Multiple Compton scattering in the forward direction*

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We prove that, to lowest order in α, the amplitude for the reaction $\gamma + e \rightarrow n\gamma + e$, where $n = 2,3,4,\ldots$, vanishes when all n final photons have momenta parallel to that of the incident photon. We generalize this result to scalar electrodynamics, to the scattering of massless pseudoscalar (scalar) mesons off massive fermions, and to the scattering of massless scalar mesons off massive scalar mesons.

I. INTRODUCTION

In this paper we consider the process

$$\gamma + e \rightarrow n\gamma + e \quad (n = 2, 3, 4, \ldots), \tag{1}$$

where γ is a photon and e an electron. We will refer to this process as n-Compton scattering (for $n = 1$ it is simply Compton scattering). We will specifically be interested in the case when the n final photons have momenta parallel to that of the incident photon and will refer to this as n-Compton forward. We work in the laboratory frame of reference where the initial electron is at rest and use the following notation[1]:

$k_1 = (\vec{k}_1, i\omega_1)$ incident photon 4-momentum;

$k_j = (\vec{k}_j, i\omega_j)$ $(j = 2, 3, \ldots, n+1)$, 4-momentum of final photons;

$p = (0, im)$ 4-momentum of target electron; the electron mass is m;

$p' = (\vec{p}\,', iE')$ 4-momentum of final electron.

By energy-momentum conservation it is easy to check that when the final n photons each have momenta parallel to the incident photon momentum (we will refer to this as "forward scattering" from hereon)

$$p = p', \tag{2}$$

and

$$k_1 = k_2 + k_3 + \cdots + k_{n+1}. \tag{3}$$

We will find it convenient to introduce

$$k_1' = -k_1$$
$$k_j' = k_j \quad (j = 2, 3, \cdots n+1), \tag{4}$$

since in terms of the k_i' the invariant amplitude for process (1) is symmetric under permutations of *all* $n+1$ *incident and final* photons. In terms of the primed variables, relation (3) becomes

$$\sum_{j=1}^{n+1} k_j' = 0 \tag{5}$$

In Fig. 1 we show the Feynman diagrams for double Compton scattering[2] to lowest order in α, i.e., including only the so-called "tree" diagrams.[3]

In Sec. II we will show that, in the tree approximation, the amplitude for reaction (1) vanishes in the forward direction, i.e., when the final photons have momenta parallel to that of the incident photon. When radiative corrections are included the theorem no longer seems to hold. In Sec. III we show that the vector nature of the photon and the fact that the electron has spin $\frac{1}{2}$ are not responsible for the vanishing of the amplitude in the forward direction. We do this by considering the scattering of massless pseudoscalar (or scalar) mesons off massive fermions and the scattering of massless mesons off massive mesons, and showing that, in the tree approximation, the re-

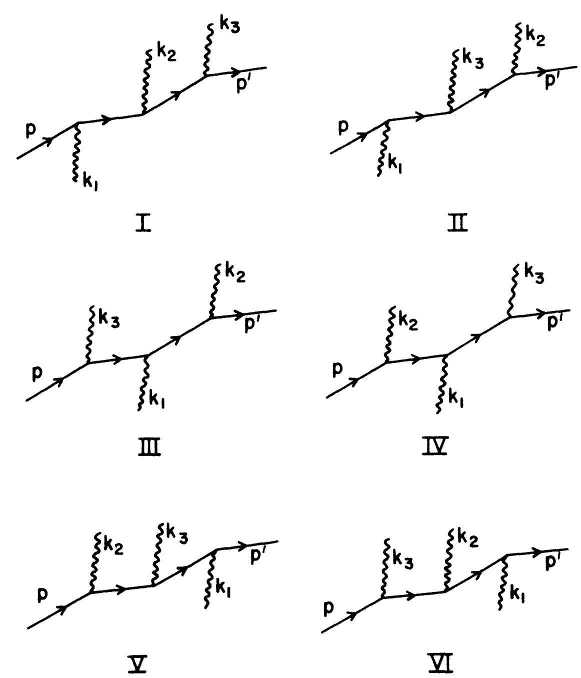

FIG. 1. Feynman diagrams for double Compton scattering to lowest order in α.

