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Non-Abelian gauge fields as Nambu-Goldstone fields*

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Gauge invariance is obtained as a *consequence* of the spontaneous breaking of a larger symmetry. The Yang-Mills gauge fields are the corresponding Nambu-Goldstone fields. This involves the study of field theories in a space with more than four dimensions. In a "cylindrical" sector these field theories reduce to new non-Abelian generalizations of Jordan-Brans-Dicke theory. The nonobservability of the excess dimensions (while a difficulty for theories in which these dimensions are bosonic) should cause no problems if the higher dimensions are fermionic. In the latter case, field theories in which all basic fields are at the same time Nambu-Goldstone and Yang-Mills fields become possible. The relation of this work to recent work on gravitation theory by Yang is explored.

I. INTRODUCTION

Gauge fields and Nambu-Goldstone fields are the basic ingredients of unified quantum field theories of the electromagnetic, weak, and possibly even strong and gravitational interactions. With the advent of supersymmetries[1] that shuffle bosons and fermions, even Fermi fields may belong to the classes of gauge and Nambu-Goldstone fields, originally both massless (all masses in the theory are expected to originate through spontaneous symmetry breaking). One is then tempted to conjecture that these two basic categories of fields are equivalent in some sense. Maybe *every gauge field is also a Nambu-Goldstone field and/or vice versa. A priori,* the later half of this statement seems to be flawed, as for instance, the pion fields of the σ model are Nambu-Goldstone fields while they certainly are not gauge fields. Nevertheless the first half of the statement has some remarkable illustrations:

(i) The gauge field of the Abelian group U(1) or O(2) can be viewed as the Nambu-Goldstone field corresponding to the spontaneous breaking of Lorentz invariance.[2]

(ii) The gauge field of the Lorentz group (the graviton) can be viewed also as the Nambu-Goldstone field for the spontaneous breaking of general covariance.[3-5]

We shall set up a field theory within which the Yang-Mills-type gauge fields for a (non-Abelian) internal-symmetry group become Nambu-Goldstone fields for the spontaneous breaking of general covariance in a higher-dimensional space. When restricted to a 4-dimensional world this theory turns into a non-Abelian counterpart of a field theory constructed in the Abelian case by Kaluza and by Jordan.[6-8] This theory is interesting in its own right, and we work it out in detail in Secs. IV and V.

We also compare in Sec. VI our approach with Yang's recent work[9] on gravitation. Yang's theory may be more "natural" if one views gravitation as a geometric copy of Yang-Mills theory; however, if we insist on all massless fields being Nambu-Goldstone fields then Einstein's theory obtains and not Yang's.

Finally, we discuss the possibility that the higher dimensions are fermionic (in the manner of the Salam-Strathdee-Volkov-Akulov superspace[1]). In theories of this type scalar and spin-$\frac{1}{2}$ fields can appear as gauge fields, and the possibility arises that even the second half of our earlier statement holds and the concepts of gauge and Nambu-Goldstone fields become completely equivalent.

II. GRAVITATION AS A NAMBU-GOLDSTONE FIELD: A BRIEF REVIEW

Our argument that Yang-Mills fields are Nambu-Goldstone fields will exhibit a close similarity to the work of Isham, Salam, and Strathdee[3] and of Ogievetsky and Borisov[4,5] in which the Nambu-Goldstone nature of the gravitational field is established. We therefore briefly review here the major steps in their reasoning.

The best starting point is an observation due to Ogievetsky.[4] It states that any field theory in $4+N$ dimensions invariant under the general affine (i.e., inhomogeneous linear: linear + translations) group in $4+N$ dimensions $IGL(4+N,R)$ and under the standard $(4+N)$-dimensional nonlinear realization of the $(4+N)$-dimensional conformal group $O(4+N, 2)$ is $4+N$ generally covariant. This theorem is proved by noting that the generators of $GL(4+N,R)$ and of the $(4+N)$-dimensional conformal group do not close on a finite algebra, on account of the nonlinear coordinate dependence of the conformal boosts. Rather, they close on the infinite-dimensional algebra of the general covariance group in $4+N$ dimensions.

To identify gravitation with a Nambu-Goldstone boson one therefore starts from the case $N=0$ and considers the simultaneous nonlinear realization of the groups IGL$(4,R)$ and O$(4,2)$ which becomes linear for the Poincaré subgroup. Breaking O$(4,2)$ to the Poincaré subgroup one picks up $15-10=5$ Nambu-Goldstone fields: a scalar dilaton χ and a vector field C_m for the conformal boosts. Without loss of consistency one can, however, identify C_m with the gradient of the dilaton field, so that one picks up essentially a single scalar field this way. When spontaneously breaking IGL$(4,R)$ down to its Poincaré subgroup one picks up further $20-10=10$ fields which are essentially the vierbein fields in a gauge in which the vierbein matrix is symmetric. Group theory then guides the construction of what ultimately turns out to be Einstein's Lagrangian.

Alternatively one can start directly from the group of general coordinate transformations in 4-space [once one knows that this is equivalent with the GL$(4,R)$ – O$(4,2)$ procedure by Ogievetsky's theorem] and represent in a unique way any element $L_m^{\bar{m}}$ of GL$(4,R)$ in the form (Latin indices run from 1 to 4 and a bar on an index indicates that the index is "active" under Lorentz transformation)

$$L_m^{\bar{m}}=A_{\bar{a}}^{\bar{m}}S_m^{\bar{a}}. \tag{2.1}$$

Here $L_m^{\bar{m}}$ is any regular 4×4 matrix, S (which plays the role of vierbein) is the exponential of a symmetric matrix, and A is a Lorentz transformation. Now consider a general (invertible) coordinate transformation

$$x'^m=x'^m(x^n). \tag{2.2}$$

At every point P (of old coordinates x^m and new coordinates x'^m) we associate to the transformation (2.2) the GL$(4,R)$ element T defined by the matrix

$$T_m^n=\frac{\partial x^n}{\partial x'^m}. \tag{2.3}$$

To obtain the transformation law of the vierbein fields S we perform the linear transformation T on L, then decompose anew in the manner (2.1), and find the new vierbein S'. In detail,

$$L'^{\bar{m}}_m=L_n^{\bar{m}}T_m^n=A'^{\bar{m}}_{\bar{p}}S'^{\bar{p}}_m, \tag{2.4}$$

so that using (2.1) we have

$$S'^{\bar{m}}_m=A'^{-1\bar{m}}_{\bar{p}}A_{\bar{a}}^{\bar{p}}S_n^{\bar{a}}T_m^n, \tag{2.5}$$

with T_m^n given by Eq. (2.3). We stress that all fields in Eqs. (2.4) and (2.5) are evaluated at the same physical point having old coordinates x and new coordinates x', but to simplify the notation we have not explicitly written out the argument (x or x') in the various fields. Equation (2.5) establishes the transformation law of the S fields and these indeed play the role of vierbein. One then uses this vierbein to construct a metric tensor and then establishes the integral of the corresponding curvature density, i.e., the Einstein-Hilbert action

$$I_E=-\frac{1}{16\pi G}\int d^4x\sqrt{-g}R$$

to be an acceptable generally invariant action for the Nambu-Goldstone fields.

