New narrow boson resonances and SU(4) symmetry: Selection rules, SU(4) mixing, and mass formulas

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General SU(4) sum rules are obtained for bosons in the theoretical framework of asymptotic SU(4), chiral SU(4) ⊗ SU(4) charge algebra, and a simple mechanism of SU(4) and chiral SU(4) ⊗ SU(4) breaking. The sum rules exhibit a remarkable interplay of the masses, SU(4) mixing angles, and axial-vector matrix elements of 16-plet boson multiplets. Under a particular circumstance (i.e., in the "ideal" limit) this interplay produces selection rules which may explain the remarkable stability of the newly found narrow boson resonances. General SU(4) mass formulas and inter-SU(4)-multiplet mass relations are derived and SU(4) mixing parameters are completely determined. Ground state 1^{--} and 0^{-+} 16-plets are especially discussed and the masses of charmed and uncharmed new members of these multiplets are predicted.

I. INTRODUCTION

The discovery of new narrow resonances[1] aroused an interest in classifying bosons in terms of the $15\oplus 1$ representation of the SU(4) group.[2] The 16-plet boson multiplet will be denoted by $B_{\alpha,s}$, where s denotes J^{PC} and α denotes the physical SU(4) index, i.e., $\alpha=\pi$, K, η, η_c, D, F, and η'. (D_s^+, D_s^0) and F_s^+ denote the *charm*-carrying $I=\frac{1}{2}$ and $I=0$ members and η_s, η_{cs}, and η_s' denote the $I=0$ nonstrange and uncharmed members of the 15-plet and singlet, respectively.

One of the purposes of this paper is to give a *general* derivation of SU(4) sum rules, previously utilized[3] to explain the new narrow resonances, which hold for any 16-plet SU(4) multiplet and exhibit the following remarkable *interplay* of the masses of a 16-plet multiplet $B_{\alpha,s}$, SU(4) η_s-η_s'-η_{cs} mixing angles, and the axial-vector matrix elements[3]; namely, if a 16-plet multiplet satisfies the so-called "ideal"[4] nonet mass constraints (we use the notation $\pi_s^2=m_{\pi_s}^2$, etc.), $\pi_s^2=\eta_s'^2$ and $\eta_s^2-K_s^2=K_s^2-\pi_s^2$, our sum rules *require* that the η_s-η_s'-η_{cs} mixing angles take the "ideal" values and the η_s, η_{cs}, and η_s' will then belong to the "ideal" configurations, $s\bar{s}$, $c\bar{c}$, and $(1/\sqrt{2})(u\bar{u}+d\bar{d})$, respectively, in the $q\bar{q}$ description of bosons. Furthermore, our sum rules contain the following selection rules: The couplings $\eta_s\to$ any *nonstrange* meson + pseudoscalar meson (π,K) and $\eta_{cs}\to$ any *uncharmed* meson + pseudoscalar meson (π,K,η) are forbidden in our "ideal" limit, $\pi_s^2=\eta_s'^2$ and $\eta_s^2-K_s^2=K_s^2-\pi_s^2$. The leakage from the "ideal structure" of any 16-plet $B_{\alpha,s}$ may be measured crudely by the degree of *deviation* of the quantities $\eta_s'^2-\pi_s^2$ and $\Delta_s^2\equiv\eta_s^2-2K_s^2+\pi_s^2$ from *zero*.

Therefore, if we assign, for the sake of argument, the recently discovered narrow resonances $\psi(3105)$ and $\psi(3695)$ to the η_c members of the ground state 1^{--} (including ρ, K^*, ω, and ϕ) and its excited 1^{--} state, respectively, we may explain the narrow[5,6,7] widths of $\psi(3105)$ and $\psi(3695)$, *provided* that these 16-plet 1^{--} mesons satisfy the "ideal" mass constraints *well*. Experimentally the "ideal" constraints for the ground state 1^{--}, $\rho^2=\omega^2$ and $\phi^2-K^{*2}=K^{*2}-\rho^2$, are indeed well satisfied, and one may even suspect that the small violation in this case can be blamed for the SU(2) breaking which we have to neglect at present. Therefore, the stability of $\psi(3105)$ may be explained reasonably well by the above-mentioned selection rules obtained in our approach.[3]

Our SU(4) sum rules also predict the SU(6)-[perhaps now SU(8)-] like[8] (but more general) intermultiplet mass relations among the 16-plet boson spectra. Our sum rules determine the masses of the charmed members D^* and F^* of the ground state 1^{--} multiplet, once $\psi(3105)$ is assigned to its η_c member. Then for any "ideal" 16-plet multiplet $B_{\alpha,s}$, one can determine the mass of each member of the $B_{\alpha,s}$ if the mass of π_s is given. Even for *non-"ideal"* multiplet $B_{\alpha,s}$, the masses of η_{cs}, D_s, and F_s can be determined if the masses of π_s, η_s, and η_s' are given. The η_s-η_{cs}-η_s' mixing parameters are *completely* determined from our sum rules. In Sec. III B we predict an intermultiplet mass relation among the "ideal" 16-plets. Some speculation is added to the 1^{--} multiplet involving the $\psi(3695)$. We especially discuss the masses and mixing angles of the ground state 1^{--} and 0^{-+} mesons, which were also discussed recently by Mathur, Okubo, and Borchardt[5,9] by using an entirely different SU(4) approach.

II. DERIVATION OF GENERAL SUM RULES IN SU(4)

Our theoretical ingredients are simply as follows[3]: (i) asymptotic SU(4), (ii) chiral SU(4)⊗SU(4) charge algebra, and (iii) simple mechanism of SU(4) and chiral SU(4)⊗SU(4) symmetry breaking.

To cope with broken SU(4) [which is certainly more broken than SU(3)] we use asymptotic SU(4)[3]: "The annihilation operator $a_\alpha(\vec{k},\lambda)$ of $B_{\alpha,s}$ with physical SU(4) index α and helicity λ does transform *linearly* [including SU(4) mixing] under SU(4), but only in the limit $\vec{k}\to\infty$."