spective amplitudes vanish in the forward direction, i.e., when the final n massless mesons each have momenta parallel to that of the incident massless meson. In Sec. IV we generalize these results to scalar electrodynamics. All this seems to indicate that our theorem may be more general than was at first thought and is not limited to the scattering of photons off electrons. For all five models we considered we have found that, in the tree approximation, the amplitude for the process

$$A+B\to nA+B \quad (n=2,3,\ldots), \tag{6}$$

where A is a massless particle and B is a massive one, vanishes in the forward direction. As we shall see from the proofs of our theorem for the five models considered, the only important common features seem to be (i) the masslessness of the A particles implying $k_A{}^2=0$ for real A particles, and (ii) the restriction to tree diagrams which only involve real A particles. It is not possible to generalize the theorem to include radiative corrections since for *virtual* A particles $k_A{}^2\neq 0$. In fact, as we discuss later, the theorem *does not* hold when radiative corrections are included.

II. A SIMPLE THEOREM IN QUANTUM ELECTRODYNAMICS

We assume that the interaction responsible for reaction (1) has the usual minimal form

$$\mathcal{L}_\gamma = :\, ie\bar\psi(x)\gamma_\mu\psi(x)A_\mu(x):\quad (e<0), \tag{7}$$

where $\psi(x)$ and $A_\mu(x)$ are the fermion and electromagnetic fields, respectively, and γ_μ $(\mu=1,2,3,4)$ are the Dirac γ matrices. Consider first the case $n=2N+1$ where $N=1,2,\ldots$. It is quite straightforward to show that, in the tree approximation, "the invariant amplitude" for reaction (1) when the final $2N+1$ photons are produced in the forward direction is proportional to[4]

$$T^{(2N+1)}=\sum_P \frac{\bar S_{\vec p',\lambda'}\not\epsilon^{n_1}\not\epsilon^{n_2}\cdots\not\epsilon^{n_{2N+1}}\not\epsilon^{n_{2N+2}}\hat{\not k}S_{\vec p,\lambda}}{(\omega'_{n_1}+\omega'_{n_2})(\omega'_{n_1}+\omega'_{n_2}+\omega'_{n_3}+\omega'_{n_4})(\omega'_{n_1}+\cdots+\omega'_{n_6})\cdots(\omega'_{n_1}+\cdots+\omega'_{n_{2N}})}, \tag{8}$$

where ϵ^{n_m} is the linear polarization vector of the photon appearing at the m'th vertex of the Feynman diagram. From relation (4)

$$\omega'_1=-\omega_1,\qquad \omega'_m=\omega_m\quad (m=2,3,\ldots,2N+2), \tag{9}$$

and we have introduced

$$\hat k=\frac{k_1}{\omega_1}. \tag{10}$$

$S_{\vec p,\lambda}$, the electron spinor corresponding to momentum $\vec p$ and polarization λ, satisfies

$$(\not p-m)S_{\vec p,\lambda}=0, \tag{11}$$

and

$$\bar S_{\vec p,\lambda}(\not p-m)=0, \tag{12}$$

where

$$\bar S_{\vec p,\lambda}=S^\dagger_{\vec p,\lambda}\gamma_4.$$

In deriving relation (8) we made use of the following:

$$p\cdot\epsilon^m=0\quad (m=1,2,\ldots,n+1); \tag{13}$$

$$k_l\cdot\epsilon^m=0\quad (l,m=1,2,\ldots,n+1); \tag{14}$$

$$k_l\cdot k_m=0\quad (l,m=1,2,\ldots,n+1); \tag{15}$$

$$p'=p. \tag{16}$$

Relations (13)–(16) are readily derived by making use of the following facts:

(i) We work in the laboratory frame of reference in which the target electron is initially at rest;

(ii) The linear polarization vectors ϵ^m are real and perpendicular to $\vec k_m$;

(iii) all photon momenta are parallel;

(iv) energy-momentum is conserved.

