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Associated production by weak charged and neutral currents

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Cross-section estimates are presented for the charged- and neutral-current associated-production reactions $\nu_\mu N \to lK\Lambda$ and $\nu_\mu N \to lK\Sigma$, where $l = \mu$ or ν_μ, respectively. The neutral-current reactions are considered within the framework of the Weinberg-Salam theory and theories in which the hadronic neutral current is proportional to the electromagnetic current. The calculations are performed using the Born approximation for the range of incident neutrino energies relevant for present experiments, from threshold to 3 GeV. We find flux-averaged cross sections much smaller than those indicated by preliminary Argonne data. Finally, calculations of the longitudinal polarization of the hyperons are presented.

Low-energy exclusive neutrino reactions have the capability of yielding much detailed information about the structure of weak interactions which complements that obtainable from high-energy inclusive processes. Recently, data on several such exclusive reactions have become available from a bubble-chamber experiment at Argonne involving neutrinos incident upon hydrogen and deuterium.[1] Further data will soon be forthcoming from similar experiments at Brookhaven National Laboratory and CERN. It is thus of immediate interest to estimate the total and differential cross sections predicted by the standard $V-A$ theory for the charged current reactions and by various gauge theory (and other) models for the neutral current processes. The pion production reactions have been analyzed in considerable detail by Adler and other authors.[2] Here we consider the associated production reactions $\nu_\mu N \to lK\Lambda$ and $\nu_\mu N \to lK\Sigma$, where $l=\mu, \nu_\mu$ for charged and neutral currents, respectively. In a set of 945 000 pictures the Argonne experiment observed two $\nu_\mu n \to \mu^- K^+\Lambda$ events, one $\nu_\mu n \to \nu_\mu K^0\Lambda$ event, and no $\nu_\mu p \to \nu_\mu K^+\Lambda$ or $\nu_\mu N \to lK\Sigma$ events.[1] These are the data, admittedly a very small sample, with which we shall compare our cross-section estimates.

These reactions are important for several reasons. The charged-current processes afford a new test of the Cabibbo theory, while the neutral-current processes give information on matrix elements of this current between nucleon and $K\Lambda$ or $K\Sigma$ states, which will be valuable in testing the various gauge theories of weak interactions and in providing a guide for further model building. Moreover, the $K\Lambda$ reactions have the special attraction that, since the Λ analyzes its polarization in its decay, it is experimentally feasible to measure this polarization.[3] In contrast, such a measurement in the $\nu N \to lN'\pi$ reactions would necessitate the rescattering of the final nucleon, which would be quite difficult with the present neutrino beam intensities and the small magnitude of exclusive weak-interaction cross sections. However, the KY ($Y=\Lambda, \Sigma$ hyperon) reactions are not enhanced by any strong resonances, in contrast to weak pion production, at least by charged currents, which is dominated by the $\Delta(1232)$.[4] For this reason, and also because the KY reactions have higher thresholds and consequently less phase space available at a given energy, the cross sections for associated production are smaller than those for pion production at the same energy. What is actually measured is, of course, not a cross section at a given energy, but rather a cross section integrated over the neutrino flux. The energy range in which the Argonne neutrino beam has a reasonably large flux is shown in Fig. 1. The flux $\phi(E)$ is sharply peaked at an incident neutrino energy $E \simeq 0.5$ GeV and falls rapidly at higher energies; for $E = 3$ GeV it is less than 1% of its maximum value.[5] The flux at Brookhaven and CERN is similarly restricted to low energy, and is peaked at $E \simeq 1-2$ GeV. This has the effect of further suppressing the KY reactions relative to the πN ones, because in the energy range in which the flux is maximal the latter reactions are well above threshold and have sizable cross sections, whereas the former reactions are either below threshold (the case at Argonne) or only slightly above. The importance of the KY reactions should, however, more than compensate for their smaller cross sections.

The specific reactions of the type $\nu N \to lKY$ are the charged-current processes

$$\nu n \to \mu^-(K^+\Lambda, K^+\Sigma^0, K^0\Sigma^+), \tag{1}$$

$$\nu p \to \mu^- K^+\Sigma^+, \tag{2}$$

and the neutral-current processes

$$\nu n \to \nu(K^0\Lambda, K^0\Sigma^0, K^+\Sigma^-), \tag{3}$$

$$\nu p \to \nu(K^+\Lambda, K^+\Sigma^0, K^0\Sigma^+). \tag{4}$$

There are also corresponding antineutrino reactions, but we shall concentrate primarily on the reactions of Eqs. (1)–(4) because the antineutrino flux at these accelerators is considerably smaller than the neutrino flux. For example, as is evident from Fig. 1, at Argonne the flux of antineutrinos is consistently more than an order of magnitude smaller than that of neutrinos. Furthermore, we shall focus on the $K\Lambda$ reactions, since they have cross sections larger than those for the $K\Sigma$ reactions.

The calculation is performed using the method of the generalized Born approximation, which consists of including only tree diagrams, but using full form factors for the current-hadron vertices. This is a reasonable approximation to use near threshold if there are no strong nearby resonances in a particular reaction. This condition is met for both the $K\Lambda$ and $K\Sigma$ reactions; the lowest-lying resonances in these channels are the $N^*\,S_{11}(1700)$ and $N^*\,P_{11}(1780)$, which decay mainly into $N\pi$ and $N\pi\pi$. They are not observed as strong resonances even in these dominant channels and, moreover, their branching ratios into $K\Lambda$ and $K\Sigma$ are quite small ($\lesssim 7\%$). Of course, the Born approximation would not be accurate near resonances in the $K\Lambda$ and $K\Sigma$ channels. It also becomes less accurate as one gets farther and farther away from the Born poles, and fails at high energies, among other reasons, because of the nonrenormalizable Pauli-magnetic-moment couplings which it includes (even given the damping due to the form factors). Accordingly, our most reliable results are the differential cross sections $\partial\sigma(E,W)/\partial W$, where W is the invariant mass of the KY system, for W near threshold $W_{\text{th}}=(m_Y+m_K)$, independent of E. Thus in order to test the predictions given in this paper it would be best for experimentalists to measure the W distribution.

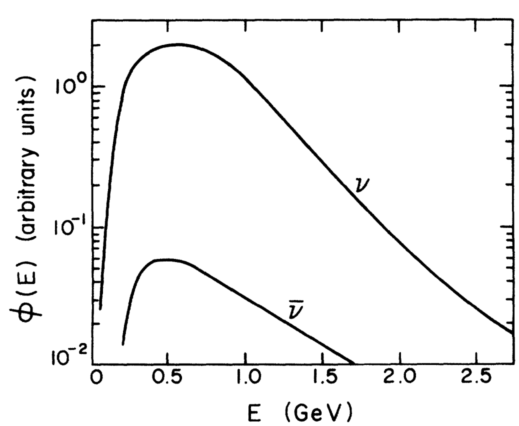

FIG. 1. Argonne neutrino flux $\phi(E)$. The units are arbitrary since flux-averaged quantities are defined as $\bar{f}=\int_0^\infty f(E)\phi(E)dE\Big/\int_0^\infty\phi(E)dE$.

As regards the total cross section $\sigma(E)$, the Born approximation is most accurate at low E, since this also forces W to be small. Although experimentally, only the flux-averaged cross section,

$$\bar{\sigma}=\int_{E_{\text{th}}}^{\infty}\sigma(E)\phi(E)dE\Big/\int_0^\infty\phi(E)dE\,, \tag{5}$$

is presently accessible, the flux distribution of Argonne is ideally suited for an application of the Born approximation since it weights small E and hence W so heavily. Quantitatively 90% of the Argonne flux is in the range $E<1.5$ GeV ($W<1.9$ GeV), while 96% is in the range $E<2$ GeV ($W<2.2$ GeV).[6] For reference, $W_{\text{th}}(K\Lambda)\simeq 1.6$ GeV and $W_{\text{th}}(K\Sigma)\simeq 1.7$ GeV. If much more data were available, it would be appropriate to employ the more careful methods used in photoproduction, electroproduction, and weak pion production, in which one projects out multipole amplitudes from the hadron center-of-mass Born amplitude and then uses dispersion relations or simply a fit to a phenomenological resonance-plus-background form to solve for the full amplitude. However, given the very small data sample of KY events, this seems premature at present.

