# *np* charge exchange and the $\sigma$ model

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The shape of the $np$ charge-exchange forward spike implies striking properties for the amplitudes. These are shown to indicate the existence of a polelike singularity located near the physical threshold ($u=4m^2$) for the $u$ channel $\bar{p}p\to\bar{n}n$. If this singularity is due to the exchange of a single particle, it must be an isospin-zero, $G$ parity +, scalar meson, and its coupling to the nucleons has to be equal to that of the $\pi$; one is thus led naturally to the $\sigma$ model as giving a simple interpretation of the very special properties of the amplitudes.

## I. INTRODUCTION

The behavior of the differential cross section for $np$ charge exchange

$$n+p\to p+n \tag{1}$$

near the forward direction is very striking: One observes[1-4] a very narrow spike, the cross section dropping by a factor $\simeq 2$ between $t=0$ and $t=-\mu^2$ ($\mu$ is the pion mass). This leads[5,6] to the following double conclusion: Pion exchange in the $t$ channel plays a dominant role near $t=0$, which is not surprising since the position $t=\mu^2$ of the pion pole is very near to the physical region; however, an equally important contribution comes from a slowly varying "background," which is responsible for the nonvanishing value of the forward cross section and possesses very special properties.

To be more precise, let us consider the $s$-channel helicity amplitudes $\varphi_i$ ($i=1,2,\ldots,5$) for reaction (1). In the very forward region ($0\leq -t\lesssim 2\mu^2$), they can be taken in the form[7]

$$\begin{aligned}
\varphi_1 &\simeq m^2[\hat{G}_A(s)+\hat{G}_C(s)],\\
\varphi_2 &\simeq \frac{g^2}{4\pi}\frac{t}{t-\mu^2}+2m^2\hat{G}_B(s),\\
\varphi_3 &\simeq m^2[\hat{G}_C(s)-\hat{G}_A(s)],\\
\varphi_4 &\simeq \frac{g^2}{4\pi}\frac{t}{t-\mu^2},\\
\varphi_5 &\simeq 0,
\end{aligned} \tag{2}$$

where $m$ is the nucleon mass and the functions $\hat{G}_A, \hat{G}_B, \hat{G}_C$ are linear combinations of the invariant amplitudes $\hat{G}_i(s,t)$ ($i=1,2,\ldots,5$) taken at $t=0$:

$$\hat{G}_A(s)=\hat{G}_2(s,0)+\left(1-\frac{s}{2m^2}\right)\hat{G}_3(s,0), \tag{3a}$$

$$\hat{G}_B(s)=\left(1-\frac{s}{2m^2}\right)\hat{G}_2(s,0)+\hat{G}_3(s,0), \tag{3b}$$

$$\hat{G}_C(s)=\left(1-\frac{s}{2m^2}\right)\hat{G}_4(s,0)+\hat{G}_5(s,0). \tag{3c}$$

(The set of invariant amplitudes used here is defined as the $G_i$ of Ref. 8, but using the $t$ channel instead of the $s$ channel.) Now a very simple analysis of the forward spike[7] shows that these three functions are practically independent of $s$ and given by

$$\hat{G}_A(s)\simeq 0, \tag{4a}$$

$$\hat{G}_B(s)\simeq -\frac{1}{2}\frac{1}{m^2}\frac{g^2}{4\pi}, \tag{4b}$$

$$\hat{G}_C(s)\simeq 0. \tag{4c}$$

This is confirmed by numerical fits,[9] and corresponds to a prescription[10,11] which gives approximate results of lengthier absorption-model calculations.

Two points are worth emphasizing at this stage:

(i) *The approximate values (4) hold not only at high energies ($s\gg m^2$), but in a wide energy range,* say for an incident laboratory momentum $p_L$ verifying

$$600\ \text{MeV}/c\lesssim p_L\lesssim 29\ \text{GeV}/c \tag{5}$$

(the upper bound being just an experimental one), i.e., as soon as

$$s-4m^2\gg\mu^2. \tag{6}$$

(ii) $t$-channel $\pi$ exchange does not contribute to $\hat{G}_2$ and $\hat{G}_3$, so that the remarkable value (4b) found for $\hat{G}_B$ appears generally as a mere coincidence: In any $t$-channel exchange model, such as a Regge model with or without absorption,[12-14] *the fact that $\hat{G}_B(s)$ is a simple factor times the $\pi N$ coupling constant is not explained,* but just taken as the result of a fitting procedure where this value is a free parameter. (Actually, the value of $\hat{G}_B$ came out more naturally in the "conspirator model,"[15,16] but this model has been abandoned because of apparently unsolvable difficulties with experiment.[17,18])

