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Hadronic spectroscopy for a linear quark containment potential*

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We investigate the hadronic spectroscopy of a linear confining potential acting between quarks. The resulting systematics are quite satisfactory. In particular the decreasing spacing between radially excited states of a given orbital angular momentum has substantial support in both meson and baryon spectra.

INTRODUCTION

A major problem of particle physics in recent years has been that of quark confinement. Since confinement will only occur for a binding interaction which becomes increasingly strong at large distances, perturbative investigations of the phenomenon are not generally possible. Nonetheless, some progress has been made. In particular a binding potential between quarks which increases linearly with separation has a number of attractive features and arises naturally in several different theoretical approaches.[1,2] More recently, this type of potential has been used to interpret[3-5] the recently observed[6] resonances at 3.1 and 3.7 GeV, the latter presumably being an excited radial recurrence of the former. The two parameters of the linear potential model (charmed quark mass and potential strength) are determined from these resonances, and further radial excitations are predicted, beginning with one at about 4.19 GeV. This mass prediction appears to be in accord with experiment.[7] In contrast, a harmonic-oscillator potential implies equal spacing between the various excited resonances. Thus it is clearly of interest to pursue the linear potential approach.

One possible test of this picture is to look vigorously for the host of additional resonances belonging to the J family. In particular, the masses of further radial recurrences will be of great interest. However, a more immediate experimental confrontation is possible. This paper will attempt to describe the observed "ordinary" mesons and, more sketchily, baryons in terms of a linear potential model. In so doing, it is important to separate the unique features of such a potential from the usual complexities of quark-model spectrum analyses.[8] As we have already said, decreasing separation between radially excited recurrences (which we label by the quantum number $n=1, 2, 3, \ldots,$ $n-1$ being the number of radial nodes) of a given ground state is the hallmark of the linear potential. This spacing will not be overly sensitive[3-5] to the usual short-range potentials common to all quark-model spectroscopy. These include:

(a) short-range attractive forces (present in most field-theoretical models, including asymptotically free ones) responsible for decreasing the over-all energy level of mesons with low orbital angular momentum L relative to mesons with larger values of L;

(b) short-range forces yielding $\vec{L}\cdot\vec{S}$ splitting or "fine" structure, separating resonances of the same L and n but different J; and

(c) short-range $\vec{S}_1\cdot\vec{S}_2$ spin-interaction forces.

Our treatment will be nonrelativistic. We will comment in greater detail on possible relativistic modifications in the next section; here we merely state that as long as it is the lowest few energy levels that are of interest, these relativistic modifications are relatively slight. In a typical relativistic treatment the potential strength and quark masses which must be chosen to fit the first two levels of a radial sequence are different from the values required to fit the same levels in a nonrelativistic calculation. Then, further radial excitations begin to differ significantly from those calculated nonrelativistically only above the fourth or fifth level with the details depending somewhat on the relativistic treatment employed.

The results of the analysis are very encouraging. There is considerable evidence in the observed hadron spectrum for the sort of level spacings expected from a linear binding potential. (This statement assumes that certain of the not-yet firmly established resonances, listed in the Review of Particle Properties,[9] will be promoted to established status.)

MASS FORMULA FOR MESONS

We begin by reviewing the basic features of a linear potential. The reduced center-of-mass radial equation for $\phi\ [=rR(r)]$,

$$\left[-\frac{1}{m}\frac{d^2}{dr^2}+kr-\overline{E}+\frac{L(L+1)}{mr^2}\right]\phi(r)=0 \qquad (1)$$

for $L=0$, has as its solution an Airy function

$$\phi(r)=\text{const}\times\text{Ai}((mk)^{1/3}(r-\overline{E}/k))\ . \qquad (2)$$

Energy levels are obtained by the requirement that $\phi(r)$ vanish at the origin (so that R is finite) yielding

$$\bar{E} = bx_i\,,\quad b=(k^2/m)^{1/3} \tag{3}$$

where the x_i are the (negative) zeros of the Airy function. (For convenience we list[10] a few: $x_1=2.34$, $x_2=4.09$, $x_3=5.52$, $x_4=6.79$, and $x_5=7.94$.) In the absence of other potentials, the total energy E is the sum of the quark masses (m is the quark mass) and $\bar{E}$. However, in general, there will be one or more short-range potentials V_{sr} present. If we treat these perturbatively, the energy shift resulting will be, for a sufficiently short-range effect,

$$\Delta E = \int \frac{\phi}{r} V_{sr} \frac{\phi}{r} r^2 dr$$

$$\approx |\phi'(0)|^2 \int V_{sr}\, r^2 dr \tag{4}$$

for the $L=0$ case. Since the wave function at the origin, $\phi'(0)$, for the case of a linear potential, is independent of the principle quantum number n we take ΔE as a constant independent of n. For instance, for the attractive short-range force (a), we write

$$\Delta_{(a)} E_{L=0} = -A_{L=0} \tag{5}$$

independent of n. [Computer checks indicate that this approximation is not too good (though not too bad) for potentials like the Coulomb, which have very long-range tails such that the second integral in (4) diverges.]

We do not have any settled opinion as to the nature of V_{sr}. One currently popular approach is to use a Coulomb-type force (which is supposed to arise from an asymptotically free gauge theory of the hadrons). The simple form $[-\frac{4}{3}(\alpha_s/r)]$ for the potential, proposed on this basis, should be considered valid only for small $q\bar{q}$ separations The basic assumption, both here and in the gauge-theory approaches, is that at larger separations the potential has a form appropriate to quark confinement (taken here to be kr). In the present perturbative approach the distance by which the changeover must have occurred (in order for our procedure to be valid) may be estimated as the point at which the linear expansion employed in (4) for ϕ breaks down, $r_0 \simeq (mb)^{-1/2}$, the first maximum of the Airy function. Then

$$\Delta E \simeq -\tfrac{4}{3}\alpha_s |\phi'(0)|^2 \int_0^{r_0} \frac{1}{r}\, r^2 dr\,. \tag{4$'$}$$

Our phenomenological fit to the observed meson spectrum (given in detail later) requires this ΔE to be of order 0.3 GeV, which is produced by this integral for $\alpha_s \simeq 0.3$.

For $L\neq 0$ we may solve the equation (1) by computer, again determining the energy eigenvalues by the requirement that the wave function ϕ vanish at the origin (like $\gamma_L r^{L+1}$). Table I presents the unperturbed level spacings X_{nL} for two choices of the parameter b: one appropriate to the mesons not containing charmed quarks; the second for the new mesons composed, in this picture, of $\mathcal{P}'$ and $\bar{\mathcal{P}}'$ quarks.

Note that the spectrum is similar to that of a harmonic-oscillator potential in that $L=0$ (1), $n=N$ is very nearly degenerate with $L=2i$ $(2i+1)$, $n=N-i$. However, the equidistant level spacing rule no longer holds; in particular the $L=1$, $n=1$ level is closer to the $L=0$, $n=2$ than to the $L=0$, $n=1$ level, and the higher n recurrences for a given L have progressively smaller spacings. For $L=0$, $X_{n0}=b(x_n-x_1)$. For $L\neq 0$, the level spacings are only approximately linear in the parameter b. The energies of the above table are accurate to within 0.01 GeV.

It is perhaps worth making a few comments concerning Regge trajectories in the present context. A plot of $\bar{E}^2$ [$\bar{E}=(bx_1+X_{nL})$ is the resonance mass minus the sum of the quark masses] as a function of L (holding n fixed at 1) reveals almost perfect linearity in L. An asymptotic prediction requires a decision as to the correct relativistic generalization of our considerations.

For $L\neq 0$ the perturbative effects of the short-range potentials will again be taken to be independent of n, at least for small n (< 4 or 5). This seems fairly well justified as the coefficient γ_L of the r^{L+1} part of ϕ is roughly independent of n. Computer calculations in a number of explicit cases indicate that the variation with n of the short-range perturbation is not likely to be greater than 30%.

The next important question concerns the mass dependence of the A_L and of the spin-spin interaction strength. In general, there are only two

TABLE I. I. $X_{nl}(b)$. Relative positions of energy levels.

