Nonleptonic decays of charmed mesons: Implications for e^+e^- annihilation

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Using a generalization of the $|\Delta I| = \frac{1}{2}$ rule, we analyze the nonleptonic weak decays of pseudoscalar charmed mesons. We call attention to the likelihood that the dominant decays will be into many-body channels. Special attention is devoted to the properties of charmed final states just above charm threshold in e^+e^- annihilation. The questions of charged multiplicity, copious kaon production, and energy carried by neutrals are explored.

I. INTRODUCTION

The recent discovery[1,2] of narrow resonances at masses of 3.1 and 3.7 GeV/c^2 has stimulated an enormous amount of theoretical speculation. That the new particles appear to respect G parity in their decays[3] and can be photoproduced diffractively[4] suggests strongly that in spite of their very small widths, the newly discovered objects are hadrons. Among the speculations which interpret the narrow states as hadrons, one attractive possibility is that they are composed of a fourth, massive quark which bears a new quantum number called charm. This suggestion carries a great many implications,[5] not the least of which is the prediction of a rich spectrum of additional particles awaiting discovery. The conditions necessary for the discovery of the conjectured states depend in detail upon their decay modes, since the least massive charmed particles would be stable against strong decays. Thus, one needs to know, in addition to the rudiments of SU(4) spectroscopy, how the charmed particles participate in weak and electromagnetic interactions.

Perhaps the most elegant way to represent the weak and electromagnetic currents of the four quarks is by analogy with the leptonic currents, as first discussed by Bjorken and Glashow[6] and elaborated by Glashow, Iliopoulos, and Maiani.[7] Indeed, from the viewpoint of renormalizable theories of the weak and electromagnetic interactions, this representation of currents supplies the charmed quark with its *raison d'être*. In the Bjorken-Glashow-Iliopoulos-Maiani scheme, the Cabibbo-favored weak transition ($\propto \cos\theta_C$) of the charmed quark is to a strange quark. This is the basis for the oft-repeated observation[8] that the onset of charmed particle production would be signaled by a substantial increase in the multiplicity of strange particles. No such strange-particle avalanche has been observed, in spite of the fact that if charmed particles exist on the expected mass scale, they have almost surely been produced.

It is likely that ψ $(J)(3100)$ production in hadron-hadron collisions will be accompanied by particles of nonzero charm.[9] Because diffraction is primarily a reflection of inelastic (nondiffractive) processes, the apparent rise[10] in the cross section for $\gamma N \to \psi(3100)N$ from 11.1 GeV to approximately 100 GeV may indicate that charmed hadrons are being photoproduced at Fermilab. To set a mass scale for the charmed mesons, we assume that $\psi(3100)$ and $\psi(3700)$ are vector states of hidden charm. Then the small width of $\psi(3700)$ suggests that the charmed-meson masses exceed 1.85 GeV/c^2, and first-order SU(4) breaking[5] of a quadratic mass formula yields 2.2 GeV/c^2 for the mass of the charmed pseudoscalar mesons D^+, D^0, $\bar{D}^0$, D^-, F^+, F^-. On the basis of these estimates it is expected that the threshold for production of a pair of charmed mesons in e^+e^- annihilation is $\gtrsim$ 4 GeV. Indeed, the considerable width of the enhancement observed at 4.15 GeV may indicate that this state lies above charm threshold. However, not even in the SPEAR experiments (where the ratio of charmed to uncharmed hadrons is likely to be largest) has there been any report of copious kaon production. This has led to considerable anxiety among theorists that the charm interpretation is incorrect.[11] We shall comment below on the validity of these misgivings.

Rough estimates[5] suggest that the dominant decays of the least-massive charmed hadrons should be the nonleptonic weak transitions. Since the positive, unambiguous identification of a single charmed particle is of immediate importance, we have explored in detail the systematics of decays into some two-body and quasi-two-body channels, and several three-body channels. The mean multiplicity of charged particles in e^+e^- annihilation is only about 4.3 at 4.8 GeV.[12] Therefore, if charmed particles are being produced as copiously as expected, they do not decay into large numbers of particles unless there are a great many neutral products.

An essential complication in the discussion of nonleptonic weak decays is the issue of a possible enhancement of certain modes over others. Recall that in strangeness-changing decays the $|\Delta I| = \frac{1}{2}$ rule ("octet dominance") works extremely well. Typically, a $|\Delta I| = \frac{3}{2}$ amplitude is less than about 5% of the corresponding $|\Delta I| = \frac{1}{2}$ amplitude. The conventional wisdom, especially following the work of Gaillard and Lee[13] and of Altarelli and Maiani,[14] is that the strong interactions work to enhance the $|\Delta I| = \frac{1}{2}$ piece of the nonleptonic weak Hamiltonian by a factor of roughly 5, whereas matrix elements of the $|\Delta I| = \frac{3}{2}$ piece are reduced. The enhancement mechanism has yet to be fully understood. Unfortunately, for the charm-changing decays we do not yet have the benefit of observation, but there is strong theoretical motivation (reviewed in Sec. III below) to anticipate a similar enhancement-suppression phenomenon. We feel it is essential to take account of it in discussing final states.

The plan of this article is as follows. In Sec. II, we illustrate the sort of difficulties one might anticipate in the search for charm by asking a hypothetical question: How would the kaon have been found if the strange quark had been very much more massive than the nonstrange quarks? In Sec. III, we review the SU(3) and SU(4) structure of the hadronic weak currents and consider the algebraic properties of the nonleptonic Hamiltonian. We review the motivation for sextet dominance of charm-changing transitions and discuss the general consequences for the decays of charmed pseudoscalar mesons. In Sec. IV, we discuss in detail the charmless final states resulting from the weak decay of a charmed pseudoscalar into two pseudoscalars, or two vectors, or a pseudoscalar and a vector, or three pseudoscalars in a totally symmetric state, or a baryon and an antibaryon. Our expectations for the gross properties of the final states in e^+e^- annihilation just above charm threshold make up Sec. V. These differ significantly from what is commonly believed. Section VI contains a summary of our principal conclusions. In a mathematical Appendix we summarize some of the properties of the representations of SU(4) and other aspects of that not-yet-familiar algebra. Section III and the Appendix are somewhat technical and may be omitted by the reader concerned chiefly with results.

Although we find the charm scheme aesthetically attractive, we offer the present discussion not in a spirit of advocacy, but in the hope that the issues can more sharply be defined.

II. IF THE KAON MASS WERE $\gtrsim 2$ GeV/c^2

Before engaging in a calculation of the amplitudes for charmed-meson decays, we will find it instructive to develop some insight into the properties of a massive, weakly decaying system. To do so, we invent an artificial problem which has a well-defined solution, free of the ambiguities of charmed particle decay: We imagine the kaon mass to be greater than about 2 GeV/c^2, and compute the relative rates for its allowed decays. The results of our calculation suggest that if strange particles were as massive as we expect the charmed mesons to be, they too might still be awaiting discovery.

We shall assume, as discussed in the Introduction and elaborated in Sec. III, that an enhancement of the octet part of the nonleptonic weak Hamiltonian underlies the $|\Delta I| = \frac{1}{2}$ rule. The evaluation of the matrix elements is then a standard exercise, which we have carried out for the four classes of decays

$$K \to \mathcal{P}\mathcal{P}, \tag{2.1}$$

$$K \to \mathcal{V}\mathcal{V}, \tag{2.2}$$

$$K \to \mathcal{P}\mathcal{V} \text{ in a symmetric octet}, \tag{2.3}$$

$$K \to (\mathcal{P}\mathcal{P}\mathcal{P})_{\text{symmetric octet}}, \tag{2.4}$$

where $\mathcal{P}$ and $\mathcal{V}$ denote (nonstrange) pseudoscalar and vector mesons, respectively. In this fictitious world, it seems likely that η–η' mixing would be ideal,[15] so that

$$\eta = (2)^{-1/2}(u\bar{u} + d\bar{d})$$

would be lighter than the kaon, whereas

$$\eta' = s\bar{s}$$

would be heavier. The absolute squares of the enhanced matrix elements for K^+ decay[16] are collected in Table I. Each of the classes of decays

TABLE I. Relative decay rates for a heavy K^+ meson.

Decay mode	Squared matrix element $\sin^2\theta_C \cos^2\theta_C \times$
$\pi^+\eta$	1
$\rho^+\omega$	1
$\rho^+\eta$	$\frac{1}{2}$
$\pi^+\omega$	$\frac{1}{2}$
$\pi^+\eta\eta$	$\frac{9}{14}$
$\pi^+\pi^+\pi^-$	$\frac{2}{7}$
$\pi^+\pi^0\pi^0$	$\frac{1}{14}$

(2.1)–(2.4) is characterized by a distinct strength parameter, which we cannot estimate without making further detailed assumptions.

The only point we wish to make with this calculation is that there are many decay modes which may be of comparable importance. Nearly all of them involve one or more neutral particles, and would be difficult to observe without good sensitivity to both charged and neutral particles. The familiar $K^+ \to \pi^+\pi^0$ mode, which signals a small violation of the $|\Delta I| = \frac{1}{2}$ rule, is unlikely to compete with the enhanced decays. Although this example has been rather contrived, it does accurately mimic our subsequent conclusions about the difficulty of finding charmed mesons in effective (or missing) mass distributions. It also demonstrates that the origin of this difficulty is kinematical, i.e., simply reflects the large number of open channels, and does not depend sensitively upon the form to be chosen for the effective charm-changing weak Hamiltonian.