For our purposes, the important lesson is that the Nambu-Goldstone fields play the role of vierbein in a curved manifold and that the standard nonlinear-realization argument suffices to establish their transformation law under general coordinate transformations.

III. GAUGE FIELDS AS NAMBU-GOLDSTONE FIELDS

We want to construct a theory which exhibits manifest local gauge invariance under some N-dimensional internal symmetry G. Yet we do not want to achieve this by deliberately introducing Yang-Mills gauge fields. Rather we want these massless vector fields to emerge automatically as Nambu-Goldstone fields corresponding to the spontaneous breaking of a larger symmetry $\bar{G}\supset G$. Moreover, we want gauge invariance under local G transformations not to be a separate input but rather a consequence of the larger symmetry.

In implementing this program the first step must obviously be the choice of the larger group $\bar{G}$. To this end we first embed the N-dimensional compact group G into O(N), which is always possible. We then embed O(N) into IGL$(4+N,R)$ on one hand and the conformal group O$(4+N,2)$ on the other hand. At the same time we extend 4-dimensional spacetime to a $(4+N)$-dimensional space with one timelike and $3+N$ spacelike dimensions. By Ogievetsky's theorem a field theory in $(4+N)$-dimensional space invariant under IGL$(4+N,R)$ and O$(4+N,2)$ is invariant under the general coordinate transformations in the $(4+N)$-dimensional space. Now, let us follow the 4-dimensional procedure and look for a nonlinear realization of $(4+N)$-general covariance which becomes linear for the $(4+N)$-dimensional Poincaré group P_{4+N}. The dimensions of IGL$(4+N,R)$, O$(4+N,2)$, and P_{4+N} are respectively $(4+N)(5+N)$, $(5+N)(6+N)/2$, and $(4+N)(5+N)/2$. From the breaking of the conformal group we then pick up $(5+N)(6+N)/2-(4+N)(5+N)/2=N+5$ Nambu-Goldstone fields: one scalar and one $(4+N)$-vector. The latter is again the $(4+N)$ gradient of the scalar (dilatonlike) field. Conformal symmetry breaking thus yields again one scalar field. The breaking of IGL$(4+N,R)$ yields $(4+N)(5+N)\times(1-1/2)=(4+N)(5+N)/2=10+4N+N(N+1)/2$ Nambu-Goldstone fields. Of these, 10 are again the components of the gravitational vierbein, $4N$ will turn out to be the Yang-Mills fields, and $N(N+1)/2$ are Lorentz-scalar Nambu-Goldstone bosons with tensorial transformation properties under G. As before, the scalar dilaton field is related to a combination (the trace) of the vierbein fields and therefore, modulo this technical point, we restrict our considerations to the breaking of GL$(4+N,R)$. As in the 4-dimensional case we write the general element of GL$(4+N,R)$ in the form of a $(4+N)\times(4+N)$ matrix L

$$L_A^{\bar{A}}=\hat{O}_{\bar{B}}^{\bar{A}}S_A^{\bar{B}}, \tag{3.1}$$

where again we made the unique polar decomposition of the type (2.1), i.e., S is the exponential of a symmetric matrix and $\hat{O}$ a pseudo-orthogonal matrix. In Eq. (3.1) and in the following we adopt these notations: Capital Latin indices run from 1 to $4+N$, small Latin indices run from 1 to 4, small Greek indices from 5 to $4+N$. Indices that are "active" under pseudo-orthogonal $[\mathrm{O}(3+N),1)]$ transformations are marked with a bar ($\bar{A}, \bar{B}, \bar{a}, \bar{\alpha}$, etc.,). Any capital barred or unbarred index can always be replaced by a Latin-Greek pair of small indices e.g., $A\equiv(a,\alpha)$, $M\equiv(m,\mu)$, $\bar{N}\equiv(\bar{n},\bar{\nu})$, etc.

Finally, to pass to an explicit matrix notation it is convenient to regard the upper (lower) index as giving the line (column) of the respective entry. Thus for any matrix (with barred or unbarred indices)

$$M_B^A=\begin{pmatrix} M_b^a & M_\beta^a \\ M_b^\alpha & M_\beta^\alpha \end{pmatrix}, \tag{3.2}$$

and in matrix form Eq. (3.1) becomes

$$L=\hat{O}S\ . \tag{3.1$'$}$$

Now consider a matrix with a null right $4\times N$ corner

$$\tilde{N}_B^A=\begin{pmatrix} \tilde{N}_b^a & 0 \\ \tilde{N}_b^\alpha & \tilde{N}_\beta^\alpha \end{pmatrix}. \tag{3.3}$$

The exponential of such a matrix

$$N=\exp\tilde{N} \tag{3.4a}$$

again has null right $4\times N$-corner

$$N=\begin{pmatrix} N_b^a & 0 \\ N_b^\alpha & N_\beta^\alpha \end{pmatrix}. \tag{3.4b}$$