Relations (13) and (14) imply that

$$\not p\not\epsilon^m+\not\epsilon^m\not p=0, \tag{17}$$

and

$$\not k_l\not\epsilon^m+\not\epsilon^m\not k_l=0. \tag{18}$$

In relation (8), $\sum_P$ indicates summation over *all* photon permutations—incident as well as final ones. It is very important to note that $\omega'_{n_{2N+1}}$ and $\omega'_{n_{2N+2}}$ do not appear in relation (8). We will now show that

$$T^{(2N+1)}=0. \tag{19}$$

Divide $n_1,n_2,\ldots,n_{2N+2}$ into $N+1$ pairs as follows: $(n_1,n_2),(n_3,n_4),\ldots,(n_{2N+1},n_{2N})$. Note that the denominator in relation (8) is symmetric under interchange of members within any pair. By summing first over permutations of terms *within* pairs and using

$$\not\epsilon^{n_i}\not\epsilon^{n_j}+\not\epsilon^{n_j}\not\epsilon^{n_i}=-2(\epsilon^{n_i}\cdot\epsilon^{n_j}), \tag{20}$$

we find that

$$T^{(2N+1)}=(-2)^{N+1}\sum_{P'}(\hat\epsilon^{n_1}\cdot\hat\epsilon^{n_2})(\hat\epsilon^{n_3}\cdot\hat\epsilon^{n_4})\cdots(\hat\epsilon^{n_{2N+1}}\cdot\hat\epsilon^{n_{2N+2}})$$
$$\times\frac{\bar S_{\vec p',\lambda'}\hat{k}S_{\vec p,\lambda}}{(\omega'_{n_1}+\omega'_{n_2})(\omega'_{n_1}+\omega'_{n_2}+\omega'_{n_3}+\omega'_{n_4})\ldots(\omega'_{n_1}+\omega'_{n_2}+\cdots+\omega'_{n_{2N}})},$$

where $\sum_{P'}$ denotes summation over all possible photon pairings (n_1,n_2), $(n_3,n_4),\ldots,(n_{2N+1},n_{2N+2})$ excluding permutations within any given pair. It is easy to see that

$$T^{(2N+1)}=(-2)^{N+1}\sum_{P''}(\hat\epsilon^{n_1}\cdot\hat\epsilon^{n_2})(\hat\epsilon^{n_3}\cdot\hat\epsilon^{n_4})\cdots(\hat\epsilon^{n_{2N+1}}\cdot\hat\epsilon^{n_{2N+2}})$$
$$\times\left[\frac{1}{(\omega'_{n_1}+\omega'_{n_2})(\omega'_{n_1}+\omega'_{n_2}+\omega'_{n_3}+\omega'_{n_4})\cdots(\omega'_{n_1}+\omega'_{n_2}+\cdots+\omega'_{n_{2N}})}\right.$$
$$\left.+\frac{1}{(\omega'_{n_3}+\omega'_{n_4})(\omega'_{n_1}+\omega'_{n_2}+\omega'_{n_3}+\omega'_{n_4})\cdots(\omega'_{n_1}+\omega'_{n_2}+\cdots+\omega'_{n_{2N}})}\right]\bar S_{\vec p',\lambda'}\hat{k}S_{\vec p,\lambda}$$
$$=(-2)^{N+1}\sum_{P''}(\hat\epsilon^{n_1}\cdot\hat\epsilon^{n_2})(\hat\epsilon^{n_3}\cdot\hat\epsilon^{n_4})\cdots(\hat\epsilon^{n_{2N+1}}\cdot\hat\epsilon^{n_{2N+2}})$$
$$\times\frac{\bar S_{\vec p',\lambda'}\hat{k}S_{\vec p,\lambda}}{(\omega'_{n_1}+\omega'_{n_2})(\omega'_{n_3}+\omega'_{n_4})(\omega'_{n_1}+\omega'_{n_2}+\cdots+\omega'_{n_6})\cdots(\omega'_{n_1}+\omega'_{n_2}+\cdots+\omega'_{n_{2N}})}$$

