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Model of the bare Pomeron*

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We present a model of the bare Pomeron. By bare Pomeron we mean a mechanism which accounts for constant total cross sections, zero real parts of scattering amplitudes, and limiting fragmentation (or Feynman scaling). No attempt is made to estimate higher-order corrections, which will presumably generate logarithmic growth, finite real parts, and violation of Feynman scaling. The model can be most clearly formulated in terms of the bag of Chodos *et al.*, although it is probably more general. It consists in the exchange of confined, colored Yang-Mills gluons between confined quarks and appears to account qualitatively for the properties listed above, as well as Gottfried's model of multiplicities in hadron-nucleus collisions. The model in its present form is unable to account for Pomeron factorization. The nonrationalized coupling constant $g^2/4\pi$ is approximately $\frac{1}{3}$.

I. INTRODUCTION

A most remarkable feature of total hadronic cross sections is that over a very large energy range (say, laboratory energy from 10 to 300 GeV) they are approximately constant. Physicists have come to associate with this behavior of the total cross section the following experimental facts.

1. Elastic cross sections are also approximately constant over the same energy range, and are considerably smaller than the corresponding total cross sections (for the pp system, by about a factor of 6). Indeed, in the first instance, the elastic amplitudes appear as the diffraction due to multiparticle production processes, and only secondarily reflect the elastic processes themselves.

2. Real parts of forward scattering amplitudes are small compared to imaginary parts, where small means of the order of 10% or less. Note that the real part associated via dispersion relations with an exactly constant σ_t is indeed zero and only becomes nonvanishing by virtue of the deviation of σ_t from constancy. However, it is possible to have large real parts and constant total cross sections because of the existence of odd signature amplitudes, and any believable model must forbid these.

3. The diffraction peak is somewhat narrower (in momentum transfer) than that associated with exchange processes, and is roughly Gaussian in $\sqrt{-t}$.

4. There may be approximate factorization of diffraction amplitudes and total cross sections (this is not too well founded experimentally; rather, it appears to be a strong item of folklore which is not contradicted by experiment).

5. Approximate factorization and Feynman scaling (or limiting fragmentation)[1] hold in inclusive processes. That is, in the region where c is a fragment of b, $(E_c/\sigma_{ab})d\sigma_{ab}/d^3q_c = F_{bc}(t_{bc}, x_c)$, independent of a, with t_{bc} the invariant momentum transfer from b to c, and x Feynman's scaling variable, $x \simeq 1 - M^2/s$, with M the missing mass, $M^2 = -(p_a+p_b-q_c)^2$, and s the center-of-mass energy squared, $s=-(p_a+p_b)^2$. Note our metric: $-p^2 = p_0^2 - \vec{p}^2$.

The model to be described in Sec. II appears to account qualitatively for all the above, as well as the constant σ_t, with the exception of factorization, which would appear to be accidental.

We take seriously the approximate constancy of the total cross section and, in particular, ask for a bare, or lowest-order Pomeron which leads in a natural way to an exactly constant cross section, but whose coupling is sufficiently weak so that the logarithms brought in by higher-order effects will not appreciably spoil the constancy over the 10–300 GeV energy range. Thus, all questions connected with rising cross sections and by implication logarithmic energy dependence will be viewed as higher-order corrections and ignored in this paper. Although one frequently hears the question: "Why do the cross sections rise?" it seems obvious to this author that the first question should be: "Why are they approximately constant?" That is the question for which an answer will presently be conjectured.

The possibility that seems most attractive does not work. This would be that in a confined quark model such as the bag[2] there would be natural absorption which remained constant as $s \to \infty$. This does not seem to be the case, as can be seen by the following argument. Imagine two bags approaching each other, with velocities $\pm v$, where $v \sim c$. As the boundary of one bag enters the other bag, it becomes a surface source for radiation of quarks. This radiation can go forward or backward. If forward, it is confined to a narrow interval of length $(c-v)t$ between the source and the wave front, where t is the time the collision lasts. As $v \to c$, this interval goes to zero and should give rise to no excitation. If the radiation is backward, it is infrared-shifted to low frequency, and, correspondingly, its amplitude approaches zero, since it is radiated by a rapidly receding source. This qualitative argument is borne out by perturbative calculations in three dimensions. In one space dimension, there is an exact solution of the problem of scattering of bags with scalar quarks given by Wu, McCoy, and Cheng,[3] which may be trivially generalized to spinor quarks. In both these cases one finds many solutions, depending on when the bags choose to come apart; however, in none of the solutions is there a trace of excitation, whatever the velocity: The bags separate in their ground state. This is a much stronger result then would have been guessed by the argument given above since it happens at all energies. It clearly depends intimately on the nonlinear and classical character of the bag equations, whereas the general argument given above is probably more reliable, although qualitative. It is therefore hard to avoid the conclusion that a specific interaction is required to maintain a finite absorption at high energy.

We choose the specific interaction following a simple observation of Gell-Mann, Goldberger, and this author,[4] to wit, that the exchange of an elementary vector meson leads in lowest nonvanishing order to a constant elastic cross section. It seems evident that it will therefore lead to a constant absorption in a confined quark model such as the bag.

Consider first an Abelian gauge field corresponding to a particle of mass μ coupled to a conserved vector current with a (rationalized) coupling constant g. To lowest order in $g^2/4\pi=\alpha$ the elastic scattering amplitude is real, and approaches

$$f \cong -\frac{W\alpha}{\mu^2 - t} \tag{1.1}$$

as $W=\sqrt{s} \to \infty$ at fixed momentum transfer, t.

In next order, the total cross section is *constant* (but elastic)

$$\begin{aligned}\sigma_t &= \sigma_e \\ &= \int d\Omega\, |f|^2 \cong \frac{4\pi\alpha^2}{\mu^2}\,,\end{aligned} \tag{1.2}$$

corresponding to a forward scattering amplitude, to order α^2,

$$f = -\frac{W\alpha}{\mu^2} + \frac{iW\alpha^2}{2\mu^2} \tag{1.3}$$

as $W \to \infty$.