The aim of this work is to try to see whether these two very simple and striking properties of the amplitudes can be given an equally simple explanation, instead of just appearing as fortuitous consequences of a complicated mechanism. Section II shows that the first of these properties indicates the existence of a particlelike singularity in the $u$ channel, located around the physical threshold for $\bar{N}N$ scattering. Section III then discusses the properties of such a "particle" (the details being given in the Appendix): It is an isospin zero, scalar meson, whose coupling to the nucleon is equal to that of the pion. Finally, Sec. IV considers a particularly attractive possibility,[19] the $\sigma$ model, in which point (ii) above also appears as a natural consequence of more fundamental ideas.

## II. POSITION OF THE SINGULARITIES

If one calculates $\hat{G}_2(s,0)$ and $\hat{G}_3(s,0)$ from the definitions (3) and the values (4), one easily finds

$$\hat{G}_2(s,0) \simeq \frac{g^2}{4\pi}\frac{s-2m^2}{s(s-4m^2)},$$
$$\hat{G}_3(s,0) \simeq \frac{g^2}{4\pi}\frac{2m^2}{s(s-4m^2)}. \tag{7}$$

Now it is well known that the invariant amplitudes $\hat{G}_i$ have no kinematical singularity. Formulas (7) then suggest the existence of *dynamical* singularities at $s \simeq 4m^2$ and $s \simeq 0$. It is clear from condition (6) that the positions of these singularities can only be given approximately, but the approximation is quite good, since the terms neglected are of order $\mu^2/m^2$.

The presence of dynamical singularities near $s=4m^2$ is no surprise: In the $s$ channel, the deuteron contributes a pole at $s=m_d^2 \simeq 4m^2$; the $u$-channel $\pi^0$ pole lies at $u=\mu^2$, i.e., (remember we are working at $t=0$) at $s=4m^2$; furthermore, one knows from the total $pp$ and $np$ cross sections[1] that the absorptive parts of the amplitudes in the $s$ channel are sizable at very low energies (again around $s=4m^2$) and should become rapidly negligible as energy increases, because of the exotic nature of the $s$ channel.

What about the singularity near $s=0$? It can only come from a polelike singularity in the $u$ channel, at

$$u_0 = 4m^2. \tag{8}$$

Since this is much less conservative than the previous one, it is worth questioning its existence and position. For instance, could it be merely due to the approximate nature of Eqs. (4)? It is easy to show that this is indeed not the case.

To see this,[20] let us go back to the $s$-channel helicity amplitudes $\varphi_i$. It is well known that they must satisfy certain constraints at $s=4m^2$ and $s=0$ (see Refs. 15 and 21); these constraints precisely express the absence of kinematical singularities in the invariant amplitudes $\hat{G}_i$. Since we know (see above) that there exist dynamical singularities lying (within our approximations) at $s=4m^2$, the corresponding constraints will not be verified by the $\varphi_i$ and we need not consider them. On the other hand, the $s=0$ constraint reads simply

$$\varphi_1-\varphi_2-\varphi_3+\varphi_4 \underset{s\to 0}{\propto} s. \tag{9}$$

Using relations (2) this is easily translated into

$$\hat{G}_A-\hat{G}_B \underset{s\to 0}{\propto} s. \tag{10}$$

Now Eqs. (4) are far from verifying this condition. It is quite true that the values (4) have been obtained in the physical region of the $s$ channel, and not near $s=0$, where they surely no longer hold. Nevertheless, because of the point (i) emphasized in Sec. I, these values indicate the existence of a *dynamical* singularity near $u=4m^2$; furthermore, a small modification of Eqs. (4) would not alter this result, nor shift the position of this singularity.

