PHYSICAL REVIEW D VOLUME 12, NUMBER 7 1 OCTOBER 1975

Inelastic eikonal phenomenology in a stationary-phase approximation*

Hector Moreno

Departamento de Física, Centro de Investigacion y de Estudios Avanzados del Instituto Politecnico Nacional, Apartado Postal 14-740, Mexico 14, D. F.

H. M. Fried†

Physics Department, Brown University, Providence, Rhode Island 02912

(Received 7 May 1975)

A stationary-phase approximation to a functional-integral representation of model elastic and inelastic pion amplitudes is shown to yield a phenomenology corresponding to the simplest possible unitary model. Possible relevance at CERN ISR energies may be easily tested.

I. INTRODUCTION

One of the outstanding theoretical problems of high-energy eikonal physics is the incorporation of some measure of t-channel unitarity. An interesting attempt in this direction is the recent paper of Botke *et al.*,[1] wherein a version of elastic t-channel unitarity is used as an input in writing an equation for the functional Fourier transform whose integral produces the elastic scattering amplitude. Another and partially related problem is the choice of specific model, any one of which is guaranteed to satisfy s-channel unitarity at asymptotic energies.[2] The present remarks were motivated by those of Refs. 1 and 2, in particular by the desire to find an alternate point of departure for the insertion of t-channel unitarity, including sums over all t-channel thresholds. The present, heuristic development leads quite naturally to a choice between versions of eikonal phenomenology, with that model containing the simplest formulation of s-channel unitarity appearing as the preferred one, in the limit of asymptotic energies. Such arguments immediately provide a phenomenology for inelastic reactions, given in terms of an experimentally determined elastic eikonal function, and predictions can be written for inelastic experiments yet to be performed at CERN ISR energies. It is also interesting to observe that the forms obtained have a generic resemblance to those found in recent studies of phase transitions in quantum field theories,[3] and in older calculations using a mean-field approximation in statistical problems.[4]

The starting point of these remarks is the observation that there already exists, in any nontrivial field theory, a concise functional statement of the corresponding eikonal function. For definiteness, this quantity will be illustrated by the eikonal of that theory previously derived,[5] approximated,[6] and employed[7] elsewhere. For simplicity, no attempt will be made to incorporate diffractive effects, although that particular generalization could be estimated by allowing neutral-vector meson production in the model. The functional statement of this eikonal, which includes all tower graphs and their higher t-channel threshold (checkerboard) generalizations may be put in the form of an equivalent functional integral. At asymptotic energies it is then argued that, in effect, all inelastic pion emissions from a given multiperipheral chain are replaced by a single pion satisfying a classical-like equation of motion, in which rapidity and impact-parameter variables enter in an essential way. The elastic eikonal is treated similarly, as the shadow of such effective single-pole-per-chain emission. At least a rudimentary form of elastic t-channel unitarity is maintained, while s-channel unitarity is guaranteed by the eikonal formalism. We briefly discuss, in Sec. III, a phenomenology for these heuristic forms, and present a brief summary in the final section.

II. FORMALISM

We begin with the briefest possible review of the expressions describing elastic and inelastic reactions in that eikonal model where a pair of fast nucleons ψ exchange massive, neutral vector mesons (NVM) W_μ, which in turn exchange and emit arbitrary numbers of (scalar) pions π. The coupling of ψ to W_μ is represented by the constant g, with the coupling of W_μ to π by the constant λ, according to the interaction Lagrangian $\mathcal{L}' = ig\bar{\psi}\gamma\cdot W\psi - (\lambda/2)\pi W^2$. Derivations have been presented elsewhere,[5-7] in detail, and it is therefore only necessary to explain each of the quantities entering into the expressions for eikonal versions of relevant quantities. At large c.m. energies, $s = -(p_1+p_2)^2$, and relatively small momentum transfers, $t = -(p_1 - p_1')^2$, the elastic, nonspinflip scattering amplitude is represented by

$$T(s,t)=\frac{is}{2}\int d^2b\, e^{i\vec{q}\cdot\vec{b}}[1-e^{i\chi(b,s)}], \tag{1}$$

where $\vec{q}^2=-t$, $-t/s \ll 1$, and normalization has been chosen such that $d\sigma_{EL}/dt=(1/\pi s^2)|T|^2$. One also has

$$\sigma_{tot}=\frac{4}{s}\,\mathrm{Im}\,T(s,0)$$
$$=2\,\mathrm{Re}\int d^2b[1-e^{i\chi}], \tag{2}$$

$$\sigma_{el}=\int d^2b|1-e^{i\chi}|^2. \tag{3}$$

and

$$\sigma_{in}=\int d^2b[1-e^{-2\rho}], \tag{4}$$

with $\rho=\mathrm{Im}\chi$. The statement of unitarity, in this eikonal context, is given by the relation $\sigma_{tot}=\sigma_{el}+\sigma_{in}$.