The η_s-η_{cs}-η'_s mixing parameters will then be defined,[3,10] among the physical operators $a^s_\eta(\vec{k})$, $a^s_{\eta'}(\vec{k})$, and $a^s_{\eta_{cs}}(\vec{k})$ and the hypothetical representation operators $a^s_8(\vec{k})$, $a^s_0(\vec{k})$, and $a^s_{15}(\vec{k})$, in the limit $\vec{k}\to\infty$ by

$$\begin{pmatrix} a^s_\eta \\ a^s_{\eta'} \\ a^s_{\eta_c} \end{pmatrix} = \begin{pmatrix} \alpha^s & \alpha'^s & \alpha^s_c \\ \beta^s & \beta'^s & \beta^s_c \\ \gamma^s & \gamma'^s & \gamma^s_c \end{pmatrix} \begin{pmatrix} a^s_8 \\ a^s_0 \\ a^s_{15} \end{pmatrix}, \tag{1}$$

where

$$\begin{pmatrix} \alpha^s & \alpha'^s & \alpha^s_c \\ \beta^s & \beta'^s & \beta^s_c \\ \gamma^s & \gamma'^s & \gamma^s_c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\psi_s & -\sin\psi_s \\ 0 & \sin\psi_s & \cos\psi_s \end{pmatrix} \begin{pmatrix} \cos\theta_s & -\sin\theta_s & 0 \\ \sin\theta_s & \cos\theta_s & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_s & \sin\phi_s \\ 0 & -\sin\phi_s & \cos\phi_s \end{pmatrix}. \tag{2}$$

With the imposition of the commutation relations $[V_i, V_j] = if_{ijk}V_k$ and $[V_i, A_j] = if_{ijk}A_k$ $(i,j,k = 1,2,\ldots,15)$, our asymptotic SU(4) leads to the following simple result [hereafter denoted by result (a)][10]: "The vector and axial-vector matrix element but taken only between the states all of which have infinite momentum, such as $\langle B_{\alpha,s}(\vec{k},\lambda)|V_i$ and $A_i|B_{\beta,s}(\vec{k},\lambda)\rangle$ with $\vec{k}\to\infty$, can still be parametrized in *broken* SU(4) by the prescription of *exact* SU(4) plus mixing."

As the algebraic expressions of SU(4) and chiral SU(4)⊗SU(4) breaking, we assume[3,10] the presence of the *exotic* commutation relations of the form $[\dot{V}_\alpha = (d/dt)V_\alpha]$, $[\dot{V}_\alpha, V_\beta] = 0$, and $[\dot{V}_\alpha, A_\beta] = 0$, where (α,β) stands for such combinations[11] as (K^0,K^0), (K^0,D^0), (K^0,F^+), (F^-,D^-), (D^0,π^-), (F^-,π^-), (K^0,π^-), etc., i.e., the combined SU(4) structure of the SU(4) indices α and β does not belong to a 15-plet of SU(4). $[\dot{V}_\alpha, V_\beta] = 0$ and $[\dot{V}_\alpha, A_\beta] = 0$ are weaker assumptions than the usual pure $(4,4^*)+(4^*,4)$ breaking.

The realization of all the exotic commutation relations $[\dot{V}_\alpha, V_\beta] = 0$ (in the limit $\vec{k}\to\infty$) among all the possible single particle states of $B_{\alpha,s}$ leads to the following four *independent* SU(4) constraints[3] involving the masses and SU(4) mixing angles:

$$(\alpha^s)^2\eta_s^{\,2} + (\beta^s)^2\eta_s'^{\,2} + (\gamma^s)^2\eta_{cs}^{\,2} = \tfrac{1}{3}(4K_s^{\,2} - \pi_s^{\,2}), \tag{3}$$

$$\alpha^s\alpha^s_c\eta_s^{\,2} + \beta^s\beta^s_c\eta_s'^{\,2} + \gamma^s\gamma^s_c\eta_{cs}^{\,2} = \frac{-\sqrt{2}}{3}(K_s^{\,2} - \pi_s^{\,2}), \tag{4}$$

$$(\alpha^s_c)^2\eta_s^{\,2} + (\beta^s_c)^2\eta_s'^{\,2} + (\gamma^s_c)^2\eta_{cs}^{\,2} = \tfrac{3}{2}(D_s^{\,2} + \tfrac{1}{9}K_s^{\,2} - \tfrac{4}{9}\pi_s^{\,2}), \tag{5}$$

$$F_s^{\,2} - K_s^{\,2} = D_s^{\,2} - \pi_s^{\,2}. \tag{6}$$

These SU(4) mass relations are *exact* (they are not the *first*-order perturbation-theoretic formulas) and should hold in mass-squared form.[12] Equation (3) is the direct extension of the familiar SU(3) mass formula. We need, however, one more constraint to determine completely the η_s-η'_s-η_{cs} mixing parameters from the masses $\pi_s^{\,2}$, $K_s^{\,2}$, $\eta_s'^{\,2}$, and $\eta_{cs}^{\,2}$.

We now realize the exotic commutation relations $[V_\alpha, A_\beta] = 0$ and $[\dot{V}_\alpha, A_\beta] = 0$ among all the possible states $\langle B_s(\vec{k},\lambda)|$ and $|B_t(\vec{k},\lambda)\rangle$ with $C_sC_t = -1$ and also among $\langle B_s(\vec{k},\lambda)|$ and $|B_u(\vec{k},\lambda)\rangle$ with $C_sC_u = 1$ in our asymptotic limit $\vec{k}\to\infty$.