Needless to say, one knows that the $NN$ amplitudes have a more complicated singularity structure; although, as recalled above, one expects the $s$-channel absorptive parts to be small except at very low energies, the $u$-channel ones surely have non-negligible contributions coming, for instance, in the unphysical part of the cut, from boson exchanges. The approximate relations (4), however, show that, *for* $t=0$, these contributions practically cancel locally all along the $u$-channel cut [at least for $\hat{G}_2$ and $\hat{G}_3$ and the linear combination (3c) of $\hat{G}_4$ and $\hat{G}_5$], so that the absorptive parts remain small, except around the crossed channel threshold $u=4m^2$ [see Ref. 7, Sec. IV].

## III. NATURE OF THE SINGULARITIES

We have indicated that the singularity near $s=4m^2$ can have three different origins. It is very easy to see that, for $\hat{G}_2$ and $\hat{G}_3$, the over-all strength (and sign) needed is that of the $u$-channel $\pi$ pole: The low-energy $s$-channel absorptive part practically cancels the deuteron pole.

As for the singularity near $u=4m^2$, Eqs. (7) show that its contribution to $\hat{G}_2$ and $\hat{G}_3$ is

$$\hat{G}_2^{(S)}(u,t=0) \simeq -\frac{g^2}{4\pi}\frac{1}{2}\frac{1}{u-4m^2},$$
$$\hat{G}_3^{(S)}(u,t=0) \simeq \frac{g^2}{4\pi}\frac{1}{2}\frac{1}{u-4m^2}. \tag{11}$$

Because the combination $\hat{G}_c$ of $\hat{G}_4(s,0)$ and $\hat{G}_5(s,0)$ should remain small compared to $\hat{G}_B$ for all values of $s>4m^2$ [see Eqs. (4)], the $u=4m^2$ singularity has to be approximately opposite in $\hat{G}_4$ and $\hat{G}_5$:

$$\hat{G}_4^{(s)}(u,t=0)\simeq -\frac{g^2}{4\pi}\frac{\alpha}{2}\frac{1}{u-4m^2},\qquad \hat{G}_5^{(s)}(u,t=0)\simeq \frac{g^2}{4\pi}\frac{\alpha}{2}\frac{1}{u-4m^2}. \tag{12}$$

The value of $\alpha$ cannot be found at this stage: Only one linear combination of $\hat{G}_4(s,0)$ and $\hat{G}_5(s,0)$ is known, and, furthermore, $\hat{G}_5$ does not necessarily vanish for $s$ tending to infinity (see Ref. 7, Sec. IV). Finally, the strength of the singularity is completely unknown in $\hat{G}_1$, and we will let

$$\hat{G}_1^{(s)}(u,t=0)=\frac{g^2}{4\pi}\frac{\beta}{2}\frac{1}{u-4m^2}. \tag{13}$$

It is not evident nor necessary that the singularity we are studying should come from a single-particle intermediate state in the $u$ channel. However, the absorptive part is concentrated in a small $u$ interval, and it actually looks like a one-particle-exchange term. Let us suppose that this is indeed the case, and see whether Eqs. (11) and (12) are sufficient to characterize the properties of this particle.

This investigation is performed in the Appendix, with the following result: If the contributions (11), (12), and (13) to the invariant amplitudes indeed come from the exchange of a particle $S$ in the $u$ channel, *this particle has isospin* $I_S=0$, *G parity* +, *and spin-parity* $J^P=0^+$. Strictly speaking, because of the fact that the above values are only known at $t=0$, one cannot completely exclude other natural-parity, isospin-zero exchanges; however, higher spins are much less likely because of centrifugal barrier effects, which are very important so near to the physical $u$-channel threshold (see the Appendix).

## IV. AN INTERESTING POSSIBILITY: THE $\sigma$ MODEL

If the singularity near $u=4m^2$ is due to an isospin-zero, scalar particle, it is easy to see from the results of the Appendix that *the coupling of this particle to the nucleons is the same as that of the* $\pi$. More precisely, formulas (11), (12), and (13) with (as found in the Appendix) $\alpha=\beta=1$ imply that the effective Lagrangian for the coupling of the nucleons with the $\pi$ and the new "particle" $S$ is

$$\mathcal{L}_I=-g\bar{N}[S+i\gamma_5\vec{\pi}\cdot\vec{\tau}]N. \tag{14}$$

This is precisely the nucleon part of the interaction Lagrangian in the $\sigma$ model.[22–24] So, as already indicated in Ref. 19, the experimental facts (i) and (ii) underlined in Sec. I can be very simply understood in the framework of the $\sigma$ model: The forward spike observed in $np$ charge exchange appears as the result of the interplay between $\pi$ exchange in the $t$ channel and $\pi$ and $\sigma$ exchanges in the $u$ channel, provided the mass of the $\sigma$ "particle" is

$$m_\sigma\simeq 2m. \tag{15}$$

Besides its simplicity, this possible explanation possesses a number of attractive features.

