PHYSICAL REVIEW D VOLUME 12, NUMBER 6 15 SEPTEMBER 1975

Quantization of supersymmetric gauge theories*

B. de Wit†

Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11794

(Received 14 April 1975)

We consider the quantization of supersymmetric gauge theories in a rather general class of gauges. A generating functional is constructed that is supersymmetric, for which we formulate the Ward-Takahashi identities of the local gauge invariance. The existence of a supermultiplet of ghost fields is established. We present several examples to demonstrate the consistency of our approach.

I. INTRODUCTION

Supersymmetric field theories were first proposed by Wess and Zumino.[1] The supersymmetry consists of a set of infinitesimal transformations that transform bosons into fermions and vice versa. Such theories, which generally have a small number of free parameters, turn out to exhibit several interesting features. One of them is that the quantum corrections are much less divergent than could have been expected on general grounds,[2,3] or are even completely absent in certain cases.[4] Another one is the emergence of a massless Goldstone fermion when the supersymmetry is spontaneously broken.[5]

In a supersymmetric theory we have representations that contain both Bose and Fermi fields, which are called supermultiplets. The behavior of the fields under supersymmetry transformations involves space-time derivatives of the fields, which is related to the fact that Bose and Fermi fields usually have different dimensions. This is also apparent from the algebra of the supersymmetry generators. This algebra, which was first considered from a rather different point of view by Akulov and Volkov,[6] involves both commutators and anticommutators, and is closed if we include the energy-momentum operators. Also connected to the presence of derivatives in the transformation properties of the fields is the fact that this algebra is not preserved in the presence of an external or spontaneous symmetry breaking.[7]

More recently, Wess and Zumino,[8] and later Salam and Strathdee,[9,10] and Ferrara and Zumino[11] have considered the construction of supersymmetric theories that are locally gauge invariant. Obviously, the presence of derivatives in the supersymmetry transformations makes it nontrivial to implement a local gauge symmetry. In order for the gauge symmetry to be compatible with the supersymmetry it turns out that one has to extend the conventional gauge group into a gauge group that involves both commuting and anticommuting parameters, which transform into each other under supersymmetry transformations. Obviously, such supersymmetric gauge theories have a much more complicated structure than the conventional gauge theories, and the restrictions that are imposed by both supersymmetry and gauge invariance make it far from trivial to find physical applications for these ideas. Nevertheless, Fayet[12] has recently succeeded in constructing a semirealistic model of this type that describes weak and electromagnetic interactions of leptons.

In this paper we want to address ourselves to the quantization of those theories. In principle, the quantization program is identical to the one that was carried through for the conventional gauge theories,[13] but there are a number of technical complications because of the presence of the commuting and anticommuting quantities. There have been some preliminary calculations in the Abelian model of Wess and Zumino[8] in a special gauge in which the theory is renormalizable by power counting.[14] However, before the renormalization can be established the quantization must be studied in a wider class of gauge conditions. In order for this program to make sense one must show the gauge independence of the S matrix, something that can be done by exploiting the Ward-Takahashi identities. In these theories the formulation for a wider variety of gauges is even more urgent, because the special gauges that are renormalizable by power counting break the supersymmetry.

We will discuss the quantization for a rather general class of gauges, with special attention to supersymmetric gauge conditions. This will be done in the context of the path-integral method, as it was formulated by Faddeev and Popov,[15] and by 't Hooft.[16] The Ward-Takahashi identities of the local gauge invariance will be formulated in a supersymmetric gauge, and we will work out some examples. We establish the existence of Faddeev-Popov ghost fields, which will occur in anticommuting supermultiplets, i.e., there are anticommuting spinless and commuting spinor ghost fields. However, we should point out that this program can presently not be carried through with the same rigor as in the conventional gauge theories, because a regularization procedure for most of these gauges is lacking.[17] Our results are mostly formulated for the Abelian gauge model, but the generalization to the non-Abelian case is usually obvious.

In Sec. II we will briefly review the formalism of superfields and the construction of supersymmetric gauge theories. In Sec. III the quantization procedure is described. Supersymmetric gauge conditions are discussed in Sec. IV where we construct a supersymmetric generating functional for the Green's functions. We will also work out an explicit example of a supersymmetric gauge condition. Section V deals with the generalized Ward-Takahashi identities of the local gauge invariance. In Sec. VI we demonstrate the gauge independence of S-matrix elements in the one-loop approximation in an explicit example, where we make use of the previously introduced supersymmetric gauge condition. Finally, our conclusions are given in Sec. VII. Some useful formulas are collected in an Appendix.

II. PRELIMINARIES

A. Superfields

We will first briefly discuss the technique of superfields and at the same time establish our notation. The notion of a superfield was first introduced by Salam and Strathdee[10,18] and was subsequently also developed by Ferrara, Wess, and Zumino.[19] A superfield is a function of both the four-vector of space and time, x_μ, and a constant four-component, anticommuting Majorana spinor θ_α. Each superfield can be decomposed in the following way[20]:

$$\Phi(x,\theta)=A(x)+\bar\theta\psi(x)+\tfrac{1}{2}\bar\theta\theta F(x)+\tfrac{1}{2}\bar\theta i\gamma_5\theta G(x)+\tfrac{1}{2}\bar\theta i\gamma_\mu\gamma_5\theta B_\mu(x)+(\bar\theta\theta)\bar\theta\chi(x)+\tfrac{1}{4}(\bar\theta\theta)^2D(x)\,. \qquad (1)$$

The degree of each component is defined by the number of independent θ_α to which it is proportional in the superfield expansion (1). The components of a general superfield will generally be referred to by $A, \psi, \ldots, D$.

In the eight-dimensional space spanned by x_μ and θ_α we define two operations:

$$G_\alpha=\frac{\partial}{\partial\bar\theta_\alpha}-(\not\partial\theta)_\alpha\,,\qquad D_\alpha=\frac{\partial}{\partial\bar\theta_\alpha}+(\not\partial\theta)_\alpha\,, \qquad (2)$$

where $\partial/\partial\bar\theta$ is defined as the *left* derivative with respect to $\bar\theta$. G_α is the generator of the infinitesimal supersymmetry transformations, and D_α is the so-called covariant derivative. One can easily verify that

$$\{G_\alpha,\bar G_\beta\}=2(\not\partial)_{\alpha\beta}\,,\qquad \{D_\alpha,\bar D_\beta\}=-2(\not\partial)_{\alpha\beta}\,,\qquad \{G_\alpha,D_\beta\}=0\,. \qquad (3)$$

Notice that both G and D satisfy the Majorana constraint.