We note that the real part of f is nonzero, and, for weak coupling, larger than the imaginary part. It is not related to the imaginary part by a dispersion relation, since the imaginary part has even signature and the real part odd signature. The idea of Ref. 4 was that the even signature part might generate the Pomeron. In Regge language, the second term in Eq. (1.3) represents a fixed J plane pole at $J=1$ in the even signature amplitude. Since the two intermediate vectors in the $\bar{n}n$ channel have a nonsense state at $J=1$ [the $^5(J-2)_J$ state], it was conjectured in Ref. 4 that this situation might be analogous to the fixed pole at $J=-1$ in the lowest-order Schrödinger equation, which in higher orders moves into the complex J plane as a Regge pole. As shown by Cheng and Wu,[5] this is not what happens, at least in the leading logarithm approximation of conventional field theory. Rather, a fixed branch point develops to the right of $J=1$; the branch point is then brought back to $J=1$ by unitarization. In the Cheng and Wu calculation, the Froissart bound is eventually saturated, and $\sigma_t \propto (\log s)^2 \propto 2\sigma_e$ for sufficiently large s.

We ask first (rhetorically) whether the model described above can be made realistic by including strong interaction corrections to the bare Pomeron, but *not* higher-order processes in the Pomeron coupling, $\alpha = g^2/4\pi$. A realistic theory would assign a mass $\mu > 1$ GeV to the exchanged vector. The lower limit on the mass is probably conservative and rests on the apparent Regge character of the known vector mesons, which makes them unsuitable as Pomeron generators. The elastic scattering would be modified by $F^4(t)$, where $F(t)$ is the appropriate vector form factor (we ignore spin complications here). The elastic cross section would thus be suppressed by a factor of roughly M_d^2/μ^2, where M_d is the dipole mass in the form factor. The inelastic cross section can be estimated from SLAC data, excluding, consistent with our lowest-order Pomeron calculation, the Pomeron contribution to the structure functions themselves. The result of all this effort is total failure. The coupling constant α and mass μ can be adjusted to approximate two of the three quantities Ref, σ_t, and σ_{el}; the third is then off by orders of magnitude. Furthermore, *any* choice leads to enormous values of α—at least 25—and of μ so that the lowest-order calculation is completely unbelievable. Finally, whatever values of these quantities one is willing to accept, the inelastic cross section is dominated by single and double diffraction, which is in qualitative disagreement with experiment.

All of these problems are removed by considering exchange of a colored Yang-Mills[6] gauge field between color singlet states. A variety of models of this kind have been suggested to achieve quark confinement; the simplest and most straightforward of these is the bag model of Chodos *et al.*,[2] to which we shall, for definiteness, confine our discussion, although it is probable that any confined quark model would have similar properties at high energy.

II. QUALITATIVE DISCUSSION OF HIGH-ENERGY SCATTERING IN THE BAG MODEL

In the bag model, quark quantum numbers are forbidden and symmetric quark statistics guaranteed by introducing a confined colored gauge field, whose boundary conditions on the bag surface, together with Gauss's theorem, require the vanishing of all eight color generators, so that only color singlet states are allowed. The gauge coupling constant is not determined by this requirement. The gauge field may serve a second purpose in splitting SU(6) multiplets. In the following a third function is proposed for this field: to produce the constant hadronic cross section.

We describe the process qualitatively: Two bags, α and β, approach each other in their center-of-mass system with a definite impact parameter, $\vec{b}$, as shown in Fig. 1. We assume for simplicity that α and β have the same mass and radius. Since the colored gluon field is confined to the interior of the bags, if $b > 2R$ the bags will pass each other without interacting, classically; in a future quantum theory, the boundaries will presumably be blurred, and sharp edges will be softened.

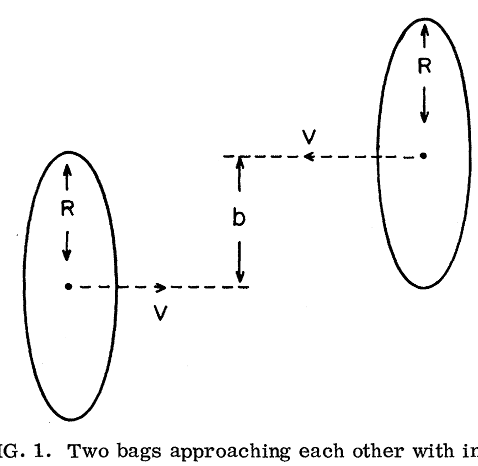

FIG. 1. Two bags approaching each other with impact parameter b.

If $b < 2R$ as the bags interpenetrate they may exchange a colored gluon. Cheng and Wu have shown that at very high energy the gluon will carry only transverse momentum. The color singlet contents of the bags will have become octets, racing away from each other with relative velocity $2v \sim 2c$ and small transverse momentum as in Fig. 2. The bag stretches as shown in order to keep the colored lines of force confined and avoid large momentum transfer processes. Clearly, the initial state cannot reemerge from the configuration shown in Fig. 2 without enormous momentum transfer. Therefore, the lowest-order real part of the elastic scattering amplitude $f(b)$ will vanish.

The actual decay of the configuration of Fig. 2 can occur via a cascade, as follows: The width r is determined by minimizing $\mathcal{E}$, the sum of the volume and field energies, where

$$\mathcal{E} = ALB + AL\,\frac{\vec{E}^2}{8\pi}\ ; \tag{2.1}$$

$\vec{E}$ is the effective "electric" field, $|\vec{E}| = g/A$, g is the effective color charge, and A is the area πr^2 of the connecting tube. The minimum is

$$\mathcal{E}_{\rm min} \sim L(2\alpha B)^{1/2}$$

and (2.2)

$$A_{\rm min} \sim \left(\frac{\alpha}{2B}\right)^{1/2}.$$

When $\mathcal{E}_{\rm min} \sim 2\omega$, where ω is the smallest diquark or gluon energy permitted by the transverse dimension,

$$\omega \sim \left(\frac{\alpha}{2\pi^2 B}\right)^{-1/4}, \tag{2.3}$$

the stretched bag may break by producing a pair of minimum energy diquarks or gluons near the center of the collision. The configuration will now be as shown in Fig. 3, with each bag an overall color singlet.

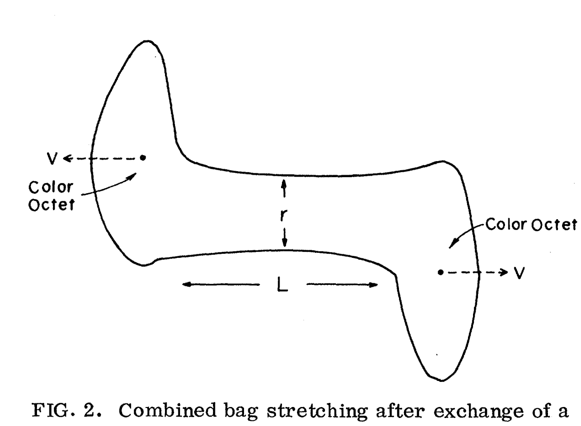

FIG. 2. Combined bag stretching after exchange of a colored gluon.