First, its validity is not restricted to high energies: It remains the same in the whole energy interval (5). When the energy increases, it points towards the existence of a fixed pole at $J=0$ in the complex angular momentum plane[25,26]: although this conclusion is not unambiguous (in particular, because we have been studying only $t=0$), the special role played by Born terms compared with continuous absorptive parts suggests that the high-energy version of the present explanation does contain a fixed pole.[27,28]

Secondly, a link is established between the observed behavior of $np$ charge exchange and the more profound and general ideas which the $\sigma$ model is devised to express, namely approximate chiral symmetry, the Goldberger-Treiman relation, and partial conservation of axial-vector current (PCAC). For instance, the equality of the $\bar{N}N\sigma$ and $\bar{N}N\pi$ coupling constants [as in Eq. (14)] appears in the $\sigma$ model as a consequence of chiral symmetry, and yields the Goldberger-Treiman relation when this symmetry is broken by a nonvanishing vacuum expectation value of the $\sigma$. In this respect, because of the high value (15) needed for the mass, the $\sigma$ model advocated here will give practically the same results as if $m_\sigma$ were infinite, i.e., the usual current-algebra results. For instance, the $\pi\pi$ scattering lengths calculated from the $4\pi$ contact term and the $\sigma$-exchange graphs in the three channels are given by formula (10-6) of Ref. 24:

$$a_0=\frac{1}{\mu}\frac{7}{32\pi}\left(\frac{\mu}{f_\pi}\right)^2\left(1+\frac{29}{7}\frac{\mu^2}{m_\sigma^2}+\cdots\right),\qquad a_2=-\frac{1}{\mu}\frac{1}{16\pi}\left(\frac{\mu}{f_\pi}\right)^2\left(1-\frac{\mu^2}{m_\sigma^2}+\cdots\right). \tag{16}$$

The corrections to Weinberg's values[29] are of order $\mu^2/m_\sigma^2$, i.e., completely negligible here. In the same way, the $\pi N$ scattering lengths given by the two nucleon Born terms plus $\sigma$ exchange in the $t$ channel are

$$a_{1/2}=\frac{1}{\mu}\,\frac{1}{4\pi}\left(\frac{\mu}{m}\right)^2\frac{g_{\pi N}{}^2}{G_A{}^2}\left(1-\frac{\mu m}{m_\sigma{}^2}-\frac{5\mu}{4m}+\cdots\right),$$

$$a_{3/2}=-\frac{1}{\mu}\,\frac{1}{8\pi}\left(\frac{\mu}{m}\right)^2\frac{g_{\pi N}{}^2}{G_A{}^2}\left(1+2\,\frac{\mu m}{m_\sigma{}^2}+\frac{\mu}{2m}+\cdots\right). \qquad (17)$$

The correction is here of order $\mu/m$, i.e., an order of magnitude larger than in the $\pi\pi$ case, but still quite small ($\lesssim 7\%$).

Another interesting property of the simple model considered here lies in the analogy between the *np* charge-exchange spike and the similar one observed in charged-pion photoproduction. It is indeed well known that the differential cross section for the reactions

$$\gamma+p\to\pi^+ +n,$$
$$\gamma+n\to\pi^- +p \qquad (18)$$

also shows a pronounced very narrow forward peak in a wide energy interval.[30] Here too, if one naively calculates the $\pi$-exchange graph, one finds that its contribution vanishes in the forward direction. But this graph is not gauge invariant by itself. Adding the charged-nucleon Born term (with only the $\gamma_\mu$ coupling to the photon) restores gauge invariance. (This is the so-called "electric Born" model.) Now this is precisely what is needed in order to explain the existence of the spike, its absolute magnitude, and more generally all properties of reactions (18) near the forward direction.[31] In this case, then, gauge invariance forces one to include the nucleon-exchange graph besides the $\pi$-exchange one, and the equality of the $\pi^+$ and $p$ charges yields a quantitative explanation of the behavior of the amplitudes near $t=0$. Something quite similar seems to be at work in *np* charge exchange: $t$-channel $\pi$ exchange has to be complemented by the $u$-channel $\pi$ and $\sigma$ Born terms, the equality between the $\bar{N}N\pi$ and $\bar{N}N\sigma$ couplings being essential to obtain a quantitative explanation of the magnitude and shape of the spike.