We now define a real vector field by

$$V(x,\theta)=A(x)+\bar\theta\psi(x)+\tfrac{1}{2}\bar\theta\theta F(x)+\tfrac{1}{2}\bar\theta i\gamma_5\theta G(x)+\tfrac{1}{2}\bar\theta i\gamma_\mu\gamma_5\theta B_\mu(x)+(\bar\theta\theta)\bar\theta[\chi(x)+\tfrac{1}{2}\not\partial\psi(x)]+\tfrac{1}{4}(\bar\theta\theta)^2[D(x)+\tfrac{1}{2}\partial^2A(x)]\,, \qquad (4)$$

where $V^\dagger(x,\theta)=V(x,\theta)$, and V transforms as a scalar under Lorentz transformations: $V(x',\theta')=V(x,\theta)$. This implies that A, F, G, and D are real spinless fields, whereas ψ and χ are Majorana fields. Because the covariant derivatives D commute with G, we can also impose a supersymmetric constraint on a general superfield:

$$(1\mp\gamma_5)_{\alpha\beta}D_\beta S_\pm(x,\theta)=0\,.$$

This leads to the definition of chiral scalar superfields:

$$S_\pm(x,\theta)=\exp(\mp\tfrac{1}{2}\bar\theta\not\partial\gamma_5\theta)[\tfrac{1}{2}H_\pm(x)+\bar\theta\Psi_\pm(x)+\tfrac{1}{4}\bar\theta(1\pm\gamma_5)\theta M_\pm(x)]$$
$$=\tfrac{1}{2}H_\pm(x)+\bar\theta\Psi_\pm(x)+\tfrac{1}{4}\bar\theta(1\pm\gamma_5)\theta M_\pm(x)\mp\tfrac{1}{4}\bar\theta\gamma_\mu\gamma_5\theta\partial_\mu H_\pm(x)+\tfrac{1}{2}(\bar\theta\theta)\bar\theta\not\partial\Psi_\pm(x)+\tfrac{1}{16}(\bar\theta\theta)^2\partial^2H_\pm(x)\,. \qquad (5)$$

Such fields have three components: two complex fields H and M, and one chiral Dirac spinor Ψ. One can require in addition that both chiral components are related by complex conjugation, $S_+^\dagger=S_-$, which implies that Ψ is a Majorana spinor and $H_+^\dagger=H_-$, $M_+^\dagger=M_-$. Unless stated otherwise, this constraint is to be understood.

It is obvious that a product of superfields is again a superfield. One can also show that the product of right- or left-handed chiral superfields is again a right- or left-handed superfield, respectively. Finally we notice that the component with highest degree (D) of a superfield, or the F and G components of a chiral superfield, transform under a supersymmetry transformation by a total space-time derivative. This is a useful observation if one wants to construct Lagrangians that lead to a supersymmetric action. One can construct the components of a superfield by taking the appropriate derivatives with respect to θ, or by an integral over components of θ. Integrals over anticommuting numbers θ_α are defined by[21]

$$\int d\theta_\alpha = 0; \quad \int \theta_\alpha d\theta_\beta = \delta_{\alpha\beta}\,. \tag{6}$$

We have collected some useful formulas for dealing with superfields in an Appendix.

B. Construction of supersymmetric gauge models

For future purposes we review the construction of supersymmetric gauge theories, with the emphasis on the Abelian model of Wess and Zumino.[8] Following Salam and Strathdee,[9,10] and Ferrara and Zumino[11] we introduce a real vector field $V(x,\theta)$ which transforms under a local transformation in the following way:

$$e^{gV(x,\theta)} \to e^{ig\Lambda_-(x,\theta)}\, e^{gV(x,\theta)}\, e^{-ig\Lambda_+(x,\theta)}\,. \tag{7}$$

$\Lambda_+(x,\theta)$ is a chiral left-handed superfield, with components $\delta(x)$, $\lambda_+(x)$, and $\omega(x)$, and $\Lambda_- = \Lambda_+^\dagger$. If the underlying group structure is SU(N) both V and $\Lambda_\pm$ are to be considered as $N\times N$ Hermitian matrices. From the quantity $\exp(gV)$ we can construct the analog of a gauge field:

$$V_\mu(x,\theta) = \frac{1}{8g}[\gamma_\mu(1+\gamma_5)]_{\alpha\beta}\bar{D}_\alpha(e^{-gV(x,\theta)}D_\beta e^{gV(x,\theta)})\,, \tag{8a}$$

which transforms under the local gauge group according to

$$V_\mu \to e^{ig\Lambda_-}V_\mu e^{-ig\Lambda_+} + ig^{-1}e^{ig\Lambda_-}\partial_\mu e^{-ig\Lambda_+}\,. \tag{8b}$$

From $V_\mu(x,\theta)$ we can then construct a supersymmetric and gauge-invariant Lagrangian:

$$\mathcal{L}(x) \propto \int d^4\theta\, \mathrm{Tr}\{V_\mu V_\mu + V_\mu^\dagger V_\mu^\dagger\}\,. \tag{9}$$

For an Abelian gauge group this construction is much simpler. Equation (7) corresponds to

$$V(x,\theta) \to V(x,\theta) - i[\Lambda_+(x,\theta) - \Lambda_-(x,\theta)]\,. \tag{10a}$$

For the various components of $V(x,\theta)$ this implies

$$\begin{aligned} &\delta A = -\tfrac{1}{2}i(\delta - \delta^\dagger), && \delta B_\mu = \tfrac{1}{2}\partial_\mu(\delta+\delta^\dagger)\,,\\ &\delta\psi = -i\gamma_5\lambda, && \delta\chi = 0\,,\\ &\delta F = -\tfrac{1}{2}i(\omega - \omega^\dagger)\,, && \delta D = 0\,.\\ &\delta G = -\tfrac{1}{2}(\omega+\omega^\dagger)\,, && \end{aligned} \tag{10b}$$

The Abelian gauge field can be chosen as

$$V_\mu(x,\theta) = -\tfrac{1}{8}\,i\bar{D}\gamma_\mu(1+\gamma_5)DV(x,\theta)\,.$$

The corresponding invariant Lagrangian according to Eq. (9) is then given by

$$\mathcal{L}_V = -\tfrac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \tfrac{1}{2}\bar{\chi}\partial\!\!\!/\chi + \tfrac{1}{2}D^2\,. \tag{11}$$