where $\sum_{P''}$ denotes summation over all possible photon pairings, excluding those that just correspond to the permutations of the pairs (n_1,n_2) and (n_3,n_4) and also excluding permutations within pairs. Continuing this process it is easy to see that $T^{(2N+1)}$ can be reduced to

$$T^{(2N+1)}=(-2)^{N+1}\sum_{\pi}(\hat\epsilon^{n_1}\cdot\hat\epsilon^{n_2})(\hat\epsilon^{n_3}\cdot\hat\epsilon^{n_4})\cdots(\hat\epsilon^{n_{2N+1}}\cdot\hat\epsilon^{n_{2N+2}})\frac{\bar S_{\vec p',\lambda'}\hat{k}S_{\vec p,\lambda}}{(\omega'_{n_1}+\omega'_{n_2})(\omega'_{n_3}+\omega'_{n_4})\cdots(\omega'_{n_{2N-1}}+\omega'_{n_{2N}})},$$

where by $\sum_\pi$ we mean summation over all possible pairings excluding those that just correspond to permutations of the pairs (n_1,n_2), $(n_3,n_4),\ldots,(n_{2N-1},n_{2N})$ and also excluding permutations within pairs. Finally, using the notation $\sum_{\pi'}$ to indicate summation over all photon pairings excluding those that simply correspond to permutations of the photon pairs (n_1,n_2), $(n_3,n_4),\ldots,(n_{2N+1},n_{2N+2})$ and also excluding interchange of members within a given pair, we can write

$$T^{(2N+1)}=(-2)^{N+1}\sum_{\pi'}(\hat\epsilon^{n_1}\cdot\hat\epsilon^{n_2})(\hat\epsilon^{n_3}\cdot\hat\epsilon^{n_4})\cdots(\hat\epsilon^{n_{2N+1}}\cdot\hat\epsilon^{n_{2N+2}})\frac{\bar S_{\vec p',\lambda'}\hat{k}S_{\vec p,\lambda}\sum_{j=1}^{2N+2}\omega'_{n_j}}{(\omega'_{n_1}+\omega'_{n_2})(\omega'_{n_3}+\omega'_{n_4})\cdots(\omega'_{n_{2N+1}}+\omega'_{n_{2N+2}})}$$
$$=0,$$

since by energy-momentum conservation [see relation (5)]

$$\sum_{j=1}^{2N+2}\omega'_{n_j}=0. \tag{21}$$

The above proof applied to the case $n=2N+1$. For $n=2N$ the proof that the amplitude for forward scattering vanishes is actually much simpler since *each* of the terms in the amplitude corresponding to the Feynman diagram of Fig. 2 *separately* vanishes. This follows directly by using relations (11), (12), (15), (17), and (18).

As pointed out in Sec. I, when the invariant amplitude is expressed in terms of the k'_j [see relation (4)] it remains unchanged under permutations of all $n+1$ incident and final photons. In this respect, there is little difference between an incoming and an outgoing photon. It is therefore easy to see that the amplitude for the reaction

$$l\gamma+e\to n\gamma+e\quad(l+n=3,4,5,\ldots), \tag{22}$$

also vanishes when all $(l+n)$ photons have parallel

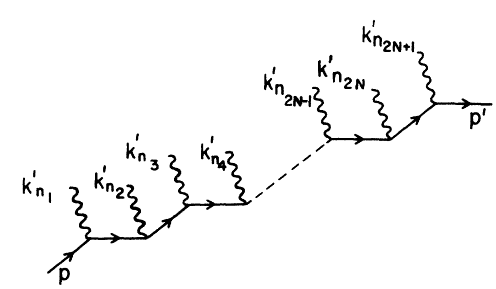

FIG. 2. Feynman diagram for n-Compton scattering when $n=2N$. Note that one of the photons in the diagram should be the incoming one.