Perhaps it is worth emphasizing again that such a Born-term model should actually be understood as the result of a local (approximate) cancellation of the various contributions to the $u$-channel absorptive parts (see the end of Sec. II).

What about the value (15) of the $\sigma$ mass? Basically, the general properties of the $\sigma$ model are the same whatever the $\sigma$ mass. More precisely (see, for instance, Ref. 24), fixing the nucleon and $\pi$ masses, the $\bar{N}N\pi$ coupling constant and the Goldberger-Treiman relation leaves one free parameter, which one may take as the $\sigma$ mass. It is worth noting that although the $\sigma\pi\pi$ coupling constant strongly depends on this free parameter [see, for instance, Eqs. (3d-1) of Ref. 24], the results one is generally interested in, such as (16) and (17), are practically independent of $m_\sigma$ as soon as it is not too low. Furthermore, early studies of spontaneous breakdown of chiral symmetry[32] gave, besides a $\pi$ with vanishing mass, an $I=0$ scalar particle having precisely mass $2m$.[33] This might not be just a coincidence: Perhaps the $\sigma$ model can actually be considered as the Yukawa-type version of a more fundamental Fermi-type theory, the $\pi$ and $\sigma$ appearing as $\bar{N}N$ bound states with vanishing renormalization constants[34]; although a Fermi interaction is highly unrenormalizable, this could be the origin of the value (15) for the $\sigma$ mass.

On the other hand, from a phenomenological point of view, the value (15) may look incompatible with the results of the $\pi\pi$ phase-shift analyses, which show the presence of an $I=0$ $S$-wave object at a much lower mass.[35] It is not evident, though, that the "particle" of the $\sigma$ model has to be identified with this object. Of course, such an identification is perfectly reasonable and natural, and the results of Ref. 36 give it substantial support. These results are based, however, on a particular approximation, and are perhaps not so compelling in what concerns the $\sigma$ mass as they look at first sight.[37] Furthermore, in other situations, similar calculations have been performed directly in the nonlinear $\sigma$ model, i.e., with $m_\sigma$ infinite, and good agreement with experiment is also claimed.[38] Within the framework of the above discussion, the $S$-wave $\pi\pi$ resonance near 700 MeV should participate, like the $\rho, \omega$, etc., that can be exchanged in the $u$ channel, to the approximate cancellation of the absorptive parts, whereas the singularity at the $\bar{N}N$ threshold is associated with the scalar "particle" of the $\sigma$ model.

## IV. CONCLUSION

The existence and shape of the forward spike observed in *np* charge exchange implies very special properties for the three amplitudes contributing at $t=0$ (besides the $t$-channel $\pi$-exchange term): These amplitudes are practically independent of energy; two of them remain negligible and the third one is proportional to the $\pi N$ coupling constant, although it receives no contribution from $t$-channel $\pi$ exchange. These striking properties reflect a very simple singularity structure for the invariant amplitudes at $t=0$: The absorptive parts are small everywhere except around the physical thresholds for the $s$ channel and $u$ channel. The singularity near $s=4m^2$ comes essentially from the $u$-channel $\pi^0$ Born term. The one around $u=4m^2$ is more surprising. If one analyzes it as being due to a one-particle exchange in the $u$-channel, one finds that this "particle" should have isospin 0, $G$ parity +, and spin parity $0^+$; furthermore, its coupling to the nucleon is equal to that of the pion. Thus, the $np$ charge-exchange forward spike can be given a very simple interpretation within the framework of the $\sigma$ model: It appears as being due to the "tree graphs," i.e., to the $\pi$ and $\sigma$ Born terms.

## APPENDIX

We assume here that Eqs. (11), (12), and (13) above result from the $u$-channel exchange of a particle $S$ with mass $M_S \simeq 2m$, and we look for the intrinsic properties of this particle.