Subsequently we consider two chiral superfields S^1 and S^2 with $(S_+^{1,2})^\dagger = S_-^{1,2}$. The components of $S_+^{1,2}$ are denoted by $H_{1,2}$, $\Psi_{1,2}$, and $M_{1,2}$. Under the gauge group the fields $S^{1,2}$ transform according to

$$\begin{aligned} S_+^1(x,\theta) &\to e^{ig\Lambda_+(x,\theta)}S_+^1(x,\theta)\,,\\ S_+^2(x,\theta) &\to e^{-ig\Lambda_+(x,\theta)}S_+^2(x,\theta)\,. \end{aligned} \tag{12a}$$

For the various components this implies the following transformation properties:

$$\begin{aligned} \delta H_1(x) &= \tfrac{1}{2}ig\delta(x)H_1(x)\,,\\ \delta\Psi_1(x) &= \tfrac{1}{4}ig\{[\delta(x)+\delta^\dagger(x)]\Psi_1(x) + [\delta(x)-\delta^\dagger(x)]\gamma_5\Psi_1(x)\\ &\quad + \lambda(x)[H_1(x)+H_1^\dagger(x)]\\ &\quad + \gamma_5\lambda(x)[H_1(x) - H_1^\dagger(x)]\}\,,\\ \delta M_1(x) &= \tfrac{1}{2}ig[\delta(x)M_1(x) + \bar{\lambda}(x)(1+\gamma_5)\Psi_1(x) + \omega(x)H_1(x)]\,. \end{aligned} \tag{12b}$$

For S^2 the transformation properties can be obtained by replacing $1\to 2$ and $g\to -g$ in Eq. (12b).

As follows from Eqs. (12) the F component of $S_+^1S_+^2$ and $S_-^1S_-^2$ is both supersymmetric and gauge invariant. This gives rise to the following invariant mass term for S^1 and S^2:

$$\mathcal{L}_m = \tfrac{1}{2}m(H_1M_2 + H_2M_1 + H_1^\dagger M_2^\dagger + H_2^\dagger M_1^\dagger - 2\bar{\Psi}_1\Psi_2)\,. \tag{13}$$

The supersymmetric and gauge-invariant interaction between the chiral superfields and the vector field, which at the same time provides a kinetic term for the chiral fields, is given by the following expression:

$$\begin{aligned} \mathcal{L}_g \propto \int d^4\theta[&S_+^1(x,\theta)S_-^1(x,\theta)e^{gV(x,\theta)}\\ &+ S_+^2(x,\theta)S_-^2(x,\theta)e^{-gV(x,\theta)}]\,. \end{aligned} \tag{14a}$$

A somewhat tedious but straightforward calculation leads to a more explicit result for $\mathcal{L}_g$:

$$
\begin{aligned}
\mathcal{L}_g = {} & e^{gA}(-\tfrac{1}{2}|\partial_\mu H_1|^2 - \tfrac{1}{2}\bar{\Psi}_1 \not{\partial} \Psi_1 + \tfrac{1}{2}|M_1|^2) \\
& + \tfrac{1}{4} g e^{gA}\{|H_1|^2(D+\partial^2 A) + H_1^\dagger M_1(F-iG) + H_1 M_1^\dagger(F+iG) - \bar{\Psi}_1[M+M^\dagger+\not{\partial}(H+H^\dagger)]\psi + \bar{\Psi}_1\gamma_5[M-M^\dagger+\not{\partial}(H-H^\dagger)]\psi \\
& \qquad - (H_1+H_1^\dagger)\bar{\Psi}_1(\chi+\not{\partial}\psi) + (H_1-H_1^\dagger)\bar{\Psi}_1\gamma_5(\chi+\not{\partial}\psi) + iB_\mu(H_1\overleftrightarrow{\partial}_\mu H_1^\dagger + \bar{\Psi}_1\gamma_\mu\gamma_5\Psi_1)\} \\
& + \tfrac{1}{8} g^2 e^{gA}\{|H_1|^2[(\partial_\mu A)^2 - 2\bar{\chi}\psi - \bar{\psi}\not{\partial}\psi + (F^2+G^2-B_\mu{}^2)] - \tfrac{1}{2}(H_1^\dagger M_1 + H_1 M_1^\dagger)\bar{\psi}\psi \\
& \qquad + \tfrac{1}{2}(H_1^\dagger M_1 - H_1 M_1^\dagger)\bar{\psi}\gamma_5\psi - (H_1+H_1^\dagger)\bar{\Psi}_1(F - i\gamma_5 G - i\not{B}\gamma_5 + \not{\partial}A)\psi \\
& \qquad - (H_1-H_1^\dagger)\bar{\Psi}_1(\gamma_5 F + iG + i\not{B} + \gamma_5\not{\partial}A)\psi + \tfrac{1}{2}(\bar{\psi}\gamma_\mu\gamma_5\psi)(H_1\overleftrightarrow{\partial}_\mu H_1^\dagger + \bar{\Psi}_1\gamma_\mu\gamma_5\bar{\Psi}_1)\} \\
& + \tfrac{1}{16} g^3 e^{gA}\{-|H_1|^2\bar{\psi}(F - i\gamma_5 G - i\not{B}\gamma_5)\psi + \tfrac{1}{6}(H_1+H_1^\dagger)\bar{\Psi}_1[(\bar{\psi}\psi) + (\bar{\psi}i\gamma_5\psi)i\gamma_5 \\
& \qquad - (\bar{\psi}i\gamma_\mu\gamma_5\psi)i\gamma_\mu\gamma_5]\psi - \tfrac{1}{6}(H_1-H_1^\dagger)\bar{\Psi}_1[(\bar{\psi}\psi)\gamma_5 + i(\bar{\psi}i\gamma_5\psi) + (\bar{\psi}i\gamma_\mu\gamma_5\psi)i\gamma_\mu]\psi\} \\
& + \tfrac{1}{384} g^4 e^{gA}|H_1|^2[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2 - (\bar{\psi}i\gamma_\mu\gamma_5\psi)^2] \\
& + (1 \to 2; g \to -g) . \qquad (14b)
\end{aligned}
$$

Hence, we have constructed the invariant Lagrangian for a supersymmetric gauge field theory:

$$\mathcal{L}_{\text{inv}} = \mathcal{L}_V + \mathcal{L}_m + \mathcal{L}_g . \qquad (15)$$

$\mathcal{L}_V$, $\mathcal{L}_m$, and $\mathcal{L}_g$ are given in Eqs. (11), (13), and (14), respectively. Regarding the uniqueness of this Lagrangian we wish to point out that $\mathcal{L}_m + \mathcal{L}_g$ is the most general parity-conserving Lagrangian that is quadratic in the chiral superfields S^1 and S^2. Concerning the vector superfield V there is no such requirement, since $\mathcal{L}_g$ contains already all powers of V. Consequently, there is no fundamental reason to confine ourselves to quadratic terms in $\mathcal{L}_V$. One such reason could be that only the Lagrangian given in Eq. (15) is renormalizable, something which still remains to be proven at this moment.