A. $C_sC_t = -1$

We obtain[13] asymptotic SU(4) relations [i.e., special cases of the result (a)] of the matrix elements of A_{π^-}, i.e.,

$$-\langle K^0_s(\vec{k})|A_{\pi^-}|K^+_t\rangle = \sqrt{\tfrac{1}{2}}\langle \pi^0_s(\vec{k})|A_{\pi^-}|\pi^+_t\rangle$$

and

$$\frac{1}{\sqrt{2}}\langle \pi^0_s(\vec{k})|A_{\pi^-}|\pi^+_t\rangle = \langle D^0_s(\vec{k})|A_{\pi^-}|D^+_t\rangle, \quad \vec{k}\to\infty,$$

and also the intermultiplet mass relations

$$K_s^{\,2} - \pi_s^{\,2} = K_t^{\,2} - \pi_t^{\,2}$$

and

$$D_s^{\,2} - \pi_s^{\,2} = D_t^{\,2} - \pi_t^{\,2}.$$

By repeating the same procedure among 16-plets, sometimes in a hybrid way, we obtain *general* intermultiplet mass relations,

$$K_s^2 - \pi_s^2 = \text{const} \quad (s \text{ is arbitrary}), \tag{7}$$

and

$$D_s^2 - \pi_s^2 = \text{const} \quad (s \text{ is arbitrary}). \tag{8}$$

These are the SU(6)- [now perhaps SU(8)-] like mass relations which should be valid among 16-plet mass spectra. We note that $K^2 - \pi^2 = K^{*2} - \rho^2 = K^{**2} - A_2^2$ are well satisfied.[14] No fruther constraints are obtained from $[\dot{V}_\alpha, A_\beta] = 0$ for the case $C_s C_t = -1$.

B. $C_s C_u = 1$

Define $A_{su} \equiv \langle \eta_s(\vec{k}) | A_{\pi^-} | \pi_u^+ \rangle$, $B_{su} \equiv \langle \eta_s'(\vec{k}) | A_{\pi^-} | \pi_u^+ \rangle$, and $C_{su} = \langle \eta_{cs}(\vec{k}) | A_{\pi^-} | \pi_u^+ \rangle$ with $\vec{k} \to \infty$ and $X_\alpha^s = \alpha^s - \sqrt{2}\,\alpha_c^s$ and $X_\beta^s \equiv \beta^s - \sqrt{2}\,\beta_c^s$ and $X_\gamma^s \equiv \gamma^s - \sqrt{2}\,\gamma_c^s$. We then obtain[15] the following *constraints* upon the matrix elements of axial charge $A_{\pi^\pm}$ involving the states η_s, η_s', and η_{cs}:

$$\frac{A_{su}}{B_{su}} \equiv \frac{\langle \eta_s | A_{\pi^-} | \pi_u^+ \rangle}{\langle \eta_s' | A_{\pi^-} | \pi_u^+ \rangle} = -\left(\frac{X_\beta^s}{X_\alpha^s}\right)\left(\frac{\eta_{cs}^2 - \eta_s'^2}{\eta_{cs}^2 - \eta_s^2}\right), \tag{9}$$

$$\frac{C_{su}}{B_{su}} \equiv \frac{\langle \eta_{cs} | A_{\pi^-} | \pi_u^+ \rangle}{\langle \eta_s' | A_{\pi^-} | \pi_u^+ \rangle} = -\left(\frac{X_\beta^s}{X_\gamma^s}\right)\left(\frac{\eta_s^2 - \eta_s'^2}{\eta_s^2 - \eta_{cs}^2}\right), \tag{10}$$

and another mass constraint[16] which should be added to Eqs. (3)–(6),

$$\alpha^s X_\beta^s X_\gamma^s (\eta_s^2 - \pi_s^2)(\eta_s'^2 - \eta_{cs}^2) + \beta^s X_\alpha^s X_\gamma^s (\eta_s'^2 - \pi_s^2)(\eta_{cs}^2 - \eta_s^2) + \gamma^s X_\alpha^s X_\beta^s (\eta_{cs}^2 - \pi_s^2)(\eta_s^2 - \eta_s'^2) = 0. \tag{11}$$

We call attention to the *general* nature of our constraints, Eqs. (9) and (10). The right-hand sides of these equations do not depend on u and both s and u are *arbitrary*, provided $C_s C_u = +1$. No further independent constraints are obtained.

While Eqs. (3), (4), (5), (6), and (11) represent the constraints upon the masses and η_s-η_{cs}-η_s' mixing parameters of the 16-plet B_s and Eqs. (7) and (8) are the intermultiplet mass constraints, Eqs. (9) and (10) provide the constraints, involving the masses and mixing angles of the 16-plet $B_{\alpha,s}$, imposed upon the axial-vector matrix elements, A_{su}, B_{su}, and C_{su} involving the states η_s, η_s', and η_{cs}. Other matrix elements, $\langle B_s | A_i | B_u(\vec{k}) \rangle$ with $\vec{k} \to \infty$, are related to A_{su}, B_{su}, and C_{su} by SU(4) rotation according to our general result (a).

III. "IDEAL" 16-PLET MESONS—SELECTION RULES AND MASSES

A. "Ideal" structure and selection rules

Our set of constraints exhibits the following remarkable interplay between the masses, SU(4) mixing angles, and the axial-vector matrix elements: If the "ideal" nonet mass constraints[17] for the 16-plet $B_{\alpha,s}$, $\eta_s'^2 = \pi_s^2$ and $\eta_s^2 - 2K_s^2 + \pi_s^2 = 0$, are *satisfied* or *imposed*, we can easily see[3] from our sum rules that

$$X_\beta^s \equiv \beta^s - \sqrt{2}\,\beta_c^s = 0, \tag{12}$$

$$A_{su} \equiv \langle \eta_s | A_{\pi^-} | \pi_u^+(\vec{k}) \rangle = 0 \quad (\vec{k} \to \infty), \quad C_s C_u = 1, \tag{13}$$

$$C_{su} \equiv \langle \eta_{cs} | A_{\pi^-} | \pi_u^+(\vec{k}) \rangle = 0 \quad (\vec{k} \to \infty), \quad C_s C_u = 1, \tag{14}$$

and

$$\sin\theta = \sqrt{\tfrac{1}{3}}, \quad \sin\phi = \tfrac{1}{2}, \text{ and } \psi = 0. \tag{15}$$

We call the mixing angles which satisfy Eq. (15) "ideal" mixing angles, θ_i, ϕ_i, and ψ_i.