The starting point is the $t=0$ values of the invariant amplitudes $\hat{G}_i$, which, however, depend on 2 unknown parameters:

$$\hat{G}_1^{(S)}(u, t=0) = \frac{\beta}{2m^2} F(u), \tag{A1a}$$

$$\hat{G}_2^{(S)}(u, t=0) = -\frac{1}{2m^2} F(u), \tag{A1b}$$

$$\hat{G}_3^{(S)}(u, t=0) = \frac{1}{2m^2} F(u), \tag{A1c}$$

$$\hat{G}_4^{(S)}(u, t=0) = -\frac{\alpha}{2m^2} F(u), \tag{A1d}$$

$$\hat{G}_5^{(S)}(u, t=0) = \frac{\alpha}{2m^2} F(u), \tag{A1e}$$

where

$$F(u) = \frac{g^2}{4\pi}\frac{m^2}{u - M_S{}^2}. \tag{A2}$$

It is easy to calculate the $u$-channel helicity amplitudes from Eqs. (A1). First, the invariant functions $\overline{G}_i$ defined in Ref. 8 are linear combinations of the $\hat{G}_i$, so that

$$\overline{G}_i^{(S)} = \frac{\epsilon}{2}\sum_{j=1}^{5} (\Delta^{su}\Delta^{st})_{ij}\hat{G}_j^{(S)}. \tag{A3}$$

In this formula, $\epsilon=+1$ if the isospin of the particle is $I_S=0$, $\epsilon=-1$ if $I_S=1$; $\Delta^{su}$ and $\Delta^{st}$ are crossing matrices, $\Delta^{su}$ being identical with the matrix $\Delta$ defined in Ref. 8 [Eq. (4-28)], while $\Delta^{st}$ is such that

$$(\Delta^{st})_{ij} = (-1)^{i+j+1}(\Delta^{su})_{ij}. \tag{A4}$$

The "parity-conserving"[39] $u$-channel helicity amplitudes $\overline{f}_i$ are then obtained from the $\overline{G}_i$ by formulas similar to Eqs. (4–23) of Ref. 8. The final results are

$$\overline{f}_1^{(S)}(u, t=0) = \frac{\epsilon}{2}\overline{p}^2\frac{1-\alpha}{2m^2}F(u), \tag{A5a}$$

$$\overline{f}_2^{(S)}(u, t=0) = -\frac{\epsilon}{2}\overline{p}^2\frac{3+\alpha}{2m^2}F(u), \tag{A5b}$$

$$\overline{f}_3^{(S)}(u, t=0) = \frac{\epsilon}{8}\overline{p}^2\frac{2-3\alpha+\beta}{2m^2}F(u), \tag{A5c}$$

$$\overline{f}_4^{(S)}(u, t=0) = \frac{\epsilon}{8}\overline{p}^2\frac{2-\alpha-\beta}{2m^2}F(u), \tag{A5d}$$

$$\overline{f}_5^{(S)}(u, t=0) = 0, \tag{A5e}$$

where $\overline{p}$ is the $u$-channel center-of-mass momentum.

Now the $\overline{f}_i$ can be expanded in partial waves (see, for instance, Appendix A of Ref. 39):

$$\overline{f}_1(u, \overline{z}) = \sum_J (2J+1)P_J(\overline{z})\overline{f}_0^{J-}(u), \tag{A6a}$$

$$\overline{f}_2(u, \overline{z}) = \sum_J (2J+1)P_J(\overline{z})\overline{f}_{00}^{J+}(u), \tag{A6b}$$

$$\overline{f}_3(u, \overline{z}) = \sum_J \frac{2J+1}{J(J+1)}\{[P_J'(\overline{z}) + \overline{z}P_J''(\overline{z})]\overline{f}_1^{J-}(u) - P_J''(\overline{z})\overline{f}_{11}^{J+}(u)\}, \tag{A6c}$$

$$\overline{f}_4(u, \overline{z}) = \sum_J \frac{2J+1}{J(J+1)}\{[P_J'(\overline{z}) + \overline{z}P_J''(\overline{z})]\overline{f}_{11}^{J+}(u) - P_J''(\overline{z})\overline{f}_1^{J-}(u)\}, \tag{A6d}$$

$$\overline{f}_5(u, \overline{z}) = -\frac{2m}{\sqrt{u}}\sum_J \frac{2J+1}{[J(J+1)]^{1/2}}P_J'(\overline{z})\overline{f}_{10}^{J+}(u). \tag{A6e}$$