The functional expression for the eikonal may be stated in terms of the quantity

$$e^{F[\pi]}=\exp\left(-\frac{i}{2}\int\frac{\delta}{\delta\pi}D_c\frac{\delta}{\delta\pi}\right)\exp\left(ig^2\int\mathfrak{F}_{\rm I}\overline{\Delta}_c[\pi]\mathfrak{F}_{\rm II}\right), \tag{5}$$

where $i\chi=F[\pi]_{\pi\to 0}$. The Feynman graphs contained in (5) correspond to the virtual exchange of arbitrary numbers of pions between all possible virtual NVM's, including self-effects given by virtual pions emitted by each NVM. In (5), $D_c(x-y)$ denotes the pion propagator, and $\pi(z)$ represents a fictitious, c-number, pion source whose only function is to provide the appropriate pion linkages between the NVM propagators, $\overline{\Delta}_c(x,y|\pi)$; the latter are to represent relativistic (Feymann gauge: $\delta_{\mu\nu}\Delta_c$) NVM propagation in the presence of the external field $\pi(z)$. The quantities

$$\mathfrak{F}^{\mu}_{\rm I,II}(W)=p^{\mu}_{1,2}\int_{-\infty}^{+\infty}d\xi\,\delta^{(4)}(W-z_{1,2}+\xi p_{1,2})$$

denote classical currents of the fast nucleons, of position $z_{1,2}$ and momenta $p_{1,2}$, elastically scattering according as $p_1+p_2\to p_1'+p_2'$, and with transverse impact parameter $\vec{b}=(\vec{z}_1-\vec{z}_2)_T$.

It may be noted that (5) is that particular functional representation which makes no reference to the concept of functional integration. It stands as an expression in its own right, and has spawned a sequence of applications defined in terms of particular approximations.[6] One easily sees that all the varied pion linkages are achieved by expansion of the D_c exponential, while expansion of the $\overline{\Delta}_c$ exponential corresponds to the inclusion of higher t-channel thresholds, in the sense of connected contributions to the eikonal. At the cost of having specified the interactions of a particular theory, (5) provides a representation for the eikonal which, in principle, contains full s- and t-channel unitarity.

Inelastic pion production within the eikonal context—every pion so produced takes off a very small fraction of the available 4-momentum—may also be specified in terms of the $F[\pi]$ of (5). For example, it is not difficult to obtain an expression[7] for the total cross section involved in the production of n pions,

$$\sigma_n(s)=\frac{1}{n!}\int d^2b\left(i\int\frac{\delta}{\delta\pi}D_{(+)}\frac{\delta}{\delta\pi'}\right)^n e^{F[\pi]}e^{F^*[\pi']}\Big|_{\pi'=\pi=0}, \tag{6}$$

where $D_{(+)}$ denotes the positive frequency, mass-shell, pion function, $\tilde{D}_{(+)}(k)=-i(2\pi)^{-3}\theta(k_0)\delta(k^2+\mu_\pi^2)$.

With (6), one may conveniently introduce the particle partition function (ppf)

$$\sigma_{in}(z)=\sum_{n=1}^{\infty}z^n\sigma_n, \tag{7}$$

so that

$$\sigma_{in}(z)=\int d^2b\left[\left(\exp\, iz\int\frac{\delta}{\delta\pi}D_{(+)}\frac{\delta}{\delta\pi'}\right)-1\right]e^{F[\pi]}e^{F^*[\pi']}\Big|_0$$
$$=\int d^2b\left[\exp\left(iz\int\frac{\delta}{\delta\pi}D_{(+)}\frac{\delta}{\delta\pi'}\right)e^{F[\pi]}e^{F^*[\pi']}\Big|_0 - e^{-2\rho(b,s)}\right]. \tag{8}$$