	$b=0.2743$				$b=0.3429$			
n	$L=0$	$L=1$	$L=2$	$L=3$	$L=0$	$L=1$	$L=2$	$L=3$
1	0	0.3	0.52	0.72	0	0.35	0.66	0.94
2	0.48	0.7	0.9	1.09	0.6	0.87	1.13	1.37
3	0.87	1.06	1.25	1.405	1.09	1.33	1.555	1.775
4	1.22	1.39	1.56	1.71	1.53	1.74	1.95	2.15
5	1.53	1.69	1.85	2.00	1.925	2.12	2.31	
6	1.83	1.98	2.11	2.27	2.29			
7	2.1	2.25	2.38	2.52				
8	2.38	2.51	2.64					
9	2.63							

simple choices consistent with the observed masses of ρ, K^*, and ϕ and of their $L=1, 2$ partners. Because of the equal spacing rule, $\phi - K^* = K^* - \rho$, and because the same spacing applies to the $L=1, 2$ nonets, these parameters must be either m-independent or they must scale according to the average quark mass. The latter, of course, amounts to a redefinition of quark mass.

Our final resonance energy formula for mesons will thus be of the form

$$E_{nLJ} = m_q + m_{\bar{q}} - A_L + c\delta_{L0}\vec{S}_1\cdot\vec{S}_2 + (X_{nL} + bx_1) + a_L\vec{L}\cdot\vec{S}\,. \tag{6}$$

The parameters are:

(i) the quark masses m_q and $m_{\bar{q}}$ (2 parameters required, m_λ and $m_{\mathcal{P}} = m_{\mathfrak{N}}$, for mesons not containing charmed quarks);

(ii) the attractive, n- and mass-independent energy shifts, A_L, with $A_0 > A_1 > A_2 = A_3 = \cdots = 0$ (2 parameters);

(iii) the n- and mass-independent spin-spin perturbation coefficient c [in analogy with positronium it is assumed to be present only for the $L=0$ states (1 parameter)];

(iv) the level-spacing parameter b; we assume it to be quark-mass-independent for the $\mathcal{P}$, $\mathfrak{N}$, and λ quarks [this allows linear quark-mass formulas to hold, and implies a potential strength k which depends on quark mass (1 parameter)]; and

(v) the n- and mass-independent $\vec{L}\cdot\vec{S}$ splitting strengths, a_L; $a_1 > a_2 > a_3 \cdots$ [in the case of positronium, for instance, $a_L \propto 1/(L+1)$ (1 parameter for each $L \neq 0$)].

It will be noted that we do not include tensor force contributions. This is a phenomenological decision. The ^3P_J states $A_2(1300)$, $A_1(1100)$, and $\delta(970)$ are sufficiently well fitted by pure $\vec{L}\cdot\vec{S}$ splitting with diagonal matrix elements $(1, -1, -2)$ that the contribution from the tensor force S_{12} with matrix elements $(-\frac{2}{5}, 2, -4)$ should be relatively small. We do not have any deep understanding of why this should be so, and indeed it only follows phenomenologically after accepting the nominal A_1 mass (1100 MeV). In fact, the present uncertainty in this mass prevents one from putting any hard limit on the possible tensor force contribution. An A_1 mass 70 to 100 MeV higher would allow a tensor force contribution comparable to the $\vec{L}\cdot\vec{S}$ splitting.

The mass formula is not sufficient for the 0^{-+} nonet. Among other things the η and η' are not pure nonstrange $q\bar{q}$ and pure $\lambda\bar{\lambda}$ states, respectively, unlike the situation for the 1^{--} and 2^{++} nonets. Thus some mixing has to be introduced. In the context of a colored quark model this is, perhaps, most naturally achieved by introducing[11] the following:

(vi) an annihilation term in the mass matrix. We also adopt the argument[11] based on asymptotically free gauge theories that this term will only be important for the lowest-mass meson nonets, i.e., the pseudoscalar nonets.

Then $M_{ab,\alpha\beta}$, the mass matrix, takes the form

$$M_{ab,\alpha\beta} = E_{nLJ}\delta_{a\alpha}\delta_{b\beta} + B\delta_{ab}\delta_{\alpha\beta}\,. \tag{7}$$

Before turning to a detailed discussion of the nonrelativistic phenomenology we discuss possible alterations of the energy spectrum due to relativistic effects. A full relativistic field-theoretic treatment (e.g., exact Bethe-Salpeter equation) is beyond the reach of present technique. We have examined two other approaches: (a) the Kadyshevsky version of the "quasipotential" approach to the relativistic two-body problem,[5] and (b) the Klein-Gordon and Dirac potential equations.[12] In both cases, even for light-quark systems, as long as the quark masses and long-range force strength are adjusted to fit the first two energy levels ($L=0$), essentially the same energy values for the next two or even three levels are obtained as in the nonrelativistic calculation. In case (a) computer calculation[13] gives energy levels 0.77, 1.25, 1.67, 2.02, 2.40, ..., compared to the nonrelativistic results 0.77, 1.25, 1.64, 1.99, 2.30. As mentioned previously, rather different values of the effective quark mass and long-range force are required in the relativistic calculation and in the nonrelativistic calculation. The analagous results for case (b) will be discussed in detail in a forthcoming report, but again the level spectrum shows significant differences from the nonrelativistic calculation only above the fourth level. We conclude that the main effect of relativistic calculations is to decrease the significance of the values of the parameters (e.g., effective quark mass and long-range force strength) found by fitting the mass spectrum with a nonrelativistic calculation. However, because the spectrum itself is not greatly affected (once the parameters are readjusted), one can sensibly study the question of whether or not the observed hadron spectrum shows radial excitations typical of a kr potential using the nonrelativistic results. We also remark that the virial theorem for a linear potential implies that twice as much potential energy goes into making up a given resonance mass as kinetic energy. Thus the system tends to be less relativistic than one first expects from a comparison of the quark masses with the resonance masses.

Returning to the question of the significance of the parameters, case (b) is amenable to a WKB approach, and one finds a significant difference in the dependence of the energy on the parameters as one goes from a nonrelativistic calculation to the relativistic calculation. We have already mentioned that in the nonrelativistic calculation linear mass formulas such as $\phi - K^* = K^* - \rho$ require $b = (k^2/m)^{1/3}$ to be independent of the effective quark mass. This implies that k must depend on the effective quark mass as $m^{1/2}$. In contrast, for the relativistic case (b), one finds that the energy levels depend primarily on k, rather than on b; thus in this case one expects k to be roughly independent of the quark mass.

A possibly more serious problem is the neglect of coupled decay channel effects for the resonances calculated in a simple quark potential model. In common with other detailed quark-model energy level fits we are forced to simply ignore this problem.

S=1 MESONS

We first discuss the $S=1$ mesons not containing charmed quarks. Here we have only 7 parameters to determine (only one combination of A_0 and c is needed for $S=1$). We discuss them in turn.

(1) The radial excitation spacing parameter b is determined by our choice for the first two levels of the $L=0$, ρ sequence. Candidates from the Particle Data Group (PDG) tables[9] are $\rho(0.77)$, $[\rho'(1.25)]$, $\rho'(1.6)$, $[\rho'(2.1)]$, and $[\rho(2.275)]$. One possibility is to discard the unconfirmed $\rho'(1.25)$ and use the $\rho'(1.6)$ to set the scale. This fixes $b=0.474$ and all of the X_{nL} of the mass formula (6). With this large value of b, the $L=0$ ground state in the linear potential is at $2m_{\mathcal{P}} + 1.1$ GeV. There is also a positive contribution of about 0.1 GeV from the $\vec{S}_1 \cdot \vec{S}_2$ term. Thus to get the $\rho(0.77)$ one requires a strong attractive energy shift $-A_0 = -2m_{\mathcal{P}} - 0.43$ GeV. We may accept this large attractive energy shift, but then we find that it is not possible to understand the observed $L \neq 0$ meson spectra using the mass formula (6), subject only to the theoretical constraint that the short-range attractive forces are less effective in higher L states (implying $A_0 > A_1 > A_2 > A_3 > \cdots \geq 0$). This (negative) result follows from the observations that the centrum of the ${}^3P_J(A_2, A_1, \delta)$ and the ${}^1P_1(B)$ lies at 1.2 GeV, that the centrum of the ${}^3D_J$ states must lie below 1.65 GeV so that the positive $\vec{L} \cdot \vec{S}$ contribution will give the $g(1.68)$, and, if the recent analysis[14] showing that the $S(1.93)$ is the 4^{++} state is confirmed, that the unsplit ${}^3F_J$ level cannot lie above 1.9 GeV. With $b=0.474$ and $m_{\mathcal{P}}=0.15$, the A_L required to fit these levels are $A_0=0.73$, $A_1=0.65$, $A_2 \gtrsim 0.63$, $A_3 \gtrsim 0.83$. (By decreasing $m_{\mathcal{P}}$, one can decrease each A_L by this amount, but the violations of the inequalities remain.)