We may calculate the mass of each bag, roughly, by neglecting the surface interaction and the volume and field energies (which are essentially independent of the initial collision energy) and count only the energies and momenta of the quark and gluons.

We let $\vec{p}', E'$ be the momentum and energy of the rapidly moving quarks, in, say, bag α, and $\vec{q}, \omega$ be the momentum and energy of the produced, slowly moving diquark or gluon in bag α. We can, if we wish, allow ω to include field energy. Then, by energy conservation,

$$E' = E + \omega$$

and

$$M_\alpha^2 = (E' + \omega)^2 - (\vec{p}' + \vec{q})^2 \sim 2E'(\omega - \hat{v}\cdot\vec{q}) \sim 2E(\omega - \hat{v}\cdot\vec{q}),$$

whereas

$$M_0 = \sqrt{2s} = 2E .$$

Thus, the initial configuration of mass $M_0 = 2E$ has split into two configurations, each of mass

$$M_1 \sim c_1\sqrt{M_0} ,$$

where

$$c_1^2 \sim \omega - \hat{v}\cdot\vec{q} .$$

If we work in hadronic mass units, c_1 is a number of order unity.

Now each of the two bags reproduces a configuration very like the initial one, and must split again. Clearly,

$$M_2 \sim c_2\sqrt{M_1} ,$$

etc. This process will continue until M_n is hadronic in magnitude, or of order unity.

We easily calculate M_n, since

$$\ln M_1 = \ln c_1 + \tfrac{1}{2}\ln M_0 ,$$

$$\ln M_2 = \ln c_2 + \tfrac{1}{2}\ln M_1 ,$$

so that

$$\ln M_{n_f} = \ln c_{n_f} + \tfrac{1}{2}\ln c_{n_f-1} + \cdots + \frac{1}{2^{n_f-1}}\ln c_1 + \frac{1}{2^{n_f}}\ln M_0 .$$

Thus,

$$2^{n_f} \sim k\ln M_0 ,$$

where the constant k is a number of order unity. Since 2^{n_f} is just the multiplicity of this event, we find a multiplicity proportional to $\ln s$. Of course, if N particles instead of two are shaken off at or between each splitting, we will have a multiplicity proportional to a small power of $\ln s$, the power being $\sim \ln\overline{N}/\ln 2$.

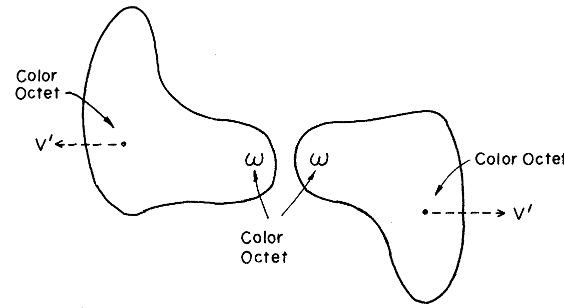

FIG. 3. Two bags reseparating by creation of a pair of colored octets.

According to the above argument, the length L_0 (in the center-of-mass system) to which the initial system will grow before splitting is, according to Eqs. (2.2) and (2.3),

$$L_0 \sim \left(\frac{\pi^2}{2B\alpha^3}\right)^{1/4} ; \tag{2.4}$$

the corresponding time after collision is $\tau_0 \approx L_0$, since $v \sim c = 1$.

In the lab system the corresponding event happens at a time after collision

$$t_L = \frac{\tau_0}{(1 - v_L^2)^{1/2}}$$

and distance from the collision point

$$x_L = \frac{v_c\tau_0}{(1 - v_L^2)^{1/2}} ,$$

where $-v_L$ is the velocity of the transformation from the center-of-mass to the lab system,

$$v_L = \frac{P_L}{m + E_L}$$

and

$$\frac{1}{(1 - v_L^2)^{1/2}} \sim \left(\frac{E_L}{2m}\right)^{1/2} .$$

At 300 GeV, for example, $1/(1 - v_L^2)^{1/2} \sim 12$ so that the combined state holds together for a long time before breakup. This property may furnish a natural explanation for the Gottfried model[7] of particle production in nuclei.

Finally, we note that limiting fragmentation and hence Feynman scaling follows naturally from the model we have described, since the object which is at rest in the lab frame may clearly choose to fragment color singlets while it is waiting to detach itself from the stretched bag of Fig. 2. Since, as we shall see in the next section, the excitation spectrum of each bag is energy-independent as $E \to \infty$, and since the bag stretching depends on v (which approaches c), a limiting amplitude for the fragmentation may be expected.

III. FORMULA FOR THE ELASTIC AMPLITUDE

In order to proceed, we assume that the exchange of the colored gluon excites the quarks in the two bags into cavity resonant states; these are analogous to the doorway states of nuclear excitation. Since the decay time given by Eq. (2.4) is long in the weak-coupling limit, it is not unreasonable to ignore the decay mechanism in calculating the cross section. A second, more doubtful assumption is that we may replace the gluon exchange potential by the free gluon propagator, with an effective mass μ replacing the appropriate boundary conditions. The justification here is that because of the vanishing of the color charge in the incident states the final formula is relatively insensitive to an infrared cutoff.

With these caveats in mind, we write the S matrix for the scattering of a bag α moving with velocity v_α in the plus z direction by a bag β moving with velocity v_β in the minus z direction, considering the bags as carrier cavities containing an appropriate number of quarks in color singlet states. Since we do not have an appropriate quantum theory for these systems, we treat the carriers classically. They are separated by an impact parameter $\vec{b}$, but are undeflected by the collision. The S matrix will then be a function of $\vec{b}$, $S=S(\vec{b})$, and the elastic scattering amplitude will be given by

$$f(\vec{\Delta})=\frac{ip}{2\pi}\int d\vec{b}\,e^{i\vec{b}\cdot\vec{\Delta}}[1-S(\vec{b})], \tag{3.1}$$

where $\vec{\Delta}$ is the momentum transfer and p the incident momentum.