III. REPRESENTATION CONTENT OF THE WEAK-INTERACTION HAMILTONIAN

In preparation for the calculations to follow, we shall now review the SU(4) structure of the hadronic weak currents. We represent the four quark fields as a composite spinor

$$\psi^\alpha = \begin{bmatrix} \psi^0 \\ \psi^1 \\ \psi^2 \\ \psi^3 \end{bmatrix} = \begin{bmatrix} c \\ u \\ d \\ s \end{bmatrix} \tag{3.1}$$

(We suppress any dependence on a color degree of freedom.) The three quarks u, d, s form the familiar basis for the fundamental representation of SU(3), and the c quark is an SU(3) singlet which carries the new quantum number charm. We take the charged weak current to be[7]

$$J = \bar{u}(d\cos\theta_C + s\sin\theta_C) + \bar{c}(s\cos\theta_C - d\sin\theta_C). \tag{3.2}$$

Since we are not concerned with the space-time structure of the current, we adopt an abbreviated notation in which, for example, $\bar{u}d$ represents $\bar{u}\gamma^\mu(1-\gamma_5)d$. It is convenient to write the current as[7]

$$J = \bar{\psi}\mathcal{O}\psi, \tag{3.3}$$

where $\mathcal{O}$ is the 4×4 matrix

$$\mathcal{O} = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix}, \tag{3.4}$$

and

$$U = \begin{pmatrix} -\sin\theta_C & \cos\theta_C \\ \cos\theta_C & \sin\theta_C \end{pmatrix}. \tag{3.5}$$

Denoting the Hermitian conjugate field $\psi^{\alpha\dagger}$ by ψ_α, we may write the current as

$$J = \psi_\alpha \mathcal{O}_{\alpha\beta}\psi^\beta, \tag{3.6}$$

which is a linear combination of states transforming as the direct product $\underline{4}^* \otimes \underline{4} = \underline{15} \oplus \underline{1}$. Because the matrix $\mathcal{O}$ is traceless, the singlet does not appear in (3.6), i.e., the weak current transforms like a member of a 15-dimensional representation of SU(4).

We take the familiar current-current form for the nonleptonic weak Hamiltonian,

$$\mathcal{H}_W = \tfrac{1}{2}(JJ^\dagger + J^\dagger J). \tag{3.7}$$

It will be useful for the subsequent discussion to understand the SU(4) transformation properties of $\mathcal{H}_W$. Clearly, it transforms as the symmetric product $(\underline{15} \otimes \underline{15}^*) \oplus (\underline{15}^* \otimes \underline{15})$. The pentadeciment is self-conjugate and the Clebsch-Gordan series is

$$\underline{15} \otimes \underline{15} = \underline{1}_S \oplus \underline{15}_S \oplus \underline{15}_A \oplus \underline{20}_S \oplus \underline{45}_A \oplus \underline{45}_A^* \oplus \underline{84}_S, \tag{3.8}$$

where the subscript S or A indicates that the representation occurs in a symmetric or antisymmetric product. (Explicit wave functions for the irreducible representations are listed in the Appendix.) Because of the symmetric nature of $\mathcal{H}_W$, only symmetric representations occur. Therefore, in general, we should expect

$$\mathcal{H}_W \cong \underline{1} \oplus \underline{15}_S \oplus \underline{20} \oplus \underline{84}, \tag{3.9}$$

but in fact the $\underline{15}$ does not occur for the current of interest (3.2). We show in the Appendix that the absence of the $\underline{15}$ is a consequence of (i) trace $\mathcal{O} = 0$ and (ii) the anticommutator $\{\mathcal{O}, \mathcal{O}^+\} \propto 1$. Of course, for charm- or strangeness-changing transitions the singlet also does not enter, so for these

$$\mathcal{H}_W \cong \underline{20} \oplus \underline{84}. \tag{3.10}$$

As Kingsley *et al.*,[17] have remarked, a direct application of the decomposition (3.10) to matrix elements of physical interest is likely to be unrewarding because SU(4) is a badly broken spectroscopic symmetry. However, SU(3) invariance is a much better first approximation, so it may be more lucrative to exploit the SU(3) transformation properties of $\mathcal{H}_W$. Decomposing the SU(4) representations with respect to SU(3) subgroups, one finds

$$\underline{20}=[\underline{6}]\oplus[\underline{8}]\oplus[\underline{6}^*], \tag{3.11}$$

$$\underline{84}=[\underline{6}^*]\oplus\{[\underline{3}^*]+[\underline{15}_M^*]\}\oplus\{[\underline{1}]\oplus[\underline{8}]\oplus[\underline{27}]\} \oplus\{[\underline{3}]+[\underline{15}_M]\}\oplus[\underline{6}], \tag{3.12}$$

where we have used square brackets to distinguish representations of SU(3) from those of SU(4), and the subscript M denotes a representation of mixed symmetry. In the $\underline{20}$, the octet is charm conserving ($\Delta C=0$), and $[\underline{6}]$ and $[\underline{6}^*]$ correspond to $\Delta C=\pm 1$, respectively. In the $\underline{84}$, the singlet, octet, and $[\underline{27}]$ are charm conserving, $\{[\underline{3}^*]\oplus[\underline{15}_M^*]\}$ and $\{[\underline{3}]\oplus[\underline{15}_M]\}$ correspond to $\Delta C=\pm 1$, and $[\underline{6}^*]$ and $[\underline{6}]$ correspond to $\Delta C=\pm 2$. The $|\Delta C|=2$ pieces do not contribute to $\mathcal{H}_W$, and we show in the Appendix that the $[\underline{3}]$ and $[\underline{3}^*]$ do not occur in the Hamiltonian (3.10).

It might be expected *a priori* that the reduced matrix elements of every contributing SU(3) representation would be of the same order. However, recalling the familiar situation for strangeness-changing decays, in which matrix elements of the octet operators are enhanced compared to those in the $[\underline{27}]$, we must ask what is the analog of the $|\Delta I|=\frac{1}{2}$ rule for the charm-changing decays. It has been shown[13,14] that, in an asymptotically free, SU(3)-invariant gauge theory of strong interactions, the gluons enhance the $|\Delta I|=\frac{1}{2}$ part of $\mathcal{H}_W$ relative to the $|\Delta I|=\frac{3}{2}$ terms. If these arguments were applied to an SU(4)-invariant theory, one would find that the $\underline{20}$ is enhanced relative to the $\underline{84}$. This is easy to see because the suppressed $[\underline{27}]$ is contained in the $\underline{84}$. Looking back to (3.11), we find that $\underline{20}$ dominance means octet enhancement for $\Delta C=0$ transitions and sextet enhancement for $|\Delta C|=1$ terms.[18] The conclusion that $[\underline{6}]$ and $[\underline{6}^*]$ are enhanced relative to $[\underline{15}_M^*]$ and $[\underline{15}_M]$ undoubtedly does not rest on the assumption of SU(4) invariance. Had we followed the line of reasoning of Ref. 13, we should have found that the term in $\mathcal{H}_W$ antisymmetric in quarks is enhanced relative to the symmetric term, which implies sextet dominance.

Hereafter, then, we shall assume sextet dominance of the nonleptonic decays of charmed particles. Because SU(4) is a grossly broken symmetry, we cannot reliably compare the magnitude of the sextet enhancement factor with that of the octet enhancement factor. Let us consider the decays of the SU(3) triplet of $C=1$ pseudoscalar mesons D^+, D^0, F^+, which are expected to be the lightest charmed particles. We wish to evaluate the matrix element $\langle P_C|\mathcal{H}_W|h\rangle$, where $|P_C\rangle$ denotes a charmed pseudoscalar and $|h\rangle$ is a charmless hadronic final state. The object $\langle P_C|\mathcal{H}_W$ transforms under SU(3) like $[\underline{6}]\otimes[\underline{3}]=[\underline{8}]\oplus[\underline{10}]$. Consequently, in the limit of SU(3) invariance, the final state $|h\rangle$ must transform like an octet or decimet. Suppose first that $|h\rangle$ consists of a pair of particles, each a member of an SU(3) octet. The product of two octets includes a decimet $[\underline{10}]$, as well as a symmetric and antisymmetric octets $[\underline{8}_S]$ and $[\underline{8}_A]$, so in general such a decay will involve three SU(3)-invariant reduced amplitudes. A considerable simplification occurs when the two particles are identical bosons since, according to the Pauli principle, they must be in a state which is symmetric under particle interchange. Since they must couple to zero total angular momentum, and therefore be symmetric in space $\times$ spin, they must also be in a state symmetric in SU(3) indices. This requirement excludes the $[\underline{10}]$ and $[\underline{8}_A]$. As a result, there is a *single* reduced amplitude for the enhanced decays of a charmed meson into two identical bosons.

Results for decays into a pair of pseudoscalars, a pair of vectors, a pseudoscalar-vector pair, three pseudoscalars in a totally symmetric state, or a baryon and antibaryon will be presented in Sec. IV. The foregoing analysis is modified somewhat by octet-singlet mixing, at least for decays involving vector mesons. In the quark model, a nonet symmetry holds approximately, so we shall allow the final-state bosons to be all members of a nonet. For the pseudoscalars, we add to our considerations $\eta'(958)=(3)^{-1/2}(u\bar{u}+d\bar{d}+s\bar{s})$, ignoring the small η-η' mixing. For the vectors, we assume that $\omega=(2)^{-1/2}(u\bar{u}+d\bar{d})$ and $\phi=s\bar{s}$ are ideally mixed.

We have not analyzed the weak decays of other charmed mesons, such as the vector triplet, because it seems likely that the dominant decays of these presumably heavier particles will be strong or electromagnetic. For the same reason, we have not considered the decays of charmed baryons.