We note that for an oscillator potential, kr^2, the situation is not essentially different. The unperturbed higher L states ($\bar{n}=0$) lie a little lower—relative to the unperturbed $L=0$ states ($\bar{n}=0, 1, \ldots$)—than for the linear kr potential, so the difficulty is not quite so acute; but one still finds it necessary to have short-range attraction which is essentially unchanged as one goes from $L=0$ to $L=1$, 2, and 3. We do not believe that any smooth short-range potential will give rise to the pattern (L dependence) of attractive energy shifts described above. Thus it appears substantially more difficult to interpret the meson spectrum if the excitation scale is set by the difference between $\rho'(1.6)$ and $\rho(0.77)$.

We have therefore decided to present the results of an analysis of the meson spectrum based on the excitation scale determined by the $\rho(0.77)$, $[\rho'(1.25)]$ difference. This determines $b=0.2743$, and the three higher proposed radial recurrences are well fitted. This procedure also determines the zero point energy, $2m_{\mathcal{P}} - (A_0 - \frac{1}{4}c) = 0.126$ GeV.

(2) The $\vec{L} \cdot \vec{S}$ splitting coefficient a_1 we obtain from the $2^{++} - 1^{++}$ ($A_2 - A_1$) mass splitting. This requires $a_1 = 0.1$ GeV.

(3) Given a_1 we can determine the combination $\Delta_{01} = (A_0 - \frac{1}{4}c) - A_1$ from the position of the 2^{++} (A_2) relative to the ρ. We obtain $\Delta_{01} = 0.13$ GeV.

(4) a_2 and $\Delta_{12} = A_1 - A_2$ must be chosen (subject to their natural constraints) to give correctly the position of the 3^{--} g relative to the ρ. We take $A_2 = 0$, $a_2 = 0.07$ ($< a_1$), $A_1 = 0.07$.

(5) Finally, the quark mass difference $m_\lambda - m_{\mathcal{P}} = 0.123$ GeV follows from the $K^* - \rho$ mass difference.

Note that given $A_2 = 0$, Δ_{01}, and A_1 we can determine $2m_{\mathcal{P}} = 0.326$ GeV.

Using the above seven parameters (or combinations of parameters) we obtain the $S=1$ spectra of Figs. 1, 2, and 3. (For notation see the figure captions.) As in any quark-model spectrum, there are a number of predicted SU(3) multiplets (e.g., the 2^{--}) for which there is yet little or incomplete experimental evidence. If we concentrate on the observed mesons, there are a number of notable successes of the present approach.

$L=0$.

(a) The five ρ radial recurrences are very well fit.

(b) Two possible[15] ϕ-like mesons are easily interpretable as ϕ radial recurrences.

(c) Several K^*-like objects have masses appropriate to K^* radial recurrences.

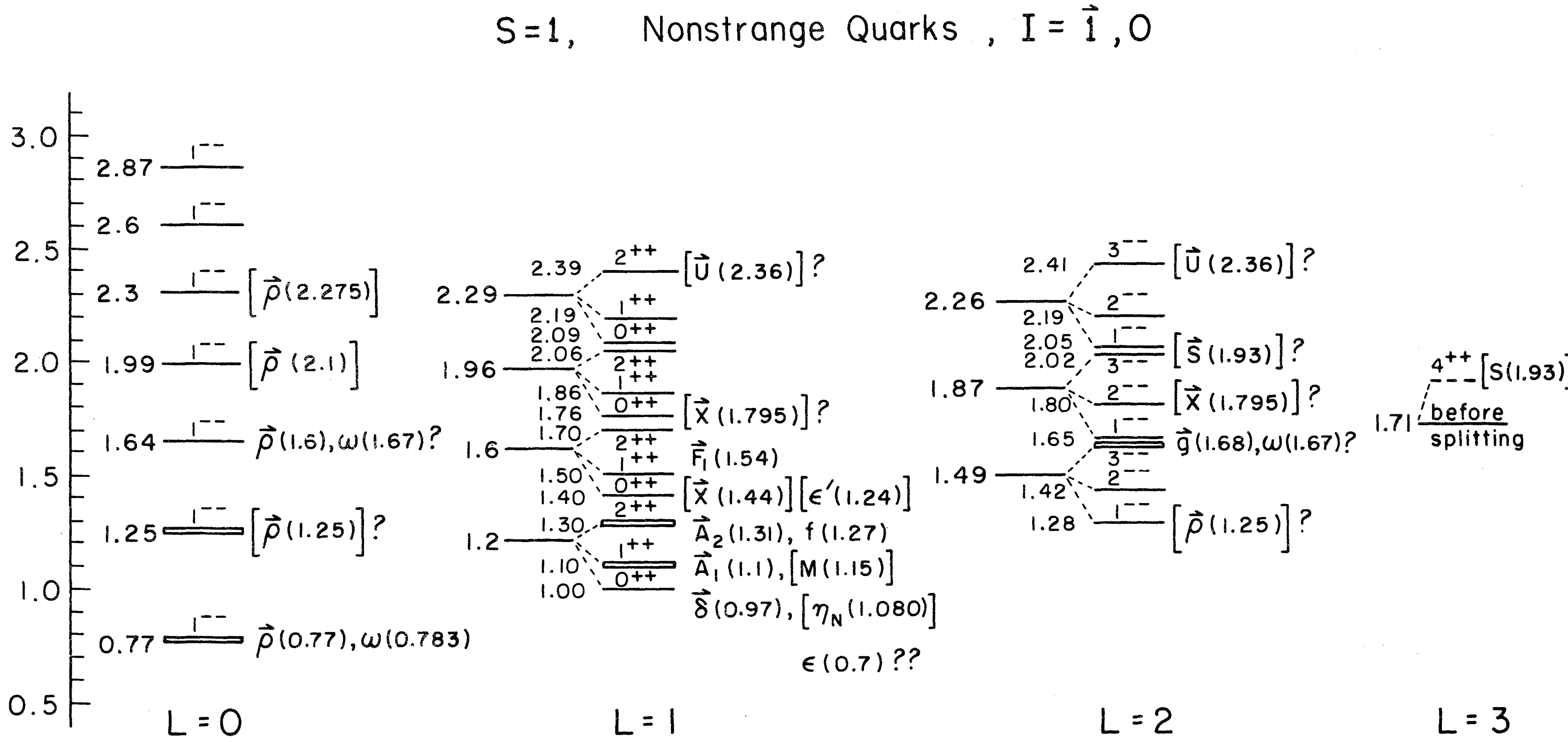

FIG. 1. $S=1$ mesons consisting of $\mathcal{P}\bar{\mathcal{P}}$ or $\mathfrak{N}\bar{\mathfrak{N}}$ quarks. Our notation is as follows:

(a) Established resonances are indicated without brackets, while those regarded as uncertain (i.e., requiring additional experimental confirmation) by the Particle Data Group are bracketed, [ ].

(b) Resonances which can be easily placed in more than one place in the spectra, either because of degeneracy or because of inadequate quantum number information, have a single question mark (?). (Usually, but not always, at least two of the alternative assignments are indicated.)

(c) Resonances which are poorly fit, i.e., cannot be easily placed in the spectra, have a double question mark (??).