momenta. Except for minor changes in notation, the proof of this result goes through unchanged as for the case of reaction (1). An interesting special case of reaction (2) is

$$n\gamma + e \to n\gamma + e \quad (n = 2, 3, 4, \ldots), \tag{23}$$

corresponding to $l = n$. The fact that the amplitude for reaction (23) also vanishes in the tree approximation when all $2n$ photon momenta are parallel is a clear indication that a symmetry principle is not involved in our theorem. Furthermore, using the optical theorem, one can readily prove that when radiative corrections are included the amplitude for reaction (23) no longer vanishes.[5]

III. GENERALIZATION OF THEOREM TO SCATTERING OF MASSLESS PSEUDOSCALAR (SCALAR) MESONS OFF MASSIVE FERMIONS AND TO SCATTERING OF MASSLESS SCALAR MESONS OFF MASSIVE MESONS.

In order to study whether or not the theorem we proved in Sec. II depends on the vector nature of the photon we consider here the reaction

$$\phi + e \to N\phi + e \quad (N = 2, 3, 4, \ldots), \tag{24}$$

where ϕ is a massless pseudoscalar meson and e a massive spin-$\frac{1}{2}$ particle. We assume that the interaction responsible for reaction (24) is given by

$$\mathcal{L}_{PS} = :g\bar{\psi}(x)\gamma_5\psi(x)\phi(x): . \tag{25}$$

$\phi(x)$ is the massless pseudoscalar field and $\psi(x)$ is the massive spin-$\frac{1}{2}$ field. We use the same notation as in Secs. I and II with k_j $(j = 1, 2, \ldots, N+1)$ referring to the massless ϕ-meson 4-momenta. In place of photon polarization vectors we now have a γ_5 matrix occurring at every vertex. Since however,

$$\gamma_5 \not{p} + \not{p}\gamma_5 = 0, \tag{26}$$

and

$$\gamma_5 \not{k}_j + \not{k}_j \gamma_5 = 0 \quad (j = 1, 2, \ldots, N+1), \tag{27}$$

in analogy to relations (17) and (18), the proof that, in the tree approximation, the amplitude for reaction (24) vanishes in the forward direction carries through just as in the case of quantum electrodynamics. The denominators occurring in the invariant amplitude are identical to those occuring in relation (8). The numerator is simpler, however, since $\gamma_5^2 = 1$.

We have also considered the case when the ϕ particles are massless scalar mesons and the interaction is given by

$$\mathcal{L}_S = :g\bar{\psi}(x)\psi(x)\phi(x): . \tag{28}$$

In this case we have shown that the forward amplitude vanishes in the tree approximation for $N = 2, 3, 4$. The generalization to larger N is somewhat more complicated here, but we have no doubt that it is quite feasible.

To see that our result is independent of the spin-$\frac{1}{2}$ nature of the target particle we consider the reaction

$$\phi + \varphi \to N\phi + \varphi \quad (N = 1, 2, 3, \ldots), \tag{29}$$

where ϕ now represents a massless scalar particle and φ a massive one. We assume that the interaction responsible for this process is given by

$$\mathcal{L}_{SS} = :g\varphi^\dagger(x)\varphi(x)\phi(x): . \tag{30}$$

The notation is self explanatory. In the tree approximation, the invariant amplitude for reaction (29) in the forward direction, i.e., when the final ϕ's have momenta parallel to the incident one, is proportional to

$$A^{(N)} = \sum_P \frac{1}{\omega'_{n_1}(\omega'_{n_1} + \omega'_{n_2}) \cdots (\omega'_{n_1} + \omega'_{n_2} + \cdots + \omega'_{n_N})}, \tag{31}$$

where $\sum_P$ indicates summation over all possible permutations of the $N+1$ ϕ's (incident and final ones). Noting that $\omega'_{n_{N+1}}$ is missing from the above expression and that

$$\sum_{j=1}^{N+1} \omega'_j = 0, \tag{32}$$

it is easy to show that

$$A^{(N)} = 0. \tag{33}$$

We therefore see that, in the tree approximation, the amplitude for reaction (29) vanishes when the final ϕ's are produced with momenta parallel to that of the incident one. It is interesting to note that, while the forward amplitude for reaction (1) with $n = 1$ did not vanish, the forward amplitude for reaction (29) also vanishes when $N = 1$.