Any such matrix N can be written in the form

$$N=\tilde{O}K\ , \tag{3.5}$$

where K, besides having a null right $4+N$ corner

$$K=\begin{pmatrix} K_b^a & 0 \\ K_b^\alpha & K_\beta^\alpha \end{pmatrix}, \tag{3.6a}$$

also obeys

$$K_b^a=K_a^b,\quad K_\beta^\alpha=K_\alpha^\beta, \tag{3.6b}$$

and $\tilde{O}$ is an element of $\mathrm{O}(3,1)\times\mathrm{O}(N)$, i.e., a matrix of the type

$$\tilde{O}=\begin{pmatrix} \tilde{\theta}_4 & 0 \\ 0 & \tilde{\theta}_N \end{pmatrix},\quad \tilde{\theta}_4\tilde{\theta}_4^T=\eta_4,\quad \tilde{\theta}_N\tilde{\theta}_N=\eta_N \tag{3.7}$$

where $\eta_4=\mathrm{diag}(---+)$ and $-\eta_N$ is the $N\times N$ unit matrix (this specifies our metric convention). For reasons that will become clear in the next section we shall call a matrix of the type (3.6a) and (3.6b) a *Kaluza basis.* Now, any regular $(4+N)\times(4+N)$ matrix L can be written in a unique way also in the form

$$L_A^{\bar{A}}=O_{\bar{B}}^{\bar{A}}K_A^{\bar{B}}$$

or (3.8)

$$L=OK\ ,$$

where O is a pseudo-orthogonal matrix and K a Kaluza basis.[10] Comparing Eqs. (3.1) and (3.8) we find the matrix relation

$$K=(O^{-1}\hat{O})S\ . \tag{3.9}$$

There is thus a one-to-one correspondence between the fields S and the Kaluza basis K. For our purposes it will be more convenient to choose the Kaluza basis K rather than the symmetric matrix S to represent the Nambu-Goldstone fields for the spontaneous breaking of GL$(4+N,R)$ invariance.

As in Sec. II, we can now derive the transformation law of the Nambu-Goldstone fields $K_B^{\bar{A}}$ under general coordinate transformations (alternatively one could consider the related fields $S_B^{\bar{A}}$ but they are less convenient to us). For the general regular coordinate transformations

$$x^A\to x'^A=x'^A(x^B), \tag{3.10}$$

with

$$T_N^M=\frac{\partial x^M}{\partial x'^N}, \tag{3.11}$$

we have

$$L'=LT=O'K', \tag{3.12}$$

so that from Eqs. (3.12) and (3.1$'$) we obtain

$$K'=\Omega KT,\quad \Omega=O'^{-1}O \tag{3.13}$$

the desired transformation law.

We now consider the special class of infinitesimal transformations for which the matrix of Eqs. (3.11)–(3.13) is given by

$$T(\theta)_N^M = \eta_N^M + \delta T(\theta)_N^M$$
$$= \eta_N^M + \begin{pmatrix} 0 & 0 \\ \kappa\partial_n\theta^\mu & f^\mu_{\nu\rho}\theta^\rho \end{pmatrix}, \qquad (3.14)$$

where θ^ρ $(\rho = 5, \ldots, 4+N)$ are N arbitrary infinitesimal functions of the x_a $(a = 1, \ldots, 4)$, κ is for the time being, an arbitrary constant, and $f^\mu_{\nu\rho}$ are the structure constants of the group G. The transformations (3.14) close since

$$[\delta T(\theta), \delta T(\theta')] = -\delta T(\theta\times\theta'), \qquad (3.15a)$$

with

$$(\theta\times\theta')^\mu = f^\mu_{\nu\rho}\theta^\nu\theta'^\rho . \qquad (3.15b)$$

Under the transformations (3.14) the coordinates change according to

$$x'^a = x^a$$
$$x'^\alpha = x^\alpha + \kappa x^b\partial_b\theta^\alpha + f^\alpha_{\beta\gamma}x^\beta\theta^\gamma \qquad (3.16)$$

Note that space-time (x^a) remains unaffected and only the internal-space coordinates undergo space-time-dependent transformations. This is suggestive of gauge transformations and we will now show that these transformations are indeed gauge transformations.

From Eqs. (3.6a) and (3.14),

$$(KT)_N^{\bar M} = K_R^{\bar M} T_N^R$$
$$= \begin{pmatrix} K_n^{\bar m} & 0 \\ K_n^{\bar\mu} + \kappa K_\rho^{\bar\mu}\partial_n\theta^\rho & K_\nu^{\bar\mu} + f^\rho_{\nu\sigma}\theta^\sigma K_\rho^{\bar\mu} \end{pmatrix}.$$

Since $(KT)_\nu^{\bar\mu}$ is not symmetric, we must correct this with an O$(N+3, 1)$ transformation

$$\Omega_{\bar M}^{\bar A} = \begin{pmatrix} \eta_{\bar m}^{\bar a} & 0 \\ 0 & \omega_{\bar\mu}^{\bar\alpha} \end{pmatrix}, \quad \omega\omega^T = \underline{1} .$$

We thus have to lowest order in the infinitesimal functions θ^α

$$K'^{\bar M}_N = (\Omega KT)_N^{\bar M}$$
$$= \begin{pmatrix} K_n^{\bar m} & 0 \\ \omega_{\bar\rho}^{\bar\mu}K_n^{\bar\rho} + \kappa K_\sigma^{\bar\mu}\partial_n\theta^\sigma & \omega_{\bar\rho}^{\bar\mu}K_\nu^{\bar\rho} + f^\rho_{\nu\sigma}\theta^\sigma K_\rho^{\bar\mu} \end{pmatrix}.$$

We define new fields A_a^β by

$$K_a^{\bar\gamma} = e\kappa K_\beta^{\bar\gamma} A_a^\beta , \qquad (3.17)$$

where e is a new constant. The constant κ [introduced in Eq. (3.1)] will be ultimately related to the gravitational coupling constant and e to the Yang-Mills coupling constant. We end up finding the transformation laws

$$K'^{\bar m}_n = K_n^{\bar m},$$
$$K'^{\bar\mu}_\nu = \omega_{\bar\rho}^{\bar\mu}K_\nu^{\bar\rho} + f^\rho_{\nu\sigma}\theta^\sigma K_\rho^{\bar\mu}, \qquad (3.18)$$
$$A'^\alpha_a = A_a^\alpha + \frac{1}{e}\partial_a\theta^\alpha + f^\alpha_{\beta\gamma}\theta^\beta A_a^\gamma .$$

We now unambiguously recognize the transformations (3.18) for what they are: local gauge transformations of the group G provided e is the Yang-Mills coupling constant. The $K_n^{\bar m}$ are invariant, the $K_\nu^{\bar\mu}$ transform nonlinearly, and the A_a^α fields transform precisely like the Yang-Mills gauge potentials. We thus conclude that *any generally-invariant theory in 4 +N dimensions is also G gauge-invariant* (i.e., invariant under G transformations with x-dependent parameters). We have, so to speak, derived G gauge invariance from the spontaneous breaking of general invariance in $4+N$ dimensions. *Some of the corresponding Nambu-Goldstone fields become the gauge fields of the group G.* These Yang-Mills gauge fields are thus also Nambu-Goldstone fields for the breaking of GL$(4+N,R)$ symmetry. There is, however, one difficulty: All these fields depend not on 4 but on $4+N$ coordinates x^A. To *circumvent* (not remove) this problem we will provide in the next section a geometrical interpretation of this field theory. The more radical solution of considering supergauge theory will be discussed later.