Other quantities, such as inclusive cross sections, may be defined in terms of functional derivatives with respect to $\tilde{D}_{(+)}(k)$, but we shall not consider these here. From (5), (6), and (8) it is clear that knowledge of the particle properties in the high-energy reactions of this theory depends upon the evaluation, or estimation, of $F[\pi]$, and subsequently of the inelastic quantity

$$\exp\left(iz\int\frac{\delta}{\delta\pi}D_{(+)}\frac{\delta}{\delta\pi'}\right)e^{F[\pi]}e^{F^*[\pi']}\Big|_0. \tag{9}$$

We now attempt to evaluate (9) and (5) by the replacement of functional differential operations by those of functional integration. This is sensible only if one has a prescription for the evaluation of these functional integrals; and since the most complicated form of integration known is Gaussian integration, the method is basically restricted to an approximation such as stationary phase. It then remains to argue that this mathematical restriction has a reasonable physical interpretation, one that may be expected to be valid in the very large energy limit.

A convenient way of introducing the appropriate functional integral is via the relation

$$\int d[\varphi]\exp\left(i\int j\varphi \mp \frac{i}{2}\int \varphi B\varphi\right) = C_{(\pm)}\exp\left(-\frac{1}{2}\mathrm{Tr}\ \ln B\right)\exp\left(\pm\frac{i}{2}\int jB^{-1}j\right), \quad (10)$$

where unpleasant questions concerning measure are lumped into the constants $C_{(+)}=C^*_{(-)}$, which automatically cancel in the final result. In this way, (5) may be rewritten as

$$e^{F[\pi]} = C^{-1}_{(+)}\exp(\tfrac{1}{2}\mathrm{Tr}\ \ln K)\int d[\varphi]\exp\left(-\frac{i}{2}\int\varphi K\varphi\right) \times\exp\left(\int\varphi\frac{\delta}{\delta\pi}\right)e^{\mathfrak{F}[\pi]}, \quad (11)$$

where $\mathfrak{F}[\pi]=ig^2\int\mathfrak{F}_{\mathrm{I}}\overline{\Delta}_c[\pi]\mathfrak{F}_{\mathrm{II}}$, and $K=(\mu_\pi{}^2-\partial^2)=D_c^{-1}$. Performing the indicated displacement, and then a redefinition of variable, this becomes

$$e^{F[\pi]} = C^{-1}_{(+)}\exp(\tfrac{1}{2}\mathrm{Tr}\ \ln K) \times\int d[\varphi]\exp\left[-\frac{i}{2}\int(\varphi-\pi)K(\varphi-\pi)\right]e^{\mathfrak{F}[\varphi]},$$

or

$$C^{-1}_{(+)}C^{-1}_{(-)}\int d[\varphi]\int d[\alpha]\exp\left[i\int(\varphi-\pi)\alpha\right] \times\exp\left(\frac{i}{2}\int\alpha D_c\alpha\right)e^{\mathfrak{F}[\varphi]}, \quad (12)$$

where (10) has been used a second time, in order to acheive the simplest functional dependence upon the source π. An identical representation exists for

$$e^{F^*[\pi']} = C^{-1}_{(+)}C^{-1}_{(-)}\int d[\theta]\int d[\beta]\exp\left[-i\int(\theta-\pi')\beta\right] \times\exp\left(-\frac{i}{2}\int\beta D_c^*\beta\right)e^{\mathfrak{F}^*[\theta]}, \quad (13)$$

and both (12) and (13) may be substituted into (9) to obtain the representation of that quantity,

$$|C_{(+)}|^{-2}\exp[-\tfrac{1}{2}\mathrm{Tr}\ \ln D_c-\tfrac{1}{2}\mathrm{Tr}\ \ln D_c^*-\tfrac{1}{2}\mathrm{Tr}\ \ln(1+z^2KD_{(+)}KD_{(+)})] \times\int d[\varphi]\int d[\theta]\exp\left\{\mathfrak{F}[\varphi]+\mathfrak{F}^*[\theta]-\frac{i}{2}\int\varphi K[1+z^2D_{(+)}KD_{(+)}K]^{-1}\varphi\right\}. \times\exp\left\{\frac{i}{2}\int\theta K[1+z^2D_{(+)}KD_{(+)}K]^{-1}\theta+iz\int\varphi K[1+z^2D_{(+)}KD_{(+)}K]^{-1}D_{(+)}K\theta\right\}, \quad (14)$$

again employing (10) to perform the α,β functional integrals. Because $D_{(+)}$ is on the mass shell, (14) simplifies to