Since the invariant amplitude remains unchanged under permutations of all incident and final ϕ's when it is expressed in terms of k'_j, it is easy to see that the amplitude for the reaction

$$l\phi + \varphi \to n\phi + \varphi \quad (l + n = 2, 3, 4, \ldots), \tag{34}$$

also vanishes in the tree approximation when all $(l+n)$ ϕ's have parallel momenta. We have explicitly verified, however, that when radiative corrections are included the amplitude for reaction (34) with $l = n = 1$ does not vanish even though it vanishes in the tree approximation. In fact, we can use the optical theorem again[5] to show that the radiative corrections to (34) cannot vanish when $l = n$.

IV. GENERALIZATION OF THEOREM TO SCALAR ELECTRODYNAMICS

We now show that our theorem holds also in scalar electrodynamics. This is a case of particular interest since, unkike the previous models discussed, it involves an interaction with derivative coupling.

We consider the process

$$\gamma + \pi \to N\gamma + \pi \quad (N = 2, 3, 4, \ldots), \tag{35}$$

where π is a massive charged scalar particle. We shall also here demonstrate that, in the tree approximation, the amplitude for reaction (35) vanishes in the forward direction.

As usual, we assume that the interaction between photons and charged pions is given by the minimal electromagnetic interaction[6]

$$\mathcal{L}_\gamma = : -ie\{\varphi^\dagger(x)\partial_\mu\varphi(x) - [\partial_\mu\varphi^\dagger(x)]\,\varphi(x)\}A_\mu(x) \\ -e^2\varphi^\dagger(x)\varphi(x)A_\mu(x)A_\mu(x): \quad (e<0). \tag{36}$$

$\varphi(x)$ is a complex scalar field whose quanta are associated with the charged mesons. Our notation remains essentially the same as in previous sections with p and p' referring to the massive charged meson's initial and final 4-momentum, respectively. We work in the target-meson rest system and consider only tree diagrams so that relations (2), (3), (13), (14), (15), and (16) still hold.

In the forward direction, diagrams with *any* number of *single-photon vertices* corresponding to the derivative coupling term in the interaction (36) give no contribution to the amplitude. To see this we note that the factor corresponding to a single-photon vertex associated with polarization vector ϵ^{n_i} and incoming and outgoing meson momenta p_{n_i} and $p_{n_{i+1}}$ (see Fig. 3) is proportional to

$$\epsilon^{n_i}\cdot(p_{n_i}+p_{n_{i+1}}) = \epsilon^{n_i}\cdot\Big(2p - 2\sum_{j=1}^{i-1} k'_{n_j} - k'_{n_i}\Big) = 0,$$

where we have used

$$p_{n_i} = p - \sum_{j=1}^{i-1} k'_{n_j},$$

and

$$p_{n_{i+1}} = p_{n_i} - k'_{n_i},$$

as well as relations (13) and (14).

We now finally show that, in the tree approximation, the term quadratic in $A_\mu(x)$ gives no contribution to the amplitude for reaction (35) in the forward direction. We can restrict ourselves to odd N since for even N we must also have at least one single-photon vertex. Odd N means that we have to consider diagrams with $\frac{1}{2}(N+1)$ two-photon vertices. The proof that, in the tree approximation, the forward amplitude corresponding to such diagrams vanishes is essentially identical to the one we produced when we considered reaction (29). The only minor difference is that now we have to deal with pairs of photons at each vertex.