Correspondence to the quark picture is as follows.[3] From the configuration $\phi_8 = \sqrt{\frac{1}{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$, $\phi_{15} = \sqrt{\frac{1}{12}}(u\bar{u} + d\bar{d} + s\bar{s} - 3c\bar{c})$, and $\phi_0 = \frac{1}{2}(u\bar{u} + d\bar{d} + s\bar{s} + c\bar{c})$, the first rotation by the angle ϕ_i brings ϕ_{15} to the pure $c\bar{c}$ state, while the subsequent rotation by θ_i brings ϕ_8 and ϕ_0 to the pure $s\bar{s}$ and $(1/\sqrt{2})(u\bar{u} + d\bar{d})$ states, respectively. This configuration is called "ideal" and in this configuration the mass relations $\pi_s^2 = \eta_s'^2$ and $\eta_s^2 - 2K_s^2 + \pi_s^2 = 0$ are satisfied, and furthermore, the particular axial-vector matrix elements, A_{su} and C_{su}, vanish.

With PCAC, $A_{su} = 0$ implies the vanishing of $\eta_s \to \pi_u^+ + \pi^-$ couplings.[10] For $s = 1^{--}$, in the ideal nonet limit $\rho^2 = \omega^2$ and $\phi^2 - 2K^{*2} + \rho^2 = 0$, $A_{su} = 0$ implies that $\phi \to \rho\pi(u = 1^{--})$ and $\phi \to B\pi(u = 1^{+-})$ couplings, etc., are zero. The smallness of these couplings is impressively indicated by experiment.[14] For $s = 2^{++}$ and $u = 0^{-+}$, the approximate experimental[14] realization of ideal constraints, $A_2^2 \simeq f^2$ and $f'^2 - 2K^{**2} + A_2^2 \simeq 0$, implies the vanishing of the $f' \to \pi\pi$ coupling consistent with experiment.[14]

The vanishing of the matrix element C_{su} leads to the vanishing of the $\eta_{cs} \to \pi_u^+ + \pi^-$ couplings. If we assign $\psi(3105)$ to the η_c member of the ground state 1^{--} meson, then $\psi(3105) \to \rho\pi(s = u = 1^{--})$ and $\psi(3105) \to B\pi(s = 1^{--}, u = 1^{+-})$ couplings, etc., vanish in the "ideal" limit. By SU(4) rotation we demonstrate that the $\eta_{cs} \to$ uncharmed B_u + pseudoscalar (π, K, η) coupling also vanishes in our ideal limit as follows.

(i) $C_s C_u = 1$

According to our result (a) (in the limit $\vec{k} \to \infty$),

$$\langle \eta_{cs} | A_{K^0} | \bar{K}_u^0 \rangle = C_{su} - \tfrac{3}{2}\gamma_s(\alpha_s A_{su} + \beta_s B_{su} + \gamma_s C_{su}), \tag{16}$$

$$\langle \eta_{cs}|A_\eta|\eta_u\rangle = \frac{1}{\sqrt{2}}(\gamma_s\langle K_s^0|A_{K^0}|\eta_u\rangle + \alpha_u\langle\eta_{cs}|A_{K^0}|\bar{K}_u^0\rangle)\,, \tag{17}$$

$$\langle \eta_{cs}|A_\eta|\eta_u'\rangle = \frac{1}{\sqrt{2}}(\gamma_s\langle K_s^0|A_{K^0}|\eta_u'\rangle + \beta_u\langle\eta_{cs}|A_{K^0}|\bar{K}_u^0\rangle)\,, \tag{18}$$

$$\langle \eta_{cs}|A_\eta|\eta_{cu}\rangle = \frac{1}{\sqrt{2}}(\gamma_s\langle K_s^0|A_{K^0}|\eta_{cu}\rangle + \gamma_u\langle\eta_{cs}|A_{K^0}|\bar{K}_u^0\rangle)\,. \tag{19}$$

All these matrix elements *vanish* in our "ideal limit," because then $C_{su}=0$ and $\gamma_s=0$. For the $\psi(3105)$ this implies that its decays into K_B^*K (K_B^* is the $I=\frac{1}{2}$ counterpart of the B meson), $\phi\eta$, $\omega\eta$, $\phi_B\eta$, and $\omega_B\eta$ (ϕ_B and ω_B are the η and η' members of 1^{+-}), etc., are forbidden in our "ideal" limit.[18]

(ii) $C_sC_t=-1$

Analogous to Eqs. (16)–(19), we obtain ($\vec{k}\to\infty$)

$$\langle\eta_{cs}|A_{K^+}|K_t^-(\vec{k})\rangle = \sqrt{\tfrac{3}{2}}\gamma_s\langle K_s^0|A_{\pi^+}|K_t^-(\vec{k})\rangle\,. \tag{20}$$

Thus in the "ideal" limit for the 16-plet $B_{\alpha,s}$ (i.e., $\gamma_s=0$), $\langle\eta_{cs}|A_{K^+}|K_t^-\rangle$ always vanishes for any t. In the case of $\eta_{cs}=\psi(3105)$, this implies that the $\psi(3105)$ decays into KK, $KK^{**}(1420)$, KK_A (K_A is the $I=\frac{1}{2}$ member of 1^{++} meson), etc., are all forbidden.

Therefore, we have demonstrated that the couplings, $\eta_{cs}\to$ any uncharmed boson + pseudoscalar (π, K, η), are forbidden[18] in our theoretical framework, as long as the 16-plet $B_{\alpha,s}$ belongs to an "ideal" nonet. We note that this selection rule is a *theoretical* consequence and is not a consequence of particular postulate[4] or a rule.[6]

Since the *ground* state 1^{--} seems to satisfy the ideal nonet mass constraints very well [after making an allowance for the SU(2) breaking in masses], its η_c member may exhibit a surprisingly narrow width in spite of its large mass, according to the selection rules derived above. Therefore, if SU(4) is correct we may expect the existence of such high-mass narrow resonances as $\psi(3105)$ and $\psi(3695)$, if some low-lying 16-plet mesons satisfy the "ideal" mass constraints well.