The calculations of cross sections and polarizations divide into those for the charged-current reactions and those for the neutral-current reactions. In the former case the theory, the standard Cabibbo $V-A$ theory, is presumably reliable, and our purpose is test the applicability of the Born approximation. In the neutral-current case we assume the validity of the Born approximation and test a minimal gauge theory which has so far been in reasonable agreement with experiment (although the agreement is only fair in places), namely, the Weinberg-Salam theory.[7] This theory is based on the gauge group SU(2)×U(1) and has a weak hadronic neutral current of the form

$$J_z^\mu=(V^\mu-A^\mu)_{(3)}-2\sin^2\theta_W J_{\text{em}}^\mu-\tfrac{1}{2}J_s^\mu+\tfrac{1}{2}J_c^\mu\,. \tag{6}$$

Here $(V^\mu-A^\mu)_{(3)}=F_3^\mu-F_3^{5\mu}$ is the isospin rotated weak $V-A$ current, J_{em}^μ is the electromagnetic current, and θ_W is the Weinberg angle. The current J_s^μ is a ($\Delta S=0$) isoscalar $V-A$ strangeness current, while the last current, J_c^μ, is a charm current, $V-A$, SU(3) singlet, Δcharm $=0$. In terms of the usual three quarks, $\mathcal{P}$, $\mathfrak{N}$, and λ, plus a fourth charmed quark $\mathcal{P}'[Q=\frac{2}{3}$, SU(3) singlet$]$, these currents are[8]

$$F_3^\mu-F_3^{5\mu}=\tfrac{1}{2}\bar{\mathcal{P}}\gamma^\mu(1-\gamma_5)\mathcal{P}-\tfrac{1}{2}\bar{\mathfrak{N}}\gamma^\mu(1-\gamma_5)\mathfrak{N}\,, \tag{7}$$

$$J_s^\mu=\bar{\lambda}\gamma^\mu(1-\gamma_5)\lambda\,, \tag{8}$$

$$J_c^\mu=\bar{\mathcal{P}}'\gamma^\mu(1-\gamma_5)\mathcal{P}'\,. \tag{9}$$

Equation (6) is the Glashow-Iliopoulos-Maiani

(GIM) modification of the original Weinberg-Salam neutral current which is necessary for adequate suppression of strangeness-changing neutral-current processes.[9] We shall neglect J_s^μ and J_c^μ for the purposes of our estimates, since there is no very dependable model for their matrix elements. One could, indeed, go beyond gauge theories and consider scalar, pseudoscalar, and tensor neutral "currents."[10] But one would have the same problem; unless the matrix elements of the neutral current can be related to experimentally measured matrix elements of the electromagnetic or charged weak currents, or calculated from some reliable model, it is difficult to obtain more than just bounds on the cross sections.

CALCULATIONS

The general associated-production reaction under consideration is $\nu(l_1)+N(p_1)\to l(l_2)+Y(p_2)+K(k)$ with $Y=\Lambda$ or Σ, and momenta as labeled. Since we neglect muon mass, the general kinematics are the same for neutral- and charged-current reactions.[6] The Born diagrams for typical reactions of Eqs. (1) and (2) are shown in Figs. (2) and (3), respectively. In general they include a nucleon pole in the s channel, a Σ or a Σ and a Λ pole in the u channel, and K and K^* poles in the t channel. In the spirit of the simple Born approximation we do not include the contributions of N^* and Y^* resonances in the s and u channels or higher meson resonances in the t channel. Various reactions have certain features worth noting; for example, the reactions $\nu n\to\mu^-K^+\Lambda$ and $\bar{\nu}p\to\mu^+K^0\Lambda$ have only a Σ u-channel contribution, whereas the reactions $\nu n\to\nu K^0\Lambda$ and $\nu p\to\nu K^+\Lambda$ receive contributions from both Σ and Λ exchange in the u channel. In both cases the u channel makes only a small contribution to the total amplitude. The u-channel Σ exchange amplitude is negligible since it is proportional to $g_{NK\Sigma}$ (the strong-interaction coupling constant for the $NK\Sigma$ vertex), while the s-channel, kaon t-channel, and Λ u-channel amplitudes are proportional to the considerably larger coupling constant $g_{NK\Lambda}$.[11] The term arising from Λ exchange in the u channel is also small, although it is not suppressed by a small coupling constant dependence. The reason for this is that the Z boson-$\Lambda\Lambda$ vertex receives no contribution from the isovector part of the neutral hadronic current and, in the Weinberg model or models with $J_Z^\mu \propto J_{\text{em}}^\mu$, it involves only the (isoscalar part of the) electromagnetic current. Hence the γ_μ and also, since the $K\Lambda$ reactions which include this term are neutral-current ones, the $\sigma_{\mu\nu}q^\nu$ part of the vertex give a small contribution for the small values of q^2 which are most important for the cross section.

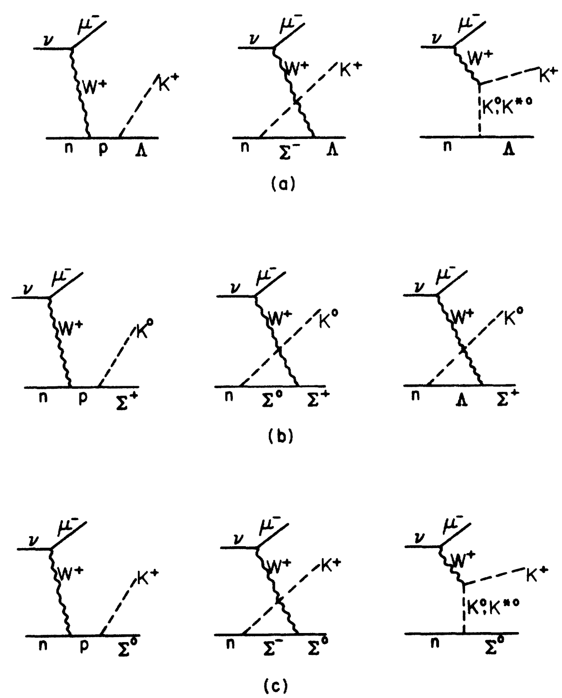

FIG. 2. Born diagrams for $K\Lambda$ and $K\Sigma$ charged-current reactions (a) $\nu n\to\mu^-K^+\Lambda$, (b) $\nu n\to\mu^-K^0\Sigma^+$, and (c) $\nu n\to\mu^-K^+\Sigma^0$.

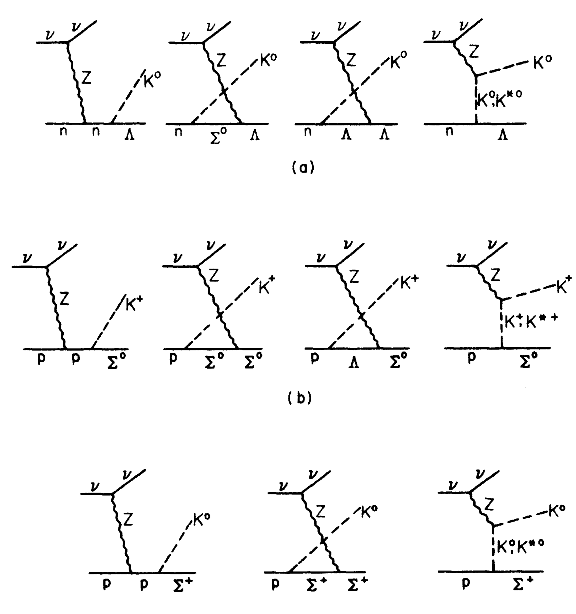

FIG. 3. Born diagrams for $K\Lambda$ and $K\Sigma$ neutral-current reactions (a) $\nu n\to\nu K^0\Lambda$, (b) $\nu p\to\nu K^+\Sigma^0$, and (c) $\nu p\to\nu K^0\Sigma^+$.