IV. GEOMETRICAL INTERPRETATION OF THE FIELD THEORY

To construct a usual 4-dimensional field theory, we provide our massless fields $K_A^{\bar B}$ with the geometrical interpretation of components of the "$4+N$-bein" corresponding to the bundle of orthonormal frames of a Riemann manifold. The metric of this manifold is given by

$$\tilde\gamma_{AB} = K_A^{\bar M}K_B^{\bar N}\eta_{\bar M\bar N}, \text{ or } \tilde\gamma = K^TK . \qquad (4.1)$$

The matrix $O^{-1}\hat O$ of Eq. (3.9) being pseudo-orthogonal, the metric $\tilde\gamma$ is also equal to S^TS when expressed in terms of the symmetric Nambu-Goldstone matrix S of Eq. (3.1′), and thus analogous to the usual 4-dimensional definition of Refs. 3–5. Using Eqs. (3.6) and (3.17)

$$\tilde\gamma_{AB} = \begin{pmatrix} g_{ab} + e^2\kappa^2 g_{\gamma\delta}A_a^\gamma A_b^\delta & e\kappa g_{\beta\gamma}A_a^\gamma \\ e\kappa g_{\alpha\gamma}A_b^\gamma & g_{\alpha\beta} \end{pmatrix}, \qquad (4.2a)$$

where

$$g_{ab} = K^{\bar m}_a K^{\bar n}_b \eta_{\bar m \bar n}\,, \quad g_{\alpha\beta} = K^{\bar\mu}_\alpha K^{\bar\nu}_\beta \eta_{\bar\mu\bar\nu}\,. \tag{4.2b}$$

We immediately notice the striking similarity of the metric (4.2) with that of the non-Abelian generalizations[11,12] of Kaluza-type higher-dimensional unified field theories. The differences are that (i) the $g_{\alpha\beta}$ rather than being the Killing metric of the group G, are fields themselves, and (ii) all fields depend in a (at this point) nontrivial way on the extra "internal" coordinates $x^5, \ldots, x^N$. We could now write down an action principle of the Einstein-Hilbert type starting from the $(4+N)$-dimensional metric (4.2). This would lead, however, to field equations yielding explicitly $x_5, \ldots, x_N$-dependent fields. This is quite different from having a usual field theory. This $(4+N)$-dimensional theory may yet have its own meaning and relevance for physics. At present, however, we want to confine ourselves to fields that depend only on the usual 4 space-time coordinates. To this end we have to somehow restrict ourselves to "cylindrical" solutions just as in Kaluza's original theory and in the subsequent work of Klein.[13] This appears to spoil the original program of constructing a theory that maintains exact $4+N$ general covariance in the Nambu-Goldstone manner. We rather wish to think of the cylindrical theory as a subsector of the full generally covariant theory. The reason for which the $x_5, \ldots, x_N$ dependence of fields is unobservable could possibly be blamed, according to Klein,[13] on a much smaller "typical scale" in these dimensions say of the order of Planck's length $\sqrt{G} \sim 10^{-33}$ cm.

There is an alternative way out: that the additional dimensions are unobservable by being fermionic as in local supersymmetric theories. We shall further elaborate on this important alternative in Sec. VII.

Here we achieve independence of the N internal coordinates by appropriately restricting the $(4+N)$-dimensional manifold. Rather than let this manifold be a general $(4+N)$-dimensional Riemann space, we require it to be a fiber bundle[11,12] $P(M, G, \Pi)$, physical 4-space M (with metric g_{ab}) being its base manifold, and the internal-symmetry group G its structure group. A^β_a, as we had seen, transform as gauge potentials under gauge transformations. Therefore, they can be related to the connection form ω on the bundle.[12] In detail let $\xi_a(x)$ be commuting basis vectors of the tangent space $T_x(M)$ to the base manifold M at the point $x \in M$. Let $\sigma(x)$ be a cross section of the bundle (i.e., a homeomorphic mapping of M into P such that $\Pi(\sigma(x)) = x$. Further, let the mapping σ map the vector ξ_a into the vector $\sigma \cdot \xi_a$ in the tangent space $T_{\sigma(x)}(P)$ to P at $\sigma(x)$. Finally, let ω^α be the components of the connection form of the bundle (the connection form maps vectors in the tangent space to the bundle into elements of the Lie algebra of G). Then the desired relation is

$$e\kappa A^\alpha_a(x) = \omega^\alpha(\sigma \cdot \xi_a(x)), \tag{4.3}$$

with the constants e and κ as defined above. As was shown in detail in Ref. 12, $\tilde\gamma_{AB}$ is a metric of the bundle in the noncoordinate basis $\tilde D_A$, the commutation relations of which are

$$[\tilde D_A, \tilde D_B] = \tilde C^C_{AB} \tilde D_C\,,$$
$$\tilde C^C_{AB} = \begin{cases} \kappa^{-1} f^\gamma_{\alpha\beta} & \text{for } A=\alpha,\ B=\beta,\ C=\gamma \\ 0 & \text{otherwise.} \end{cases} \tag{4.4}$$

We then write the action

$$I_{4+N} = -\frac{1}{16\pi G}\,\frac{1}{V}\int d^N\theta\, d^4x (R_{4+N} - \lambda)\sqrt{\gamma}\,, \tag{4.5a}$$

where

$$\gamma = (-1)^{N+1}\det(\tilde\gamma_{AB}), \tag{4.5b}$$

R_{4+N} is the scalar curvature of the Riemann space P, the integrals $d^N G$ and d^4x run over a fiber and over a cross section of the bundle, respectively, and G, V_N are the gravitational constant and the volume of a fiber, respectively. The fiber integration is trivial and we find

$$I_{4+N} = I_4 = -\frac{1}{16\pi G}\int d^4x (R_{4+N} - \lambda)\sqrt{\gamma}\,. \tag{4.5c}$$