B. Mass formulas for "ideal" 16-plets

With the "ideal" mixing angles [Eq. (15)] and the "ideal" mass constraints, $\eta_s'^2=\pi_s^2$ and $\eta_s^2-2K_s^2+\pi_s^2=0$, Eq. (5) leads to an ideal 16-plet mass formula,

$$\eta_{cs}^2 = 2D_s^2-\pi_s^2 \quad (B_s \text{ is ideal})\,. \tag{21}$$

This serves to determine the mass of charmed meson D_s once π_s and η_{cs} are given. Equation (6) then predicts the mass of F_s by $F_s^2=D_s^2+(K_s^2-\pi_s^2)$. Note that $K_s^2-\pi_s^2=K^2-\pi^2$ [Eq. (7)]. Suppose that another 16-plet B_t is also "ideal." Then $\eta_{ct}^2=2D_t^2-\pi_t^2$ and we obtain a simple "inter-ideal multiplet" mass relation using Eq. (8),

$$\eta_{cs}^2-\eta_{ct}^2=\pi_s^2-\pi_t^2 \quad (B_s \text{ and } B_t \text{ are "ideal"})\,. \tag{22}$$

Since 2^{++} mesons satisfy the "ideal" mass constraints reasonably well,[14] we can determine the mass of the η_c member of the 2^{++} 16-plet by using Eqs. (21) and (22), i.e., $\eta_c^2(2^{++})-\psi^2(3105)\simeq A_2^2-\rho^2$. We predict $\eta_c(2^{++})=3.28$ GeV.

Suppose now that $\psi(3695)$ belongs to the η_c member of an excited state of the ground state 1^{--} 16-plet. Then this multiplet should satisfy a fairly good "ideal" configuration. However, the assignment of the observed $\rho'(1600)$ to the $I=1$ member of this 16-plet leads to a noticeable contradiction with Eqs. (21) and (22), namely

$$\frac{\psi^2(3695)-\psi^2(3105)}{\rho'^2(1600)-\rho^2(770)}\simeq 2\,. \tag{23}$$

Therefore, this assignment of ρ' implies that the multiplet deviates significantly from the "ideal" configuration and $\psi(3695)$ cannot be stable. Then it seems more realistic to assume the existence of the third ρ meson corresponding to the narrow resonance $\psi(3695)$. Our "inter-ideal multiplet" mass relation, Eqs. (21) and (22), places the mass of the above-mentioned third ρ at around 2.127 GeV. It is amusing to notice that $\rho_1\equiv\rho_0(770)$ and $\rho_2\equiv\rho'(1600)$ and $\rho_3\equiv\rho(2127)$ satisfy an equal mass-squared spacing

$$\rho_2^2-\rho_1^2\simeq\rho_3^2-\rho_2^2\,. \tag{24}$$

We may then speculate that the $I=1$ members of vector-meson multiplets $(\rho_1,\rho_2,\ldots)$ satisfy the equal mass spacing,[19] $\rho_n^2=\rho_1^2+a^2(n-1)$, $n=1,2,\ldots$, where a^2 is a constant and $\simeq 1.97$ GeV^2. Recently a rather broad resonance $\psi(4150)$ has also been reported in the e^+e^- reactions. We note $\psi^2(4150)-\psi^2(3695)\simeq\psi^2(3695)-\psi^2(3105)$ is also satisfied reasonably well. It may then be natural to identify $\psi(4150)$ as the η_c member of the multiplet involving $\rho_5(3907)$. Then the assignments may be as follows:

$$\begin{array}{ccccc}
\psi_1(3105) & \psi_2(3407?) & \psi_3(3684) & \psi_4(3941?) & \psi_5(4183)\cdots \\
\updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow \\
\rho_1(770) & \rho_2(1600) & \rho_3(2127) & \rho_4(2547) & \rho_5(3907)\cdots\,.
\end{array}$$

While the (ρ_1,ψ_1) and (ρ_3,ψ_3) families should have the "ideal" structure, other families will deviate from the "ideal" structure rather significantly. The mass values of ψ_2 and ψ_4 are obtained assuming "ideal" structure from Eqs. (21) and (22) and thus they only serve to give some feeling about the mass values. ψ_2 and ψ_4 will have broad widths.[20]

IV. REALISTIC 16-PLET MESONS—FORMULAS FOR THE MIXING ANGLES AND MASSES

A. Approximately "ideal" case

We now solve the constraints equation, Eqs. (3), (4), and (11) in the case when the 16-plet $B_{\alpha,s}$ approximately satisfies the "ideal" nonet mass constraints. We thus seek to compute the *first-order* correction to the "ideal" structure. The *first-order* deviations from the "ideal" mixing angles given by Eq. (15) are

$$\delta\theta_s \simeq \frac{1}{2\sqrt{2}}\left(\frac{2\eta_s^2+\eta_s'^2-4K_s^2+\pi_s^2}{\eta_s^2-\eta_s'^2}\right), \quad (25)$$

$$\delta\phi_s \simeq \sqrt{3}\left[\left(\frac{\eta_s^2-2K_s^2+\pi_s^2}{\eta_{cs}^2-\eta_s^2}\right) - \frac{1}{4}\left(\frac{4K_s^2+\eta_s'^2-2\eta_s^2-3\pi_s^2}{\eta_s^2-\pi_s^2}\right)\right], \quad (26)$$

and

$$\psi_s \simeq \sqrt{2}\left[\left(\frac{\eta_s^2-2K_s^2+\pi_s^2}{\eta_{cs}^2-\eta_s^2}\right) - \frac{1}{2}\left(\frac{\eta_s'^2-\pi_s^2}{\eta_{cs}^2-\eta_s'^2}\right)\right]. \quad (27)$$

Substituting the values of θ, ϕ, and ψ corrected by Eqs. (25)–(27) into Eq. (5), we then obtain a mass formula for the *approximately* "ideal" 16-plet B_s,

$$\eta_{cs}^2-\eta_s^2 = \frac{(D_s^2-K_s^2)(\eta_s^2-\pi_s^2)}{(K_s^2-\pi_s^2)+\frac{1}{4}(\eta_s'^2-\pi_s^2)}, \quad (28)$$

which serves to determine D_s, when π_s, η_s, η_s', and η_{cs} are given. The term $\frac{1}{4}(\eta_s'^2-\pi_s^2)$ in Eq. (28) is, of course, very small.