We write for the fourth-order S matrix a conventional formula,

$$S(\vec{b})=1+\frac{g^4}{2}\int\langle\alpha|(j_\mu(x)j_\nu(y))_+|\alpha\rangle\Delta_F(x-z)\,\Delta_F(y-w)\langle\beta|(j_\mu(z-\vec{b})j_\nu(w-\vec{b}))_+|\beta\rangle\,dx\,dy\,dz\,dw, \tag{3.2}$$

where the j's are color currents, with color indices suppressed, and Δ_F is the propagation function

$$\Delta_F(x)=\frac{1}{(2\pi)^4 i}\int\frac{e^{ik\cdot x}\,dk}{k^2+\mu^2}, \tag{3.3}$$

with μ the mass that simulates the confinement properties of the colored vector field. Since the interaction turns out to be almost local, the error made in so doing is probably small. The integrations are four-dimensional. Two- and three-dimensional integrations are indicated by arrows, thus: $d\vec{b}, d\vec{r}$.

We proceed by transforming the states α and β to rest. We have

$$S(\vec{b})-1=\frac{g^4}{2}a^\alpha_{\mu\lambda}a^\alpha_{\nu\eta}a^\beta_{\mu\delta}a^\beta_{\nu\rho}\int dx\,dy\,dz\,dw\,\Delta_F(x-y)\,\Delta_F(z-w)$$
$$\times\langle\alpha_0|(j_\lambda(a^{\alpha-1}x)j_n(a^{\alpha-1}y))_+|\alpha_0\rangle\langle\beta_0|(j_\delta(a^{\beta-1}z)j_\rho(a^{\beta-1}w))_+|\beta_0\rangle, \tag{3.4}$$

where we have temporarily replaced $z-\vec{b}$ and $w-\vec{b}$ by z and w, respectively, inside the β matrix element; thus we allow the state β to carry the impact-parameter information. Also, a^α denotes a Lorentz transformation along the z axis with velocity $-v_\alpha$, a^β one with velocity v_β. A change of variable $x\to a^\alpha x$, $y\to a^\alpha y$, $z\to a^\beta z$, and $w\to a^\beta w$ changes (3.4) to

$$S(\vec{b})-1=\frac{g^4}{2}a^\alpha_{\mu\lambda}a^\alpha_{\nu\eta}a^\beta_{\mu\delta}a^\beta_{\nu\rho}\int dx\,dy\,dz\,dw\,\Delta_F(a^\alpha x-a^\beta z)\,\Delta_F(a^\alpha y-a^\beta w)$$
$$\times\langle\alpha_0|(j_\lambda(x)j_\eta(y))_+|\alpha_0\rangle\langle\beta_0|(j_\delta(z)j_\rho(w))_+|\beta_0\rangle. \tag{3.5}$$

Equation (3.5) is manifestly Lorentz-invariant. It is, however, convenient to work in a frame in which both v_α and v_β are close to one—for example, the center-of-mass system. As each $v\to 1$, then, the components a_{33}, a_{30}, a_{03}, and a_{00} all go as $1/(1-v^2)^{1/2}$, and dominate at high energy. In this limit we have, neglecting terms of order $(1-v^2)^{1/2}$,

$$S(\vec{b})-1=\frac{2g^4}{(1-v_\alpha{}^2)(1-v_\beta{}^2)}\int dx\,dy\,dz\,dw\,\Delta_F(a^\alpha x-a^\beta z)\,\Delta_F(a^\alpha y-a^\beta w)$$
$$\times\langle\alpha_0|([j_3(x)+j_0(x)][j_3(y)+j_0(y)])_+|\alpha_0\rangle\langle\beta_0|([j_3(z)-j_0(z)][j_3(w)-j_0(w)])_+|\beta_0\rangle. \tag{3.6}$$

We next write out the components of $a^{\alpha}x - a^{\beta}z$ explicitly. These are

$$a^{\alpha}x - a^{\beta}z = \left(x_1 - z_1, x_2 - z_2, \frac{x_3 + v_{\alpha}x_0}{(1-v_{\alpha}{}^2)^{1/2}} - \frac{z_3 - v_{\beta}z_0}{(1-v_{\beta}{}^2)^{1/2}}, \frac{x_0 + v_{\alpha}x_3}{(1-v_{\alpha}{}^2)^{1/2}} - \frac{z_0 - v_{\beta}z_3}{(1-v_{\beta}{}^2)^{1/2}}\right). \tag{3.7}$$

We transform from x_0 and z_0 to new variables σ and τ which are respectively the third and fourth components of Eq. (3.7):

$$\sigma = \frac{x_3 + v_{\alpha}x_0}{(1-v_{\alpha}{}^2)^{1/2}} - \frac{z_3 - v_{\beta}z_0}{(1-v_{\beta}{}^2)^{1/2}}, \quad \tau = \frac{x_0 + v_{\alpha}x_3}{(1-v_{\alpha}{}^2)^{1/2}} - \frac{z_0 - v_{\beta}z_3}{(1-v_{\beta}{}^2)^{1/2}}. \tag{3.8}$$

The Jacobian of the transformation is

$$\frac{\partial(\sigma, \tau)}{\partial(x_0, z_0)} = \frac{v_{\alpha} + v_{\beta}}{(1-v_{\alpha}{}^2)^{1/2}(1-v_{\beta}{}^2)^{1/2}}. \tag{3.9}$$

We may also solve for x_0 and z_0. We find

$$\begin{aligned} x_0 &= -\frac{x_3(1+v_{\alpha}v_{\beta})}{v_{\alpha}+v_{\beta}} + \frac{(1-v_{\alpha}{}^2)^{1/2}}{v_{\alpha}+v_{\beta}}[\sigma + v_{\beta}\tau + (1-v_{\beta}{}^2)^{1/2})z_3], \\ z_0 &= \frac{z_3(1+v_{\alpha}v_{\beta})}{v_{\alpha}+v_{\beta}} + \frac{(1-v_{\beta}{}^2)^{1/2}}{v_{\alpha}+v_{\beta}}[\sigma - v_{\alpha}\tau - (1-v_{\alpha}{}^2)^{1/2}x_3], \end{aligned} \tag{3.10}$$

or, as $v_{\alpha}, v_{\beta} \to 1$ at fixed σ and τ, $x_0 \to -x_3$ and $z_0 \to z_3$.

The integral over σ and τ in Δ_F may now be carried out, and converges:

$$\int d\sigma\, d\tau\, \Delta_F(\vec{x}_{\perp} - \vec{z}_{\perp}, \sigma, \tau) = \frac{\Delta_2(\vec{x}_{\perp} - \vec{z}_{\perp})}{i}, \tag{3.11}$$

where

$$\Delta_2(\vec{x}_{\perp}) = \int \frac{d\vec{k}_{\perp}}{(2\pi)^2} \frac{e^{i\vec{k}_{\perp}\cdot\vec{x}_{\perp}}}{\vec{k}_{\perp}{}^2 + \mu^2}$$

and $\vec{x}_{\perp}$ is purely transverse.