V. CONCLUSIONS

For all five models considered we have essentially shown that, in the tree approximation, the amplitude for the process

$$lA + B \to nA + B \quad (l+n = 3, 4, 5, \ldots), \tag{37}$$

where A is a massless particle and B is a massive one, vanishes in the forward direction, i.e., when all initial and final A particles have parallel momenta. Since this is also true when $l = n = 2, 3, \ldots,$ it is clear that a selection rule resulting from some symmetry principle *cannot* be involved. The fundamental reason for the vanishing of the amplitude in the forward direction still remains a mystery. The only essential ingredients common to all models discussed were (i) the masslessness of the A particle, and (ii) our restriction to tree diagrams so that all A particle 4-momenta satisfied $k_A^2 = 0$.

The proof of the theorem is quite elaborate and subtle and it does not seem that the result could simply be guessed by observation. Note, for example, that the theorem does not hold for ordinary Compton scattering corresponding to $n = 1$ in relation (1). The theorem provides an important general statement concerning the amplitude for process (37) when the momentum transfer to the B particle is zero.

In applying the theorem to quantum electrodynamics the following points should be noted:

(1) The theorem applies to free electrons. If the incident photon energy is large compared to electron binding energies (the usual Compton scattering regime) it is reasonable to ignore binding and the electrons can essentially be treated as free.

(2) The theorem does not state that the process

$$\gamma + e \to n\gamma + e \quad (n = 2, 3, 4, \ldots), \tag{1}$$

is absolutely forbidden for zero momentum transfer to the electron. It just states that the amplitude

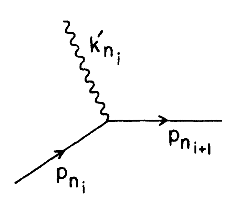

FIG. 3. Vertex for single-photon–charged-meson interaction.

vanishes to lowest order in α, the fine structure constant. The process can and will occur, but only through higher-order (radiative corrections) effects. *The theorem therefore implies that at zero momentum transfer the cross section for reaction (1) is smaller than what one would normally expect by a factor of* $\alpha^2 \sim 10^{-4}$.

We have also shown that when radiative corrections are included the amplitude for reaction (37) no longer vanishes in the forward direction.

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[1]We use a Minkowski metric in which the space part of our 4-vectors is real and the fourth component pure imaginary. We have adopted natural units $\hbar = c = 1$.

[2]L. M. Brown and R. P. Feynman, Phys. Rev. 85, 231 (1952); F. Mandl and T. H. R. Skyrme, Proc. R. Soc. London 215, 497 (1952); Michael Ram and P. Y. Wang, Phys. Rev. Lett. 26, 476 (1971); 26, 1210 (1971).

[3]"Tree" diagrams are the subset of those Feynman diagrams that do not include any radiative corrections.

[4]If q is a 4-vector, then $\not{q} = -i\gamma_u q_u$, where summation over repeated indices is implied.

[5]I am grateful to Professor T. T. Wu for pointing this out to me. The argument goes as follows: From the optical theorem

$$\mathrm{Im}\, T(n\gamma + e \to n\gamma + e) = c\sigma_t\,(n\gamma + e \to e + \text{anything}),$$

where $T(n\gamma + e \to n\gamma + e)$ is the forward amplitude for reaction (23) and c is a proportionality factor. $\sigma_t\,(n\gamma + e \to e + \text{anything})$ is the total cross section for $n\gamma + e \to e + \text{anything}$. The lowest-order *nonvanishing* contribution to $\sigma_t\,(n\gamma + e \to e + \text{anything})$ is proportional to α^{n+1}, so that the lowest *nonvanishing* contribution to $\mathrm{Im}\, T(n\gamma + e \to n\gamma + e)$ is also proportional to α^{n+1}.

[6]P. T. Matthews, Phys. Rev. 76, 684 (1949); 80, 292 (1950); F. Rohrlich, *ibid.*, 80, 666 (1950); A. Salam, *ibid.* 86, 731 (1952).