In all the $K\Lambda$ reactions the s channel is the dominant one, as will be seen from the numerical results. In contrast, in the $K\Sigma$ reactions this is not the case; in those with no Λ exchange in the u channel all the channels except K^* exchange are proportional to $g_{NK\Sigma}$. When the Λ u-channel exchange occurs it gives the dominant part of the amplitude.[12]

The invariant matrix element for the reaction $\nu N\to lKY$ is

$$\mathfrak{M}=(G/\sqrt{2})j_\mu\langle K(K)Y(p_2)|J^\mu|N(p_1)\rangle\ , \qquad (10)$$

where

$$j_\mu=\bar{u}_l(l_2)\gamma_\mu(1-\gamma_5)u_\nu(l_1)\ , \qquad (11)$$

and the momentum dependence of the W or Z boson propagator has been neglected since the momentum transfer Q^2 $(=-q^2$, where $q=l_1-l_2)$ is much smaller than $m_W{}^2$ or $m_Z{}^2$. J^μ is the weak hadronic charged or neutral current. The Weinberg-Salam neutral current has been discussed above, and the charged current is the standard $\Delta S=0$ V-A Cabibbo current

$$J^\mu=\cos\theta_C[F^\mu_{1+i2}-F^{5\mu}_{1+i2}]\ , \qquad (12)$$

where $\cos\theta_C=0.97$. It is first necessary to determine the vertices to use in the Born diagrams. We shall consider the charged-current reactions first; the neutral-current ones are similarly treated and will be dealt with only briefly. We make use of the SU(3) relations

$$\langle i|F^\mu_j|k\rangle=d_{ijk}D^\mu_V+if_{ijk}F^\mu_V\ , \qquad (13)$$

$$\langle i|F^{5\mu}_j|k\rangle=d_{ijk}D^\mu_A+if_{ijk}F^\mu_A, \qquad (14)$$

where i, j, and k are SU(3) labels in the range $1,\ldots,8$, and d_{ijk} and f_{ijk} are the symmetric and antisymmetric structure constants of SU(3). D^μ_V and F^μ_V can be expressed in terms of matrix elements of the electromagnetic current:

$$D^\mu_V=-\tfrac{3}{2}\langle n|J^\mu_{\rm em}|n\rangle\ , \qquad (15)$$

$$F^\mu_V=\langle p|J^\mu_{\rm em}|p\rangle+\tfrac{1}{2}\langle n|J^\mu_{\rm em}|n\rangle\ . \qquad (16)$$

For the axial-vector current,

$$D^\mu_A=\left(\frac{D}{D+F}\right)\langle p|F^{5\mu}_{1+i2}|n\rangle\ , \qquad (17)$$

$$F^\mu_A=\left(\frac{F}{D+F}\right)\langle p|F^{5\mu}_{1+i2}|n\rangle\ , \qquad (18)$$

where

$$\langle p|F^{5\mu}_{1+i2}|n\rangle=(D+F)F_A(q^2)\bar{u}_p\gamma^\mu\gamma_5u_n \qquad (19)$$

with[13]

$$F_A(q^2)=\frac{1}{(1-q^2/m_A{}^2)^2}\ , \qquad (20)$$

$$m_A\simeq 0.95\ \text{GeV}\ , \qquad (21)$$

and

$$D=0.78\pm0.02\ , \qquad (22)$$

$$F=0.45\pm0.02\ . \qquad (23)$$

We have used the fact that the $\Delta S=0$ vector current is conserved and the assumption of no second-class currents, so that no terms of the form q_μ or $i\sigma_{\mu\nu}q^\nu\gamma_5$ appear in the above matrix elements of the charged weak current. Furthermore, the induced pseudoscalar term $F_p(q^2)q_\mu\gamma_5$ is neglected since it gives a contribution proportional to m_μ when contracted with the charged lepton current (and zero with the neutral lepton current). For the t-channel graphs the matrix elements $\langle K^+|J^\mu|K^0\rangle=\langle K^0|J^{\dagger\mu}|K^+\rangle$ and $\langle K^+|J^\mu|K^{*0}\rangle=\langle K^0|J^{\dagger\mu}|K^{*+}\rangle$ are needed. In the former case only the vector current contributes and consequently,

$$\langle K^+|J^\mu|K^0\rangle=\langle K^+|J^\mu_{\rm em}|K^+\rangle-\langle K^0|J^\mu_{\rm em}|K^0\rangle\ . \qquad (24)$$

There are very few data on the kaon electromagnetic form factor in the spacelike region. For the K^+ we have used a pole-dominated form factor, $F(q^2)=1/(1-q^2/m_\rho{}^2)$ and for comparison also the nucleon isovector form factor $F^V_1(q^2)$, by analogy with the pion electromagnetic form factor which can be fitted by either of these.

In calculating the Born amplitude one must also know the strong-interaction coupling constants $g_{NK\Lambda}$ and $g_{NK\Sigma}$ for the nucleon-hyperon-kaon Yukawa vertices. Unfortunately, these have not been measured very precisely. Rough experimental bounds are[14]

$$-14<g_{NK\Lambda}<-6\ , \qquad (25)$$

$$0<g_{NK\Sigma}<5 \qquad (26)$$

(where $g_{\pi NN}$ is taken to be positive).

From a combination of recent data on $K\Lambda$ photoproduction and KN scattering we take the values $g_{NK\Lambda}=-10$ and $g_{NK\Sigma}=1.3$. We have tested the dependence of the cross sections on these coupling constants and have found, as expected, a roughly quadratic dependence upon $g_{NK\Lambda}$ in the $K\Lambda$ reactions, which would be exactly quadratic if the Σ exchange in the u channel and the K^* exchange in the t channel were absent. The $K\Sigma$ reactions which have a Λ pole in the u channel are also roughly proportional to $g_{NK\Lambda}{}^2$. However, in the other $K\Sigma$ reactions the s, u, and kaon t channels are all proportional to $g_{NK\Sigma}$ and hence small. These reactions are more model dependent, because the K^* exchange plays a relatively greater role than for the $K\Lambda$ reactions.