The scalar curvature is most conveniently evaluated in a different basis: the horizontal lift basis[12] D_A with commutation relations

$$[D_A, D_B] = C^C_{AB} D_C\,,$$
$$C^C_{AB} = \begin{cases} \kappa^{-1} f^\gamma_{\alpha\beta} & \text{for } A=\alpha,\ B=\beta,\ C=\gamma \\ -e\kappa F^\gamma_{ab} & \text{for } A=a,\ B=b,\ C=\gamma \\ 0 & \text{otherwise,} \end{cases} \tag{4.6a}$$

where

$$F^\gamma_{ab} = \partial_a A^\gamma_b - \partial_b A^\gamma_a + e f^\gamma_{\alpha\beta} A^\alpha_a A^\beta_b\,. \tag{4.6b}$$

Again e appears as the Yang-Mills coupling constant. In the basis (4.6), the metric is

$$\gamma_{AB} = \begin{pmatrix} g_{ab} & 0 \\ 0 & g_{\alpha\beta} \end{pmatrix}, \tag{4.7}$$

and we have

$$
\begin{aligned}
&D_\alpha g_{ab}=0\\
&D_\alpha g_{\beta\gamma}=\kappa^{-1}(f^\delta_{\alpha\beta}g_{\gamma\delta}+f^\delta_{\alpha\gamma}g_{\beta\delta}),\\
&D_a g_{bc}=\partial_a g_{bc},\\
&D_a g_{\beta\gamma}=\partial_a g_{\beta\gamma}-e\kappa A^\delta_a D_\delta g_{\beta\gamma}.
\end{aligned}
\tag{4.8}
$$

The second of these equations shows that $g_{\beta\gamma}$ transforms like a symmetric tensor under the group G. In geometric language we have chosen a right invariant metric on the group manifold.

Introducing the reciprocal of the metric tensor

$$\gamma^{MN}=\begin{pmatrix} g^{mn} & 0\\ 0 & g^{\mu\nu}\end{pmatrix},\quad \gamma^{MN}\gamma_{NP}=\delta^M_P \tag{4.9a}$$

the Christoffel symbols are given in this noncoordinate basis by[14]

$$
\begin{aligned}
\Gamma^C_{AB}=\tfrac{1}{2}\gamma^{CD}(&D_A\gamma_{BD}+D_B\gamma_{AD}-D_D\gamma_{AB}\\
&-C^E_{AD}\gamma_{BE}-C^E_{BD}\gamma_{AE})+\tfrac{1}{2}C^C_{AB},
\end{aligned}
\tag{4.9b}
$$

and the Riemann tensor is[14]

$$
\begin{aligned}
R^A{}_{BCD}=&C^E_{DC}\Gamma^A_{EB}-D_D\Gamma^A_{CB}+D_C\Gamma^A_{DB}\\
&-\Gamma^A_{DE}\Gamma^E_{CB}+\Gamma^A_{CE}\Gamma^E_{DB},
\end{aligned}
\tag{4.9c}
$$

so that

$$R_{4+N}=\gamma^{MN}R^A{}_{MAN}. \tag{4.9d}$$

Equations (4.6)–(4.9) yield the Lagrangian

$$
\begin{aligned}
\mathfrak{L}&=-\frac{1}{16\pi G}(R_{4+N}-\lambda)\sqrt{\gamma}\\
&=\mathfrak{L}_T+\mathfrak{L}_V+\mathfrak{L}_{S1}+\mathfrak{L}_{S2},
\end{aligned}
\tag{4.10a}
$$

where

$$
\begin{aligned}
&\mathfrak{L}_T=-\frac{1}{16\pi G}R_4\sqrt{\gamma},\\
&\mathfrak{L}_V=-\frac{1}{4}\frac{e^2\kappa^2}{16\pi G}(-g_{\alpha\beta})g^{mp}g^{nq}F^\alpha_{mn}F^\beta_{pq}\sqrt{\gamma},\\
&\mathfrak{L}_{S1}=-\frac{1}{16\pi G}[\tfrac{1}{4}g^{\alpha\beta}g^{\gamma\delta}g^{mn}(\nabla_m g_{\alpha\gamma}\nabla_m g_{\beta\delta}-\nabla_m g_{\alpha\beta}\nabla_n g_{\gamma\delta})-\tfrac{1}{2}g^{mn}\nabla_m g^{\alpha\beta}\nabla_n g_{\alpha\beta}-g^{\alpha\beta}g^{mn}\nabla_m\nabla_n g_{\alpha\beta}]\sqrt{\gamma},\\
&\mathfrak{L}_{S2}=\frac{\kappa^{-2}}{16\pi G}(\tfrac{1}{2}f^\gamma_{\alpha\delta}f^\delta_{\beta\gamma}g^{\alpha\beta}+\tfrac{1}{4}f^\mu_{\alpha\gamma}f^\nu_{\beta\delta}g^{\alpha\beta}g^{\gamma\delta}g_{\mu\nu}+\kappa^2\lambda)\sqrt{\gamma},
\end{aligned}
\tag{4.10b}
$$

with

$$\sqrt{\gamma}=(-\det g_{mn})^{1/2}[(-1)^N\det g_{\alpha\beta}]^{1/2} \tag{4.10c}$$

and ∇_m the simultaneous Yang-Mills and gravitational convariant derivative, so that

$$
\begin{aligned}
&\nabla_m g_{\alpha\beta}=D_m g_{\alpha\beta},\\
&\nabla_m\nabla_n g_{\alpha\beta}=D_m D_n g_{\alpha\beta}-\Gamma^p_{mn}D_p g_{\alpha\beta},
\end{aligned}
\tag{4.10d}
$$

with the Yang-Mills covariant derivative D_m as defined in Eqs. (4.8) and the Christoffel symbols Γ^p_{mn} given by Eqs. (4.9b).

We now turn to a detailed discussion of the unified field theory built on this Lagrangian. In the process, we will understand the significance of the constants κ, e, and λ introduced above.