Finally, let us make some comments about the parity assignments. The vector field $V(x, \theta)$ has an intrinsic negative parity, so that its vector component B_μ is a vector field. Under parity, the gauge transformation $\exp[i\Lambda_+(x, \theta)]$ changes into $\exp[i\Lambda_-(x, \theta)]$, and consistency requires that $S^1(x, \theta)$ and $S^2(x, \theta)$ are connected by a parity transformation.

III. QUANTIZATION

As was shown explicitly in the previous section, supersymmetric gauge theories are invariant under local gauge transformations that contain both commuting (spinless) and anticommuting (Majorana spinor) parameters. We will now formulate the quantization in those theories, following the procedure that was proposed by Faddeev and Popov[15] and by 't Hooft[16] for theories with the conventional gauge invariance.[13] For simplicity we will mainly consider the Abelian model, but the generalization to non-Abelian models is usually straightforward.

The starting point is the Feynman[22] vacuum-to-vacuum amplitude expressed as a functional integral:

$$\langle \text{out} | \text{in} \rangle \propto \int [dV][dS^1][dS^2] \exp\{iS[V, S^1, S^2]\} . \qquad (16)$$

$S[V, S^1, S^2] = \int d^4x\, \mathcal{L}_{\text{inv}}(x)$ is the invariant action, and the measure $[dV][dS^1][dS^2]$ of the functional integration runs over all independent components of the superfields V, S^1, and S^2 at various space-time points. For instance, $[dV]$ is given by

$$[dV] = \prod_x \left[dA(x) \prod_\alpha d\psi_\alpha(x)\, dF(x)\, dG(x) \times \prod_\mu dA_\mu(x) \prod_\beta d\chi_\beta(x)\, dD(x) \right] .$$

The action is invariant under the transformations that were described in Eqs. (10) and (12):

$$
\begin{aligned}
& V(x, \theta) \to V(x, \theta) - i[\Lambda_+(x, \theta) - \Lambda_-(x, \theta)] , \\
& S_+^1(x, \theta) \to \exp[ig\Lambda_+(x, \theta)] S_+^1(x, \theta) , \\
& S_+^2(x, \theta) \to \exp[-ig\Lambda_+(x, \theta)] S_+^2(x, \theta) .
\end{aligned}
$$

In other words, the action is constant over an orbit of this complicated gauge group. Because of this the integral (16) will be proportional to an infinite factor, and as usual we will extract this factor before considering the quantization. In order to do this we impose a gauge condition, and because the gauge group parameters have the structure of a chiral superfield, such a condition can be written in the form

$$
\begin{aligned}
& C_+[V, S^1, S^2] = 0 , \\
& C_-[V, S^1, S^2] = 0 .
\end{aligned} \qquad (17)
$$

C_+ and C_- are functionals of the superfields V, S^1, and S^2 which are written as chiral superfields.

However, we do not wish to imply that C transforms as a chiral superfield under supersymmetry transformations. If that is the case the gauge conditions will preserve the supersymmetry. Gauge conditions of this type will be studied in more detail in the next sections. In this section we arrange the various components of C in a superfield form purely for notational reasons. In that formulation we have $C_+^\dagger = C_-$. The functionals $C_\pm$ have to be chosen such that the gauge conditions can be obtained by a suitable gauge transformation. Such permissible gauges are called *nonsingular* for reasons that will be explained below.

Imposing the constraint equations (17) in the functional integral will obviously eliminate the infinite factor, but this must be done in such a way that the gauge invariance of this integral is maintained. As is well known,[15] that can be achieved in the following way:

$$\langle \text{out} | \text{in} \rangle \propto \int [dV][dS^1][dS^2]\, \delta(C_+[V,S^1,S^2]) \times \delta(C_-[V,S^1,S^2]) \times \Delta_C[V,S^1,S^2] \exp\{iS[V,S^1,S^2]\}. \tag{18}$$

The two δ functions are to be understood as δ functions over all the independent components of $C_\pm$: one Majorana spinor and four real spinless components. The factor $\Delta_C[V,S^1,S^2]$ is to be defined such that the functional integral does not depend on the choice of the gauge conditions $C_\pm = 0$. Such a factor is given by the Jacobian of the transformation that changes C into the canonical parameters Ω of the gauge group. Hence Δ_C depends on the quantities $\delta C/\delta\Omega$, which are also given by the change of the various components of C under an infinitesimal gauge transformation, which we will denote formally by $\delta C/\delta\Lambda$. In the conventional gauge theories, where the functionals C and the parameters of the gauge group are basically commuting quantities, the Jacobian is given by the determinant, and we have

$$\Delta_C = \det\left(\frac{\delta C}{\delta \Lambda}\right).$$

However, in supersymmetric models both the gauge condition and the gauge transformations contain commuting as well as anticommuting parameters.

Let us therefore first generally define the Jacobian for transformations that contain both commuting and anticommuting parameters.[21] Consider a set of commuting and anticommuting quantities $x_1, \ldots, x_n$ and $\theta_1, \ldots, \theta_k$, respectively. On the basis of the anticommuting objects θ_α we can define a Grassmann algebra with k generators. Every element of the algebra which depends on x_i and θ_α can be decomposed as follows:

$$P(x,\theta) = p^{(0)}(x) + \sum_\alpha p_\alpha^{(1)}(x)\theta_\alpha + \sum_{\alpha<\beta} p_{\alpha\beta}^{(2)}(x)\theta_\alpha\theta_\beta + \cdots + p^{(k)}(x)\theta_1\theta_2\cdots\theta_k .$$

The upper index of the various components is referred to as the *degree*.