IV. ENHANCED NONLEPTONIC DECAYS OF THE CHARMED MESONS

We now discuss nonleptonic weak decays of the SU(3) triplet of charmed pseudoscalar mesons D^+, D^0, F^+, assuming sextet dominance of the charm-changing Hamiltonian. In the Appendix we show that the enhanced part of the charm-lowering Hamiltonian transforms as

$$\mathcal{H}_{\text{eff}}=[\underline{6}]^{22}\cos^2\theta_C+2\,[\underline{6}]^{23}\sin\theta_C\cos\theta_C +[\underline{6}]^{33}\sin^2\theta_C. \tag{4.1}$$

As already remarked in Sec. III, $\langle P_C|\mathcal{H}_{\text{eff}}$ transforms as $[\underline{3}]\otimes[\underline{6}]=[\underline{8}]\oplus[\underline{10}]$. If we wish to discuss decays into pairs of particles, each of which resides in an octet or nonet, we need only know the coupling of the two octets (or nonets) to $[\underline{8}]$ and $[\overline{\underline{10}}]$. This can be obtained from tables of SU(3) Clebsch-Gordan coefficients or computed using the tensor formalism contained in the Appendix. There we also indicate how the U(3) nonet structure can be accommodated.

Decays into two pseudoscalars or two vectors each depend upon a single reduced matrix element (which may be different for $\mathcal{P}\mathcal{P}$ and $\mathcal{V}\mathcal{V}$). The squared matrix elements (which measure the branching ratios up to phase-space corrections) for each decay mode are displayed in Table II, for D^+ and F^+ decays, and in Table III, for D^0 decays.[19] For completeness we have included rates proportional to $\sin^2\theta_C \cos^2\theta_C$ and to $\sin^4\theta_C$.

A striking feature of Table II is the absence of any enhanced D^+ decay which is proportional to $\cos^4\theta_C$. It is understood most simply by a V-spin conservation argument presented in the Appendix. The only Cabibbo-favored nonleptonic decays of D^+ into $\mathcal{P}\mathcal{P}$ or $\mathcal{V}\mathcal{V}$ are $\bar{K}^0\pi^+$ and $\bar{K}^{*0}\rho^+$, which proceed via $[\underline{15}^*_M]$. *Observation of these modes would provide a direct measure of the magnitude of the unenhanced amplitude.* This is not to say that there are no enhanced nonleptonic decays of order $\cos^4\theta_C$. They can in principle occur in final states which couple to $[\overline{\underline{10}}]$ in, e.g., the $\mathcal{P}\mathcal{V}$ mode or the baryon-antibaryon mode to be discussed below. However, from the perspective of the quark model, $[\underline{10}]$ is an exotic channel so it may well be that, at least in the $\mathcal{P}\mathcal{V}$ case, this would-be enhanced amplitude is not enhanced at all.[20] If all $\cos^4\theta_C$ nonleptonic modes are sufficiently suppressed, the squared matrix elements of order $\sin^2\theta_C \cos^2\theta_C$ may accurately reflect the branching ratios. If the magnitude of the sextet enhancement is comparable to that of the observed octet enhancement, we expect the nonleptonic rates proportional to $\sin^2\theta_C \cos^2\theta_C$ to be of the same order as the semileptonic decays proportional to $\cos^2\theta_C$, such as $D^+ \to \bar{K}^0 l^+ \nu$. Thus for D^+, but apparently not for D^0 or F^+, the possibility exists that semileptonic modes might compete favorably with nonleptonic decays. With such a large Q value for the decay, it is likely that many different semileptonic decays may occur, no one of which will have a large branching ratio.

Another remark to be made is that the only dominant decay of D^0 into two charged particles is the

TABLE II. Relative rates for F^+ decay. Decays of D^+ can be obtained by multiplying each mode by $\tan^2\theta_C$.

$\cos^4\theta_C\times$		$\cos^2\theta_C \sin^2\theta_C\times$	
$\pi^+\eta'$	$\frac{4}{9}$	$K^+\eta'$	$\frac{4}{9}$
$K^+\bar{K}^0$	$\frac{1}{3}$	$K^0\pi^+$	$\frac{1}{3}$
$\pi^+\eta$	$\frac{2}{9}$	$K^+\pi^0$	$\frac{1}{6}$
		$K^+\eta$	$\frac{1}{18}$
$\rho^+\omega$	$\frac{2}{3}$	$K^{*+}\phi$	$\frac{1}{3}$
$K^{*+}\bar{K}^{*0}$	$\frac{1}{3}$	$K^{*0}\rho^+$	$\frac{1}{3}$
		$K^{*+}\rho^0$	$\frac{1}{6}$
		$K^{*+}\omega$	$\frac{1}{6}$

TABLE III. Relative rates for D^0 decay.

$\cos^4\theta_C\times$		$\cos^2\theta_C \sin^2\theta_C\times$		$\sin^4\theta_C\times$	
$\bar{K}^0\eta'$	$\frac{4}{9}$	$\eta\eta'$	$\frac{2}{3}$	$K^0\eta'$	$\frac{4}{9}$
$K^-\pi^+$	$\frac{1}{3}$	$\pi^-\pi^+$	$\frac{1}{3}$	$K^+\pi^-$	$\frac{1}{3}$
$\bar{K}^0\pi^0$	$\frac{1}{6}$	K^-K^+	$\frac{1}{3}$	$K^0\pi^0$	$\frac{1}{6}$
$\bar{K}^0\eta$	$\frac{1}{18}$	$\pi^0\eta'$	$\frac{2}{9}$	$K^0\eta$	$\frac{1}{18}$
		$\eta\eta$	$\frac{1}{6}$		
		$\pi^0\pi^0$	$\frac{1}{6}$		
		$\pi^0\eta$	$\frac{1}{9}$		
$K^{*-}\rho^+$	$\frac{1}{3}$	$\phi\phi$	$\frac{2}{3}$	$K^{*+}\rho^-$	$\frac{1}{3}$
$\bar{K}^{*0}\phi$	$\frac{1}{3}$	$K^{*+}K^{*-}$	$\frac{1}{3}$	$K^{*0}\phi$	$\frac{1}{3}$
$\bar{K}^{*0}\rho^0$	$\frac{1}{6}$	$\omega\rho^0$	$\frac{1}{3}$	$K^{*0}\rho^0$	$\frac{1}{6}$
$\bar{K}^{*0}\omega$	$\frac{1}{6}$	$\rho^+\rho^-$	$\frac{1}{3}$	$K^{*0}\omega$	$\frac{1}{6}$
		$\rho^0\rho^0$	$\frac{1}{6}$		
		$\omega\omega$	$\frac{1}{6}$		

$K^-\pi^+$ mode, which accounts for $\frac{1}{3}$ of the $\mathcal{P}\mathcal{P}$ decays. There is no way reliably to estimate what fraction of all decays are $\mathcal{P}\mathcal{P}$, but we should be surprised if the branching ratio into $K^-\pi^+$ exceeded 10%.[21] We note that unlike the D^+ or F^+, the D^0 will always (in the dominant decays) yield a $\bar{K}$.

Turning to the pseudoscalar-vector decay modes, the situation is immediately more complicated, since, as noted in Sec. III, there will be three reduced amplitudes instead of only one. We list the general case in Table IV, but this is clearly not very useful for phenomenological purposes. For illustrative purposes, we have tabulated the decays into symmetric octet states in Tables V and VI, about which we shall have more to say in Sec. IV.

As for the general case, we have little to say until such time as these mesons are found. Then it would be exceedingly interesting to obtain the relative magnitudes of these enhanced decays. Thus, observation of $F^+\to\pi^+\phi$, $D^+\to\bar{K}^0\rho^+$, and $D^+\to\pi^+\bar{K}^{*0}$ would establish the strength of the decimet amplitude as well as providing two tests of sextet dominance. Even if, as we suspect, the decimet amplitude is suppressed, it will be important to determine the relative magnitude of the decimet to the "enhanced" pentadecimet contribution.

TABLE IV. Decay amplitudes for the pseudoscalar-vector mode. T, S, and A are the reduced matrix elements for decay into a decimet, a symmetric octet, and an antisymmetric octet, respectively.

	$\cos^2\theta_C\times$		$\sin\theta_C\cos\theta_C\times$		$\sin^2\theta_C\times$
			$F^+\to\mathcal{P}\mathcal{V}$		
$\pi^+\phi$	T	$K^+\rho^0$	$\frac{1}{\sqrt{2}}(2T-S-A)$	K^+K^{*0}	$-3T$
$\pi^+\rho^0$	$\frac{1}{\sqrt{2}}(T-2A)$	$K^+\omega$	$-\frac{1}{\sqrt{2}}(2T+S+A)$	K^0K^{*+}	$3T$
$\pi^+\omega$	$-\frac{1}{\sqrt{2}}(T+2S)$	$K^+\phi$	$2T-S+A$		
$\eta\rho^+$	$\frac{1}{\sqrt{6}}(3T-2S)$	π^0K^{*+}	$-\frac{1}{\sqrt{2}}(2T+S-A)$		
$\pi^0\rho^+$	$-\frac{1}{\sqrt{2}}(T-2A)$	ηK^{*+}	$\frac{1}{\sqrt{6}}(6T+S+3A)$		
$K^+\bar{K}^{*0}$	$T-S+A$	π^+K^{*0}	$-2T-S+A$		
$\bar{K}^0K^{*+}$	$-(T+S+A)$	$K^0\rho^+$	$2T-S-A$		
$\eta'\rho^+$	$-\frac{2}{\sqrt{3}}S$	$\eta'K^{*+}$	$-\frac{2}{\sqrt{3}}S$		
			$D^+\to\mathcal{P}\mathcal{V}$		
$\bar{K}^0\rho^+$	$-3T$	$\pi^+\phi$	$2T$	$K^+\rho^0$	$\frac{1}{\sqrt{2}}(T+S+A)$
$\pi^+\bar{K}^{*0}$	$3T$	$\pi^+\rho^0$	$\sqrt{2}\,(T+A)$	$K^+\omega$	$\frac{1}{\sqrt{2}}(-T+S+A)$
		$\pi^+\omega$	$\sqrt{2}(-T+S)$	$K^+\phi$	$T+S-A$
		$\eta\rho^+$	$(\frac{2}{3})^{1/2}(3T+S)$	π^0K^{*+}	$\frac{1}{\sqrt{2}}(-T+S-A)$
		$\pi^0\rho^+$	$-\sqrt{2}(T+A)$	ηK^{*+}	$\frac{1}{\sqrt{6}}(3T-S-3A)$
		$K^+\bar{K}^{*0}$	$2T+S-A$	π^+K^{*0}	$-T+S-A$
		$\bar{K}^0K^{*+}$	$-2T+S+A$	$K^0\rho^+$	$T+S+A$
		$\eta'\rho^+$	$\frac{2}{\sqrt{3}}S$	$\eta'K^{*+}$	$+\frac{2}{\sqrt{3}}S$