B. Mixing angles and masses when both ψ and θ are small

A well-known exception to the approximately "ideal" structures of observed bosons is the ground state 0^{-+} meson. One of the "ideal" constraints, $\eta_s'^2=\pi_s^2$, is badly violated. The mass structure of 0^{-+} meson suggests that both ψ and θ [which corresponds to the SU(3) η-η' mixing angle] are small.

In this section we study the mixing angles and mass formulas when *both* ψ_s and θ_s are small. We obtain in a self-consistent manner

$$\sin^2\theta_s \simeq \frac{4K_s^2-3\eta_s^2-\pi_s^2}{3(\eta_s'^2-\eta_s^2)}, \quad (29)$$

$$\sin(\phi_s-\psi_s) \simeq -\frac{\theta_s}{\sqrt{2}}\left[\frac{(\eta_s'^2-\eta_s^2)(\eta_{cs}^2-\pi_s^2)}{(\eta_{cs}^2-\eta_s'^2)(\eta_s^2-\pi_s^2)}\right], \quad (30)$$

$$\psi_s \simeq \frac{-\sqrt{2}(K_s^2-\pi_s^2)}{3\theta_s\cos(\phi_s-\psi_s)(\eta_{cs}^2-\eta_s^2)} - \frac{\tan(\phi_s-\psi_s)(\eta_s'^2-\eta_s^2)}{(\eta_{cs}^2-\eta_s^2)}. \quad (31)$$

Again from Eq. (5) we obtain a mass formula for the 16-plet B_s, when *both* θ_s and ψ_s are reasonably *small*,

$$(\eta_{cs}^2-\eta_s'^2)[\eta_{cs}^2-\tfrac{3}{2}(D_s^2+\tfrac{1}{9}K_s^2-\tfrac{4}{9}\pi_s^2)] = \frac{1}{6}\frac{(4K_s^2-3\eta_s^2-\pi_s^2)(\eta_s'^2-\eta_s^2)(\eta_{cs}^2-\pi_s^2)^2}{(\eta_s^2-\pi_s^2)^2}. \quad (32)$$

V. NUMERICAL RESULT—MASSES AND MIXING ANGLES OF THE GROUND STATE 1^{--} AND 0^{-+} 16-PLETS

In all our calculations we have neglected SU(2) violation. Since the result involves leakage factors, $\eta_s'^2-\pi_s^2$ and $\Delta_s^2=\eta_s^2-2K_s^2+\pi_s^2$, for the approximately "ideal" case, our numerical result is sensitive even to the SU(2) breaking. Another source of difficulty in obtaining accurate results is that the center mass values of the input broad resonances (ρ, K^*, etc.) are not well known. When comparing our result with experiment some allowance should be made to take into account the theoretical and experimental uncertainties mentioned above. In the following computation, we have adopted (rather arbitrarily)[21] the following mass values of 1^{--} mesons (see also Sec. VI A); $\phi=1.015$, $K^*=0.898$, $\rho=0.772$, $\omega=0.780$, and $\phi_c=3.105$ GeV.

From Eqs. (25)–(27) we then obtain $\delta\theta\simeq 1.9°$, $\delta\phi=0.97°$, and $\psi=1.2\times10^{-3}$ rad. The deviations from "ideal" angles are indeed very small and we obtain (ideal values are $\theta\simeq 35°$, $\phi=30°$, and $\psi=0$)

$$\theta=37.2°,\ \phi=31.0°,\ \text{and}\ \psi=0.069°. \quad (33)$$

From Eq. (28) we obtain the mass of a charmed 1^{--}, D^*, in terms of the masses of ρ, K^*, ϕ, ω, and $\psi(3105)$. From Eq. (6), the mass of another charmed 1^{--}, F^*, is then determined,

$$D^*=2.245\ \text{GeV and}\ F^*=2.29\ \text{GeV}. \quad (34)$$

By now using the intermultiplet mass relation, Eq. (7),

$$D^{*2}-\rho^2=D^2-\pi^2, \quad (35)$$

we can predict the mass of a charmed ground state 0^{-+} meson D from Eq. (35), and Eq. (6) then determines the mass of its $I=0$ counterpart F. We obtain[22]

$$D=2.11\ \text{GeV},\quad F=2.16\ \text{GeV}. \quad (36)$$

Finally, from the value of D, the mass of η_c of the 0^{-+} 16-plet can be determined by using Eq. (32).

At present we have two candidates for the $\eta'(0^{-+})$. The usual candidate is the $X(958)$ but its J^{PC} cannot at present exclude[13] the possibility of 2^-. $E(1420)$ can also be a 0^{-+} meson. Actually $E(1420)$ satisfies the Schwinger's mass formula[23] better than $X(958)$. We thus consider the two alternatives.[24]

We take $\pi = 0.137$, $K = 0.496$, $\eta = 0.549$, and $\eta' = X = 0.958$ or $\eta' = E(1420) = 1.416$ GeV. θ, ϕ, and ψ are determined from Eqs. (29)–(31). The mass of $\eta_c(0^{-+})$ is determined from the mass formula Eq. (32) using the value of $D(0^{-+})$ obtained in Eq. (36). Equation (32) gives two values of η_c (since it is quadratic in η_c) and we select the physically acceptable solution. The result is shown in Table I. In Table I the ± signs should be taken in the same order and the values of D^*, F^*, D, and F are fixed, once $\psi(3105)$ is assigned to the η_c member of 1^{--} 16-plet (ground state) as discussed above. We note that the alternative assignment of $\eta' \equiv X(958)$ or $\eta' \equiv E(1420)$ produces a large difference in the predicted mass of $\eta_c(0^{-+})$ which will be of experimental and theoretical interest. [See also the concluding remark VI B.]