Over the Grassmann algebra one can define integrals, as is prescribed by Eq. (6), and we are interested in the integral

$$I = \int dx_1 \cdots dx_n\, d\theta_k \cdots d\theta_1\, P(x,\theta).$$

The x integration goes over the interval $-\infty < x < \infty$, and $P(x,\theta)$ is supposed to go to zero as $|x| \to \infty$ fast enough so that the integral converges. According to the integration rules over anticommuting parameters, only the element of highest degree will survive the integration, so that the answer is

$$I = \int dx_1 \cdots dx_n\, p^{(k)}(x).$$

Let us now consider the corresponding Jacobian for a transformation of the parameters (x_i, θ_α) into (x_i', θ_α'):

$$x_i = x_i(x_i', \theta_\alpha'),$$
$$\theta_\alpha = \theta_\alpha(x_i', \theta_\alpha').$$

Of course, this transformation should be such that x' and θ' are still, respectively, commuting and anticommuting quantities, and the transformation may contain additional anticommuting c numbers. For such transformations the Jacobian can be expressed by[23]

$$J(x(x',\theta'), \theta(x',\theta')) = \frac{\det\left(\dfrac{\partial x_i}{\partial x_j'}\right)}{\det\left[\dfrac{\partial\theta_\alpha}{\partial\theta_\beta'} - \dfrac{\partial\theta_\alpha}{\partial x_i'}\left(\dfrac{\partial x}{\partial x'}\right)^{-1}_{ij}\dfrac{\partial x_j}{\partial\theta_\beta'}\right]}. \tag{19}$$

The derivatives $\partial\theta/\partial\theta'$ and $\partial x/\partial x'$ are commuting, whereas $\partial\theta/\partial x'$ and $\partial x/\partial\theta'$ are anticommuting quantities. $\partial x/\partial\theta'$ is defined to be the *right* derivative. For the remaining partial derivatives right and left derivatives are simply equivalent. With this Jacobian the following identity holds:

$$\int dx_1 \cdots dx_n \, d\theta_k \cdots d\theta_1 P(x, \theta)$$

$$= \pm \int dx_1' \cdots dx_n' \, d\theta_k' \cdots d\theta_1'$$

$$\times J(x(x', \theta'), \theta(x', \theta')) P(x(x', \theta'), \theta(x', \theta')) .$$

Hence in a supersymmetric gauge theory the so-called Faddeev-Popov factor Δ_C is much more complicated than a single determinant, but it is proportional to the Jacobian $J(x(x', \theta'), \theta(x', \theta'))$ where x and θ represent, respectively, the commuting and anticommuting independent components of $C_\pm$, and x' and θ' represent, respectively, the commuting and anticommuting parameters of the infinitesimal gauge transformation, δ, $\delta^\dagger$, ω, $\omega^\dagger$, and λ. It is obvious that permissible gauge conditions are those for which the Jacobian factor is finite. For this reason these gauges are called nonsingular, i.e., the inverse transformation $(x', \theta') \to (x, \theta)$ must exist.

Before formulating the gauge conditions that are manifestly supersymmetric, we consider the special gauge given by Wess and Zumino.[8] The importance of this gauge stems from the fact that the theory becomes renormalizable by power counting. The gauge conditions are given by

$$A = F = G = \psi = 0 . \tag{20}$$

In addition there is a gauge condition for the vector field B_μ, which can be chosen in the standard way. However, in this gauge (20) the Faddeev-Popov term Δ_C is necessarily a field-independent factor, because the change of A, F, G, and ψ under an infinitesimal gauge transformation does not involve the fields anymore. In the non-Abelian models based on SU(N) one can consider a gauge condition similar to (20), which also makes the theory renormalizable by power counting.[14] In that case the changes of these components under an infinitesimal gauge transformation do depend on the fields. However, one can show that, except for the vector fields, these fields are the same fields that were required to vanish by the gauge condition, so that Δ_C reduces to the Faddeev-Popov factor of the conventional gauge theories.

It is obvious that these special gauge conditions (20) will break the manifest supersymmetry of the functional integral. Supersymmetric gauge conditions are, however, very important for the following reason. Provided that the generalized Ward-Takahashi (or Slavnov-Taylor) identities of the local gauge invariance can be proved with the same rigor as for the conventional gauge theories, something which requires the existence of a suitable regularization scheme, then the S matrix can be shown to be independent of the choice of the gauge condition. In that case the supersymmetry of the S matrix will be preserved, provided that there is a manifestly supersymmetric quantization procedure. A proof of the renormalizability could then hopefully be carried through in a more suitable gauge which does not need to be supersymmetric. For instance, in the non-Abelian models based on SU(N) with only vector superfields there is a special, nonsupersymmetric gauge in which the Lagrangian reduces to the Lagrangian for SU(N) gauge fields coupled to Majorana spinors in the regular representation.[10,11] Since the latter case is a conventional gauge theory its renormalizability is well established. Hence if the Ward-Takahashi identities can be proved rigorously, and if there is a supersymmetric quantization procedure, which we will discuss in the next section, non-Abelian gauge models for only vector fields are supersymmetrically renormalizable.

IV. CONSTRUCTION OF A SUPERSYMMETRIC GENERATING FUNCTIONAL

As was previously explained, it is important to study a manifestly supersymmetric quantization for gauge theories. For this reason we will consider the construction of a supersymmetric generating functional for the Green's functions in this section. It is clear that for the gauge condition (17) to be supersymmetric the functionals $C_\pm$ should transform as chiral superfields under supersymmetry transformations. To construct such superfields one can make use of the operators $\overline{D}(1 \pm \gamma_5)D$ which, if applied to a general superfield, lead to left- and right-handed chiral superfields. Using this observation and the fact that S^1 and S^2 are already chiral superfields one can easily construct supersymmetric gauge conditions that lead to a nonsingular gauge. An example of such a gauge will be given at the end of this section.