V. THE UNIFIED FIELD THEORY

The Lagrangian (4.10) is composed of four types of terms: $\mathfrak{L}_T$, $\mathfrak{L}_V$, $\mathfrak{L}_{S1}$, and $\mathfrak{L}_{S2}$. If we equate G with the universal gravitational constant then $\mathfrak{L}_T$ is essentially the Einstein-Hilbert Lagrangian for the massless G-singlet tensor field of gravity corresponding to the 4×4 metric tensor g_{ab}. There is a difference in that there is the additional factor $[(-1)^N\det(g_{\alpha\beta})]^{1/2}$ which amounts to a space-time variation of the gravitational coupling just as in the Jordan-Brans-Dicke theory.[7,8]

In $\mathfrak{L}_V$ we essentially recognize the generally covariant form (i.e., coupled to gravity) of the Yang-Mills Lagrangian for the gauge field A^β_a of the group G provided

$$\frac{e^2\kappa^2}{16\pi G}=1. \tag{5.1}$$

Again there is a difference in that the Yang-Mills fields are contracted (in the internal indices) with the right-invariant metric $g_{\alpha\beta}$ which depends on space and time. Usually this contraction is made with the right- *and* left-invariant Killing metric. There is a second difference in the appearance of the over-all factor $[(-1)^N\det(g_{\alpha\beta})]^{1/2}$. All these factors have their counterparts in the Abelian Jordan theory.[7] We also note that in order for $\mathfrak{L}_V$ to have the proper sign (so that the energy is positive-definite) the internal dimensions $(x^5,\ldots,x^{4+N})$ must be *spacelike*.

$\mathfrak{L}_S=\mathfrak{L}_{S1}+\mathfrak{L}_{S2}$ is the Lagrangian of the scalar fields that self-interact and interact also with gravity and the Yang-Mills fields (on account of their tensorial G-transformation law).

Now, let us analyse the pieces $\mathcal{L}_{S1}$ and $\mathcal{L}_{S2}$ in more detail. $\mathcal{L}_{S1}$ looks quite complicated but is essentially a general and gauge covariant form of the kinetic Lagrangian of the scalar fields. To see this, expand the scalar and tensor fields around the flat metric

$$g_{ab} = \eta_{ab} + \tau T_{ab}\,,$$
$$g_{\alpha\beta} = \eta_{\alpha\beta} + \sigma S_{\alpha\beta}\,, \qquad (5.2)$$

where τ and σ are proper dimensional constants. We now insert these expansions into the Lagrangian $\mathcal{L}_{S1}$ [Eq. (4.10)] and retain only terms up to quadratic in $S_{\alpha\beta}$ and of zeroth order in T_{ab} and A_a^β. This way we find

$$\mathcal{L}_{S1} = \frac{\sigma^2}{16\pi G}\,\tfrac{1}{4}\partial_m S_{\alpha\beta}\partial_n S_{\gamma\delta}\,\eta^{mn}(\eta^{\alpha\gamma}\eta^{\beta\delta} + \eta^{\alpha\beta}\eta^{\gamma\delta})$$
$$+\text{exact divergence}$$
$$+\text{interaction terms}. \qquad (5.3)$$

In this equation the exact divergence terms arise from the expansion of the second-order derivative term in $\mathcal{L}_{S1}$ and its transformation through partial integration. We recognize (5.3) to provide the kinetic term for the scalar fields up to the arbitrary dimensionless normalization constant $\sigma^2/16\pi G$ just as in the Abelian Jordan-Brans-Dicke case. Finally the term proportional to λ is a cosmological-type term (again multiplied by the square root of the determinant of the scalar fields).

As was hinted at before, in the Abelian case the theory reduces to that proposed by Jordan[7] and further developed by Thirry, Brans, and Dicke.[8] For a non-Abelian group ($N \neq 1$) the theory still provides an unambiguous unification of the interactions of a tensor field, a set of vector gauge fields, and a well-defined set of scalar fields. Just as in the Abelian case, the unification is not complete in that the universal coupling constant e of the gauge field, the gravitational coupling constant $x \sim \sqrt{G}$, the scalar coupling σ, and the cosmological constant λ are still arbitrary. This is in part due to our cylindricity assumption.

One may wonder whether the natural scalar fields $g_{\alpha\beta}$ of the theory might not somehow automatically play the role of Higgs fields for further symmetry breaking from G to a smaller group. The classical potential for these fields is

$$V = -\mathcal{L}_{S2}\,. \qquad (5.4)$$

For simplicity, consider the special case $N=3$, $G=\mathrm{O}(3)$, so that $f^\alpha_{\beta\gamma} = \epsilon_{\alpha\beta\gamma}$. By an O(3) transformation, $g_{\alpha\beta}$ can always be diagonalized. Let $-a^{-1}$, $-b^{-1}$, $-c^{-1}$ be its diagonal entries. V is then given [using Eq. (5.1)] by

$$V = -\frac{e^2}{4\pi}\,\frac{1}{64\pi G^2}\,(abc)^{-1/2}$$
$$\times\left[2(a+b+c) - \tfrac{1}{2}(abc)^{-1}(ab+bc+ca)^2 + \frac{16\pi G}{e^2}\lambda\right], \qquad (5.5)$$

so that the equations

$$\frac{\partial V}{\partial g_{\alpha\beta}} = 0 \qquad (5.6)$$

have the unique solution

$$g_{\alpha\beta} = \frac{e^2}{4\pi}\,\frac{1}{8G\lambda}\,\delta_{\alpha\beta}\,, \qquad (5.7)$$

which is O(3)-invariant. This is a true minimum of the potential, as can be seen from the second derivatives of V. Equation (5.7) emphasizes the role of the cosmological constant in stabilizing the vacuum. Therefore, as it stands, the theory contains no in-built mechanism for further spontaneous G-symmetry breaking.

We shall not explore here other classical solutions or the problem of quantization for the unified field theory.

We want, however, to point out that the tensor field *need not* be that of gravity, but could well be a field of strong gravity.[15] In a sense the setup is very similar to that in dual models,[16] which in the zero-slope limit yields Yang-Mills gauge fields (the open strings) and a massless G-invariant tensor field (the closed strings of the Pomeron sector). In our theory as well, the G-singlet tensor necessarily accompanies the gauge fields, as do for that matter the scalar fields.