TABLE IV. *(Continued)*

$\cos^2\theta_C\times$		$\sin\theta_C\cos\theta_C\times$		$\sin^2\theta_C\times$	
		$D^0\to\mathcal{P}\mathcal{V}$			
$\bar K^0\rho^0$	$-\frac{1}{\sqrt2}(2T+S-A)$	$K^0\bar K^{*0}$	$-T+2A$	$K^0\phi$	$-T-S+A$
$\pi^0\bar K^{*0}$	$\frac{1}{\sqrt2}(2T-S-A)$	$\bar K^0K^{*0}$	$T-2A$	$K^0\rho^0$	$\frac{1}{\sqrt2}(T+S+A)$
$K^-\rho^+$	$-T+S-A$	K^-K^{*+}	$-T+S-A$	$K^0\omega$	$\frac{1}{\sqrt2}(T-S-A)$
π^+K^{*-}	$T+S+A$	K^+K^{*-}	$T+S+A$	π^0K^{*0}	$\frac{1}{\sqrt2}(-T+S-A)$
$\eta\bar K^{*0}$	$\frac{1}{\sqrt6}(-S+3A)$	$\pi^+\rho^-$	$-(T+S+A)$	ηK^{*0}	$\frac{1}{\sqrt6}(-3T+S+3A)$
$\eta'\bar K^{*0}$	$\frac{2}{\sqrt3}S$	$\pi^-\rho^+$	$T-S+A$	$K^+\rho^-$	$-(T+S+A)$
$\bar K^0\phi$	$S+A$	$\pi^0\phi$	$\sqrt2 T$	π^-K^{*+}	$T-S+A$
$\bar K^0\omega$	$\frac{1}{\sqrt2}(S-A)$	$\eta\phi$	$-2(\frac{2}{3})^{1/2}S$	$\eta'K^{*0}$	$-\frac{2}{\sqrt3}S$
		$\eta'\phi$	$\frac{2}{\sqrt3}S$		
		$\eta'\rho^0$	$-\frac{2}{\sqrt6}S$		
		$\eta'\omega$	$\frac{2}{\sqrt6}S$		
		$\eta\rho^0$	$\frac{1}{\sqrt3}(3T+S)$		
		$\eta\omega$	$-\frac{1}{\sqrt3}S$		
		$\pi^0\rho^0$	$-S$		
		$\pi^0\omega$	$-T+S$		

We would also like to get a feeling for the relative branching ratios for "direct" three-body or four-body decays, to compare with the quasi-two-body modes discussed above. Suppose we consider the decays into three pseudoscalar mesons. Clearly, there are very many invariant amplitudes in general which would be both tedious and uninformative to work out. In the pseudoscalar-vector case above, the vector decays into two pseudoscalars in an antisymmetric SU(3) state. Thus, an interesting complement to the cases already discussed would be when no two pseudoscalars are in an antisymmetric SU(3) state. Therefore, one simple case we consider is three pseudoscalars in a totally symmetric state which can be either an octet or a decimet. The decimet may be "exotic" in the usual quark-model sense so, for simplicity, we will neglect it. The case of the

TABLE V. Relative rates for $F^+\to\mathcal{P}\mathcal{V}$ (symmetric octet). Rates for D^+ decay are $\tan^2\theta_C$ times the entries.

$\cos^4\theta_C\times$		$\sin^2\theta_C\cos^2\theta_C\times$	
$\pi^+\omega$	$\frac{1}{3}$	$\eta'K^{*+}$	$\frac{2}{9}$
$\eta'\rho^+$	$\frac{2}{9}$	$K^+\phi$	$\frac{1}{6}$
$\bar K^0K^{*+}$	$\frac{1}{6}$	$K^0\rho^+$	$\frac{1}{6}$
$K^+\bar K^{*0}$	$\frac{1}{6}$	π^+K^{*0}	$\frac{1}{6}$
$\eta\rho^+$	$\frac{1}{9}$	$K^+\rho^0$	$\frac{1}{12}$
		$K^+\omega$	$\frac{1}{12}$
		π^0K^{*+}	$\frac{1}{12}$
		ηK^{*+}	$\frac{1}{36}$

TABLE VI. Relative rates for $D^0 \to \mathcal{P}\mathcal{V}$ (symmetric octet).

$\cos^4\theta_C \times$		$\sin^2\theta_C \cos^2\theta_C \times$		$\sin^4\theta_C \times$	
$\eta'\bar{K}^{*0}$	$\frac{2}{9}$	$\eta\phi$	$\frac{4}{9}$	$\eta' K^{*0}$	$\frac{2}{9}$
$K^-\rho^+$	$\frac{1}{6}$	$\eta'\phi$	$\frac{2}{9}$	$K^+\rho^-$	$\frac{1}{6}$
π^+K^{*-}	$\frac{1}{6}$	K^-K^{*+}	$\frac{1}{6}$	π^-K^{*+}	$\frac{1}{6}$
$\bar{K}^0\phi$	$\frac{1}{6}$	K^+K^{*-}	$\frac{1}{6}$	$K^0\phi$	$\frac{1}{6}$
$\bar{K}^0\rho^0$	$\frac{1}{12}$	$\pi^+\rho^-$	$\frac{1}{6}$	$K^0\rho^0$	$\frac{1}{12}$
$\pi^0\bar{K}^{*0}$	$\frac{1}{12}$	$\pi^-\rho^+$	$\frac{1}{6}$	π^0K^{*0}	$\frac{1}{12}$
$\bar{K}^0\omega$	$\frac{1}{12}$	$\pi^0\rho^0$	$\frac{1}{6}$	$K^0\omega$	$\frac{1}{12}$
$\eta\bar{K}^{*0}$	$\frac{1}{36}$	$\pi^0\omega$	$\frac{1}{6}$	ηK^{*0}	$\frac{1}{36}$
		$\eta'\rho^0$	$\frac{1}{9}$		
		$\eta'\omega$	$\frac{1}{9}$		
		$\eta\rho^0$	$\frac{1}{18}$		
		$\eta\omega$	$\frac{1}{18}$		

symmetric octet is easily worked out and is given in Table VII. (We thank Haim Harari for calling our attention to the possibility of a totally symmetric [10]. See note added.)

So long as we have come this far, it is no more work to include baryon-antibaryon decays, listed in Table VIII. Most of these channels are near or below threshold, if the mass formulas are correct or if the wide bump at 4.15 GeV is an indication that charmed particles are being produced. Consequently, with the possible exception of the nucleon-antinucleon channels, these branching ratios are likely to be in the range of tiny to nonexistent. The only nucleon-antinucleon channel which is not Cabibbo disfavored is $F^+ \to p\bar{n}$.

Prior to the discovery of charmed mesons, the utility of any of these tables is limited, except as a guide to experimenters as to what channels might be favored. Certainly primary attention should be devoted to the enhanced decays of order $\cos^4\theta_C$. However, even before their discovery, we can ask what sort of multiplicities and prong distributions would these modes lead if these charmed mesons are being pair produced in e^-e^+ annihilation. It is to such questions that we now turn.

V. FINAL STATES IN e^+e^- ANNIHILATION ABOVE CHARM THRESHOLD

One of our motives for this work has been the desire to sharpen the issues in the experimental search for charm. In this spirit, we now investigate the signatures for pair production of charmed mesons in electron-positron annihilation. Many of the suggested manifestations of a charm threshold which now enjoy currency were put forward rather casually, before the discovery of the new particles. For example, Glashow[8] has remarked that charm threshold should be marked by sudden increases in (i) the mean multiplicity of hadrons, (ii) the yield of kaons,[22] and (iii) the production of prompt muons. Others have anticipated a dramatic rise in a particular topological cross section. Having in hand the results of Sec. IV, we are in a position to make more informed speculations.

We shall discuss the properties of the charmed final states produced in the reactions

$$e^+e^- \to D^+D^-, \tag{5.1a}$$

$$e^+e^- \to D^0\bar{D}^0, \tag{5.1b}$$

$$e^+e^- \to F^+F^-, \tag{5.1c}$$

which for the purpose of discussion we shall assume to occur with equal cross sections. In the SU(4)-symmetry limit, reaction (5.1b) is forbidden, just as $e^+e^- \to K^0\bar{K}^0$ is prohibited in exact SU(3). In view of the large mass difference between charmed quarks and nonstrange quarks, we doubt that a very significant suppression will occur. Furthermore, because the allowed reaction

$$e^+e^- \to D^0\bar{D}^{*0} \tag{5.2}$$

will lead to at least one D^0 in the final state, the assumption of equal rates for reactions (5.1) appears to be a sensible guess. Finally, since we have no reliable way to judge the relative importance of the decay modes $\mathcal{P}\mathcal{P}$, $\mathcal{V}\mathcal{V}$, $\mathcal{P}\mathcal{V}$, and $\mathcal{P}\mathcal{P}\mathcal{P}$, we must treat them separately.