(d) Resonances which have been used as input to determine parameters of the mass matrix are indicated by double lines.

We omit the $H(0.99)$ and $B(1.04)$ as having essentially no experimental support as well as no natural location in our spectroscopy.

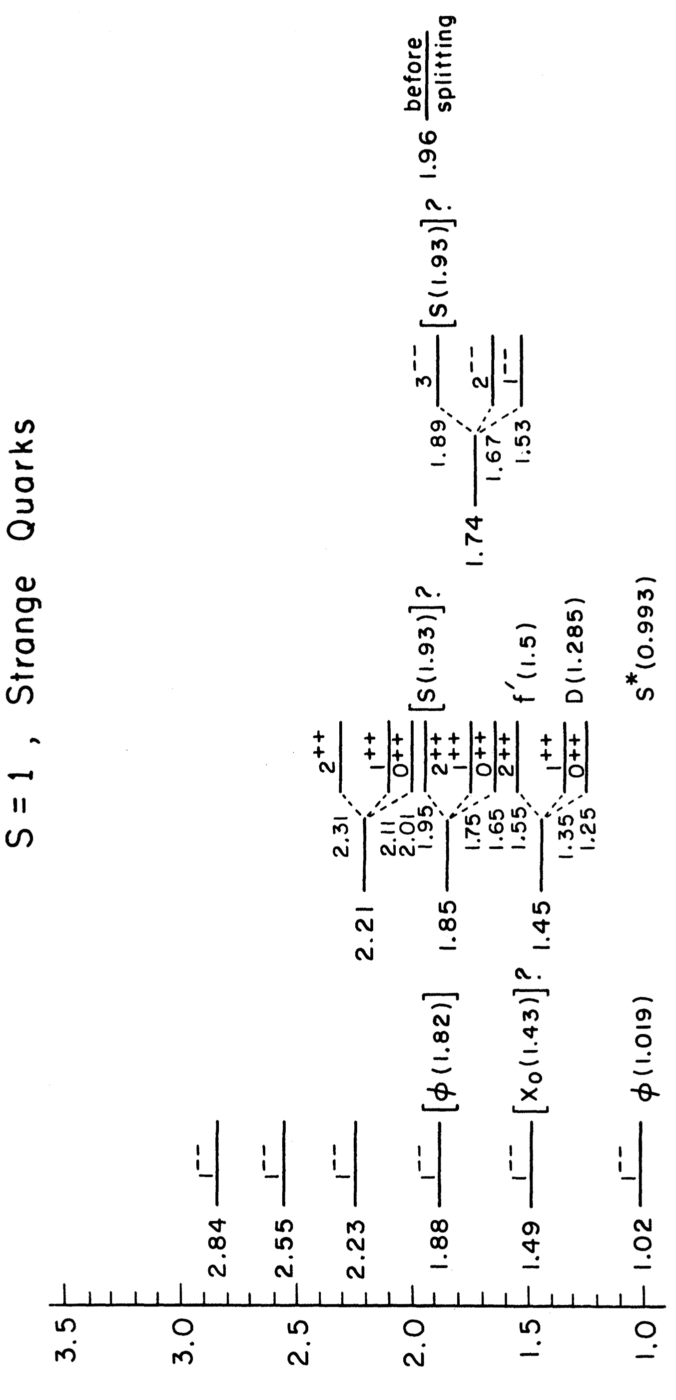

FIG. 2. $S=1$ mesons consisting of a λ and a $\bar{\lambda}$ quark. For explanation of notation, see the caption to Fig. 1.

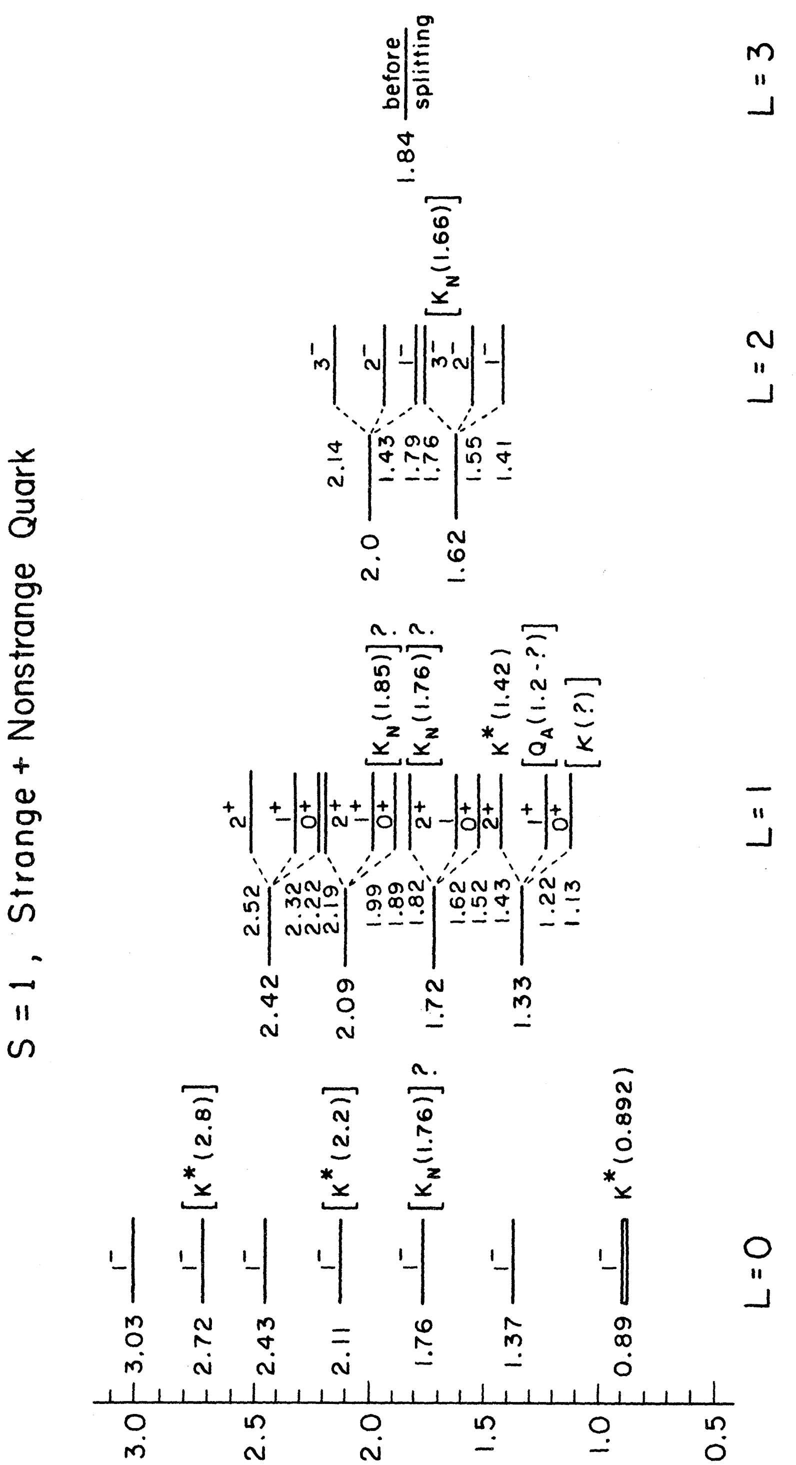

FIG. 3. $S=1$ mesons consisting of a $\mathcal{P}$ and a $\bar{\lambda}$ quark. For explanation of notation, see the caption to Fig. 1.

$L=1$.

(a) The axial-vector F_1 (1.54) fits very nicely as the recurrence of the 1^{++} A (1.1). The $\epsilon'(1.24)$, present in some $\pi\pi$ phase-shift analyses[16] could easily be the 0^{++} member of this same recurrence, though the mass is not well fit. [Note that harmonic-oscillator potentials, for which the $\rho'-\rho$ spacing of 0.48 GeV should equal the ($L=1$, $n=1$) - ($L=1$, $n=2$) spacing would predict both states still higher by about 0.1 GeV than the present model.]