It is interesting to notice that the 0^{-+} 16-plet is rather close to "ideal" with respect to the angles ϕ (i.e., $\eta_c(0^{-+})$ is almost a $c\bar{c}$ state), whereas it deviates very significantly from "ideal" with respect to the angle θ (note that $\theta_i \simeq 35°$).

TABLE I. Predicted masses, D^* and F^* of 1^{--} and D, F, and η_c of 0^{-+}, and the SU(4) mixing angles (ϕ, θ, ψ) of the 0^{-+} and 1^{--} 16-plets. The input is the mass of $\psi(3105)$. The mass of η_c and the 0^{-+} SU(4) mixing angles depend on the assignment $\eta' \equiv X(958)$ or $\eta' \equiv E(1420)$. The ± signs should be taken in the same order.

1^{--}	0^{-+} $D = 2.11$ GeV	0^{-+} $F = 2.16$ GeV
$D^* = 2.245$ GeV	$\eta' \equiv X = 0.957$ GeV	$\eta' \equiv E(1420) = 1.416$ GeV
$F^* = 2.29$ GeV	$\eta_c = 2.72$ GeV	$\eta_c = 3.04$ GeV
$\phi(1^{--}) = 31.0°$	$\phi = \mp 21.90°$	$\phi = \mp 36.1°$
$\theta(1^{--}) = 37.2°$	$\theta = \pm 10.4°$	$\theta = \pm 6.2°$
$\psi(1^{--}) = 0.069°$	$\psi = \mp 3.4°$	$\psi = \pm 0.20°$

VI. CONCLUDING REMARKS

A. Width of $\psi(3105)$

In Sec. III, we have demonstrated the existence of selection rules[18] for the couplings, $\eta_{cs} \to$ any uncharmed meson + pseudoscalar (π, K, η), in the "ideal" limit of the 16-plet $B_{\alpha,s}$. The narrow width of $\psi(3105)$ may indeed be the consequence of the well-known almost "ideal" structure of the ground state 1^{--} mesons.

As discussed in Secs. IV and V, we may actually evaluate the *deviation* from the "ideal" structure in terms of the "leakage" factors, $\eta_s'^2 - \pi_s^2$ and Δ_s^2. In Ref. 3 we have made a crude phenomenological estimate of the hadronic decays of $\psi(3105)$ which take place through the *small* leakage from the "ideal" structure. For example, we have made an estimate on the partial decays $\psi(3105) \to B\pi(\omega\pi\pi)$, $\rho\pi(\pi\pi\pi)$, KK, etc. However, the numerical values of our leakage factors are sensitive even to the SU(2) violation neglected. Moreover, the center mass values of input broad resonances such as ρ, K^*, etc., are not well known. Therefore, we should not take the numerical result listed in Ref. 3 too seriously. The numerical values of 1^{--} meson masses used in Sec. V produce, for example, the smaller values for all the partial widths of $\psi(3105)$ than those listed in Ref. 3. To achieve more accurate results, we probably need to answer the important question, i.e., what is the dynamical origin of the violation of the "ideal" structure? This may especially be the case for the ratio $\Gamma(\phi_c \to \rho\pi)/\Gamma(\phi \to \rho\pi)$ discussed in Ref. 3, since both $\Gamma(\phi_c \to \rho\pi)$ and $\Gamma(\phi \to \rho\pi)$ vanish in our "ideal" limit.

B. $\eta'(0^{-+}) \equiv X(958)$ or $E(1420)$?

At present, the choice, $\eta'(0^{-+}) \equiv X(958)$ or $E(1420)$, remains to be settled.[24] Schwinger's nonet mass relation[23] favors $E(1420)$ over $X(958)$. According to our result in Sec. V, the choice may be *settled* by studying whether the $\eta_c(0^{-+})$ has a mass close to 2.72 GeV or to 3.04 GeV. Another implication of the choice is on the $\psi(3105) \to \eta_c(0^{-+}) + \gamma$ decay. For the choice $\eta' \equiv E(1420)$, the decay $\psi(3105) \to \eta_c(0^{-+}) + \gamma$ will be much less important (by the suppression due to phase space) compared with the case $\eta' \equiv X(958)$ and will be hard to detect. Since all the mixing angles are fixed for the 0^{-+} 16-plet in Sec. V, we can treat the $1^{--} \to 0^{-+} + \gamma$ decays in detail using asymptotic SU(4). We hopt to report our result in the forthcoming paper.[25]

Notes added in proof

A. By taking a physically plausible limit involving $\eta_{cs}^2 \to \infty$ (which is consistent with the observed heavy mass of the η_{cs}) in our sum rules, we have recently found[25] that Schwinger's nonet mass formula may always be valid for *any* 16-plet, if the η_{cs} is heavy, i.e., if it is close to a $c\bar{c}$ state. The ideal nonet mass formula is valid only when the η_{cs} is *very* close to a $c\bar{c}$.

B. Therefore, the choice $\eta' \equiv E(1420)$ is now more favored than $\eta' \equiv X(958)$. For $\eta' \equiv E(1420)$ we have estimated[25] the rate of $\psi(3105) \to \eta_c(0^{-+}) + \gamma$

to be around 10 keV, which seems to be consistent with present experiment.

C. Mass formulas and mixing angles

Finally, our way of deriving the masses of 16-plet mesons is different from the approach taken by Okubo, Mathur, and Borchardt,[5] although the actual values obtained are rather close. Okubo *et al.*[5] assume that the mass splittings arise from an interaction $H_{\rm int}=T^8+\alpha T^{15}$ where T^8 and T^{15} belong to the same 15-plet of SU(4) and they assume the *same* value of α for both the 1^{--} and 0^{-+} 16-plets.