$$|C_{(+)}|^{-2}\exp(-\tfrac{1}{2}\mathrm{Tr}\ln|D_c|^2)\int d[\varphi]\int d[\theta]\exp\{\mathfrak{F}[\varphi]+\mathfrak{F}^*[\varphi]\}\exp\left(-\frac{i}{2}\int\theta K\theta+\frac{i}{2}\int\varphi K\varphi+iz\int\varphi KD_{(+)}K\theta\right), \quad (15)$$

a form which, in effect, defines the action of the products $KD_{(+)}$ upon $K\varphi$ and $K\theta$ as zero; that is, only fields which possess mass-shell singularities no worse than that of a single pole are included in the functional integrations. Just this restriction is obtained in the stationary-phase approximation to follow. There, at high energies, one may argue that many virtual pions should be present; e.g., in a simple multiperipheral model, each D_c and $D_{(+)}$ in (5) and (9) contributes to the eikonal at moderate impact parameters a factor of ln$s$. Hence the physical situation is akin to that of a strong-field approximation, with very many quanta the necessary prerequisite to classical-like states (of fixed phase). We therefore assume that the fields φ and θ should not vary appreciably from their appropriate semiclassical values, φ_0 and θ_0. The latter are each expected to be specified by a second-order differential equation, so that φ_0 and θ_0 each contain but a single mass-shell pole.[8] The particular form of classical differential equation will be given in terms of a self-consistent, mean-field approximation, where both φ and θ are specified by average φ_0 and θ_0 values of each field that can be involved in all possible pion linkages.

Consider first the φ integration, and define

$$\mathcal{G}[\varphi]=iz\int\theta K\cdot D_{(+)}\cdot K\varphi-\frac{i}{2}\int\varphi K\varphi+\mathfrak{F}[\varphi], \quad (16)$$

which is then to be approximated in the form

$$\mathcal{G}[\varphi]\simeq\mathcal{G}[\varphi_0]+\int(\varphi-\varphi_0)\frac{\delta\mathcal{G}}{\delta\varphi_0} +\tfrac{1}{2}\int(\varphi-\varphi_0)\frac{\delta^2\mathcal{G}}{\delta\varphi_0\delta\varphi_0}(\varphi-\varphi_0). \quad (17)$$

The stationary-phase condition for the φ integral of (15) then becomes

$$\frac{\delta \mathcal{G}}{\delta \varphi_0} = 0 = iz \int \theta K D_{(+)} K - iK\varphi_0 + \frac{\delta \mathcal{F}}{\delta \varphi}\bigg|_{\varphi=\varphi_0}. \tag{18}$$

If we require[8] that φ_0 and, subsequently, θ_0 are to satisfy a classical field equation such that each average field contains nothing worse than a single mass-shell pole, (18) reduces to

$$K_x \varphi_0(x) = -i \frac{\delta \mathcal{F}}{\delta \varphi(x)}\bigg|_{\varphi=\varphi_0}. \tag{19}$$

Equation (19) defines a pion field influenced by all the pions propagating into and/or from the same NVM line, and in this sense the approximation is self-consistent. The confluence of all such NVM propagators then is used to represent the effects of all t-channel thresholds. Equation (19) is a highly nonlinear relation, which one may attempt to realize in terms of a specific s- and b-dependent model for $\overline{\Delta}_c[\varphi]$; here, however, only the phenomenological consequences resulting from this interpretation and physical approximation will be explored.

One now shifts to the variable $\varphi' = \varphi - \varphi_0$, and again employs (10) to perform the φ' functional integral, yielding for (15)

$$C_{(-)}^{-1} \exp\left\{-\tfrac{1}{2}\mathrm{Tr}\ \ln D_c^* - \tfrac{1}{2}\mathrm{Tr}\ \ln\left[1 + iD_c\left(\frac{\delta^2 \mathcal{F}}{\delta\varphi_0 \delta\varphi_0}\right)\right]\right\}$$
$$\times \int d[\theta] \exp\left\{\mathcal{G}[\varphi_0] + \frac{i}{2}\int \theta K \theta + \mathcal{F}^*[\theta]\right\}. \tag{20}$$

Performing the same steps for the θ functional integral, one learns that $\theta_0 = \varphi_0^*$; and (20) then becomes

$$\exp\left(-2\rho + iz \int \varphi_0^* K D_{(+)} K \varphi_0\right), \tag{21}$$

where, in the same stationary-phase approximation, but applied to the zero-pion-source expression of (12), one obtains for the eikonal

$$\chi = -i\mathcal{F}[\varphi_0] - \tfrac{1}{2}\int \varphi_0 K \varphi_0$$
$$+ \frac{i}{2}\mathrm{Tr}\ \ln\left(1 + iD_c \frac{\delta^2 \mathcal{F}}{\delta\varphi_0 \delta\varphi_0}\right). \tag{22}$$