We make an analogous change of variables in $a^{\alpha}y - a^{\beta}w$, and also define

$$j_+(\vec{x}) = j_0(\vec{x}, x_0 = -x_3) + j_3(\vec{x}, x_0 = -x_3), \tag{3.12}$$

$$j_-(\vec{z}) = j_0(\vec{z}, z_0 = z_3) - j_3(\vec{z}, z_0 = z_3). \tag{3.13}$$

We finally substitute Eqs. (3.7)–(3.13) in Eq. (3.6) and find

$$S(\vec{b}) - 1 = -\frac{g^4}{2}\int d\vec{x}\, d\vec{y}\, d\vec{z}\, d\vec{w}\, \Delta_2(\vec{x}_{\perp} - \vec{z}_{\perp})\,\Delta_2(\vec{y}_{\perp} - \vec{w}_{\perp})\langle\alpha_0|(j_+(\vec{x})j_+(\vec{y}))_+|\alpha_0\rangle\langle\beta_0|(j_-(\vec{z})j_-(\vec{w}))_+|\beta_0\rangle. \tag{3.14}$$

Equation (3.14) for $S(\vec{b}) - 1$ evidently leads to an imaginary scattering amplitude, constant total cross section, and constant $d\sigma_{\text{elastic}}/dt$. The first statement follows from the observation that the j_+'s commute, as do the j_-'s, since $\vec{x}$ and $\vec{y}$, like $\vec{z}$ and $\vec{w}$, are separated by a spacelike interval. Thus, by completeness, Eq. (3.14) can be rewritten, for non-spin-flip,

$$1 - S(\vec{b}) = \frac{g^4}{2}\sum_{n,m}\left|\int d\vec{x}\, d\vec{z}\, \Delta_2(\vec{x}_{\perp} - \vec{z}_{\perp})\langle n|j_+(\vec{x})|\alpha_0\rangle\langle m|j_-(\vec{z})|\beta_0\rangle\right|^2, \tag{3.15}$$

showing the desired reality as well as the necessary positivity.

Some discussion of the use of Eq. (3.10) is called for, since the bag propagation function may be singular on the tangent plane to the light cone. One can study this question for the free-particle case, and one finds that the S matrix given by Eq. (3.14) is correct, although the contribution of each Feynman diagram separately is not. This comes about because each Feynman diagram is singular, corresponding to the presence of logs terms in the energy dependence of its contribution to the scattering amplitude, whereas the sum of the diagrams is nonsingular, corresponding to the absence of logs terms in the final amplitude. Furthermore, the substitution of Eq. (3.11) apparently picks out the nonsingular part of each diagram, so that both diagrams give the same contribution. Since the singular structure of the propagation functions should be similar to that of free particles, we believe the use of (3.14) is legitimate.

Actual evaluation of the matrix elements in Eq. (3.14) is extremely difficult, since, as will be shown below, the vanishing of the color charges on the states $|\alpha_0\rangle$ and $|\beta_0\rangle$ implies a sensitive cancellation between the one- and two-particle terms in the Wick expansions, so that the best we can hope for is an order-of-magnitude estimate. This estimate will be most reliable for the total cross section, and considerably less reliable for the actual $\vec{b}$ dependence of the amplitude. There are in addition zero-particle or empty bag terms, whose interpretation appears to require a genuine quantum theory for the state $|\alpha_0\rangle$, since these terms are intimately connected with the quantum theory of the quarkless state. In the absence of such a theory, we shall content ourselves with the following: We shall estimate the total cross section and forward spin-flip amplitude implied by (3.14), ignoring the zero-quark term. This will determine a value of g^2, and give us some idea of the spin-orbit coupling to be expected. We shall then estimate the $\vec{b}$ dependence and the corresponding elastic angular distribution independently by a geometric argument, which we consider more reliable in our present ignorant state.

We now attempt an evaluation of the bag propagation function

$$G_\alpha = \langle \alpha_0 | (j_+(\vec{x}) j_+(\vec{y}))_+ | \alpha_0 \rangle .$$

Since we are treating the bag as a fixed cavity, we can evaluate G_α as

$$\begin{aligned} G_\alpha &= G_{\alpha_1} + G_{\alpha_2} + G_{\alpha_0} \\ &= \psi^+_{\alpha_0}(\vec{x})(1+\alpha_3) S_Q(\vec{x}, \vec{y})(1+\alpha_3)\psi_{\alpha_0}(\vec{y}) \\ &\quad + \text{crossed term} + (\text{two-body terms}) \\ &\quad + (\text{ignored zero-body terms}), \end{aligned} \tag{3.16}$$

where the ψ_α's are the quark wave functions and where S_Q is the quark propagator in the bag:

$$\begin{aligned} S_Q(x, y) &= \sum_{n+} \psi_n(x)\psi_n^+(y), \quad x_0 > y_0 \\ &= -\sum_{n-} \psi_n(x)\psi_n^+(y), \quad x_0 < y_0 \end{aligned} \tag{3.17}$$

with $n\pm$ denoting positive- and negative-frequency solutions of the Dirac equation in the bag. We recall that the functions of the three vectors $\vec{x}$ and $\vec{y}$ are to be obtained by setting $x_0 = -x_3$ and $y_0 = -y_3$ in the wave functions occurring in Eqs. (3.16) and (3.17).

The two forms of Eq. (3.17) may be unified by the the introduction of ϵ functions:

$$\begin{aligned} S_Q(x, y) &= \frac{\epsilon(x_0 - y_0)}{2} \sum_{n\pm} \psi_n(x)\psi_n^+(y) \\ &\quad + \tfrac{1}{2} \sum_{n\pm} \psi_n(x)\psi_n^+(y)\epsilon(n) . \end{aligned} \tag{3.18}$$

Since the tangent plane is outside the light cone, the first term in Eq. (3.18), which is causal, will vanish on the tangent plane and we are left with the second.