$\overline{f}_0^{J-}$ and $\overline{f}_1^{J-}$ correspond to orbital angular momentum $l=J$ and to total spin 0 and 1, respectively: They have unnatural parity $(-1)^{J+1}$ and charge conjugation $(-1)^J$ and $(-1)^{J+1}$, respectively. $\overline{f}_{00}^{J+}$, $\overline{f}_{11}^{J+}$, and $\overline{f}_{10}^{J+}$ have $l=J\pm1$, natural parity $(-1)^J$, and charge conjugation $(-1)^J$. $t=0$ means $\overline{z}=1$, and one knows that

$$P_J(1) = 1,$$
$$P_J'(1) = \tfrac{1}{2}J(J+1), \tag{A7}$$
$$P_J''(1) = \tfrac{1}{8}J(J+1)[J(J+1)-2].$$

(The last two expressions are, for instance, easily deduced from the differential equation satisfied by the Legendre polynomials.)

Suppose first that the particle $S$ has spin $J$ and unnatural parity. In this case, either $\overline{f}_0^{J-}$ or $\overline{f}_1^{J-}$ is the only partial wave different from zero. If the only contribution to Eqs. (A6) comes from $\overline{f}_0^{J-}$, then $\overline{f}_2^{(S)}$, $\overline{f}_3^{(S)}$, and $\overline{f}_4^{(S)}$ should vanish simultaneously, which is incompatible with the expressions (A5). On the other hand, these expressions show that $\bar{f}_1^{(S)}$ and $\bar{f}_2^{(S)}$ cannot be zero at the same time, which also excludes the possibility associated with $\bar{f}_1^{J-}$. So the particle $S$ cannot have unnatural parity.

Consider then the case where $S$ would have spin $J$ and natural parity. In that case, $\bar{f}_{00}^{J+}$, $\bar{f}_{11}^{J+}$, and $\bar{f}_{10}^{J+}$ would in principle contribute all three to the $\bar{f}_i$. Now Eqs. (A5e) and (A6e) indicate that $\bar{f}_{10}^{J+}$ actually vanishes, so that, because of factorization, either $\bar{f}_{00}^{J+}$ or $\bar{f}_{11}^{J+}$ must also be zero. $\bar{f}_{00}^{J+}=0$ is excluded, because $\bar{f}_1^{(S)}$ and $\bar{f}_2^{(S)}$ cannot vanish simultaneously, so that we take

$$\bar{f}_{00}^{J+} = -\frac{a}{2J+1}\left(\frac{\bar{p}^2}{m^2}\right)^{J+1} F(u)\,, \qquad \bar{f}_{11}^{J+} = \bar{f}_{10}^{J+} = 0\,, \tag{A8}$$

where $a$ is a positive constant (since two out of these three partial waves vanish, the third one must be proportional to $\bar{p}^{2(J+1)}$ in order to avoid kinematical singularities in the invariant amplitudes). This gives

$$\bar{f}_1^{(S)} = \bar{f}_3^{(S)} = \bar{f}_4^{(S)} = \bar{f}_5^{(S)} = 0\,, \qquad \bar{f}_2^{(S)}(u, \bar{z}=1) = -a\left(\frac{\bar{p}^2}{m^2}\right)^{J+1} F(u)\,. \tag{A9}$$

These values can be identified with expressions (A5), yielding

$$\epsilon = +1\,, \tag{A10a}$$

$$\alpha = \beta = 1\,, \tag{A10b}$$

$$\left(\frac{\bar{p}^2}{m^2}\right)^J a = 1\,. \tag{A10c}$$

Since $\epsilon = +1$, the isospin is $I_S = 0$. In principle, all spins $J$ are possible. It is to be noticed, however, that $u$ should be replaced by $M_S{}^2 \simeq 4m^2$ in Eq. (A10c), which practically excludes all values of $J$ except zero: $\bar{p}^2$ being zero or very small for $u = M_S{}^2$, the coupling constant deduced from (A10c) for $J \neq 0$ would be completely unreasonable. So the only possibility left is $J^P = 0^+$.

The particle $S$ with mass $M_S \simeq 2m$ must then be an isospin zero, $G$ parity +, scalar meson. Equations (A10c), (A8), and (A2) show that its coupling to the nucleons is just $g^2/4\pi$.

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