Let us now write the functionals $C_\pm$ as a linear combination of a field-dependent part $C_\pm$ and a field-independent part $c_\pm$:

$$C_\pm \to C_\pm + c_\pm .$$

Because the functional integral (18) was independent of the choice of the gauge, it must also be independent of $c_\pm$. Following 't Hooft[16] we can then integrate the functional integral over the independent components of $c_+(x, \theta)$ and $c_-(x, \theta)$ with an arbitrary weight function. In order to maintain the supersymmetry we choose the following invariant weight function:

$$\exp\left(i\int d^4x\,\frac{\rho}{4}[c_+(x,\theta)\,c_-(x,\theta)]_D\right),$$

where ρ is an arbitrary parameter, and the subscript D denotes the D component of the superfield expression. Integrating the functional integral over c_+ and c_- removes the δ functions, and we find

$$\langle\text{out}|\text{in}\rangle\propto\int[dV][dS^1][dS^2]\,\Delta_C[V,S^1,S^2]\exp\left[i\int d^4x\left(\mathcal{L}_{\text{inv}}(x)+\frac{\rho}{4}[C_+(x,\theta)C_-(x,\theta)]_D\right)\right].$$

The next step is to describe Δ_C by loops that are generated by a set of unphysical fields, the so-called Faddeev-Popov ghost fields. In order to achieve this, we notice that the Jacobian given in Eq. (19) is proportional to the following expression:

$$J(x(x',\theta'),\theta(x',\theta'))\propto\int da_i\,db_\alpha\,da_i'\,db_\alpha'\exp\left(ia_i'\frac{\partial x_i}{\partial x_j'}a_j+ib_\alpha'\frac{\partial\theta_\alpha}{\partial\theta_\beta'}b_\beta+ia_i'\frac{\partial x_i}{\partial\theta_\alpha'}b_\alpha+ib_\alpha'\frac{\partial\theta_\alpha}{\partial x_i'}a_i\right). \tag{21}$$

The parameters a_i and a_i' are anticommuting, whereas the parameters b_α and b_α' are commuting. We have to arrange these parameters such that this expression becomes manifestly supersymmetrical and Lorentz invariant. This can be achieved by assigning the ghost fields a_i, a_i', b_α, and b_α' into chiral superfields which are anticommuting. In other words, the spinless and spinor components of these superfields are to be quantized according to Fermi-Dirac and to Bose-Einstein statistics, respectively. The presence of the commuting spinor Faddeev-Popov ghost fields is characteristic for supersymmetric gauge theories. Because the components a_i, b_α and a_i', b_α' are independent, the ghost fields have an orientation, in the manner of the ghost fields in the conventional gauge theories.[24] Hence, we introduce chiral anticommuting superfields:

$$\Phi_\pm(x,\theta)=\exp(\mp\bar\theta\not\partial\gamma_5\theta)[\tfrac{1}{2}\phi_\pm(x)\pm\bar\theta\zeta_\pm(x)+\tfrac{1}{4}\bar\theta(1\pm\gamma_5)\theta\xi_\pm(x)]. \tag{22}$$

The spinor components of anticommuting fields are not defined in the standard way (5) in order to maintain the relation $\Phi_+^\dagger=\Phi_-$. The spinor components have also different parity assignments. Hence we will introduce two such Faddeev-Popov superfields $\Phi_\pm$ and $\Phi_\pm'$ with components ϕ,ζ,ξ and ϕ',ζ',ξ', respectively. In addition, we have quantities formally denoted by $\delta C_+/\delta\Lambda_\pm$ and $\delta C_-/\delta\Lambda_\pm$, which are defined by the change of the superfields C_+ and C_- under an infinitesimal gauge transformation with parameters $\Lambda_\pm$. These quantities may contain covariant derivatives as well as ordinary space-time derivatives. Because the supersymmetry and the gauge symmetry are compatible, the expressions

$$\frac{\delta C_+}{\delta\Lambda_+}\Phi_++\frac{\delta C_+}{\delta\Lambda_-}\Phi_-$$

and

$$\frac{\delta C_-}{\delta\Lambda_+}\Phi_++\frac{\delta C_-}{\delta\Lambda_-}\Phi_-$$

are, respectively, left- and right-handed chiral superfields. The simplest way to find a supersymmetric expansion that is proportional to the Faddeev-Popov factor Δ_C as it was represented in Eq. (21) is to construct the F components of

$$\Phi_+'\frac{\delta C_+}{\delta\Lambda_+}\Phi_++\Phi_+'\frac{\delta C_+}{\delta\Lambda_-}\Phi_-$$

and

$$\Phi_-'\frac{\delta C_-}{\delta\Lambda_-}\Phi_-+\Phi_-'\frac{\delta C_-}{\delta\Lambda_+}\Phi_+.$$

Hence we have arrived at the following expression for the generating functional $W[J]$ for the Green's functions:

$$W[J]=\int[dV][dS^1][dS^2][d\Phi][d\Phi']\exp\left\{iS_{\text{eff}}[V,S^1,S^2,\Phi,\Phi']+i\int d^4x[J_V(x)V(x)+J_S^1(x)S^1(x)+J_S^2(x)S^2(x)]\right\}, \tag{23a}$$

where $S_{\text{eff}}=\int d^4x\,\mathcal{L}_{\text{eff}}(x)$, and $\mathcal{L}_{\text{eff}}$ is the effective Lagrangian given by

$$\mathcal{L}_{\text{eff}}=\mathcal{L}_{\text{inv}}+\tfrac{1}{4}\rho(C_+C_-)_D+i\left(\Phi_+'\frac{\delta C_+}{\delta\Lambda_+}\Phi_++\Phi_+'\frac{\delta C_+}{\delta\Lambda_-}\Phi_--\Phi_-'\frac{\delta C_-}{\delta\Lambda_-}\Phi_--\Phi_-'\frac{\delta C_-}{\delta\Lambda_+}\Phi_+\right)_F. \tag{23b}$$

The subscripts D and F denote, respectively, the D and F components of a superfield expression. Usually the superfield C has an intrinsic negative parity. For this reason the last two terms in Eq. (23b) have a minus sign, so that parity will be manifestly conserved. The source terms in the generating functional (23) are a formal expression for the products of all independent components of the superfields V, S^1, and S^2 with a corresponding source term. We can write this in a supersymmetric form by arranging the sources in superfields, as we will do in the next section.