In our derivation, the *total* Hamiltonian H is expressed in terms of the *physical* (i.e., "in" or "out") fields,

$$H=\sum\int \omega_{\alpha,s}(\vec{k})a^+_{\alpha,s}(\vec{k})a_{\alpha,s}(\vec{k})d^3k$$

where $\omega_{\alpha,s}(\vec{k})=(\vec{k}^2+m_{\alpha,s}{}^2)^{1/2}$, and we seek for the constraints[10,11,12] on the masses $m_{\alpha,s}$ from the exotic commutation relations, $[\dot{V}_\alpha, V_\beta]=[\dot{V}_\alpha, A_\beta]=0$, which represent the algebraic expressions of our simple mechanism of SU(4) and chiral SU(4) $\otimes$ SU(4) breaking. In our asymptotic limit $\vec{k}\to\infty$, the constraints obtained are *exact* and the mass formulas should always take the *mass-squared* form including baryons.

It is amusing to notice that our value of the mass of η_c (ground state 0^{-+}), $\eta_c(0^{-+})\simeq 3.04$ GeV for the choice $\eta'(0^{-+})\equiv E(1420)=1.416$ GeV, is close to the value 3.05 GeV of the paracharmonium I of Glashow *et al.*[26] Estimates of the partial rates of the decays of $\eta_c(0^{-+})$ into K^*K and $K^{**}(1420)K$ using our mixing angles derived in Sec. V, from the SU(4) related decays $K^*\to K\pi$ and $K^{**}\to K\pi$, yield $\Gamma(\eta_c(0^{-+})\to K^*K)\simeq 4.4$ MeV or 4.6 keV and $\Gamma(\eta_c(0^{-+})\to K^{**}K)\simeq 2.9$ MeV or 104 MeV depending on the choice of $\eta'(0^{-+})\equiv X(958)$ or $\eta'(0^{-+})\equiv E(1420)$.

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[11]For example, $V_\eta = V_8$, $V_{K^0} = V_6 + iV_7$, $V_{F^+} = V_{13} - iV_{14}$, $V_{D^+} = V_{11} - iV_{12}$, $V_{D^0} = V_9 - iV_{10}$, $V_{\eta_c} = 15$, etc.

[12]Exactly the same derivation leads to the quadratic mass formulas also for baryons. S. Oneda and E. Takasugi, Phys. Rev. D 10, 3113 (1974).

[13]Consider $\langle K_s^0(\vec{k})|[V_{K^0}(\dot{V}_{K^0}), A_{\pi^-}]|\pi_t^+(\vec{k})\rangle = 0$ and $\langle D_s^0(\vec{k})|[V_{D^0}(\dot{V}_{D^0}), A_{\pi^-}]|\pi_t^+(\vec{k})\rangle = 0$, $\vec{k}\to\infty$.

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[15]Consider $\langle F_s^+(\vec{k})|[V_{F^-}(\dot{V}_{F^-}), A_{\pi^-}]|\pi_\mu^-(\vec{k})\rangle = 0$, $\vec{k}\to\infty$. We obtain $X_\alpha^s A_{su} + X_\beta^s B_{su} + X_\gamma^s C_{su} = 0$ and $X_\alpha^s \eta_s^2 A_{su} + X_\beta^s \eta_s'^2 B_{su} + X_\gamma^s \eta_{cs}^2 C_{su} = 0$.

[16]Consider $\langle K_s^0(\vec{k})|[V_{K^0}(\dot{V}_{K^0}), A_{\pi^-}]|\pi_\mu^+(\vec{k})\rangle = 0$, $\vec{k}\to\infty$. We obtain $\alpha^s(\eta_s^2 - \pi_s^2)A_{su} + \beta^s(\eta_s'^2 - \pi_s^2)B_{su} + \gamma^s(\eta_{cs}^2 - \pi_s^2)C_{su} = 0$ when combined with the relation $K_s^2 - \pi_s^2 = K_u^2 - \pi_u^2$, Eq. (8). Eliminate A_{su}, B_{su}, and C_{su} from this equation by using Eqs. (9) and (10).

[17]If we find one more constraint, $\eta_s'^2 = \pi_s^2$ and $\eta_s - K_s^2 = K_s^2 - \pi_s^2$ may no longer be independent. A work along this line (introducing the concept of algebraic realization of SU(3) in the commutation relation $[A_i, A_j] = if_{ijk}V_k$) is found in Ref. 10.

[18]We note $A_\eta \equiv A_8$. For the η meson, we have used PCAC in the form $\partial_\mu A_\mu^\eta(x) = C_\eta \phi_\eta(x)$ together with $\partial_\mu A_\mu^\pi(x) = C_\pi \phi_\pi(x)$ and $\partial_\mu A_\mu^K(x) = C_K \phi_K(x)$ with $C_\pi \simeq C_K \simeq C_\eta$. Therefore, in the selection rules, $B_s \not\to B_u$ +pseudoscalar (π, k, η), η should be taken, strictly speaking, as ϕ_8. However, $\eta \simeq \phi_8$ is actually justified (Sec. V).

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[20]Relatively small leakage from "ideal" mass constraints can produce a large witdth for η_c, since the mass of η_c is very large.

[21]Rather large SU(2) violation is indicated by the presence of $\omega \to 2\pi$ decay and the $K^{*0} - K^{*+}$ mass difference. It is not easy to guess (from the physical masses) mass values when SU(2) breaking interactions are switched off. The values we have adopted will be reasonable but only serve as a guide.

[22]A new particle reported by K. Niu, E. Mikumo, and Y. Maeda [Prog. Theor. Phys. 46, 1644 (1971)] in a cosmic-ray event as a mass $\simeq 1.8$ GeV may possibly be one of these charmed mesons.

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