The gross properties of the charmed final states

TABLE VII. Decays into three pseudoscalars in a symmetric octet (corresponding rates for D^+ obtained by multiplying rates for F^+ by $\tan^2\theta_C$).

$\cos^4\theta_C\times$		$\cos^2\theta_C\sin^2\theta_C\times$		$\sin^4\theta_C\times$	
		F^+			
$K^+\bar{K}^0\eta'$	$\frac{1}{4}$	$K^0\pi^+\eta'$	$\frac{1}{4}$		
$\pi^+\pi^+\pi^-$	$\frac{1}{6}$	$K^+\eta'\eta'$	$\frac{1}{6}$		
$\pi^+\eta'\eta'$	$\frac{1}{6}$	$K^+K^+K^-$	$\frac{1}{6}$		
$\pi^+\eta\eta'$	$\frac{1}{6}$	$K^+\pi^0\eta'$	$\frac{1}{8}$		
$\pi^+K^+K^-$	$\frac{1}{12}$	$K^+K^0\bar{K}^0$	$\frac{1}{12}$		
$\pi^+K^0\bar{K}^0$	$\frac{1}{12}$	$K^+\pi^+\pi^-$	$\frac{1}{12}$		
$\pi^+\pi^0\pi^0$	$\frac{1}{24}$	$K^+\pi^0\pi^0$	$\frac{1}{24}$		
$\pi^+\eta\eta$	$\frac{1}{24}$	$K^+\eta\eta'$	$\frac{1}{24}$		
		$K^+\eta\eta$	$\frac{1}{24}$		
		D^0			
$K^-\pi^+\eta'$	$\frac{1}{4}$	$\pi^+\pi^-\eta'$	$\frac{1}{4}$	$K^+\pi^-\eta'$	$\frac{1}{4}$
$\bar{K}^0\eta'\eta'$	$\frac{1}{6}$	$K^+K^-\eta'$	$\frac{1}{4}$	$K^0\eta'\eta'$	$\frac{1}{6}$
$\bar{K}^0\bar{K}^0K^0$	$\frac{1}{6}$	$\eta\eta'\eta'$	$\frac{1}{4}$	$K^0K^0\bar{K}^0$	$\frac{1}{6}$
$\bar{K}^0\pi^0\eta'$	$\frac{1}{8}$	$\eta\eta\eta$	$\frac{3}{16}$	$K^0\pi^0\eta'$	$\frac{1}{8}$
$\bar{K}^0\pi^+\pi^-$	$\frac{1}{12}$	$\pi^+\pi^-\eta$	$\frac{1}{8}$	$K^0\pi^+\pi^-$	$\frac{1}{12}$
$\bar{K}^0K^+K^-$	$\frac{1}{12}$	$\pi^0\pi^0\eta'$	$\frac{1}{8}$	$K^0K^+K^-$	$\frac{1}{12}$
$\bar{K}^0\pi^0\pi^0$	$\frac{1}{24}$	$\eta\eta\eta'$	$\frac{1}{8}$	$K^0\pi^0\pi^0$	$\frac{1}{24}$
$\bar{K}^0\eta\eta$	$\frac{1}{24}$	$K^+K^-\eta$	$\frac{1}{8}$	$K^0\eta\eta$	$\frac{1}{24}$
$\bar{K}^0\eta\eta'$	$\frac{1}{24}$	$K^0\bar{K}^0\eta$	$\frac{1}{8}$	$K^0\eta\eta'$	$\frac{1}{24}$
		$\pi^0\eta'\eta'$	$\frac{1}{12}$		
		$\pi^0\eta\eta'$	$\frac{1}{12}$		
		$\pi^0\pi^0\eta$	$\frac{1}{16}$		
		$\pi^0\pi^0\pi^0$	$\frac{1}{16}$		
		$\pi^+\pi^-\pi^0$	$\frac{1}{24}$		
		$K^+K^-\pi^0$	$\frac{1}{24}$		
		$K^0\bar{K}^0\pi^0$	$\frac{1}{24}$		
		$\pi^0\eta\eta$	$\frac{1}{48}$		

in reactions (5.1) are shown in Table IX for each of the decay schemes. We have based the entries in the table on the stable particles which result from the Cabibbo-favored decays enumerated in Tables II, III, V, VI, and VII of Sec. IV.[23] For these purposes, the "stable particles" are $\pi^\pm$, π^0, $K^\pm$, K^0 and $\bar{K}^0$, and photons from sources other than π^0 decay. In addition to the mean multiplicities of the stable particles, we have tabulated the K/π ratio, the mean charged multiplicity, the prong probabilities $P_n = \sigma_n/\sigma$, and the fraction of events containing kaons. The consistency of the entries for the various decay schemes gives us confidence in the generality of our conclusions.

The only published data which may readily be compared with our expectations are the SPEAR measurements of $\langle n_{\text{ch}}\rangle$.[12] In the energy regime 4–5 GeV, the data are quite similar to the values in Table IX and show no dramatic departure from an extrapolation of the results at lower energies. Keeping in mind that significant production of uncharmed final states is likely to persist above charm threshold [the quark model suggests $\sigma(\text{uncharmed})/\sigma(\text{charmed}) = \frac{3}{2}$], we would not antici-

TABLE VIII. Amplitudes for baryon-antibaryon decays. T, S, and A are the reduced matrix elements for decay into a decimet, a symmetric octet, and an antisymmetric octet, respectively.

	$\cos^2\theta_C\times$		$\cos\theta_C \sin\theta_C\times$		$\sin^2\theta_C\times$
			$F^+\to\mathcal{B}\bar{\mathcal{B}}$		
$\Sigma^+\bar{\Lambda}$	$-\frac{1}{\sqrt{6}}(3T+2S)$	$p\bar{\Sigma}^0$	$\frac{1}{\sqrt{2}}(2T-S-A)$	$p\bar{\Xi}^0$	$-3T$
$\Sigma^+\bar{\Sigma}^0$	$\frac{1}{\sqrt{2}}(T-2A)$	$p\bar{\Lambda}$	$-\frac{1}{\sqrt{6}}(6T-S+3A)$	$n\bar{\Xi}^+$	$3T$
$\Lambda\bar{\Sigma}^+$	$\frac{1}{\sqrt{6}}(3T-2S)$	$\Sigma^0\bar{\Xi}^+$	$-\frac{1}{\sqrt{2}}(2T+S-A)$		
$\Sigma^0\bar{\Sigma}^+$	$-\frac{1}{\sqrt{2}}(T-2A)$	$\Lambda\bar{\Xi}^+$	$\frac{1}{\sqrt{6}}(6T+S+3A)$		
$p\bar{n}$	$T-S+A$	$\Sigma^+\bar{\Xi}^0$	$-2T-S+A$		
$\Xi^0\bar{\Xi}^+$	$-(T+S+A)$	$n\bar{\Sigma}^+$	$2T-S-A$		
			$D^+\to\mathcal{B}\bar{\mathcal{B}}$		
$\Xi^0\bar{\Sigma}^+$	$-3T$	$\Sigma^+\bar{\Sigma}^0$	$\sqrt{2}\,(T+A)$	$p\bar{\Sigma}^0$	$\frac{1}{\sqrt{2}}(T+S+A)$
$\Sigma^+\bar{n}$	$3T$	$\Sigma^+\bar{\Lambda}$	$(\frac{2}{3})^{1/2}(-3T+S)$	$p\bar{\Lambda}$	$\frac{1}{\sqrt{6}}(-3T-S+3A)$
		$\Lambda\bar{\Sigma}^+$	$(\frac{2}{3})^{1/2}(3T+S)$	$\Sigma^0\bar{\Xi}^+$	$\frac{1}{\sqrt{2}}(-T+S-A)$
		$\Sigma^0\bar{\Sigma}^+$	$-\sqrt{2}\,(T+A)$	$\Lambda\bar{\Xi}^+$	$\frac{1}{\sqrt{6}}(3T-S-A)$
		$p\bar{n}$	$2T+S-A$	$\Sigma^+\bar{\Xi}^0$	$-T+S-A$
		$\Xi^0\bar{\Xi}^+$	$-2T+S+A$	$n\bar{\Sigma}^+$	$T+S+A$
			$D^0\to\mathcal{B}\bar{\mathcal{B}}$		
$\Xi^0\bar{\Sigma}^0$	$-\frac{1}{\sqrt{2}}(2T+S-A)$	$n\bar{n}$	$-T+2A$	$n\bar{\Lambda}$	$\frac{1}{\sqrt{6}}(3T+S-3A)$
$\Sigma^0\bar{n}$	$\frac{1}{\sqrt{2}}(2T-S-A)$	$\Xi^0\bar{\Xi}^0$	$T-2A$	$n\bar{\Sigma}^0$	$\frac{1}{\sqrt{2}}(T+S+A)$
$\Xi^-\bar{\Sigma}^+$	$-T+S-A$	$\Xi^-\bar{\Xi}^+$	$-T+S-A$	$\Sigma^0\bar{\Xi}^0$	$\frac{1}{\sqrt{2}}(-T+S-A)$
$\Sigma^+\bar{p}$	$T+S+A$	$p\bar{p}$	$T+S+A$	$\Lambda\bar{\Xi}^0$	$\frac{1}{\sqrt{6}}(-3T+S+3A)$
$\Lambda\bar{n}$	$\frac{1}{\sqrt{6}}(-S+3A)$	$\Sigma^+\bar{\Sigma}^-$	$-(T+S+A)$	$p\bar{\Sigma}^-$	$-(T+S+A)$
$\Xi^0\bar{\Lambda}$	$-\frac{1}{\sqrt{6}}(S+3A)$	$\Sigma^-\bar{\Sigma}^+$	$T-S+A$	$\Sigma^-\bar{\Xi}^+$	$T-S+A$
		$\Sigma^0\bar{\Lambda}$	$\frac{1}{\sqrt{3}}(-3T+S)$		
		$\Lambda\bar{\Sigma}^0$	$\frac{1}{\sqrt{3}}(3T+S)$		
		$\Sigma^0\bar{\Sigma}^0$	$-S$		
		$\Lambda\bar{\Lambda}$	S		

TABLE IX. Some properties of charmed states in $e^+e^- \to$ hadrons.