Rather than continue this formal discussion we will present an example. Let us consider the following gauge condition in the Abelian gauge model:

$$C_+(x,\theta)=\tfrac{1}{4}\bar{D}(1-\gamma_5)DV(x,\theta)-\sqrt{2}\,\rho'[S^1_+(x,\theta)-S^2_+(x,\theta)],$$
$$C_-(x,\theta)=C_+(x,\theta)^\dagger\,. \tag{24}$$

Under an infinitesimal gauge transformation $C_\pm$ transform according to

$$C_+\to C_+ +\frac{i}{4}\bar{D}(1-\gamma_5)D\Lambda_- -\sqrt{2}\,i\rho' g(S^1_+ + S^2_+)\Lambda_+\,,$$
$$C_-\to C_- -\frac{i}{4}\bar{D}(1+\gamma_5)D\Lambda_+ +\sqrt{2}\,i\rho' g(S^1_- + S^2_-)\Lambda_-\,,$$

where $\Lambda_\pm$ were defined in Sec. II. Because of the field-independent terms in this result, one can easily verify that the gauge condition (24) is nonsingular, at least in perturbation theory. The Faddeev-Popov Lagrangian according to Eq. (23) is given by the F component of the following expression (remember that $\delta C/\delta\Lambda$ was defined as a right derivative):

$$-\tfrac{1}{4}[\Phi'_+\bar{D}(1-\gamma_5)D\Phi_- +\Phi'_-\bar{D}(1+\gamma_5)D\Phi_+]$$
$$+\sqrt{2}\,\rho' g[\Phi'_+(S^1_+ + S^2_+)\Phi_+ +\Phi'_-(S^1_- + S^2_-)\Phi_-]\,. \tag{25a}$$

We can write this in a more explicit form expressed in the independent components of the various fields:

$$\begin{aligned}\mathcal{L}_{\rm FP}=&-\tfrac{1}{2}\partial_\mu\phi'\partial_\mu\phi^\dagger-\tfrac{1}{2}\partial_\mu\phi'^\dagger\partial_\mu\phi-\bar{\zeta}'\not\partial\zeta+\tfrac{1}{2}\xi'\xi^\dagger+\tfrac{1}{2}\xi'^\dagger\xi\\ &+\tfrac{1}{4}\rho' g[\phi'\phi M+\phi'^\dagger\phi^\dagger M^\dagger+(\phi'\xi+\xi'\phi+2\bar{\zeta}'_-\zeta_+)H\\ &+(\phi'^\dagger\xi^\dagger+\xi'^\dagger\phi^\dagger+2\bar{\zeta}'_+\zeta_-)H^\dagger\\ &-2(\phi'\bar{\Psi}_-\zeta_+-\phi'^\dagger\bar{\Psi}_+\zeta_-\\ &-\bar{\zeta}'_-\Psi_+\phi+\bar{\zeta}'_+\Psi_-\phi^\dagger)]\,.\end{aligned} \tag{25b}$$

Here we made the following redefinitions:

$$\begin{aligned}&H=\tfrac{1}{2}\sqrt{2}\,(H_1+H_2),\ \ H'=\tfrac{1}{2}\sqrt{2}\,(H_1-H_2),\\ &\Psi=\tfrac{1}{2}\sqrt{2}\,(\Psi_1+\Psi_2),\ \ \Psi'=\tfrac{1}{2}\sqrt{2}\,(\Psi_1-\Psi_2),\\ &M=\tfrac{1}{2}\sqrt{2}\,(M_1+M_2),\ \ M'=\tfrac{1}{2}\sqrt{2}\,(M_1-M_2)\,.\end{aligned} \tag{26}$$

The gauge-fixing terms in the Lagrangian, $\frac{1}{4}\rho(C_+C_-)_D$, give only contributions that are quadratic in the various fields. Taking these terms into account and using Eqs. (11), (13), and (14), we now find the following lowest-order Lagrangian

$$\begin{aligned}\mathcal{L}_0=&-\tfrac{1}{2}|\partial_\mu H|^2-\tfrac{1}{2}\bar{\Psi}\not\partial\Psi+\tfrac{1}{2}|M|^2+\tfrac{1}{2}m(HM+H^\dagger M^\dagger-\bar{\Psi}\Psi)-\tfrac{1}{2}(1+\rho)\bar{\chi}\not\partial\chi-\tfrac{1}{2}\rho\bar{\psi}\not\partial\partial^2\psi-\rho\bar{\chi}\partial^2\psi-\tfrac{1}{2}(1+\rho\rho'^2)\bar{\Psi}'\not\partial\Psi'\\ &-\rho\rho'\bar{\Psi}'(\not\partial\chi+\partial^2\psi)+\tfrac{1}{2}m\bar{\Psi}'\Psi'-\tfrac{1}{2}(\partial_\mu B_\nu)^2+\tfrac{1}{2}(1+\rho)(\partial_\mu B_\mu)^2-\tfrac{1}{2}i\rho\rho'\partial_\mu B_\mu(M'-M'^\dagger)+\tfrac{1}{2}(1+\rho\rho'^2)(|M'|^2-|\partial_\mu H'|^2)\\ &-\tfrac{1}{2}\rho[(\partial_\mu F)^2+(\partial_\mu G)^2]-\tfrac{1}{2}\rho\rho'[\partial_\mu F\partial_\mu(H'+H'^\dagger)+i\partial_\mu G\partial_\mu(H'-H'^\dagger)]\\ &+\tfrac{1}{2}(1+\rho)D^2+\tfrac{1}{2}\rho(\partial^2A)^2+\rho D\partial^2A+\tfrac{1}{2}\rho\rho'(D+\partial^2A)(M'+M'^\dagger)-\tfrac{1}{2}m(H'M'+H'^\dagger M'^\dagger)\,.\end{aligned} \tag{27}$$

The lowest-order propagators in this gauge, which we will use in the subsequent sections, follow immediately from Eqs. (25) and (27). We will write the propagators in a matrix form. A straightforward calculation leads to the following result:

$$D(H_R,M_R;q)=\frac{1}{q^2+m^2}\begin{bmatrix}1 & -m\\ -m & -q^2\end{bmatrix},\quad D(H_I,M_I;q)=\frac{1}{q^2+m^2}\begin{bmatrix}1 & m\\ m & -q^2\end{bmatrix},$$

$$S(\Psi;q)=\frac{-i}{\not q-im}\,,$$

$$S(\chi,\psi,\Psi';q)=\begin{bmatrix}\dfrac{-i\not q}{q^2} & \dfrac{1}{q^2} & 0\\[2ex] \dfrac{1}{q^2} & \dfrac{(1+\rho)i\not q}{\rho q^4}+\dfrac{i\rho'^2}{q^2(\not q+im)} & \dfrac{\rho'\not q}{q^2(\not q+im)}\\[2ex] 0 & \dfrac{\rho'\not q}{q^2(\not q+im)} & \dfrac{-i}{\not q+im}\end{bmatrix},$$