In addition to computing the contributions from the s, u, and kaon-exchange t-channel graphs one would like to estimate how important the K^*-exchange in the t channel is. In the analogous case of t-channel vector meson exchange in weak pion production, the G parities of the charged $\Delta S=0$ vector and axial-vector currents imply that the vector current contributes only to ω exchange and the axial-vector current only to ρ exchange. Here the situation is more complicated; the K^* exchange involves both V and A parts. The general hadronic matrix element for K^* exchange is

$$\bar{u}(p_2)\left(f_1(\Delta^2)\gamma_\alpha + f_2(\Delta^2)\frac{i\sigma_{\alpha\beta}\Delta^\beta}{m_N+m_Y} + f_3(\Delta^2)\Delta_\alpha\right)u(p_1) \times \frac{g^{\alpha\lambda} - \Delta^\alpha\Delta^\lambda/m_{K^*}^2}{\Delta^2 - m_{K^*}^2}\Gamma_{\lambda\mu}, \quad (27)$$

where $\Delta_\alpha = p_{2\alpha} - p_{1\alpha}$, the f_i are form factors for the K^*NY vertex, and $\Gamma_{\lambda\mu}$ is the W-boson–K^*K vertex function. For the vector current $\Gamma^{(v)}_{\lambda\mu}$ must have the current-conserving form $\epsilon_{\lambda\mu\rho\tau}q^\rho\Delta^\tau$ and therefore the $\Delta^\alpha\Delta^\lambda$ term in the K^* propagator and the $f_3(\Delta^2)\Delta_\alpha$ term both give zero contribution. One can then use photoproduction data in conjunction with an assumed form factor for $\Gamma_{\lambda\mu}$ at $q^2\neq 0$ to estimate the K^* vector amplitude. We find that in the $K\Lambda$ reactions the change in the cross section due to this vector part of K^* exchange is about 5–20% depending on the reaction.[15] For the axial-current part of K^* exchange there is no data upon which to rely, but it seems reasonable to assume that it gives a contribution comparable to that of the vector current. Thus the three main sources of uncertainty in our cross-section estimates are: (1) the errors incurred in the use of the Born approximation, (2) the roughly quadratic dependence of the cross section on the uncertain strong NKY coupling constants, and (3) the K^* exchange in the t channel.

Essentially all the statements above are applicable also to the neutral-current processes. The matrix elements of the neutral current in the Weinberg model can be calculated in terms of those of the charged $\Delta S=0$ weak current and the electromagnetic current. In order to determine the dependence of the $K\Lambda$ cross sections on the Weinberg angle, we have used the values $\sin^2\theta_W = 0.2$ and 0.4 as well as the central value of $\sin^2\theta_W \simeq \frac{1}{3}$ indicated by present experiments.

The calculation of the differential and total cross section and the polarization proceeds in a straightforward manner once the invariant amplitude is computed. The fully differential cross section is

$$\frac{\partial^4\sigma}{\partial Q^2\partial W^2\partial\cos\theta\,\partial\varphi} = \frac{G^2}{32m_N^2E^2(2\pi)^4}\frac{|\vec{p}_2|}{W}L_{\mu\nu}W^{\mu\nu}, \quad (28)$$

where m_N is the nucleon mass,

$$L_{\mu\nu} = (l_{1\mu}l_{2\nu} + l_{2\mu}l_{1\nu} - l_1\cdot l_2 g_{\mu\nu} - i\xi\epsilon_{\mu\nu\alpha\beta}l_1^\alpha l_2^\beta), \quad (29)$$

$$\xi = \begin{cases} +1 \text{ for incident } \nu \\ -1 \text{ for incident } \bar{\nu}, \end{cases}$$

and

$$W^{\mu\nu} = \frac{1}{2}\sum_{\text{spins}}\langle KY|J^\mu|N\rangle\langle KY|J^\nu|N\rangle^*. \quad (30)$$

The vector $\vec{p}_2$ in Eq. (20) is defined in the KY center-of-mass frame $(\vec{p}_1+\vec{q}=\vec{p}_2+\vec{k}=0)$. The angle θ and the (Treiman-Yang) angle φ are defined in the same frame, as shown in Fig. 4.

The differential cross section is first decomposed with respect to the Treiman-Yang angle φ into the five terms allowed by locality[16]:

$$\frac{\partial^4\sigma}{\partial Q^2\partial W^2\partial\cos\theta\,\partial\varphi} = a_1 + a_2\cos\varphi + a_3\sin\varphi + a_4\cos 2\varphi + a_5\sin 2\varphi, \quad (31)$$

where the a_i contain no dependence upon φ. The integration over φ is then trivial; the remaining integrations necessary to obtain $\partial\sigma/\partial Q^2$, $\partial\sigma/\partial W$, and σ were then done numerically, on an IBM 360 computer. The integrated polarization $P_L(E)$ was obtained similarly. In calculating the polarization it is convenient to reexpress the invariant amplitude as a hadron center-of-mass amplitude involving Pauli spin operators sandwiched between initial and final Pauli spinors. This is a standard technique which is necessary in treating resonant reactions (for which one would then project out the multipole amplitudes). From the hadron center-of-mass amplitude one then computes the helicity amplitudes, in terms of which the polarization assumes a particularly simple form.

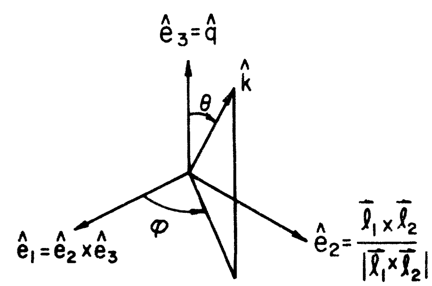

FIG. 4. Coordinate system in KY center-of-mass frame.

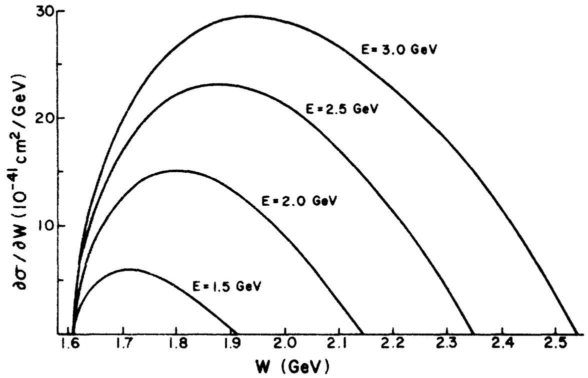

FIG. 5. $\partial\sigma(E,W)/\partial W$ for the reaction $\nu n\to\mu^- K^+\Lambda$, at $E=1.5$, 2.0, 2.5, and 3.0 GeV.

TABLE I. Mean values of kinematic quantities for $K\Lambda$ reactions, averaged over Argonne neutrino spectrum. Neutral-current reactions are calculated in the Weinberg model with $\sin^2\theta_W=\frac{1}{3}$.

Reaction	$\langle W\rangle$ (GeV)	$\langle Q^2\rangle$ (GeV^2)	$\langle E\rangle$ (GeV)
$\nu n\to\mu^- K^+\Lambda$	1.9	0.28	2.0
$\nu p\to\nu K^+\Lambda$	1.8	0.24	2.0
$\nu n\to\nu K^0\Lambda$	1.8	0.29	2.0

RESULTS AND DISCUSSION

In Fig. 5 we show the differential cross section $\partial\sigma(E,W)/\partial W$ calculated in Born approximation for the charged-current reaction $\nu n\to\mu^- K^+\Lambda$. For this and the other results to follow we neglect the K^* exchange amplitude because of the considerable uncertainty about the parameters which enter into it; this should incurr an error of less than about 10–30% in the cross sections. As was emphasized before, the W distributions are most accurate for small W (independent of E). The $\partial\sigma/\partial W$ curves for the neutral-current reactions $\nu N\to\nu K\Lambda$, calculated in the Weinberg model with $\sin^2\theta_W=\frac{1}{3}$, are quite similar in shape, although smaller in magnitude, compared to $\partial\sigma/\partial W$ for $\nu n\to\mu^- K^+\Lambda$, and are therefore not shown. This similarity indicates that the shape is determined mainly by phase space, independent of the dynamical details of the particular reaction. In the neutral-current case, $\partial\sigma/\partial W$ is not very sensitive to changes of the Weinberg angle in the range $0.2<\sin^2\theta_W<0.4$. The same is true of $\sigma(E)$, and accordingly we shall restrict ourselves to illustrating this dependence on the Weinberg angle only for the flux-averaged cross section. The W distributions, flux-averaged for the Argonne neutrino spectrum, are shown in Fig. 6. Again, the curves for all three $K\Lambda$ reactions are very similar in shape, and the charged current distribution is larger by a factor of about 4 than the two neutral current ones, which are comparable. The maximum of the $\nu n\to\mu^- K^+\Lambda$ $\partial\sigma(E,W)/\partial W$ curve moves from $W\simeq 1.7$ GeV at $E=1.5$ GeV to $W\simeq 1.9$ GeV at $E=3.0$ GeV; the maximum of the flux-averaged distribution $d\bar\sigma/dW$ is at $W\simeq 1.7$ GeV.