(b) A natural choice for the $\lambda\bar{\lambda}$-type meson $S(1.93)$, as a radial recurrence of $f'(1.5)$, exists.

(c) Several of the predicted radial recurrences with $L=1$, $n=2$ of the $K^*(1.42)$ have experimental candidates.

The most notable problems are (1) the prediction of too high a mass for the $S^*(0.993)$, a 0^{++}-($\lambda\bar{\lambda}$-) type meson (note, however, that it is seen as a highly inelastic threshold effect so that its mass position may be greatly distorted), and (2) the $\epsilon(0.7)$, which not only has too low a mass (though its large width may be part of the problem) but also tends to conflict with the assignment of $\eta_N(1.08)$ to the $I=0$, 0^{++} position. Because of the dubious quality of the $\epsilon(0.7)$ (*a* K-matrix $\pi\pi$ S-wave phase-shift analysis by Protopopescu *et al.*[17] fits their data very nearly as well with no ϵ pole as with one), this latter problem should perhaps not be taken seriously. In fact if the $\epsilon(0.7)$ really is not a resonance, the near degeneracy of the $\delta(0.970)$ and $\eta_N(1.08)$ and the KK decays of the $S^*(0.970)$ indicate the possibility that the 0^{++} mesons form an "ideal" nonet, like the 1^{--} and 2^{++} nonets. Note also that we have assigned $X_1(1.44)$ to a multiplet of nonstrange mesons because of its isospin. However, it decays to $K\bar{K}$ contrary to simple quark diagram expectations. As we have indicated, these possible problems may well not be real problems; if any of them are real problems, they would appear to be so for any quark-model potential.

At this point we review briefly the phenomenological situation that obtains if we simply ignore the theoretical constraints on the L dependence of the contributions of short-range attractive forces and make the analysis with a radial excitation scale fixed by the $\rho(0.77)$, $\rho'(1.6)$ difference. Because the theoretical constraints are slightly less acute for the kr^2 potential, and also for calculational convenience, we will use the oscillator potential for this discussion. But we emphasize that the significant feature is the greatly expanded (almost doubled) radial excitation scale, not the difference between kr and kr^2 potentials. With this expanded scale one has difficulty finding locations for a number of claimed resonances. The $\rho'(1.25)$ must be abandoned and the $\rho(2.275)$ and $\rho(2.1)$ are difficult to place. The latter is true because the $L=1$, $n=2$ unsplit level predicted in the harmonic potential approach at about 2.02 GeV lies below *both*, so that $\vec{L}\cdot\vec{S}$ splitting will not help. The $X_1(1.44)$ and certainly the $\epsilon'(1.24)$ must also be abandoned. Similarly, a host of relatively low-mass K^*-like recurrences [$K_N(1.66)$, $K_N(1.76)$, ...] become difficult to place. Thus it seems that for the $S=1$ mesons, a phenomenological decision between the spectrum predicted by a linear potential with a relatively small radial excitation spacing and an oscillator potential or linear potential with a relatively larger spacing hinges on the eventual experimental confirmation (or elimination) of some seven or eight currently claimed but not established resonances.

$S=0$ MESONS

We can now turn to the $S=0$ states. Here we encounter the complexities, already mentioned, involving the $L=0$ pseudoscalar octet, for which it is necessary to introduce an annihilation mixing term B of Eqs. (7). In this approach the kaons are unaffected so that

$$m_K=0.498$$
$$=m_{\mathcal{P}}+m_\lambda-(A_0+\tfrac{3}{4}c)+bx_1\,, \tag{8}$$

yielding

$$A_0+\tfrac{3}{4}c=0.644+0.449-0.496$$
$$=0.60\,. \tag{9}$$

This combined with

$$A_0-\tfrac{1}{4}c=A_1+\Delta_{01}$$
$$=0.20 \tag{10}$$

yields the $\vec{S}_1\cdot\vec{S}_2$ coefficient

$$c=0.40\text{ GeV}\,. \tag{11}$$

According to this approach the π mass should be 0.38 GeV. This discrepancy we attribute to the special status of the pion as the Goldstone boson for the Nambu-Goldstone mode realization of chiral SU(2)×SU(2) (partial conservation of axial-vector current). The pion and its radial excitations are treated as anomalous in this sense. [One could adjust the quark's masses and $\vec{S}_1\cdot\vec{S}_2$ coefficient c so as to describe the π and K masses. However, such adjustment requires a value for $m_\lambda-m_{\mathcal{P}}$ which disagrees with the $K^*(0.893)-\rho(0.77)$ and $K^*(1.42)-A_2(1.31)$ mass differences.] We then have one remaining parameter, B, which hopefully, can be chosen to describe the η and η' masses simultaneously. [We follow the currently accepted convention of the name η' for the 0.958-

GeV 0^{-+}, $I=0$, $Y=0$ meson, although this is awkward from our point of view—the η' not being a radial excitation of the $\eta(0.55)$.]

One, of course, diagonalizes the mass matrix in the η-η' sector with the result that

$$B=0.17 \text{ GeV}, \tag{12}$$

yielding

$$m_\eta=0.51 \text{ GeV}, \quad m_{\eta'}=0.99 \text{ GeV} \tag{13}$$

and the following mixed composition for the two objects:

$$\eta \simeq \frac{1}{\sqrt{2}} \lambda\bar{\lambda} - \tfrac{1}{2}(\mathcal{P}\bar{\mathcal{P}} + \mathfrak{N}\bar{\mathfrak{N}}),$$
$$\eta' \simeq \frac{1}{\sqrt{2}} \lambda\bar{\lambda} + \tfrac{1}{2}(\mathcal{P}\bar{\mathcal{P}} + \mathfrak{N}\bar{\mathfrak{N}}), \tag{14}$$

corresponding to a mixing angle in the octet singlet notation of about $-10°$. This is the old result for the mass squared Gell-Mann–Okubo–Sakurai mass formula. Note that their approach was based on an SU(3)-breaking term in the Hamiltonian which contributes two arbitrary constants to the η, η' mass matrix—the singlet-octet matrix element and the singlet-singlet matrix element. The present result follows from only one arbitrary constant, B, which contributes only to the singlet-singlet matrix element.

For convenience, we give Table II a summary of the parameters of the mass matrix, all of which have now been determined individually.

In drawing the $S=0$ spectra of Figs. 4, 5, and 6 we assume that this mixing is preserved in the radial recurrences of the η and η'. This is not justified by the original argument for having a non-negligible annihilation mixing only in the pseudoscalar nonet because that argument was based on the small mass of the pseudoscalar nonet relative to the other mesons. Nevertheless, the good fit to suggested higher-mass pseudoscalar mesons strongly indicates that these radial recurrences have the same mixing pattern as the ground states η and η'. We continue to assume negligible annihilation mixing for all the other nonets. Thus the $L=0$, $S=0$ state locations follow instantly from those of the unsplit $S=1$ counterpart levels. The $L=0$ π and η radial recurrences appear on the nonstrange quark meson figure, Fig. 4 while the η' recurrences are drawn with the type $S=0$, $L=0$ mesons in Fig. 5. The pion recurrences are calculated starting from the unexplained value of the pion mass.

The resulting picture is not altogether displeasing. "Successes" include the following:

$L=0$.

(a) There is degeneracy of isospin 1 and 0 objects at 1.01 GeV where $A_{1.5}$ and several "M" resonances (isospin 0 ?) are seen. Note that M-η' mixing might occur.

(b) There are natural interpretations of the $E(1.42)$, $X(1.64)$, $\rho/\eta(1.83)$, and $A_4(1.96)$ as η, η', or π recurrences, respectively.

(c) There are natural locations for the L and Q_B enhancements as $K(0.494)$ recurrences.

$L=1$ *and* $L=2$.

(a) $B(1.235)$ is properly located, though this would be true of any quark model giving the A_2 and A_1 masses correctly via $\vec{L}\cdot\vec{S}$ splitting.

(b) The $A_3(1.64)$ may require interpretation as two Breit-Wigner shapes, thus, perhaps, accounting for its large width and uncertain phase-shift status.