The ρ of (21) specifically denotes the imaginary part of (22).

s-channel unitarity, which demands the form (4) of the impact-parameter integral of (21), when used in conjunction with (8) for $z=1$, will be satisfied by the equality of ρ and $(i/2)\int \varphi_0^* K D_{(+)} K \varphi_0$. This latter condition acts as a unitarity restriction on the possible solutions of the model field Eq. (19), in much the same way as conventional unitarity delimits the possible solutions to any complete field theory. In principle, both relations should be compatible (although this has never been demonstrated exactly except for free fields); in practice one uses either unitarity plus assumed analyticity, or a succession of approximations to field equations, or some mixture of both, in order to build explicit if approximate solutions. The task is long and arduous.

Fortunately, in the present model context there is a fairly simple set of phenomenological tests which one may first apply in order to determine whether such detailed calculations of the model b, s, g, λ dependence—for this interaction Lagrangian or for any other, using the same methods—are worth the effort. Such tests follow from the relation

$$\sigma_{\mathrm{in}}(z) = \int d^2b\, e^{-2\rho(b,s)}[e^{2z\rho(b,s)} - 1], \tag{23}$$

itself generated by the above model quantities and restrictions. In the next section we discuss predictions that follow from (23), and examine the s dependence in a simple eikonal model with ρ chosen to reproduce elastic scattering data. If our semiclassical method is at all reasonable, its output must resemble appropriate measured quantities; and if any such agreement does appear to hold, experimentally, it will then be worthwhile to undertake a detailed study of the model's equations, and determine their $b, s, \ldots$ predictions. Hence, the remainder of our discussion is phenomenological, based upon the mean mulitplicity properties that any model with ppf of the form (23) must display.

It is interesting to note that (23) has just the form of inelastic emission by the simplest of all unitary mechanisms: Each multiperipheral chain emits no more than a single pion. Other unitary schemes, corresponding to varying numbers of pions emitted per chain, have been discussed in the papers of Ref. 2. What is interesting, novel, and perhaps even correct about the result (23) is the suggestion that only the simplest unitary theory is relevant at asymptotic energies. At lower energies, of course, one would have to include contributions corresponding to two or more particles emitted per chain. Equation (23) leads to the simplest sort of representation, with all asymptotic inelastic reactions given in terms of the elastic eikonal function, and we now turn to some of the obvious implications of such a phenomenology.

III. PHENOMENOLOGY

Perhaps the best empirical fit to high-energy elastic data is given by an eikonal that obeys geometric scaling,[9] and it is this form of elastic eikonal which we shall use to parametrize (23). One chooses χ as completely absorptive,

$$-i\chi=\rho(b,s)=Af[b/R(y)], \tag{24}$$

where A is a constant, and the impact-parameter scale $R(y)$ contains all the rapidity dependence of the problem. A convenient way to break geometric scaling—should it turn out to be necessary at higher energies—is to let A depend on rapidity, and such forms were long ago introduced in black-disk absorptive models.[10] Here, our remarks shall concern the estimation of (23) with the aid of (24).

The simplest quantities of interest are then

$$\sigma_{\text{tot}}=2\alpha R^2, \tag{25}$$

where $\alpha\equiv\int d^2x[1-e^{-Af(x)}]$,

$$\sigma_{\text{in}}=\beta R^2, \tag{26}$$

where $\beta=\int d^2x[1-e^{-2Af(x)}]$,

and

$$\sigma_{\text{el}}=\sigma_{\text{tot}}-\sigma_{\text{in}}=(2\alpha-\beta)R^2. \tag{27}$$

When geometric scaling is valid, A, α, and β are constants independent of energy, and each cross section increases in the same way with increasing $R(y)$. Further, the topological cross section for the production of n pions displays the same sort of energy variation, $\sigma_n=\gamma_n R^2(y)$, with

$$\gamma=\frac{1}{n!}(2A)^n\int d^2x[f(x)]^n e^{-2Af(x)}. \tag{28}$$

Such energy behavior has not been seen at Fermilab energies, and (28) is therefore a prediction for the yet-to-be-measured cross sections at CERN ISR energies, where the elastic pp scattering is quite adequately described by geometric scaling.