We point out here a convenient property for later use:

$$\begin{aligned} (1+\alpha_3) \int d\vec{y}\, S_Q(\vec{x}, \vec{y})(1+\alpha_3)\psi_{\alpha_0}(\vec{y}) \\ = \frac{\epsilon(\alpha_0)}{2}(1+\alpha_3)\psi_{\alpha_0}(\vec{x}) . \end{aligned} \tag{3.19}$$

This result follows from the observation that

$$\begin{aligned} H_D \psi_n(\vec{x}) &= \left(\frac{1}{i}\, \vec{\alpha} \cdot \vec{\nabla} + \beta m \right) e^{i\omega_n x_3} \chi_n(\vec{x}) \\ &= \omega_n (1+\alpha_3)\psi_n(\vec{x}) , \end{aligned} \tag{3.20}$$

where $\chi_n(\vec{x})$ is the time-independent stationary-state wave function satisfying the equation

$$H_D \chi_n = \omega_n \chi_n . \tag{3.21}$$

Since the bag boundary condition

$$\vec{\gamma} \cdot \hat{n}\, \psi = \psi \tag{3.22}$$

makes H_D a Hermitian operator, the appropriate orthonormality integral is

$$\int d\vec{x}\, \psi_n^+(\vec{x})(1+\alpha_3)\psi_m(\vec{x}) = \delta_{nm} \tag{3.23}$$

and hence the result Eq. (3.19). Clearly an analogous result holds for the substitution $z_0 = +z$, together with the projection $1-\alpha_3$. To proceed further, we need to make a simplifying assumption, which is motivated by the behavior of the free-quark propagation function $S_{Q_0}(\vec{x})$. One finds for the projection

$$\begin{aligned} S_{Q_0}^{(+)}(\vec{x}) &= (1+\alpha_3) S_{Q_0}(\vec{x})(1+\alpha_3) \\ &= \frac{i}{2\pi}(1+\alpha_3)\,\delta(\vec{x}_\perp)\, \mathrm{P}\frac{1}{x_3} , \end{aligned} \tag{3.24}$$

where P signifies that the principal value is to be taken upon integration. One sees that $S_{Q_0}^{(+)}(\vec{x})$ as given by Eq. (3.24) consistently satisfies the integral condition Eq. (3.19) when ψ_{α_0} is set equal to a free-quark wave function. Of course, it does not satisfy Eq. (3.19) with ψ_{α_0} taken as a bound-quark wave function, since the proper boundary conditions are not built into the function $S_{Q_0}^{(+)}$. We attempt to cure this defect by a modification of Eq. (3.24) which makes it compatible with Eq. (3.19). We assume

$$(1+\alpha_3)S_Q(\vec{x},\vec{y})(1+\alpha_3)=\delta(\vec{x}_\perp-\vec{y}_\perp)\,F(x_3,y_3)\,, \tag{3.25}$$

where the only property of F that is determined is

$$\int F(x_3,y_3)\,dy_3\psi_n(\vec{x}_\perp,y_3)=\tfrac{1}{2}(1+\alpha_3)\,\psi_n(\vec{x})\epsilon(n)\,. \tag{3.26}$$

Since in Eq. (3.14) the longitudinal integrals are completely decoupled, Eq. (3.26) is sufficient to determine S_Q for our purposes. The first two terms in Eq. (3.16), on integration over dy_3, give

$$\int G_\alpha^{(1)}dy_3=\psi_{\alpha_0}^+(\vec{x})(1+\alpha_3)\,\psi_{\alpha_0}(\vec{x})\,\delta(\vec{x}_\perp-\vec{y}_\perp)\,. \tag{3.27}$$

If the two-body terms in Eq. (3.16) were absent, Eq. (3.27) would lead to a simple Chou-Yang[8] formula, with an electromagnetic density function. The presence of color produces a significant change, however, since it requires that

$$\int d\vec{x}\,G_\alpha(\vec{x},\vec{y})=\int d\vec{y}\,G_\alpha(\vec{x},\vec{y})=0\,. \tag{3.28}$$

This last equation follows from the observation that, since $\vec{x}$ and $\vec{y}$ are spacelike, the ordering in Eq. (3.14) may be neglected. Furthermore, the orthonormality relation Eq. (3.23) shows that the space integral of j_0+j_3 will involve only $\int j_0$, which is a color charge, and hence zero. Thus the space integrals of $G_\alpha^{[2]}$ and $G_\alpha^{[1]}$ must cancel. This makes it almost possible to write down the complete answer by inspection. For the nonflip terms we find

$$\begin{aligned}\int G_\alpha^{\text{n.f.}}dy_3=&\,\psi_{\alpha_0}^+(\vec{x})\,\delta(\vec{x}_\perp-\vec{y}_\perp)\,\psi_{\alpha_0}(\vec{x})\\&-\int dy_3\{\psi_{\alpha_0}^+(\vec{x})\,\psi_{\alpha_0}(\vec{x})\,\psi_{\alpha_0}^+(\vec{y})\,\psi_{\alpha_0}(\vec{y})\\&\qquad-\tfrac{1}{6}[\,\psi_{\alpha_0}^+(\vec{x})\alpha_3\psi_{\alpha_0'}(\vec{x})\,\psi_{\alpha_0'}^+(\vec{y})\alpha_3\psi_{\alpha_0}(\vec{y})+\psi_{\alpha_0}^+(\vec{y})\alpha_3\psi_{\alpha_0'}(\vec{y})\,\psi_{\alpha_0'}^+(\vec{x})\alpha_3\psi_{\alpha_0}(\vec{x})]\}\,,\end{aligned} \tag{3.29}$$

where α_0' denotes the wave function whose spin is opposite to that of α_0. The factor $\frac{1}{6}$ holds by explicit calculation for the nucleon.

We calculate the spin-flip part of $G_\alpha^{[1]}$ by noting that

$$j_3(\vec{x})=-\vec{\nabla}\cdot\hat{n}_3\times\frac{6}{e}\,\frac{(\mu_p+\mu_N)}{2}\,\vec{\sigma}\,\mathfrak{M}(\vec{x})\,, \tag{3.30}$$

where $(\mu_p+\mu_N)/2$ is the $I=0$ magnetic moment of the nucleon, $\vec{\sigma}$ the Pauli spin matrix vector normalized to $\sigma_1{}^2=1$, $\hat{n}_3$ the unit vector in the 3 direction, and $\mathfrak{M}(x)$ the nucleon magnetization density. The factor $6/e$ arises from the isosinglet component of the nonstrange quark charges:

$$q=\frac{e}{2}\,\tau_3+\frac{e}{6}\,. \tag{3.31}$$

Equation (3.30) is sufficient to provide an explicit expression for the spin-flip contributions to G_α. To a sufficient approximation, we replace $(\mu_p+\mu_N)/e$ by $1/m$, with m the nucleon mass. We then have in all for $G_\alpha(\vec{x},\vec{y})$

