that it is not a constant, but instead varies from state to state. In a more conventional field theory the potential will presumably correspond in some way to gluon exchange between quarks. The "energy associated with the potential" will result from the purely gluonic terms in the Hamiltonian, i.e., the gluon kinetic-energy and self-interaction terms.

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[17]This is the behavior expected of a renormalization effect. So long as certain phenomonological parameters are held fixed (here the mass of the nucleon, Δ, ρ, and Ω^-), our predictions (masses of other hadrons, for example) are relatively insensitive to inclusion of the magnetic self-energy graphs. The parameters m_0, m_s, and Z_0 change in such a way as to substantially compensate the effect of these graphs. If the whole static, magnetic self-interaction were included, the observed hadron mass would be obtained by means of cancellations among several very large terms. While consistent with a renormalization effect we consider this uneconomical and prefer the simplest approach of leaving out Fig. 1(b) wherever possible.

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