Decay scheme	$\langle \pi^+ + \pi^- \rangle$	$\langle \pi^0 \rangle$	$\langle K^+ + K^- \rangle$	$\langle K^0 + \bar{K}^0 \rangle$	$\langle K \rangle / \langle \pi \rangle$	$\langle \gamma \rangle$	$\langle n_{ch} \rangle$	P_0	P_2	P_4	P_6	P_8	P_{10}	P_{12}	K events
$\mathcal{P}\mathcal{P}$	2.9	1.6	0.7	0.9	0.3	1.1	3.6	0.03	0.35	0.43	0.16	0.03			70%
$\mathcal{V}\mathcal{V}$	4.4	2.7	1.0	0.9	0.3	0.1	5.4	0	0.02	0.26	0.67	0.04			70%
$(\mathcal{P}\mathcal{V})_{8s}$	3.7	2.2	0.8	0.9	0.3	0.6	4.5	0.01	0.15	0.46	0.34	0.04			70%
$(\mathcal{P}\mathcal{P}\mathcal{P})_{\text{symmetric}}$	4.8	2.7	0.8	1.3	0.3	1.8	5.6	0.02	0.10	0.27	0.34	0.18	0.07	0.02	77%

pate any sudden change in $\langle n_{ch} \rangle$ as the energy is increased across charm threshold. Similarly, we have no reason to expect a pronounced increase in any one topological cross section, rather than a proportional increase in them all.

In charmed events, the mean multiplicity of kaons (charged plus neutral) will be approximately 1.8/ event. Whether this number is so large as to effect a sudden increase in the yield of K mesons of course depends on the properties of the charmless "background" events. In the SU(3) limit, the three-quark model suggests that roughly $\frac{4}{9}$ of the events contain strange particles,[24] and that the average kaon multiplicity is close to 1 event. Taking too seriously the hints given by the quark model, we may expect that just above charm threshold about 55% of all hadronic events will contain kaons, and the mean multiplicity of kaons will be approximately 1.3/event. Thus even the most celebrated characteristic of charmed events, the fraction yielding kaons, might not give rise to a significant change in the character of hadronic events.

As we noted in Sec. IV, the arguments of Gaillard, Lee, and Rosner[5,25] favoring nonleptonic over semileptonic decays together with the abundance of enhanced nonleptonic decays make copious muon emission very unlikely except in the decay of $D^{\pm}$. Hence the theoretical evidence seems to favor no more than a modest increase in the production of prompt muons.

We are unable to comment in detail upon the fraction of the total energy carried by neutrals. In lieu of a sophisticated analysis, we merely remark that according to Table IX the fraction of particles which are neutral

$$f_0 \equiv \frac{\langle \pi^0 \rangle + \langle K^0 + \bar{K}^0 \rangle + \langle \gamma \rangle}{\langle \pi^0 \rangle + \langle K^0 + \bar{K}^0 \rangle + \langle \gamma \rangle + \langle n_{ch} \rangle} \tag{5.3}$$

assumes the values

$$f_0(\mathcal{P}\mathcal{P}) \simeq 0.5,$$

$$f_0(\mathcal{V}\mathcal{V}) \simeq 0.4,$$

$$f_0(\mathcal{P}\mathcal{V}) \simeq 0.5,$$

$$f_0(\mathcal{P}\mathcal{P}\mathcal{P}) \simeq 0.5.$$

Although it would be premature to identify charm threshold with the onset of the decrease of the ratio of charged-particle energy to total energy in $e^+e^- \to$ hadrons[12] on the basis of these apparently large fractions, the possibility is provocative.

In summary, we expect events above charm threshold to differ only subtly from those below threshold. Most of the final states in charmed events will be complex, often with several neutrals. The great number of open channels and the presence of neutrals will make it difficult to reconstruct charmed particles in effective-mass or missing-mass distributions. As Lipkin has observed,[22] only a small fraction (we estimate $\frac{16}{15}\tan^2\theta_C$) of the events will exhibit apparent strangeness violation.

VI. CONCLUSIONS

Charmed mesons may prove to be rather elusive objects in hadronic channels because it is unlikely that a single hadronic decay mode will predominate. Although the relative rates expected for various decay modes do depend explicitly upon the form chosen for the charm-changing weak Hamiltonian, the conclusion that there should be many competing modes is implied essentially by kinematical considerations. A preponderance of the important nonleptonic modes consists of several particles including at least one neutral. Except in the case of the $D^\pm$ mesons, which may lack any enhanced, Cabibbo-favored nonleptonic decays, we do not expect the leptonic or semileptonic branching ratios to be appreciable.

In e^+e^- annihilations into hadrons the passage through charm threshold will not be marked by any spectacular qualitative change in the gross features of the final states. Quantitative increases in the fraction of events containing strange particles, in the mean multiplicity of kaons, and perhaps in the energy fraction carried by neutrals are likely to occur. The magnitudes of the expected changes were estimated in Sec. V. Our analysis makes it plausible that charm production blends more easily into the background of charmless events than has generally been recognized. This has the disappointing consequence that the hidden charm interpretation of the new narrow resonances will not easily be eliminated or established.

Note added. As indicated in the text, H. Harari has pointed out to us that three pseudoscalars can be coupled to form a totally symmetric decimet.[26] For completeness, we have added Table X, giving the Cabibbo-favored ($\cos^2\theta_C$) decay amplitudes into three pseudoscalars in a totally symmetric state. As we have seen previously in the decays into a pseudoscalar-vector or baryon-antibaryon pair, the only $\cos^2\theta_C$ decays of the $D^\pm$ allowed by sextet dominance are into a decimet. And as we remarked in connection with the $\mathcal{P}\mathcal{V}$ decays, it may be that the transition to the decimet is not enhanced. [Except for the $\pi^+\bar{K}^0\eta$ mode, the other three final states also result from a pseudoscalar-vector decay (Table IV).]

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APPENDIX: SOME PROPERTIES OF THE WEAK HAMILTONIAN IN AN SU(4) SCHEME

Only since the discovery of the new narrow resonances has the group SU(4) intruded measurably

TABLE X. Cabibbo-favored ($\cos^2\theta_C$) decay amplitudes into three pseudoscalars (totally symmetric). E and T are the reduced matrix elements for decay into a symmetric octet and symmetric decimet, respectively.

	$\cos^2\theta_C\times$
$F^+\to\{\mathcal{P}\mathcal{P}\mathcal{P}\}$	
$\bar{K}^0K^+\eta'$	$\sqrt{3}E$
$\bar{K}^0K^+\pi^0$	$\frac{3}{2}\sqrt{2}\,T$
$\bar{K}^0K^+\eta$	$-\frac{1}{2}\sqrt{6}\,T$
$\pi^+\pi^+\pi^-$	$E+T$
$\pi^+\pi^0\pi^0$	$\frac{1}{2}E+\frac{1}{2}T$
$\pi^+K^+K^-$	$E-2T$
$\pi^+K^0\bar{K}^0$	$E+T$
$\pi^+\eta'\eta'$	E
$\pi^+\eta\eta'$	$\sqrt{2}E$
$\pi^+\eta\eta$	$\frac{1}{2}E-\frac{3}{2}T$
$D^+\to\{\mathcal{P}\mathcal{P}\mathcal{P}\}$	
$\pi^+\bar{K}^0\eta$	$\frac{3}{2}\sqrt{6}\,T$
$\pi^+\pi^+K^-$	$-3T$
$\pi^+\pi^0\bar{K}^0$	$\frac{3}{2}\sqrt{2}\,T$
$K^+\bar{K}^0\bar{K}^0$	$3T$
$D^0\to\{\mathcal{P}\mathcal{P}\mathcal{P}\}$	
$\pi^+K^-\eta'$	$-\sqrt{3}E$
$\pi^+K^-\pi^0$	$-\frac{3}{2}\sqrt{2}\,T$
$\pi^+K^-\eta$	$\frac{1}{2}\sqrt{6}\,T$
$\bar{K}^0K^+K^-$	$-E+2T$
$\bar{K}^0\bar{K}^0K^0$	$-E-T$
$\bar{K}^0\pi^+\pi^-$	$-E-T$
$\bar{K}^0\pi^0\pi^0$	$-\frac{1}{2}E+T$
$\bar{K}^0\pi^0\eta$	$\sqrt{3}\,T$
$\bar{K}^0\pi^0\eta'$	$+\frac{1}{2}\sqrt{6}E$
$\bar{K}^0\eta'\eta'$	$-E$
$\bar{K}^0\eta\eta'$	$\frac{1}{2}\sqrt{2}E$
$\bar{K}^0\eta\eta$	$-\frac{1}{2}E$

upon the common consciousness of elementary-particle physicists. For this reason the rudimentary properties of representations of SU(4) are not yet familiar, although they have long been known to *virtuosi*. To enable the unversed reader to appreciate the discussion of the transformation properties of the weak Hamiltonian presented in Sec. III, we review here the necessary elements of the charm scheme.[27]