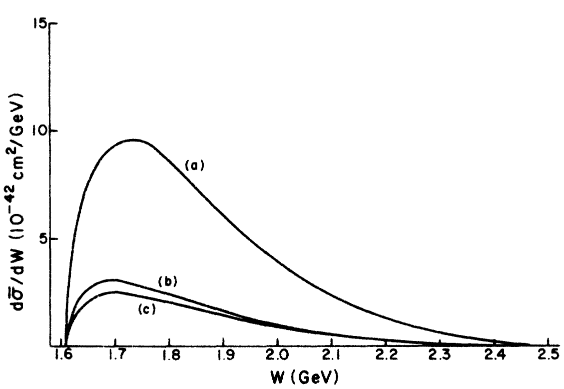

FIG. 6. $d\bar\sigma/dW$, flux-averaged for the Argonne neutrino spectrum, for the reactions (a) $\nu n\to\mu^- K^+\Lambda$, (b) $\nu n\to\nu K^0\Lambda$, and (c) $\nu p\to\nu K^+\Lambda$. The neutral-current reactions are calculated in the Weinberg model with $\sin^2\theta_W=\frac{1}{3}$.

The mean values of W for these three reactions are given in Table I; as is expected from the high-W tails on the flux-averaged $d\bar\sigma/dW$ curves, the $\langle W\rangle$ values are slightly higher than the actual maxima of these curves. The differential cross sections $\partial\sigma/\partial Q^2$ have also been calculated and exhibit a typical rapid falloff for large Q^2 due to the damping from the form factors at the current-hadron vertices. The mean values of Q^2 for the three $K\Lambda$ processes are given in Table I and are all less than 0.3 GeV^2. Clearly, the values of $\langle W\rangle$ and $\langle Q^2\rangle$ (and also $\langle E\rangle$) are quite similar for the three $K\Lambda$ reactions.

The total cross sections for the three $\nu N\to lK\Lambda$ reactions and, for comparison, also the corresponding $\bar\nu N\to \bar l K\Lambda$ reactions, are given in Table II, at three representative energies. The neutral-current processes are calculated in the Weinberg model with $\sin^2\theta_W=\frac{1}{3}$. The relationship between the sizes of the neutrino cross sections is similar to that which applies for the W distributions. The cross section can be written in the form

$$\sigma=(VV,AA)_l\,(VV,AA)_h+(VA)_l\,(VA)_h \tag{32}$$

$$\equiv\sigma_{VV,AA}+\sigma_{VA}, \tag{33}$$

where, for example, the second term in Eq. (32) denotes a product of a leptonic vector and leptonic axial-vector amplitude, multiplied by a product

TABLE II. Total cross sections for charged- and neutral-current $\nu N \rightarrow lK\Lambda$ and $\bar{\nu}N \rightarrow \bar{l}K\Lambda$ reactions. The neutral-current reactions are calculated in the Weinberg model with $\sin^2\theta_W = \frac{1}{3}$.

σ (10^{-41} cm^2)	$E = 1.2$ GeV	$E = 1.6$ GeV	$E = 2.0$ GeV
$\sigma(\nu n \rightarrow \mu^- K^+ \Lambda)$	0.14	1.8	5.4
$\sigma(\bar{\nu} p \rightarrow \mu^+ K^0 \Lambda)$	0.12	1.4	4.1
$\sigma(\nu p \rightarrow \nu K^+ \Lambda)$	3.9×10^{-2}	0.43	1.2
$\sigma(\bar{\nu} p \rightarrow \bar{\nu} K^+ \Lambda)$	5.5×10^{-2}	0.52	1.4
$\sigma(\nu n \rightarrow \nu K^0 \Lambda)$	5.4×10^{-2}	0.54	1.5
$\sigma(\bar{\nu} n \rightarrow \bar{\nu} K^0 \Lambda)$	3.5×10^{-2}	0.37	1.1

TABLE IV. Total cross sections for the neutral-current $K\Lambda$ and $K\Sigma$ reactions, flux-averaged for the Argonne neutrino spectrum, in the Weinberg model.

Reaction	$\sin^2\theta_W$	$\bar{\sigma}$ (10^{-42} cm^2)
$\nu p \rightarrow \nu K^+ \Lambda$	0.2	0.80
	0.33	0.81
	0.4	0.85
$\nu n \rightarrow \nu K^0 \Lambda$	0.2	0.88
	0.33	0.97
	0.4	1.1
$\nu p \rightarrow \nu K^+ \Sigma^0$	0.33	0.26
$\nu p \rightarrow \nu K^0 \Sigma^+$	0.33	0.017
$\nu n \rightarrow \nu K^0 \Sigma^0$	0.33	0.73
$\nu n \rightarrow \nu K^+ \Sigma^-$	0.33	0.017

of hadronic vector and axial-vector amplitudes, and similarly for the first term. Positivity requires $\sigma_{VV,AA} > |\sigma_{VA}|$. Since the hadronic matrix element in Eq. (10) is obviously the same for the ν and corresponding $\bar{\nu}$ neutral-current reactions, and also for the two charged-current reactions (because of the charge symmetry of J^μ) it follows that

$$\sigma^{\nu}_{VV,AA} = \sigma^{\bar{\nu}}_{VV,AA} \tag{34}$$

and

$$\sigma^{\nu}_{VA} = -\sigma^{\bar{\nu}}_{VA} \tag{35}$$

for each of the three pairs of $K\Lambda$ reactions. Hence, by taking sums and differences of cross sections it is possible to isolate $\sigma^{\nu}_{VV,AA}$ and σ^{ν}_{VA}. As is evident from Table III, the neutrino and corresponding antineutrino reactions have rather similar cross sections, or equivalently, σ^{ν}_{VA} is small compared to $\sigma^{\nu}_{VV,AA}$. Secondly, one can observe that σ^{ν}_{VA} is positive for the reactions $\nu n \rightarrow \mu^- K^+\Lambda$ and $\nu n \rightarrow \nu K^0\Lambda$, and negative for the reaction $\nu p \rightarrow \nu K^+\Lambda$.