$$\begin{aligned}G_\alpha(\vec{x},\vec{y})=&\,\rho(\vec{x})\,\delta(\vec{x}-\vec{y})-\left\{\rho(\vec{x})\rho(\vec{y})-\frac{3}{m^2}\,\hat{n}_3\cdot\vec{\nabla}\,\mathfrak{M}(\vec{x})\hat{n}\cdot\vec{\nabla}\,\mathfrak{M}(\vec{x})\right\}\\&-\left\{\frac{\vec{\nabla}\mathfrak{M}(\vec{x})}{m}\times\hat{n}_3\cdot\vec{\sigma}\delta(\vec{x}-\vec{y})-\frac{\vec{\nabla}}{m}\,\mathfrak{M}(\vec{x})\times\hat{n}_3\cdot\vec{\sigma}\rho(\vec{y})-\frac{\vec{\nabla}}{m}\,\mathfrak{M}(\vec{y})\times\hat{n}_3\cdot\vec{\sigma}\rho(\vec{x})\right\},\end{aligned} \tag{3.32}$$

where $\rho(\vec{x})=\psi^+(\vec{x})\psi(\vec{x})$ is the baryon density, and where we have divided the spin-flip term by 3 relative to the density, which is summed over quarks. $G_\beta(\vec{z},\vec{w})$ is given by an analogous expression, except that the $\vec{\sigma}$ terms are reversed in sign.

We note two interesting and somewhat surprising properties of Eq. (3.32) for the densities. First, although the process we are discussing (Pomeron exchange) is charge-conjugation-even, the densities are vector, so that we expect the Pomeron residue functions to be closely related to the baryon electromagnetic form factors. Second, we have helicity flip, even though the exchanged vector field is helicity conserving at high energy. This effect is due to the binding of the quarks, since it would be absent for free-particle states at rest.

We must also take into account the internal color quantum numbers in order to determine α. We choose to define the coupling constant g by

$$\mathcal{L}_I = g\,\bar{\psi}\, i\gamma_\nu \lambda_j \psi A_\nu^j , \tag{3.33}$$

where

$$\mathrm{Tr}\lambda_i^2 = 2 . \tag{3.34}$$

With this convention, we must multiply the coupling constant g^4 by the color-singlet part of each λ_α^2, i.e., $(\frac{2}{3})^2$ for each interaction, by 8, since there are eight exchanged vector mesons, and finally we must sum over all the quarks in the two bags.

Our final formula for $S(\vec{b})$ is

$$S(\vec{b}) - 1 = -\frac{g^4}{2}\, 3(\tfrac{2}{3})\times 3(\tfrac{2}{3})\times 8 \times \int d\vec{x}\, d\vec{y}\, d\vec{z}\, d\vec{w}\, G_\alpha(\vec{x},\vec{y})\, \Delta_2(\vec{x}_\perp - \vec{z}_\perp)\, \Delta_2(\vec{y}_\perp - \vec{w}_\perp)\, G_\beta(\vec{z},\vec{w}), \tag{3.35}$$

where we have reinserted the impact parameter in G_β.

IV. NUMERICAL ESTIMATES

A. The forward amplitude and total cross section

From Eq. (3.1) we learn that

$$\sigma_T = 2\int d\vec{b}\,[1 - \mathrm{Re}S(b)], \tag{4.1}$$

or, from (3.35),

$$\sigma_T = 32 g^4 \int d\vec{b}\, d\vec{x}\, d\vec{y}\, d\vec{z}\, d\vec{w}\, G_\alpha(\vec{x},\vec{y})\, G_\beta(\vec{z},\vec{w}) \times \Delta_2(\vec{x}_\perp - \vec{z}_\perp)\, \Delta_2(\vec{y}_\perp - \vec{w}_\perp). \tag{4.2}$$

For G_α and G_β we insert the spin-nonflip terms of (3.32); however, since our approximation to G_1 has clearly left out contributions corresponding to the double-flip terms, coming from G_2, we shall drop the latter, and keep only that part of G_2 which is necessary to make the integrals vanish,

$$\int G(\vec{x},\vec{y})\, d\vec{x} = \int G(\vec{x},\vec{y})\, d\vec{y} = 0 .$$

Substitution into (4.2) yields

$$\sigma_T = 128\pi^2 g^4 \int d\vec{q}_\perp [\Delta_2(\vec{q}_\perp)]^2 \{1 - [\rho(\vec{q}_\perp)]^2\}, \tag{4.3}$$

with

$$\rho(\vec{q}) = \int e^{i\vec{q}\cdot\vec{x}}\, d\vec{x}\, \rho(\vec{x}), \tag{4.4}$$

the charge form factor, and, from (3.11),

$$\Delta_2(\vec{q}) = \frac{1}{(2\pi)^2 (q^2 + \mu^2)} .$$

To sufficient accuracy, the cutoff μ is unnecessary and we have

$$\sigma_T = 128\pi \left(\frac{g^2}{4\pi}\right)^2 \int_0^\infty d(q^2) \frac{\{1 - [\rho(q)]^2\}^2}{q^4} . \tag{4.5}$$

We evaluate the integral by assuming

$$\rho(\vec{q}) = \left(\frac{m_D^2}{q^2 + m_D^2}\right)^2 , \tag{4.6}$$

and find

$$\int \frac{dq^2}{q^4} (1 - \rho^2)^2 = \frac{1}{m_D^2}\, \frac{517}{105} \sim \frac{5}{m_D^2} , \tag{4.7}$$

so that, with $\alpha = g^2/4\pi$,

$$\sigma_T \cong \frac{640\alpha^2}{m_D^2} \cong 40 \text{ mb}$$

and

$$\alpha \sim \tfrac{1}{3} .$$

The forward spin-flip amplitude is given from Eq. (3.1) by the integral

$$f_{\text{flip}}(0) = i \int \vec{b}\cdot\vec{\Delta}\, d\vec{b}\,[1 - S(\vec{b})]_{\text{flip}} \tag{4.8}$$

and the double-flip amplitude by

$$f_{\text{d.f.}}(0) = -\tfrac{1}{2} \int (\vec{b}\cdot\vec{\Delta})^2 d\vec{b}\,[1 - S(\vec{b})]_{\text{d.f.}}, \tag{4.9}$$

where $\vec{\Delta}$ is the momentum transfer. If we assume that the magnetization density $\mathfrak{M}(\vec{x})$ is equal to the charge density $\rho(\vec{x})$ (recall that the volume integrals of both are normalized to unity) the calculation is particularly simple, since the momentum integrals are identical to those of (4.3). We find, for the entire forward amplitude,

$$f(\vec{\Delta}\simeq 0)=i\,\mathrm{Im}\,f(0)\left[1+i\vec{\sigma}_\alpha\cdot n_3\times\frac{\vec{\Delta}}{m}\right]\left[1+i\vec{\sigma}_\beta\cdot\hat{n}_3\times\frac{\vec{\Delta}}{m}\right], \tag{4.10}$$

where m is the nucleon mass.