We denote the basis for the fundamental (quark) representation $\underline{4}$ by ψ^α and that for the conjugate (antiquark) representation $\underline{4}^*$ by $\psi_\alpha \equiv \psi^{\alpha\dagger}$, where the index α runs over the values (0, 1, 2, 3) corresponding to quarks (c, u, d, s), and the dagger indicates Hermitian conjugation. We introduce the Kronecker symbol δ^α_β and the totally antisymmetric four-index tensors $\epsilon_{\kappa\lambda\mu\nu} = \epsilon^{\kappa\lambda\mu\nu}$ $(\epsilon_{0123}=1)$. We shall use the symbol $\cong$ to denote isomorphism, i.e., to connect two quantities which transform in the same manner. For example, a totally antisymmetric state of three quarks transforms like an antiquark:

$$\psi_\alpha \cong \epsilon_{\alpha\kappa\lambda\mu}\psi^\kappa\psi^\lambda\psi^\mu. \tag{A1}$$

As in the usual SU(3) quark model, mesons are represented as quark-antiquark pairs $\psi^\alpha\psi_\beta$. Thus, the familiar nonet is expanded to sixteen mesons which divide under SU(4) into a singlet $\psi^\gamma\psi_\gamma$ and a pentadecimet with the (traceless) basis

$$T^\alpha_\beta \equiv \psi^\alpha\psi_\beta - \tfrac{1}{4}\delta^\alpha_\beta(\psi^\gamma\psi_\gamma). \tag{A2}$$

For what follows, it is useful to be conversant with some of the representations of low dimension,[28] which we have listed in Table XI.

As we defined it in Sec. II, the charged weak current is

$$J = \psi_\alpha \Theta_{\alpha\beta}\psi^\beta, \tag{A3}$$

where $\Theta_{\alpha\beta}$ is a 4×4 matrix with nonvanishing elements $\Theta_{03} = \Theta_{12} = \cos\theta_C$ and $\Theta_{13} = -\Theta_{02} = \sin\theta_C$ (θ_C is the Cabibbo angle). Anticipating a later need, we remark that

$$\begin{aligned}\{\Theta, \Theta^\dagger\} &= \Theta\Theta^\dagger + \Theta^\dagger\Theta\\ &= 1.\end{aligned} \tag{A4}$$

Because Θ is traceless, the current J is a sum of elements of a 15-dimensional representation

$$J = \Theta_{\alpha\beta}T^\beta_\alpha. \tag{A5}$$

Consequently the nonleptonic weak-interaction Hamiltonian is

$$\mathcal{H}_W \equiv \tfrac{1}{2}\{J, J^+\} = \tfrac{1}{2}\Theta_{\alpha\beta}\Theta^\dagger_{\sigma\rho}\{T^\beta_\alpha, T^\rho_\sigma\}. \tag{A6}$$

We require the symmetric product of two pentadecimets. For completeness, we include the general case of

$$\underline{15}\otimes\underline{15} = \underline{1}\oplus\underline{15}_S\oplus\underline{15}_A\oplus\underline{20}\oplus\underline{45}\oplus\underline{45}^*\oplus\underline{84}, \tag{A7}$$

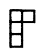
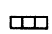
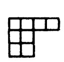
where the 20- and 84-dimensional representations which occur in the product are self-conjugate. In the $\underline{15}\otimes\underline{15}$ basis, the occurrent representations can be expressed as

$$\begin{aligned}
&(\underline{1}) = T^\beta_\alpha T^\alpha_\beta,\\
&(\underline{15}_A)^\beta_\alpha = T^\beta_\gamma T^\gamma_\alpha - T^\gamma_\alpha T^\beta_\gamma, \quad (\underline{15}_S)^\beta_\alpha = T^\beta_\gamma T^\gamma_\alpha + T^\gamma_\alpha T^\beta_\gamma - \tfrac{1}{2}\delta^\beta_\alpha(T^\gamma_\delta T^\delta_\gamma),\\
&(\underline{20})^{\beta\rho}_{\alpha\sigma} = H\left[{}^{\beta\rho}_{\alpha\sigma}\right] - \tfrac{1}{2}\left[\delta^\beta_\sigma(\underline{15}_S)^\rho_\alpha - \delta^\rho_\sigma(\underline{15}_S)^\beta_\alpha - \delta^\beta_\alpha(\underline{15}_S)^\rho_\sigma + \delta^\rho_\alpha(\underline{15}_S)^\beta_\sigma\right] + \tfrac{1}{6}(\delta^\beta_\sigma\delta^\rho_\alpha - \delta^\rho_\sigma\delta^\beta_\alpha)(\underline{1}),\\
&\text{where } H\left[{}^{\beta\rho}_{\alpha\sigma}\right] = T^\beta_\alpha T^\rho_\sigma - T^\rho_\alpha T^\beta_\sigma - T^\beta_\sigma T^\rho_\alpha + T^\rho_\sigma T^\beta_\alpha;\\
&(\underline{45})^{\beta\rho}_{\alpha\sigma} = M^{\{\beta\rho\}}_{[\alpha\sigma]} - \tfrac{1}{4}\left[\delta^\beta_\alpha(\underline{15}_A)^\rho_\sigma + \delta^\rho_\alpha(\underline{15}_A)^\beta_\sigma - \delta^\beta_\sigma(\underline{15}_A)^\rho_\alpha - \delta^\rho_\sigma(\underline{15}_A)^\beta_\alpha\right],\\
&\text{where } M^{\{\beta\rho\}}_{[\alpha\sigma]} = T^\beta_\alpha T^\rho_\sigma + T^\rho_\alpha T^\beta_\sigma - T^\beta_\sigma T^\rho_\alpha - T^\rho_\sigma T^\beta_\alpha;\\
&(\underline{84})^{\beta\rho}_{\alpha\sigma} = S^{\{\beta\rho\}}_{\{\alpha\sigma\}} - \tfrac{1}{6}\left[\delta^\beta_\alpha(\underline{15}_S)^\rho_\sigma + \delta^\beta_\sigma(\underline{15}_S)^\rho_\alpha + \delta^\rho_\sigma(\underline{15}_S)^\beta_\alpha + \delta^\rho_\alpha(\underline{15}_S)^\beta_\sigma\right] - \tfrac{1}{10}(\delta^\beta_\alpha\delta^\rho_\sigma + \delta^\beta_\sigma\delta^\rho_\alpha)(\underline{1}),\\
&\text{where } S^{\{\beta\rho\}}_{\{\alpha\sigma\}} = T^\beta_\alpha T^\rho_\sigma + T^\rho_\alpha T^\beta_\sigma + T^\rho_\sigma T^\beta_\alpha + T^\beta_\sigma T^\rho_\alpha.
\end{aligned} \tag{A8}$$

We need only consider the symmetric product which involves $\underline{1}\oplus\underline{15}_S\oplus\underline{20}\oplus\underline{84}$. We have

$$\{T^\beta_\alpha, T^\rho_\sigma\} = \tfrac{2}{15}(\delta^\beta_\sigma\delta^\rho_\alpha - \tfrac{1}{4}\delta^\beta_\alpha\delta^\rho_\sigma)(\underline{1}) + \tfrac{1}{2}(\underline{20})^{\beta\rho}_{\alpha\sigma} + \tfrac{1}{2}(\underline{84})^{\beta\rho}_{\alpha\sigma} + \tfrac{1}{3}\left[\delta^\beta_\sigma(\underline{15}_S)^\rho_\alpha - \tfrac{1}{2}\delta^\beta_\alpha(\underline{15}_S)^\rho_\sigma + \delta^\rho_\alpha(\underline{15}_S)^\beta_\sigma - \tfrac{1}{2}\delta^\rho_\sigma(\underline{15}_S)^\beta_\alpha\right], \tag{A9}$$

TABLE XI. SU(4) representations of low dimension.

Representation	Young diagram	SU(3) subgroups	Charm
4		[3]	0
		[1]	1
4*		[1]	−1
		[3*]	0
6 = 6*		[3*]	0
		[3]	1
10		[6]	0
		[3]	1
		[1]	2
15 = 15*		[3]	−1
		[1] $\oplus$ [8]	0
		[3*]	1
20 = 20*		[6*]	−1
		[8]	0
		[6]	1
20′		[8]	0
		[3*] $\oplus$ [6]	1
		[3]	2
20″		[10]	0
		[6]	1
		[3]	2
		[1]	3
45		[15_M]	−1
		[8] $\oplus$ [10]	0
		[3*] $\oplus$ [6]	1
		[3]	2
84 = 84*		[6]	−2
		[3] $\oplus$ [15_M]	−1
		[1] $\oplus$ [8] $\oplus$ [27]	0
		[3*] $\oplus$ [15_M^*]	1
		[6*]	2

which we must multiply by $\frac{1}{2}\Theta_{\alpha\beta}\Theta^\dagger_{\sigma\rho}$ to form $\mathcal{H}_W$. The last term gives rise to

$$\tfrac{1}{6}\,(\underline{15}_S)^\beta_\alpha[\{\Theta,\Theta^\dagger\}_{\alpha\beta}-\tfrac{1}{2}\Theta_{\alpha\beta}\,\mathrm{tr}\Theta^\dagger-\tfrac{1}{2}\Theta^\dagger_{\alpha\beta}\,\mathrm{tr}\Theta]$$

which vanishes because of (A4) and the tracelessness of $(\underline{15}_S)^\beta_\alpha$ and Θ. We therefore arrive at

$$\mathcal{H}_W=\tfrac{2}{15}\,(\underline{1})+\tfrac{1}{4}\Theta_{\alpha\beta}\Theta^\dagger_{\sigma\rho}\,(\underline{20})^{\beta\rho}_{\alpha\sigma}+\tfrac{1}{4}\Theta_{\alpha\beta}\Theta^\dagger_{\sigma\rho}\,(\underline{84})^{\beta\rho}_{\alpha\sigma} \tag{A10}$$

as the representation structure of the nonleptonic weak-interaction Hamiltonian. Obviously, the singlet cannot enter strangeness- or charm-changing transitions, so the $|\Delta C| = 1$ component of interest to us transforms like a linear combination of $\underline{20}$ and $\underline{84}$.