For $E \gtrsim 2$ GeV the Born approximation to the total cross section exhibits a linear rise.[17] Physically, the cross section will presumably level off at large energies, so that this linear rise is an indication of the invalidity of the Born approximation at high energies. Note that this source of error causes an overestimation of the cross section; this fact will be significant in the comparison of our results with the Argonne data. However, this overestimation should not be very large for the flux-averaged cross section, since $\phi(E)$ gives practically zero weight to $E \gtrsim 2$ GeV. In Tables III and IV we give the total cross sections, flux-averaged for the Argonne neutrino spectrum, of the charged-current and neutral-current $K\Lambda$ and selected $K\Sigma$ reactions. In the $K\Lambda$ reactions the s channel gives the dominant contribution to the cross section. For example, for the reaction $\nu n \rightarrow \mu^- K^+\Lambda$, the s channel alone gives a flux-averaged cross section of 5.9×10^{-42} cm^2, while the s, u, and t channels together give 3.4 $\times10^{-42}$ cm^2. In the neutral-current $K\Lambda$ reactions $\nu p \rightarrow \nu K^+\Lambda$ and $\nu n \rightarrow \nu K^0\Lambda$ (in the Weinberg model with $\sin^2\theta_W = \frac{1}{3}$) the s channel alone gives a flux-averaged cross section of 0.66×10^{-42} cm^2 and 1.1×10^{-42} cm^2, respectively, while the s, u, and t channels give 0.81×10^{-42} and 0.97×10^{-42} cm^2. There is thus constructive interference in the $\nu p \rightarrow \nu K^+\Lambda$ case and destructive interference in the $\nu n \rightarrow \nu K^0\Lambda$ and $\nu n \rightarrow \mu^- K^+\Lambda$ cases. The neutral $K\Lambda$ reactions do not vary strongly with the Weinberg angle; as $\sin^2\theta_W$ increases from 0.2 to 0.4, $\bar{\sigma}(\nu p \rightarrow \nu K^+\Lambda)$ increases by only about 6%, while $\bar{\sigma}(\nu n \rightarrow \nu K^0\Lambda)$ grows by ~20%. As was true of $\partial\sigma/\partial W$ and $\sigma(E)$, the charged-current cross section $\bar{\sigma}(\nu n \rightarrow \mu^- K^+\Lambda)$ is about 4 times greater than the neutral current $K\Lambda$ cross sections. The latter are comparable; for $\sin^2\theta_W = \frac{1}{3}$, $\bar{\sigma}(\nu n \rightarrow \nu K^0\Lambda)$ $\simeq 1.2\bar{\sigma}(\nu p \rightarrow \nu K^+\Lambda)$. From the two $\nu n \rightarrow \mu^- K^+\Lambda$ events and the one $\nu n \rightarrow \nu K^0\Lambda$ event, the Argonne

TABLE III. Total cross sections for charged-current $K\Lambda$ and $K\Sigma$ reactions, flux-averaged for the Argonne neutrino spectrum.

Reaction	$\bar{\sigma}$ (10^{-42} cm^2)
$\nu n \rightarrow \mu^- K^+ \Lambda$	3.4
$\nu n \rightarrow \mu^- K^0 \Sigma^+$	0.79
$\nu n \rightarrow \mu^- K^+ \Sigma^0$	0.11

experiment derives flux-averaged cross sections of $\bar{\sigma}(\nu n \to \mu^- K^+ \Lambda) \simeq 2\times 10^{-41}$ cm^2 and $\bar{\sigma}(\nu n \to \nu K^0 \Lambda) \simeq 3\times 10^{-41}$ cm^2.[18]

Thus we make the following conclusions:

(1) The Cabibbo theory predicts a flux-averaged Born-approximation cross section for the reaction $\nu n \to \mu^- K^+ \Lambda$ which is much smaller than that observed at Argonne. We stress that the basic theory is presumably dependable here.

(2) The Weinberg theory with a reasonable range of θ_W predicts $\bar{\sigma}(\nu n \to \nu K^0 \Lambda)$ to be very much smaller than that inferred from the one event at Argonne. In both cases (1) and (2) the inclusion of K^* exchange corrections could only increase the cross sections by ~30%, and the use of the maximum allowed $g_{NK\Lambda}$ ($g_{NK\Lambda} = -14$ rather than -10) would only scale the cross sections up by about a factor of 2. These changes would still leave a huge discrepancy between the predicted and observed cross sections. One may, of course, question the applicability of the Born approximation, but one should recall that the linear rise of the Born cross section tends to overestimate the actual cross section.

(3) The Weinberg theory with the same range of θ_W gives a value for $\bar{\sigma}(\nu p \to \nu K^+ \Lambda)$ comparable to that for $\bar{\sigma}(\nu n \to \nu K^0 \Lambda)$. When translated into an event rate, this indicates that no events should have been observed at Argonne. This is in agreement with the absence of any $\nu p \to \nu K^+ \Lambda$ events in the Argonne data. However, our results for these two neutral-current reactions do not agree with the inference from Argonne that the reaction $\nu p \to \nu K^+ \Lambda$ might somehow be suppressed relative to $\nu n \to \nu K^0 \Lambda$.

(4) Our calculations do not confirm the suggestion from the limited Argonne data that the neutral-current $K\Lambda$ cross sections are larger than the charged-current ones; rather, we find that

$$R = \frac{\bar{\sigma}(\nu p \to \nu K^+ \Lambda) + \bar{\sigma}(\nu n \to \nu K^0 \Lambda)}{2\bar{\sigma}(\nu n \to \mu^- K^+ \Lambda)} \simeq 0.26 \qquad (36)$$

(for $\sin^2\theta_W = \frac{1}{3}$).

For the purpose of comparison, we have also calculated the cross sections for the two neutral-current $K\Lambda$ reactions using a hadronic neutral current proportional to the electromagnetic current. This type of neutral current appears in the Bég-Zee model[19] and in the Bjorken-Llewellyn Smith 2-3 model[20] in the form

$$J_Z^\mu = a(1 - 2\sin^2\theta_Z) J_{\text{em}}^\mu \ , \qquad (37)$$

where θ_Z is a parametric angle analogous to the Weinberg angle, and where $a = m_W/(m_Z \cos\theta_Z)$ in the former model and $a = 1$ in the latter model (in which the relation $m_W = m_Z \cos\theta_Z$ holds). For $\sin^2\theta_Z = \frac{1}{3}$, $a = 1$, the cross sections are roughly an order of magnitude smaller than those in the Weinberg model:

$$\bar{\sigma}(\nu p \to \nu K^+ \Lambda)_{J_Z \propto J_{\text{em}}} = 0.58\times 10^{-43} \text{ cm}^2 , \qquad (38)$$

$$\bar{\sigma}(\nu n \to \nu K^0 \Lambda)_{J_Z \propto J_{\text{em}}} = 0.76\times 10^{-43} \text{ cm}^2 . \qquad (39)$$

Thus, models in which the neutral current is proportional to the electromagnetic current, with a moderate value of the parametric angle, are in even more serious disagreement with the Argonne results. One can assess the accuracy of the Born approximation for the neutral-current reactions with $J_Z^\mu \propto J_{\text{em}}^\mu$ by computing the corresponding electroproduction cross section and comparing it with the available data. This is a result of the fact that if $J_Z^\mu \propto J_{\text{em}}^\mu$ there is no hadronic VA interference, so that the hadronic tensor $W^{\mu\nu}$ is completely symmetric, and consequently the leptonic VA interference term $i\xi\epsilon_{\mu\nu\alpha\beta} l_1^\alpha l_2^\beta$ in $L_{\mu\nu}$ gives no contribution to the differential cross section. Thus the differential cross sections are simply proportional:

$$\left.\frac{\partial^4\sigma}{\partial Q^2 \partial W^2 \partial\cos\theta\, \partial\varphi}\right|_{eN\to eKY} = \frac{e^4}{2(Q^2)^2 G^2 a^2 (1-2\sin^2\theta_Z)^2} \times \left.\frac{\partial^4\sigma}{\partial Q^2 \partial W^2 \partial\cos\theta \partial\varphi}\right|_{\nu N\to \nu KY} . \qquad (40)$$

The $K\Lambda$ electroproduction data can be fitted reasonably well for moderate Q^2 with $g_{NK\Lambda} \simeq -11$.[21] This value of $g_{NK\Lambda}$ is near the middle of the large range [Eq. (25) above] allowed by present experimental determinations. For $Q^2 > 0.6$ GeV^2 the Born-approximation cross section falls somewhat faster with increasing Q^2 than the electroproduction data.[22] However, this is not an important source of error for the weak associated-production calculation since, as was mentioned before, the dominant contribution to the cross section comes from the small Q^2 ($Q^2 < 0.5$ GeV^2) part of phase space. Thus the comparison with $K\Lambda$ electroproduction gives one further confidence in the accuracy of the results listed in Eqs. (38) and (39); these can be scaled up or down if one uses a different value of $g_{NK\Lambda}$, a, or $\sin^2\theta_Z$ than the ones used for the calculation.