The obvious difficulty is the experimental absence of the π recurrence at 0.6 GeV. This type of difficulty is hard to avoid, regardless of the potential model employed, if one attempts to incorporate the $\rho'(1.25)$ as an $L=0$ radial recurrence of $\rho(0.77)$. Even if one goes to the larger radial excitation scale set by the $\rho(0.77)$, $\rho'(1.6)$ difference, any quark potential model will require excited pions, with the lowest probably somewhere between 1 GeV and 1.2 GeV—depending on whether one measures up 0.83 from π or down 0.4 (the hyperfine splitting) from ρ'. (The unconfirmed $[A_{1.5}(1.17)]$ is a possibility.) A more minor point is that we have been forced to assign $L(1.77)$ as a 1^+—no place in the $S=1$ K^*-like spectrum is suitable. Experiment, which prefers 2^-, does not, however, rule this out.

TABLE II. Parameters.

b (uncharmed) $=0.2743$, $b\,(\mathcal{P}'\bar{\mathcal{P}}')=0.3429$
$m_{\mathcal{P}(\mathfrak{N})}=0.163$ GeV $\quad m_\lambda=0.286$ GeV ($m_{\mathcal{P}(\mathfrak{N})}$ assumes $A_2=0$)
$A_0=0.3$ GeV, $A_1=0.07$ GeV, $A_2=0$
$a_1=0.1$ GeV, $a_2=0.07$ GeV
$c=0.4$ GeV, $B=0.17$ GeV

MESONS CONTAINING CHARMED QUARKS

Even with this phenomenology behind us, it remains difficult to make educated guesses concerning the complete spectra of mesons containing charmed quarks.[18] Of course, given the 3.1 and 3.7 J masses, the spacing b_J is determined. Thus if one were to assume all short-range forces and $\vec{L}\cdot\vec{S}$ splittings are negligible, Table I could be used to predict the masses of all other mesons composed of $\mathcal{P}'\bar{\mathcal{P}}'$. The opposite assumption, namely, that all the short-range effects remain constant in going to the charmed sector, encounters a serious difficulty. To see this, we con-

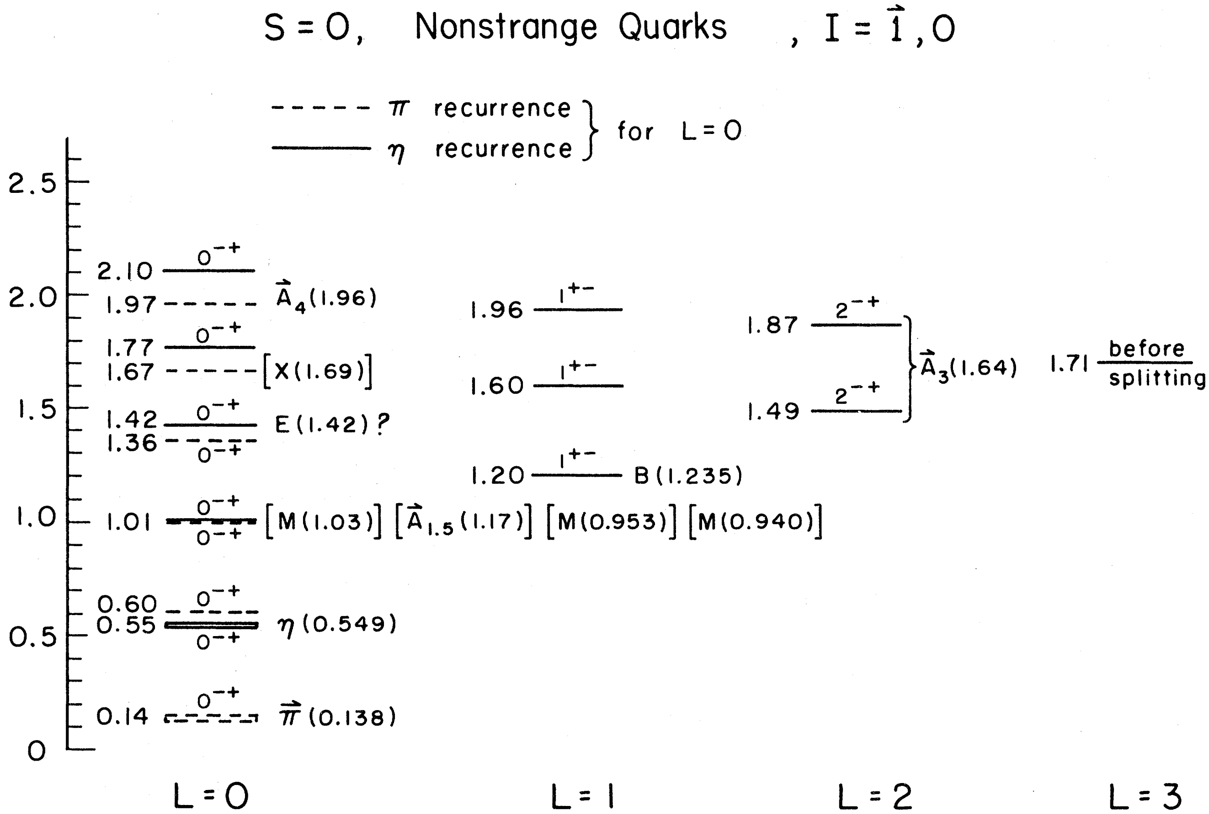

FIG. 4. $S=0$ mesons consisting of $\mathcal{P}\overline{\mathcal{P}}$ or $\mathfrak{N}\overline{\mathfrak{N}}$ quarks. For explanation of notation, see the caption to Fig. 1.

sider also the D ($S=0$) type mesons composed of a $\mathcal{P}$ and a $\overline{\mathcal{P}}'$. Consider $L=0$. Denoting the spacing parameters appropriate to the ρ, D, and J by b_ρ, b_D, and b_J, with a similar notation for the short-range energy shifts and spin-spin interaction strengths $[A_0(\rho)$, $A_0(D)$, $A_0(J)$ and c_ρ, c_D, $c_J]$, we have

$$m_D = \tfrac{1}{2}(m_J + m_\rho) - \tfrac{1}{8}(c_\rho + c_J + 6c_D) + [b_D - \tfrac{1}{2}(b_\rho + b_J)]x_1 + [\tfrac{1}{2}(A_J + A_\rho) - A_D]\,. \tag{15}$$

If we take

$$b_D = \tfrac{1}{2}(b_J + b_\rho), \quad c_\rho = c_J = c_D, \quad A_\rho = A_D = A_J\,, \tag{16}$$

we obtain

$$m_D = \tfrac{1}{2}(m_\rho + m_J) - c_\rho \simeq 1.53 \text{ GeV}\,. \tag{17}$$

This is clearly unacceptable as $J(3.1)$ would then have charmed decay modes. Even if we assume a smooth transition of the parameters in going to the charmed sector, e.g.,

$$c_D = \frac{c_J + c_\rho}{2}\,, \quad A_D = \frac{A_J + A_\rho}{2}\,, \quad b_D = \frac{b_J + b_\rho}{2}\,, \tag{18}$$

we still obtain

$$m_D = \tfrac{1}{2}(m_J + m_\rho) - \tfrac{1}{2}(c_\rho + c_J) \lesssim 1.75 \text{ GeV} \tag{19}$$

(for $c_\rho = 0.4$ as required earlier), so that $J(3.7)$ would decay into two D's. In any case, it would seem that c_J must be very much smaller than c_ρ so that orthocharmonium and paracharmonium will indeed be very closely spaced. Thus strong mass dependence of the parameters of our mass matrix, in going from the normal meson sector to mesons containing charmed quarks, would seem to be unavoidable.