The Cabibbo-favored ($\cos^2\theta_C$) charm-lowering decays are caused by the term $\cos^2\theta_C(\bar{s}c\bar{u}d + \bar{u}d\bar{s}c)$ in $\mathcal{H}_W$. As we have explained in Sec. III, we expect the matrix elements of the $\underline{20}$ to be enhanced relative to those of the $\underline{84}$. The contribution of the dominant charm-lowering term to the $\underline{20}$ is of the form

$$\cos^2\theta_C(\bar{s}c\bar{u}d - \bar{u}c\bar{s}d - \bar{s}d\bar{u}c + \bar{u}d\bar{s}c),$$

which is a singlet under V spin, the SU(2) subgroup which transforms the doublet $\binom{u}{s}$, leaving d and c unchanged. Therefore, in the limit of SU(3) symmetry, the enhanced Cabibbo-favored $|\Delta C| = 1$ transitions conserve V spin. This is the easiest way to see that the decay $D^+ \to \bar{K}^0\pi^+$ cannot proceed via an enhanced $\cos^2\theta_C$ amplitude: The initial state has $V=0$, but the (s-wave) final state has $V=1$.[29]

In Sec. III, we discuss the SU(3) properties of the nonleptonic Hamiltonian. We would like to display some of the details here. We are concerned with the product of the charm-changing and charm-conserving currents, which transform as elements of the representations $[\underline{3}]$ (or $[\underline{3}^*]$) and $[\underline{8}]$, respectively. To be explicit, the $\Delta C = 1$ product is

$$\{\bar{c}s, d\bar{u}\}\cos^2\theta_C + (\{\bar{c}s, \bar{s}u\} - \{\bar{c}d, \bar{d}u\})\cos\theta_C \sin\theta_C - \{\bar{c}d, \bar{s}u\}\sin^2\theta_C, \quad \text{(A11)}$$

which may be written as

$$T^2_{31}\cos^2\theta_C + (T^3_{31} - T^2_{21})\cos\theta_C\sin\theta_C - T^3_{21}\sin^2\theta_C, \quad \text{(A12)}$$

where

$$T^k_{ij} \equiv \{\psi^k\psi_i, \psi^0\psi_j\} - \tfrac{1}{3}\delta^k_i\{\psi^l\psi_l, \psi^0\psi_j\} \quad \text{(A13)}$$

(latin indices run over 1, 2, 3).

In general $[\underline{8}] \otimes [\underline{3}] = [\underline{3}^*] \oplus [\underline{6}] \oplus [\underline{15}^*_M]$. The states of each of these representations may be defined in this basis as

$$\begin{aligned}
[\underline{3}^*]_i &\equiv T^j_{ij}, \\
[\underline{6}]^{kl} &\equiv \epsilon^{lij}T^k_{ij} + \epsilon^{kij}T^l_{ij}, \qquad \text{(A14)} \\
[\underline{15}^*_M]^k_{ij} &\equiv T^k_{ij} + T^k_{ji} - \tfrac{1}{4}\delta^k_i T^l_{jl} - \tfrac{1}{4}\delta^k_j T^l_{il}.
\end{aligned}$$

We remark that $[\underline{6}]$ and $[\underline{15}^*_M]$ are, respectively, antisymmetric and symmetric under exchange of the quark fields $\psi_i\psi_j$. Consequently, the term of interest [Eq. (A12)] may be decomposed as

$$\tfrac{1}{2}\{[\underline{15}^*_M]^2_{31}\cos^2\theta_C + ([\underline{15}^*_M]^3_{31} - [\underline{15}^*_M]^2_{21})\cos\theta_C\sin\theta_C - [\underline{15}^*_M]^3_{21}\sin^2\theta_C\}$$
$$+\tfrac{1}{4}\{[\underline{6}]^{22}\cos^2\theta_C + 2[\underline{6}]^{23}\cos\theta_C\sin\theta_C + [\underline{6}]^{33}\sin^2\theta_C\}.$$

(Notice that $[\underline{3}^*]$ is absent.) Assuming the sextet dominates over the pentadecimet gives Eq. (4.1) of the text.

In Sec. IV, we decompose octets and decimets into products of nonets. We shall indicate here how these decompositions can be easily worked out. Let $N^i_j = \psi^i\psi_j$ ($i, j = 1, 2, 3$) denote a nonet of quark-antiquark pairs. The identification of elements of the nonet with physical states depends on the mixing scheme. As indicated in Sec. II, this is different for pseudoscalars and vectors, for example. Of course, the SU(3)-invariant subgroups of the nonet are the singlet $[\underline{1}] = N^i_i$ and octet $[\underline{8}]^i_j = N^i_j - \frac{1}{3}\delta^i_j N^k_k$. Now consider the product of two such nonets, the pseudoscalars $\mathcal{P}^i_j$ and vectors $\mathcal{V}^i_j$. We wish to extract the octets and decimet from this product. These are easily seen to be

$$\begin{aligned}
[\underline{8}_1]^i_j &= \mathcal{P}^i_k\mathcal{V}^k_j - \tfrac{1}{3}\delta^i_j(\mathcal{P}^l_k\mathcal{V}^k_l), \\
[\underline{8}_2]^i_j &= \mathcal{P}^l_j\mathcal{V}^i_l - \tfrac{1}{3}\delta^i_j(\mathcal{P}^l_k\mathcal{V}^k_l), \\
[\underline{10}]^{ijk} &= \epsilon^{klm}(\mathcal{P}^i_l\mathcal{V}^j_m + \mathcal{P}^j_l\mathcal{V}^i_m) \\
&\quad + \epsilon^{ilm}(\mathcal{P}^j_l\mathcal{V}^k_m + \mathcal{P}^k_l\mathcal{V}^j_m) \\
&\quad + \epsilon^{jlm}(\mathcal{P}^k_l\mathcal{V}^i_m + \mathcal{P}^i_l\mathcal{V}^k_m).
\end{aligned}$$

It is sometimes more convenient to rearrange the two octets into symmetric and antisymmetric combinations $[\underline{8}_S] = \frac{1}{2}([\underline{8}_1] + [\underline{8}_2])$ and $[\underline{8}_A] = \frac{1}{2}([\underline{8}_1] - [\underline{8}_2])$.

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[15]None of the qualitative conclusions of this section hinge upon the assumption of a specific mixing scheme.

[16]We have not discussed the decays of neutral heavy kaons because to properly treat the (well-understood) complications of K^0-$\bar{K}^0$ mixing would take us too far afield.

[17]R. L. Kingsley, S. B. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D 11, 1919 (1975).

[18]This argument was originally presented by G. Altarelli, N. Cabibbo, and L. Maiani, Nucl. Phys. B88, 285 (1975) and has been given independently in Ref. 17.

[19]Except for the decays involving the η', the results for the $\mathcal{P}\mathcal{P}$ modes have been given in Ref. 17, where decays of charmed vector mesons and baryons were also worked out. Nonleptonic decays of charmed mesons have also been discussed by Y. Iwasaki, Phys. Rev. Lett. 34, 1407 (1975).

[20]Gaillard *et al*., Ref. 5, have already speculated in this direction.

[21]Consequently, looking for sharp spikes in two-prong invariant-mass distributions may only produce frustration.

[22]See, e.g., the remarks by H. J. Lipkin, cited in Ref. 5.

[23]The values in the tables are simply the squares of SU(3)-invariant matrix elements and do not take into account phase-space corrections arising from using physical kinematics.

[24]In this regard, we differ with the conclusions of H. J. Lipkin, Ref. 22, who assumes that the creation of strange quarks is suppressed relative to nonstrange quarks. Experimentally the ratio $\sigma(e^-e^+ \to K^-K^+)/\sigma(e^-e^+ \to \pi^-\pi^+)$ is of order 1, suggesting that in fact SU(3) symmetry works rather well as a first approximation. For the data, see the review by K. Strauch, in *Proceedings of the Sixth International Symposium on Electron and Photon Interactions at High Energies, Bonn, Germany, 1973*, edited by H. Rollnik and W. Pfeil (North-Holland, Amsterdam, 1974).

[25]See also Altarelli *et al*., Ref. 18.

[26]See Y. Dothan and H. Harari, Suppl. Nuovo Cimento 3, 48 (1965) for the complete decomposition of $[8] \otimes [\bar{8}] \otimes [\bar{8}]$.

[27]For a pedagogical introduction to SU(4) and the properties of charmed particles, we refer the reader to the Fermilab Academic Lecture Notes by one of us (M.B.E.). [Report No. Fermilab-Lecture-75/1-THY/EXP, 1975 (unpublished)].

[28]For more information on Lie Groups, see H. J. Lipkin, *Lie Groups for Pedestrians* (North-Holland, Amsterdam, 1966) 2nd edition; D. Amati, H. Bacry, J. Nuyts, and J. Prentki, Nuovo Cimento 34, 1732 (1964); M. B. Einhorn, Ref. 27.

[29]This observation due to M. K. Gaillard was transmitted to us by B. W. Lee. See also Ref. 25.