The characteristic effects of the predicted spin flip in a single scattering experiment (polarization and right-left asymmetry) will be small, since the real part of the amplitude is small. As an example, we estimate the right-left asymmetry produced by Coulomb interference, assuming 100% initial polarization, and $\Delta\sim 100$ MeV, to be $\sim\frac{1}{80}$.

The spin-flip effects will be larger in a double scattering experiment, for which Eq. (4.10) predicts a 20% rotation of the transverse polarization of the incident beam at a momentum transfer of 100 MeV.

B. Factorization

Our model contains no natural reason for factorization, although one can easily imagine special circumstances which will make it hold. For example, if the pion and proton charge densities are not too different, as the bag model would predict, one would find factorization for pp, πp, and $\pi\pi$ total cross sections, in the ratio 9:6:4, as usual in quark models, by following the approximation scheme described in the beginning of this section. Factorization in diffractive resonance production, as well as in inclusive processes, would appear to require even more special circumstances.

C. Elastic angular distribution

To estimate the consequence of our model here, we prefer to use a geometric argument, since we believe the angular distribution to be considerably more sensitive to errors in the G's and Δ_F's than the total cross section, whereas the geometry of the spherical collision seems a somewhat safe starting point.

We refer to Eq. (3.15), and note that the vanishing of the space integrals $\int\langle n|j_\pm(\vec{x})|\alpha\rangle d\vec{x}$ implies that one can find a $k_i^{(n)}(\vec{x})$ such that

$$\langle n|j_\pm(\vec{x})|\alpha\rangle=\partial_i\langle n|k_i^{(n)}(\vec{x})|\alpha\rangle\ . \tag{4.11}$$

Indeed, this can be done, following the technique leading to Eq. (3.23). An integration by parts then shows that the effective field propagation functions are $\partial_i\,\partial_j\,\Delta_2\,(\vec{x}_\perp-\vec{z}_\perp)$, which are almost local (their circular averages are exactly local). We are thus led by Eq. (3.15) to consider a sum of terms, each of which is the square of a certain overlap function of the densities. This function is given by an integral over the transverse overlap area of the product of the longitudinal heights of the two spheres at that point. That is, we assume

$$f(\vec{b})=1-S(\vec{b})=\sum_{n,m}|F_{n,m}(\vec{b})|^2 \tag{4.12}$$

and

$$F_{n,m}=c_nI\ , \tag{4.13}$$

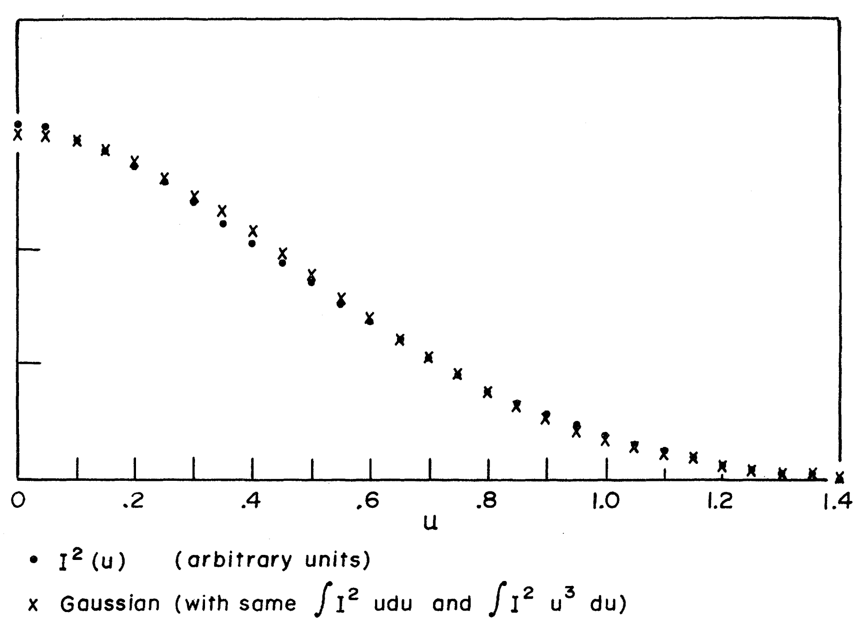

FIG. 4. Comparison of the predicted impact-parameter dependence, $I^2(u)$, with a Gaussian.

where

$$I=\frac{1}{R^4}\int_{b/2}^{R}dy\int_0^{(R^2-x^2)^{1/2}}dx(R^2-y^2-x^2)^{1/2}\times[R^2-(y-b)^2-x^2]^{1/2}. \tag{4.14}$$

Since the normalization is already established, we may ignore it in the following. We introduce the dimensionless variables $u=b/R$, y/R, and x/R. The integral then becomes

$$I(u)=\int_{u/2}^{1}dy'\int^{(1-y'^2)^{1/2}}(1-y'^2-x'^2)^{1/2}\times[1-(y'-u)^2-x'^2]^{1/2}. \tag{4.15}$$

This integral has been evaluated numerically and is shown in Fig. 4; a Gaussian with the same area

$$A=R^2\int_0^2 I^2(u)u\,du \tag{4.16}$$

and mean-square radius

$$\langle b^2\rangle=\frac{R^2\int_0^2 I^2(u)u^3du}{\int_0^2 I^2(u)u\,du}=0.47R^2 \tag{4.17}$$

is also shown in the same figure.

The value of $\langle b^2\rangle$ given above, with $R\sim 1/m_\pi$, as determined from the proton mass by Chodos *et al.*, leads to a differential cross section

$$\frac{d\sigma}{d\Omega}\bigg/\left(\frac{d\sigma}{d\Omega}\right)_0\cong e^{11t} \tag{4.18}$$

for p-p (or $p\bar{p}$) scattering.

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After the completion of this work the author learned of a similar model of the Pomeron proposed by S. Nussinov.[9]

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