VI. COMPARISON WITH YANG'S APPROACH TO THE THEORY OF GRAVITATION

Recently, on the basis of an analysis of the geometric structure of gauge theories, Yang[9] has proposed a new theory of gravitation. As we will presently show, his geometric analysis is essentially identical to the fiber-bundle approach taken here. Nevertheless our approach to gravitation is of the Einstein type and thus differs considerably from Yang's. We want to establish the reason for this difference. If a pure gauge theory over a 4-dimensional space-time is contemplated then Yang's approach is the natural one. If, on the other hand, one wishes to derive gauge invariance from a theory with spontaneous symmetry breaking, then the approach espoused above (i.e., an Einstein-Hilbert type gravity sector) is natural.

Now, to Yang's geometric interpretation of gauge theories. Yang starts from a 4-dimensional space-time manifold M and an internal-symmetry group G. To every curve γ_{xy} in M that runs between two points x and $y \in M$ he associates a group element

$g(\gamma_{xy}) \in G$. For x and $y = x + dx$ infinitesimally close he parametrizes $g(\gamma_{xy})$ in terms of what then turn out to be the gauge potentials. He then calculates the group elements corresponding to an infinitesimal parallelogram of sides dx^m, dx'^n and finds it to be $1 + F^\alpha_{mn} dx^m dx'^n H_\alpha$ (H_α = generators of G). The field strengths F^α_{mn} thus acquire the meaning of "internal" curvature. Our identification with Yang's approach is achieved if we can find the fiber-bundle meaning of the group element $g(\gamma_{xy})$ attached to the path γ_{xy}. We claim that $g(\gamma_{xy})$ is to be determined as follows. Let $\sigma(z)$ be a cross section of the bundle that intersects the fiber over x (y) at $\sigma(x)$ $[\sigma(y)]$. Let γ_{xy} be a curve in the base manifold M with end points $x, y \in M$. There exists a *unique*[17] horizontal lift γ^h_{xy} of γ_{xy} through $\sigma(x)$ [i.e., a unique curve γ^h_{xy} in the bundle P such that (i) it passes through $\sigma(x)$, (ii) its projection in M is γ_{xy}, and (iii) its tangent at any point is horizontal].

On the other hand, there is a unique curve γ^σ_{xy} situated in the cross section that projects onto γ_{xy} in M. The curve γ^σ_{xy} intersects the fiber over y at $\sigma(y)$, while γ^h_{xy} intersects this fiber at a point $h(y)$. The points $\sigma(y)$ and $h(y)$ of the bundle P, being both situated on the same fiber, are connected by a group transformation. In other words, there exists an element $g_{\sigma h} \in G$ such that

$$\sigma(y) = h(y) g_{\sigma h} .$$

The element $g_{\sigma h}$ of G is precisely Yang's $g(\gamma_{xy})$. A change in the cross section is then obviously a gauge transformation.[11,12] We leave the quite trivial proof of these statements to the reader.

In the gauge theory the potentials A^β_a are then the coefficients of the connection on the fiber bundle and the fields F^α_{mn} the components of the curvature tensor. In the case of gravity Yang proposes[18] the structure group GL($4R$); the connection coefficients are then the Christoffel symbols Γ^a_{bn} and the curvature tensor is Riemann's tensor $R^a{}_{bmn}$. So the dictionary between gauge theories and gravity theory is

$$\text{curvature,} \quad F^\alpha_{mn} \to R^a{}_{bmn} ,$$

$$\text{connection,} \quad A^\alpha_m \to \Gamma^a_{bm} .$$

The Yang-Mills Lagrangian in gauge theory is quadratic in the fields F^α_{mn}. The dictionary translates this into a Lagrangian quadratic in the components of the Riemann tensor $R^a{}_{bmn}$ and Yang proposes precisely such a Lagrangian. However, this raises certain problems. For instance, the gravitational field equations become third-order equations and no clear prescription for the inclusion of matter is provided. These difficulties disappear in the usual Einstein theory. But the perfect analogy between gravitation and Yang-Mills theories exploited by Yang appears to get lost. However, we have seen that in higher-dimensional unifications the Yang-Mills Lagrangian becomes a piece of the $(4+N)$-dimensional scalar curvature density and that its gravitational counterpart is the Einstein, not the Yang, Lagrangian. So, were one to impose the requirement of a higher-dimensional unification of gravity with a Yang-Mills gauge theory we would be led to Einstein's rather than Yang's theory. One may wonder what the requirement of the possibility of such a unification means. In line with our above discussion this requirement means that all gauge fields be at the same time Nambu-Goldstone fields associated with the breakdown of a higher symmetry. In other words, were one to only require gauge invariance and general covariance, both the Yang and Einstein type theories would be possible with Yang's theory being the more "natural" one. If, however, one requires in addition all gauge fields and gauge invariance itself to originate in the spontaneous breaking of a higher symmetry, then Einstein's theory emerges.

At this point one may wonder what would happen if in the $(4+N)$-dimensional unified theory one were to consider a Lagrangian quadratic in the $(4+N)$-dimensional Riemann tensor $R^A{}_{BCD}$ [Eq. (4.9c)]. With the metric (4.2) one then obtains as one term the 4-dimensional gravitational Yang Lagrangian, but the gauge field Lagrangian is *not* of the Yang-Mills form but rather contains terms such as $\nabla_m F^\alpha_{np} \nabla_{m'} F^\beta_{n'p'} g_{\alpha\beta} g^{mm'} g^{nn'} g^{pp'}$, so that also the gauge field equations become third-order equations just as Yang's gravitational equations.

We finally wish to emphasize that the alternative between the Einstein and Yang Lagrangians is meaningful only classically. In a quantum theory, renormalization may bring into play terms of the Yang type even if one were to start from an Einstein type theory.

VII. DISCUSSION AND PROSPECTS

We have shown how gauge fields and gauge invariance emerge from spontaneous symmetry breaking. In this context we were led to field theories in higher-dimensional spaces. A "cylindrical" subsector of these theories was identical with a usual 4-dimensional field theory. This theory is a new non-Abelian generalization of Kaluza-Jordan theory. In addition to gravitation and Yang-Mills gauge fields it contains also scalar fields. These scalar fields have no relation to the breaking of Abelian dilatation invariance as in Jordan-Brans-Dicke theories. Rather they are Nambu-Goldstone fields related to the breaking of GL(N,R) invariance. This theory is interesting in its own right.