Thus we consider as most reasonable the assumption that all short-range effects are small in the charmed sector, so that the $\mathcal{P}'\overline{\mathcal{P}}'$ charmed meson masses may be read off the second half of Table I, in which the b parameter was chosen to fit the observed mass difference between the 3.1- and 3.7-GeV resonances. The $\mathcal{P}'$ quark mass required is 1.228 GeV. From this table we find, for instance, that in addition to the $n=2$, $L=0$ and $n=3$, $L=0$ radial recurrences at 4.19 and 4.63 GeV there should be an $n=1$, $L=1$ level at about 3.45 GeV (this level may, of course, be split by $\vec{L}\cdot\vec{S}$

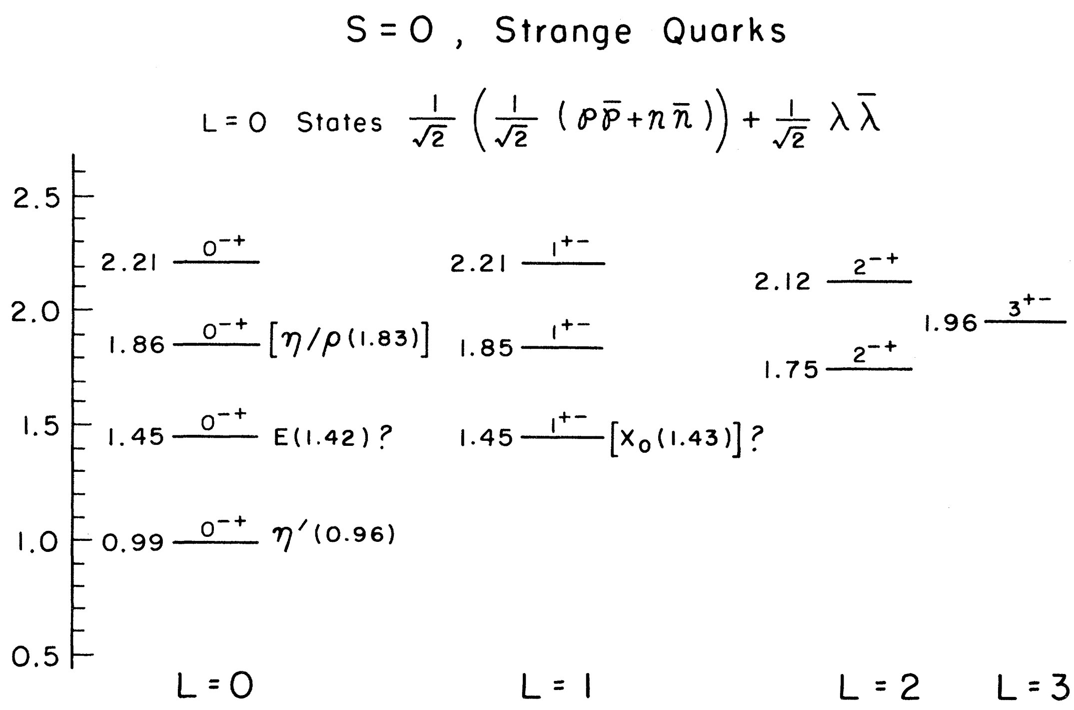

FIG. 5. $S=0$ mesons consisting of a λ and a $\bar{\lambda}$ quarks. For explanation of notation, see the caption to Fig. 1.

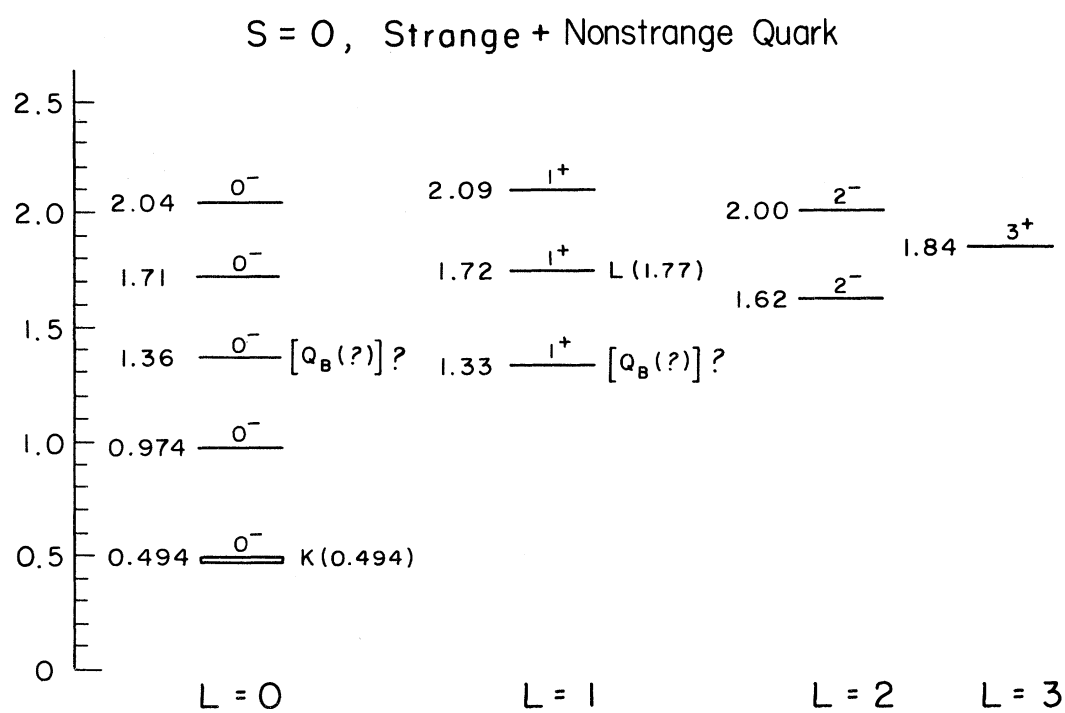

FIG. 6. $S=0$ mesons consisting of a $\mathcal{P}$ and a $\bar{\lambda}$ quark. For explanation of notation, see the caption to Fig. 1.

forces for $S=1$ mesons) and an $n=1$, $L=2$ level at about 3.76 GeV, i.e., slightly above the $n=2$, $L=0$ radial recurrence observed at 3.7 GeV. These results are essentially the same as those obtained by the Cornell group.[3] Concerning F and D mesons we only reemphasize that on the basis of smoothness [in the sense of Eq. (18)] there is some possibility that $S=0$ D mesons may be somewhat lower in mass than currently expected.

BARYONS

Let us now turn to the baryons. The analysis will be rougher than for the meson case, simply because of the increased splittings present. We adopt a model in which baryons are composed of a diquark pair and a quark. This type of structure is indicated both by proton deep-inelastic data, for which the valence $\mathcal{N}$ quark contribution—present for a model without $\mathcal{P}\mathcal{N}$ pairing in the threshold limit—seems to be absent, and by the baryon spectrum in which odd L is always associated with the mixed symmetry 70 representation of SU(6) and odd parity while even-L states have even parity and belong to the fully symmetric $\underline{56}$ SU(6) representation. Structure of this latter type is clearly also a two-body effect. We of course use a linear potential to bind the quark and diquark pair.

The experimental spectral masses we employ represent an eyeball average of the masses of baryons in a given SU(6), L representation having strangeness -1, i.e., hypercharge $Y=0$. (See Ref. 8 for an early version of such a plot.) Presumably quark mass differences account for the splitting between objects of different strangeness and other splitting forces, such as $\vec{L}\cdot\vec{S}$, $\vec{S}_1\cdot\vec{S}_2$, etc., account for the differences among members, with the same strangeness, of a given multiplet. We are concerned only with the over-all spacing between different SU(6) multiplets and their recurrences. Note that we have included an $L=0$, $n=3$ level even though no such $Y=0$ objects are experimentally seen. This level was obtained by shifting up the $L=0$, $Y=1$ N spectrum (which does contain $n=1$, 2, and 3 objects) by an amount such that the $n=1$ and $n=2$ levels coincided with the $Y=0$, $n=1$, and $n=2$ levels. The required shift is about 0.3 GeV. The $L=1$, $n=2$ level drawn was also obtained by examining the $Y=1$, N spectrum, which indicates a separation of the indicated magnitude between $L=1$, $n=1$ and $L=1$, $n=2$.