Related predictions may be made for multiplicities,

$$\langle n\rangle=\frac{2A}{\beta}\int d^2x f(x), \tag{29}$$

and

$$\langle n^2\rangle-\langle n\rangle^2=\frac{(2A)^2}{\beta}\int d^2x f^2(x), \tag{30}$$

etc. For strict geometric scaling, it is clear that all multiplicities and ratios of higher moments turn out to be independent of energy.

Phenomenological tests of the semiclassical formalism suggested in the previous section may of course be carried out in terms of the parametrizations appropriate when geometric scaling is violated, $A\to A(y)$. If A increases with rapidity [as suggested by the tower graph models,[10] where $A(y)\sim\lambda e^{\gamma y}\sim\lambda s^\gamma$, λ and $\gamma>0$, then one must also specify $f(x)$ in sufficient detail to compute the over-all y dependence of each of these quantities. Many different models have appeared in the literature,[11] and there is little need to elaborate upon them here except to observe the phenomenological tests of our semiclassical formalism will undoubtedly be more difficult, and less definitive, in the absence of strict geometric scaling.

IV. CONCLUSION

The essential point of emphasis in this paper is the natural way in which the simplest possible unitary model arises in the consideration of inelastic processes at high energies. One begins with a formalism containing full s- and t-channel unitarity, and applies a form of semiclassical averaging to obtain not just a simpler formalism, but the simplest possible version of a unitary theory. A test for the validity of this procedure is easily outlined: If strict geometric scaling holds for the elastic scattering amplitude but the CERN ISR energy variation of each σ_n turns out to be different from that of $R^2\sim\sigma_{\text{tot}}$, then the underlying, semiclassical approximation—which may be performed in any nontrivial field theory—is wrong. Conversely, if this simple σ_n test agrees with the prediction of (28), there will follow the strong suggestion that such an averaging approach is indeed sensible at high energies, a statement with ramifications in many related areas of physics.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge useful conversations with R. Blankenbecler, and with numerous colleagues at our respective institutions.

---

*Work supported by joint CONACYT-NSF travel grants, under the U. S.-Mexico Cooperative Science Program.

† Work supported in part by the U. S. Energy Research and Development Administration.

[1] J. Botke, J. Fulco, and A. Pignotti, Phys. Rev. D 11, 1673 (1975).

[2] R. Aviv, R. L. Sugar, and R. Blankenbecler, Phys. Rev. D 5, 3252 (1972); 6, 2216 (1972).

[3] L. Dolan and R. Jackiw, Phys. Rev. D 9, 3320 (1974); S. Weinberg, *ibid.* 9, 3357 (1974).

[4] L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, New York, 1962).

[5] H. M. Fried, *Functional Methods and Models in Quantum Field Theory* (MIT Press, Cambridge, Mass., 1972).

[6] R. Blankenbecler and H. M. Fried, Phys. Rev. D 8, 678 (1973).

[7] H. M. Fried, Nucl. Phys. B73, 93 (1974).

[8] One may start from the full Eq. (14), perform the stationary-phase approximation for both the φ and θ in-

tegrals keeping all terms without further approximation, and then find that the $K[1+z^2D_{(+)}KD_{(+)}K]^{-1}$ operators are algebraically replaced by K operators, leading to conventional, second-order differential equations involving a single mass-shell pole only. That is, the differential Eq. (19) has solutions with but a single mass-shell pole, in contrast to solutions of an equation of form $(K_x)^p\varphi(x)=\delta\mathfrak{F}[\psi]/\delta\psi(x)$, $p>1$, where ψ would possess a mass-shell singularity of form $(k^2+m^2)^{-p}$. We are indebted to R. Blankenbecler for the suggestion leading to this observation.

[9]V. Barger, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974*, edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. I-193. For some recent criticism, see Alex Martin, in *Proceedings of the X Rencontre de Moriond*, edited by J. Tran Thanh Van (Université de Paris-Sud, Orsay, 1975).

[10]H. Cheng and T. T. Wu, Phys. Rev. Lett. 24, 1455 (1970); S.-J. Chang and T.-M. Yan, Phys. Rev. D 4, 537 (1971).

[11]M. Le Bellac, J. L. Meunier, and G. Plaut, Nucl. Phys. B62, 350 (1973); H. Moreno, Phys. Rev. D 8, 268 (1973); A. Białas and E. Białas, Acta Phys. Pol. B5, 373 (1974).