Tables III and IV also show $\bar{\sigma}$ for certain $K\Sigma$ reactions, flux-averaged for the Argonne flux spectrum (although there are no $K\Sigma$ events observed at Argonne). The neutral current $K\Sigma$ cross sections are calculated in the Weinberg model with $\sin^2\theta_W = \frac{1}{3}$. Those which involve Λ exchange in the u channel ($\nu n \to \mu^- K^0 \Sigma^+$, $\nu n \to \nu K^0 \Sigma^0$, and $\nu p \to \nu K^+ \Sigma^0$) are, as one would expect, larger than those which have only $NK\Sigma$ strong vertices

($\nu n \to \mu^- K^+ \Sigma^0$, $\nu n \to \nu K^+ \Sigma^-$, and $\nu p \to \nu K^0 \Sigma^+$) since ${g_{NK\Lambda}}^2 \gg {g_{NK\Sigma}}^2$. Furthermore, within each of these two classes the charged-current reactions are dominant.

The $K\Lambda$ reactions are also of interest because of the experimental feasibility of measuring the polarization of the Λ. Since the $K\Lambda$ reactions are not highly resonant, the polarization normal to the hadron scattering plane (formed by $\vec{p}_2$ and $\vec{q}$ in the laboratory frame), which is time-reversal violating in the absence of final-state interactions is presumably small, especially when flux-averaged. There remain the longitudinal polarization and the polarization normal to $\vec{p}_2$, but in the hadron scattering plane, both of which are time-reversal-even and parity violating. We concentrate here on the longitudinal polarization, P_L. Figures 7, 8, and 9 show $P_L(E)$ for the three $K\Lambda$ reactions induced by neutrinos and, for comparison, also the three corresponding antineutrino reactions. For the four neutral-current reactions we again use the Weinberg model with $\sin^2\theta_W = \frac{1}{3}$.

As is evident from the graphs, the longitudinal polarization approaches constant values for $E \gtrsim 2$ GeV. The flux-averaged polarizations for the neutrino reactions are $\bar{P}_L(\nu n \to \mu^- K^+ \Lambda) \simeq 14\%$, $\bar{P}_L(\nu p \to \nu K^+ \Lambda) \simeq 18\%$, and $\bar{P}_L(\nu n \to \nu K^0 \Lambda) \simeq -30\%$. These are sufficiently large, especially in the last reaction, that it should be possible to measure the longitudinal polarization in the next few years. It should be noted that P_L arises both from leptonic VA interference (multiplied by hadronic VV, and AA terms), and from hadronic VA interference (multiplied by leptonic VV and AA terms). Consequently, there will still be longitudinal polarization present even if the neutral hadronic current is purely V or purely A.

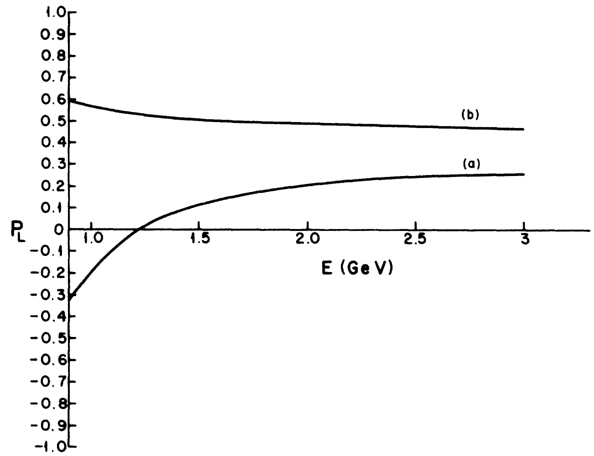

FIG. 8. Longitudinal polarization, $P_L(E)$, of Λ in the reactions (a) $\nu p \to \nu K^+ \Lambda$, and (b) $\bar{\nu} p \to \nu^- K^+ \Lambda$, calculated in the Weinberg model with $\sin^2\theta_W = \frac{1}{3}$.

The differential polarization $P_L(E, q^2, W, \theta)$ exhibits considerable structure, which will be discussed more fully in a future publication. Here we shall only note several salient features. First, in the approximation that $m_\mu = 0$ in the charged-current reactions and exactly in the neutral-current reactions, $P_L = 0$ at $q^2 = 0$. This is a consequence of the fact that for $m_\mu = 0$ and $q^2 = 0$ the amplitude is of the form $q_\mu \langle KY | J^\mu | N \rangle \sim \langle KY | \partial_\mu J^\mu | N \rangle$. Now $\partial_\mu J^\mu$ contains only the divergence of the axial-vector charged or neutral weak current since $\partial_\mu V^\mu = 0$ for the $\Delta S = 0$ charged weak current, and for the Weinberg and $J_Z^\mu \propto J_{em}^\mu$ neutral currents. Therefore, there is no leptonic or hadronic VA interference and consequently

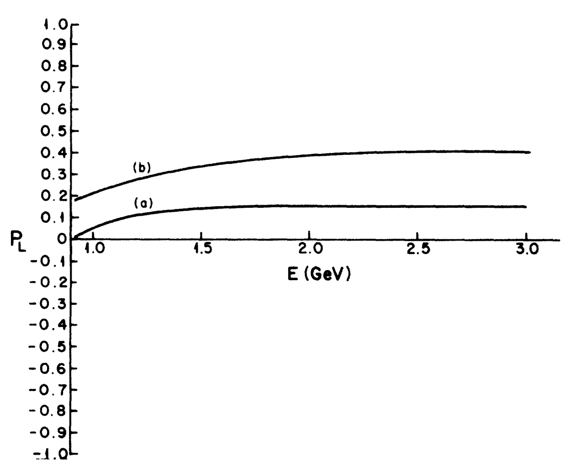

FIG. 7. Longitudinal polarization, $P_L(E)$, of Λ in the reactions (a) $\nu n \to \mu^- K^+ \Lambda$ and (b) $\bar{\nu} p \to \mu^+ K^0 \Lambda$.

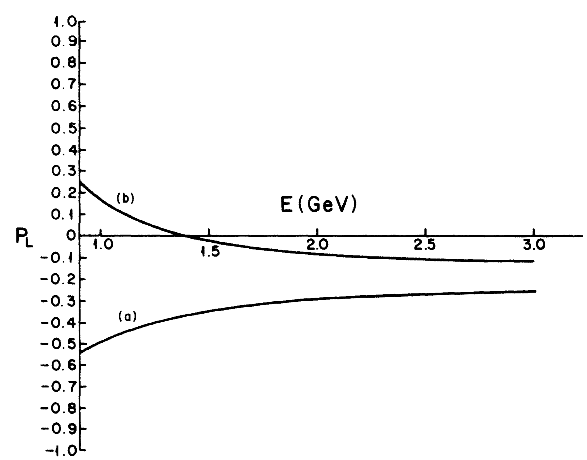

FIG. 9. Longitudinal polarization, $P_L(E)$, of Λ in the reactions (a) $\nu n \to \nu K^0 \Lambda$, and $\bar{\nu} n \to \bar{\nu} K^0 \Lambda$ calculated in the Weinberg model with $\sin^2\theta_W = \frac{1}{3}$.

no parity-violating effects, such as longitudinal polarization.[23] Secondly, as W approaches threshold, P_L becomes isotropic in terms of the hadron center-of-mass angle θ, just as the differential cross section itself does. Finally, for $Q^2 = Q_{max}^2 > 0$ at a given W, $P_L(\theta=0) = P_L(\theta=\pi) = 1\ (-1)$ for $\bar{\nu}\ (\nu)$ reactions, respectively, independent of the details of the hadronic amplitude. If $Q_{max}^2 = 0$, which occurs only for the neutral-current reactions at $W = W_{max}$, $P_L = 0$ by the argument given above. These features of the longitudinal polarization should be relatively amenable to experimental observation.