As before, we use the $L=0$ spacing to determine the parameter b. The experimental mass aver-

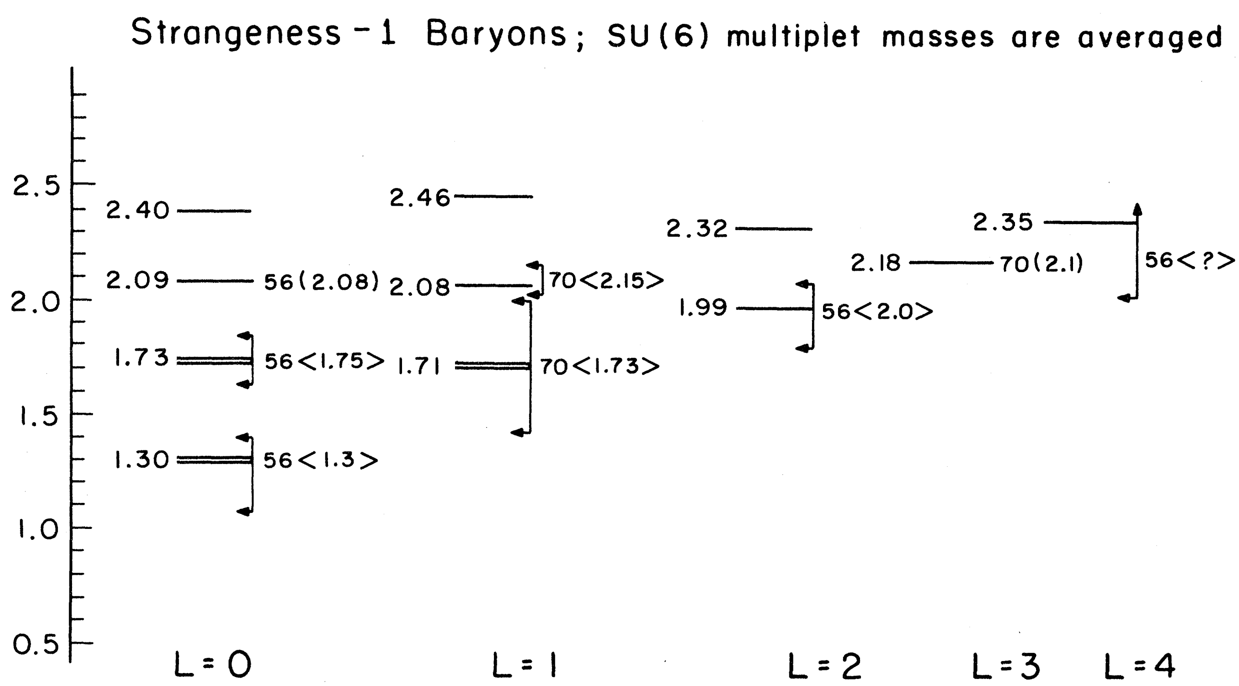

FIG. 7. Baryon spectral levels. The experimental masses represent an eyeball average of the masses of the strangeness $=-1$ members of the indicated multiplet. The brackets indicate the mass range averaged over. If only one state or degenerate state was available, no bracket is drawn. See text for details concerning the $L=0$, $n=3$ and $L=1$, $n=2$ levels which were obtained by shifting the strangeness-0 spectrum. For explanation of notation, see the caption to Fig. 1.

ages are indicated in Fig. 7 in which it is also apparent that our fit, with $b = 0.25$ GeV, works quite well. Obtaining such a fit required that the $(L=0,\ n=3)-(L=0,\ n=2)$ mass splitting be smaller than that between $L=0$, $n=2$ and $L=0$, $n=1$. This feature of the data is more difficult to interpret in a harmonic-oscillator potential approach. Assuming a separation of about 0.13 GeV between the $L=1$ and $L=0$ zero-point energies (as for the $S=1$ meson spectra) we obtain $L=1$ levels as indicated. For a separation of 0.07 GeV between the $L=2$ and $L=1$ zero-point energies we obtain the $L=2$ levels as shown, and we neglect further zero-point energy shifts, (due, recall, to short-range attractive forces), for higher L values.

The resulting description is quite satisfactory. In particular the spacing of the two $L=1$ multiplets, which is determined by the $L=0$ spacing, seems correct. Also we note that it is relatively natural to expect the $L=1$, $n=1$ multiplet to be more or less degenerate in mass with the $L=0$, $n=2$ multiplet. The zero-point energy shift required to accomplish this in the case of a harmonic potential is substantially larger than in the linear-potential case. This is, perhaps, not a problem as the zero-point energy shifts for the baryon spectra are not necessarily of the same magnitude as those appropriate to the meson spectra.

CONCLUSIONS

We have shown that in the observed hadron spectrum (mesons and baryons) there are a large number of states which can be well fitted as radial excitations, using a linear binding potential, of the meson nonets and baryon SU(3) singlets, octets, and decuplets expected in a (color) quark-diquark baryon scheme. Of course, many of these particles have the status of "claimed" rather than established, so continuing experimental work and analysis will eventually decide the validity of this approach to hadron spectroscopy. Particularly vital in this regard is the establishment of the ρ' (1.25) whose existence sets the scale for all the (uncharmed) meson radial excitations in our treatment. In particular, we have shown that it is not possible to take the radial excitation scale determined by the mass difference between the ρ' (1.6) and the ρ (0.77) and to make a consistent detailed fit to the meson spectrum using the linear potential spacings combined with possible short-range attractive forces, etc., without violating some rather basic theoretical prejudices. The one outstanding difficulty in the scheme presented here is the appearance of a radial excited pion at 600 MeV. However, the pion is an enigma in most quark-model approaches and it is not clear that an understanding of it would not eliminate the difficulty with the 600-MeV recurrence.

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[1]H. Nielsen and P. Oelsen, Nucl. Phys. B61, 45 (1973); B. Zumino, CERN Report No. CERN TH-1779, 1973 (unpublished); G. Parisi, Phys. Rev. D 11, 970 (1975); G. 't Hooft, Nucl. Phys. B79, 276 (1974); CERN Report No. TH-1902 (1974).

[2]J. Schwinger, Phys. Rev. 128, 2425 (1962); J. Kogut and L. Susskind, Phys. Rev. D 9, 3501 (1974); K. Wilson, *ibid.* 10, 2445 (1974).

[3]E. Eichten *et al.*, Phys. Rev. Lett. 34, 369 (1975).

[4]B. J. Harrington, S. Y. Park, and A. Yildiz, Phys. Rev. Lett. 34, 168 (1975).

[5]K. Jhung, K. Chung, and R. S. Willey, Univ. Of Pittsburgh Report No. PITT-139 (unpublished).

[6]J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974); J. E. Augustin *et al.*, *ibid.* 33, 1406 (1974); G. Bacii *et al.*, *ibid.* 33, 1408 (1974).

[7]J. E. Augustin *et al.*, Phys. Rev. Lett. 34, 764 (1975).

[8]See, for instance, H. Harari, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, 1968,* edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968).

[9]Particle Data Group, Phys. Lett. 50B, 1 (1974).

[10]*Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun, National Bureau of Standards Applied Mathematics Series, No. 55 (U.S.G. P.O., Washington, D. C., 1964).

[11]A. De Rújula and S. L. Glashow, Phys. Rev. Lett. 34, 46 (1975).

[12]J. F. Gunion and L.-F. Li (unpublished).

[13]These results are due to K. Jhung, using the relativistic model of Ref. 5.

[14]d'Andlau *et al.*, College of France (Paris) report (unpublished).

[15]B. R. French *et al.*, Nuovo Cimento 52A, 438 (1967).

[16]P. Estabrooks *et al.*, in *π-π Scattering—1973,* proceedings of the International Conference on π-π Scattering and Associated Topics, Tallahassee, 1973, edited by P. K. Williams and V. Hagopian (A.I.P., New York, 1973).

[17]S. D. Protopopescu *et al.*, Phys. Rev. D 7, 1279 (1973).

[18]T. Appelquist and H. D. Politzer, Phys. Rev. Lett. 34, 43 (1975); S. Borchardt *et al.*, *ibid.* 34, 38 (1975); C. G. Callan *et al.*, *ibid.* 34, 52 (1975).