In this 4-dimensional subsector of the theory, the unification is somewhat lost in that the relative strengths of the gravitational, Yang-Mills, and scalar couplings are arbitrary. This underscores the fact that while in principle interesting, this way of welding the concepts of Nambu-Goldstone and gauge fields has its obvious shortcomings. We could possibly entertain the idea of "hidden" internal dimensions of space-time. Speculations about extremely rapid variation (say with characteristic length of 10^{-33} cm) of fields in a fifth dimension have been offered by Klein.[13] Obviously one could extend them to even higher dimensions. Yet, there is a religious flavor to such ideas. One would rather like to benefit from the existence of higher dimensions, while at the same time not have to realize them physically at all. In this context the most promising avenue is that of supersymmetries.[1] In supersymmetric theories one deals with a space which along with the usual four "bosonic" space-time coordinates x_a also has additional *fermionic* coordinates θ_α. The dependence on these fermionic coordinates is *only polynomial* (on account of their anticommutative Grassmann nature). So the fields depend nontrivially only upon the space-time coordinates. It is clear that one could construct a theory invariant under superspace-dependent supersymmetry transformations. Such a "supergauge" theory would contain spin 2, $\frac{3}{2}$, 1, $\frac{1}{2}$ and even spin 0 fields (from θ_α-dependent fermionic transformations). Since even spin 0 and $\frac{1}{2}$ fields appear as gauge fields, in supergauge theories a total merger of the concepts of gauge and Nambu-Goldstone fields can occur. It is an interesting problem to prove the following conjecture: *There exist supergauge theories in which every field that appears in the Lagrangian is both a gauge field and a Nambu-Goldstone field.* We hope to return to this problem in future publications.[19]

The arguments of the present paper involved only Bose fields and usual symmetries and, as such, all fields were Nambu-Goldstone fields but not all (e.g., not the scalars) were gauge fields. It is only in the presence of supersymmetries that scalar fields can become gauge fields and correspondingly, only in supergauge theories does our conjecture have a chance of holding.

Finally it is often claimed that Higgs and Nambu-Goldstone fields are composite[20] and thus somewhat less fundamental than the Fermi and gauge fields. In the kind of theories investigated in this paper this distinction is blurred. Gauge fields are Nambu-Goldstone fields just like scalar fields. So if one wanted to derive the Nambu-Goldstone and Higgs fields dynamically one would be led to derive also the gauge fields dynamically. In supergauge theories *all* fields (including the Fermi fields) are Nambu-Goldstone (and at the same time gauge) fields, so that all fields are equally elementary or dynamical in origin. A theory that starts only from fermions is no "more fundamental" than one that starts from fermions and bosons. It is therefore an interesting question whether unified field theories are fundamental or are maybe themselves to be derived from some more fundamental "dynamics."

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[1]See, e.g., B. Zumino, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974,* edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. I-254.

[2]J. D. Bjorken, Ann. Phys. (N.Y.) 24, 174 (1963); P. G. O. Freund, Acta. Phys. Austriaca 14, 445 (1961); G. Guralnik, Phys. Rev. 136, B1404 (1964); 136, B1417(1964); Y. Nambu, Prog. Theor. Phys. Suppl. Extra No., 190 (1968).

[3]C. Isham, A. Salam, and J. Strathdee, Ann. Phys. (N.Y.) 62, 98 (1971).

[4]V. I. Ogievetsky, Lett. Nuovo Cimento 8, 988 (1973).

[5]V. I. Ogievetsky and A. B. Borisov, Dubna Report No. E2-7684 (unpublished).

[6]Th. Kaluza, Berl. Ber., 966 (1921); see also W. Thirring, in *Elementary Particle Physics*, proceedings of the XI Schladming conference on nuclear physics, edited by P. Urban (Springer, Berlin, 1972) [Acta. Phys. Austriaca, Suppl. 9 (1972)], p. 256.

[7]See P. Jordan, Z. Phys. 157, 112 (1959), where references to earlier work by Jordan are also provided.

[8]C. Brans and R. H. Dicke, Phys. Rev. 124, 925 (1961); Y. R. Thirry, C. R. Acad. Sci. Paris 226, 216 (1948).

[9]C. N. Yang, Phys. Rev. Lett. 33, 445 (1974).

[10]This is a straightforward consequence of the standard canonical and polar decomposition formulas: See I. M. Gel'fand and M. A. Neumark, *Unitäre Darstellungen der Klassischen Gruppen* (Akademie, Berlin, 1957), p. 30, and C. Chevalley, *Theory of Lie Groups* (Princeton Univ. Press, Princeton, New Jersey, 1946), p. 14.

[11]B. DeWitt, in *Relativity, Groups and Topology*, edited by B. DeWitt and C. DeWitt (Gordon and Breach, New York, 1964), p. 725; R. Kerner, Ann. Inst. H. Poincaré 9, 143 (1968); A. Trautman, Rep. Math. Phys. 1, 29 (1970).

[12]Y. M. Cho, Enrico Fermi Institute Report No. EFI 75/2 (unpublished).

[13]O. Klein, Z. Phys. 37, 895 (1926); Helv. Phys. Acta. Suppl. IV, 58 (1956).

[14]See Ref. 12 where also further references are provided.

[15]P. G. O. Freund, Phys. Lett. 2, 136 (1962); Phys. Rev. Lett. 16, 291 (1966); 16, 424 (1966); see also R. Delbourgo, A. Salam, and J. Strathdee, Nuovo Cimento 49A, 593 (1967).

[16]J. Scherk and J. Schwarz, Phys. Lett. 57B, 463 (1975).

[17]R. L. Bishop and R. J. Crittenden, *Geometry of Manifolds* (Academic, New York, 1964).

[18]As a technical point, the introduction of a Riemann geometry may reduce this group to the pseudo-orthogonal group O(3,1).

[19]Following the completion of this work we received two reports by R. Arnowitt, P. Nath, and B. Zumino in which a differential geometric field theory in curved superspace is presented. Their work is certainly closely related to ours.

[20]Otherwise, e.g., asymptotic freedom would be hard to achieve and an unpleasant proliferation of scalar fields would ensue.