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[1]S. Barish *et al.*, Phys. Rev. Lett. 33, 448 (1974); 33, 1446 (1974).

[2]S. L. Adler, Ann. Phys. (N. Y.) 50, 189 (1968); Phys. Rev. D 9, 229 (1974); Phys. Rev. Lett. 33, 1511 (1974); Phys. Rev. D (to be published); S. L. Adler *et al.*, *ibid.* (to be published). See also P. Zucker, Phys. Rev. D 4, 3350 (1971); P. Schreiner and F. von Hippel, Phys. Rev. Lett. 8, 339 (1973); B. W. Lee, Phys. Lett. 40B, 420 (1972); C. H. Albright, B. W. Lee, E. A. Paschos, and L. Wolfenstein, Phys. Rev. D 7, 2220 (1973), among other references.

[3]In the $K\Sigma$ reactions, the decays $\Sigma^\pm \to n\pi^\pm$ are, respectively, almost completely p- and s-wave, hence, effectively parity conserving and, therefore, not useful for analyzing the polarization of $\Sigma^\pm$. The $\Sigma^+ \to p\pi^0$ decay is, however, sufficient for this purpose. Polarization transverse to the hadron scattering plane is allowed in all the $K\Sigma$ reactions, but is small since they are not strongly resonant. Therefore, among these reactions those involving Σ^+ are best suited for polarization studies.

[4]The Argonne and Brookhaven data seem to indicate that in weak pion production by neutral currents the $\Delta(1232)$ resonance may not be strongly excited.

[5]I am grateful to P. Schreiner for supplying me with a copy of the computer program giving the Argonne neutrino flux as a function of energy.

[6]We neglect muon mass here and throughout the rest of the calculations (so that, e.g., $W_{max} = \sqrt{s}$, where $s = m_N^2 + 2m_N E$). This approximation is more accurate in the KY reactions than it would be if one were to apply it in the πN reactions because in the latter case there are t-channel contributions which are proportional to m_μ but are not negligible, as a result of the fact that the pion pole in the t-channel propagator is very close to the edge of the physical region. In contrast, this problem is not so severe in the KY reactions, since $m_K^2 \gg m_\pi^2$.

[7]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); 27, 1688 (1972). A. Salam, in *Elementary Particle Theory; Relativistic Groups and Analyticity* (Nobel Symposium No. 8), edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[8]We use the metric and γ-matrix conventions of J. D. Bjorken and S. D. Drell, except that spinors are normalized to $\bar{u}u = 2m$.

[9]S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[10]See, for example, the recent work of Adler and collaborators, Ref. 2.

[11]It might be noted that the (W or Z) $\Sigma\Lambda$ vertex is pure D type, and hence the vector part of it, $\propto F_1^n(q^2)\gamma_\mu$, is small for small q^2. However, the axial-vector part, $\propto DF_A(q^2)\gamma_\mu\gamma_5$, and for charged-current reactions also the magnetic moment part, $\propto \mu_n F_2^n(q^2) i\sigma_{\mu\nu}q^\nu$, are not negligible. Of course, the Σ u-channel exchange in the $K\Lambda$ reactions is negligible anyway because it is proportional to $g_{NK\Sigma}$.

[12]This is true even though the Σ u-channel exchange is pure D type for the reason given in the previous footnote, combined with the fact that in the $K\Sigma$ reactions this term is $\propto g_{NK\Lambda}$.

[13]The m_A value is taken from P. Schreiner, in *Proceedings of the 1974 Neutrino Conference, Philadelphia, Pa.* (AIP Conference Proceedings No. 22) (A.I.P., New York, 1975). The Cabibbo D and F values are from the one angle fit in L. Chounet, J. Gaillard, and M. Gaillard, Phys. Rep. 4C (1972).

[14]See H. Pilkuhn *et al.*, Nucl Phys. B65, 460 (1973); N. Samios, M. Goldberg, and B. Meadows, Rev. Mod. Phys. 46, 5 (1974). For analyses of $K\Lambda$ photoproduction data, see T. Kuo, Phys. Rev. 129, 2264 (1963); H. Thom, *ibid.* 151, 1322 (1966); F. Renard and T. Renard, Nucl. Phys. B25, 490 (1971); A. Donnachie, in *Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies*, edited by N. B. Mistry (Laboratory of Nuclear Studies, Cornell University, Ithaca, N. Y., 1972). and references therein.

[15]In the $K\Lambda$ reactions, for example, for a reasonable choice of f_1, f_2, and $\Gamma_{\lambda\mu}$, the change in the cross section due to the vector part of K^* exchange is -3% for $\nu n \to \mu^- K^+\Lambda$, $+7\%$ for $\nu p \to \nu K^+\Lambda$, and $\sim +20\%$ for $\nu n \to \nu K^0\Lambda$.

[16]A. Pais and S. B. Treiman, in *Problems in Theoretical Physics*, Anniversary Volume Dedicated to N. Bogoliubov (Nauka, Moscow, 1969) p. 257; Phys. Rev. D 1, 907 (1970).

[17]This linear rise of $\sigma(E)$ did not appear in the calculation of charged-current weak pion production by Adler

for two reasons. First, in the spirit of resonance dominance, the integral over W was cut off slightly above the $\Delta(1232)$ resonance, at $W=1.39$ GeV, or 1.47 GeV. If this cutoff were removed then the Born contribution to the cross sections would rise linearly with E, for $E \gtrsim 2$ GeV. Second, even if the cutoff were removed, the Born contribution would still be largely hidden under the very large contribution of the $\Delta(1232)$ resonance.

[18]The Argonne flux-averaged cross sections are from P. Schreiner (private communication). These cross sections are calculated from the published event rate by taking into account the detection probability for $K^+\Lambda$ and $K^0\Lambda$ events, and the event rate-to-flux-averaged cross-section conversion factors for charged- and neutral-current reactions.

[19]M. Bég and A. Zee, Phys. Rev. Lett. 30, 697 (1973); Phys. Rev. D 8, 1460 (1973).

[20]J. D. Bjorken and C. H. Llewellyn Smith, Phys. Rev. D 7, 887 (1973).

[21]C. Brown *et al.*, Phys. Rev. Lett. 28, 1086 (1972); C. Bebek *et al.*, *ibid.* 32, 21 (1974), and references contained therein. The electroproduction data are for the reaction $e^-p \rightarrow e^-K^+\Lambda$. They include a W scan at $Q^2=0.29$ GeV, $0° < \theta < 15°$, $\langle\theta\rangle \simeq 6°$, and ϵ (the polarization parameter for the virtual photon) $\simeq 0.86$; the W scan extends from $W=1.85$ GeV to $W=2.6$ GeV. In addition, they include scans in Q^2 from $Q^2=0.2$ GeV^2 to $Q^2=1.2$ GeV^2 at $W=2.17$ GeV, $\langle\theta\rangle \simeq 6°$, and $\epsilon \simeq 0.84$, and from $Q^2=0.61$ GeV^2 to $Q^2=2.0$ GeV^2, at $W=2.66$ GeV, $\langle\theta\rangle \simeq 8°$ and $\epsilon \simeq 0.86$.

[22]The full dispersion-theoretic calculation of the differential cross section for pion electroproduction in the region of the $\Delta(1232)$ resonance by S. Adler (Ref. 2, above) exhibits a similar behavior as a function of Q^2; for $Q^2 > 0.6$ GeV^2 it yields values for the differential cross section which fall somewhat below the data.

[23]The absence of parity violation in this configuration was first shown by S. Adler, Phys. Rev. 135, B936 (1964).