$$D(B_\mu, M_I', H_I', G; q) = \begin{bmatrix} \frac{1}{q^2}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) - \frac{q_\mu q_\nu}{\rho q^2}\left(\frac{1}{q^2} + \frac{\rho\rho'^2}{q^2+m^2}\right) & \frac{-\rho' i q_\mu}{q^2+m^2} & \frac{-\rho' m i q_\mu}{q^2(q^2+m^2)} & \frac{-\rho'^2 m i q_\mu}{q^2(q^2+m^2)} \\ \frac{\rho' i q_\nu}{q^2+m^2} & \frac{-q^2}{q^2+m^2} & \frac{-m}{q^2+m^2} & \frac{-\rho' m}{q^2+m^2} \\ \frac{\rho' m i q_\nu}{q^2(q^2+m^2)} & \frac{-m}{q^2+m^2} & \frac{1}{q^2+m^2} & \frac{\rho'}{q^2+m^2} \\ \frac{\rho'^2 m i q_\nu}{q^2(q^2+m^2)} & \frac{-\rho' m}{q^2+m^2} & \frac{\rho'}{q^2+m^2} & \frac{1}{\rho q^2} + \frac{\rho'^2}{q^2+m^2} \end{bmatrix},$$

$$D(D, A, M_R', H_R', F; q) = \begin{bmatrix} -1 & \frac{-1}{q^2} & 0 & 0 & 0 \\ \frac{-1}{q^2} & -\frac{1+\rho}{\rho q^4} - \frac{\rho'^2}{q^2(q^2+m^2)} & \frac{-\rho'}{q^2+m^2} & \frac{\rho' m}{q^2(q^2+m^2)} & \frac{-\rho'^2 m}{q^2(q^2+m^2)} \\ 0 & \frac{-\rho'}{q^2+m^2} & \frac{-q^2}{q^2+m^2} & \frac{m}{q^2+m^2} & \frac{-\rho' m}{q^2+m^2} \\ 0 & \frac{\rho' m}{q^2(q^2+m^2)} & \frac{m}{q^2+m^2} & \frac{1}{q^2+m^2} & \frac{-\rho'}{q^2+m^2} \\ 0 & \frac{-\rho'^2 m}{q^2(q^2+m^2)} & \frac{-\rho' m}{q^2+m^2} & \frac{-\rho'}{q^2+m^2} & \frac{1}{\rho q^2} + \frac{\rho'^2}{q^2+m^2} \end{bmatrix}, \tag{28}$$

$$D(\phi_R; q) = D(\phi_I; q) = \frac{1}{q^2}, \quad D(\xi_R; q) = D(\xi_I; q) = -1, \quad S(\zeta; q) = \frac{1}{i\not{q}}$$

The subscripts R and I denote the real and imaginary parts of the various fields.

Because these propagators (28) were constructed in a supersymmetric gauge there are relations among them which originate from the Ward identities of the supersymmetry. These can be worked out in the standard way,[3] using the supersymmetry transformations of the fields that are listed in the Appendix.

We have presented this particular gauge condition (24) because it is nontrivial in the sense that the Faddeev-Popov ghost fields are interacting. In non-Abelian gauge models the ghost fields are almost always interacting because of the gauge transformations. For practical purposes the generalized Landau gauge is very convenient. This gauge corresponds to the gauge condition $\bar{D}(1 \pm \gamma_5) DV(x, \theta) = 0$, and the propagators in this gauge can be obtained from those in Eq. (28) by taking the limits $\rho \to \infty$ and $\rho' = 0$. As one can see, the propagators simplify considerably in this limit, and the fields F and G are even eliminated.[25]

V. GENERALIZED WARD-TAKAHASHI IDENTITIES

In this section we will derive the generalized Ward-Takahashi identities of the local gauge symmetry in a supersymmetric way. For the conventional gauge theories such identities, which are also called Slavnov-Taylor identities, have been proven in various ways. In the framework of the path-integral formalism there have been several more or less general proofs by Fradkin and Tyutin,[26] Slavnov,[27] Taylor,[28] and Lee and Zinn-Justin.[29] In addition there is a rigorous combinatorial proof by 't Hooft and Veltman.[30] The Slavnov-Taylor identities contain the full symmetry contents of the gauge symmetry, and are the essential ingredient in the proof of the gauge independence and the unitarity of the S matrix.

For supersymmetric gauge theories we will formulate the generalized Ward-Takahashi identities for the supersymmetric generating functional that was constructed in the previous section. For notational reasons we will write the source terms in the generating functional in a supersymmetric form. Hence we have the following superfields $J_V(x, \theta)$, $J_S^1(x, \theta)$, and $J_S^2(x, \theta)$ and we write

$$\int d^4x (J_V V)_D + \int d^4x (J_{S+}^1 S_+^1 + J_{S-}^1 S_-^1 + J_{S+}^2 S_+^2 + J_{S-}^2 S_-^2)_F .$$

We now consider a special field-dependent infinitesimal gauge transformation that is determined by the parameters $\Lambda_\pm(x, \theta)$, which are subject to the condition

$$\tilde{\Lambda}_\pm = \frac{\delta C_\pm}{\delta \Lambda_+}\Lambda_+ + \frac{\delta C_\pm}{\delta\Lambda_-}\Lambda_- , \quad (29)$$

where $\tilde{\Lambda}_\pm$ is independent of the fields. We will assume that under this special gauge transformation the integration measure $[dV][dS^1][dS^2]\Delta_C[V,S^1,S^2]$ is invariant. For the conventional gauge theories this result has generally been proven by Lee and Zinn-Justin,[29] and we do not expect complications from the anticommuting character of some of the quantities that are involved. Performing this special gauge transformation on the integrand of the generating functional, we find

$$\int [dV][dS^1][dS^2]\Delta_C[V,S^1,S^2]\exp\left\{ i\int d^4x'[\mathcal{L}_{\rm inv}+\tfrac{1}{4}\rho(C_+C_-)_D+(J_VV)_D+\cdots](x')\right\}$$
$$\times\int d^4x\{\tfrac{1}{4}\rho[C_+(x,\theta)\tilde{\Lambda}_-(x,\theta)+C_-(x,\theta)\tilde{\Lambda}_+(x,\theta)]_D$$
$$+[J_V(x,\theta)(i\Lambda_-(x,\theta)-i\Lambda_+(x,\theta))]_D+[J^1_{S_+}(x,\theta)ig\Lambda_+(x,\theta)S^1_+(x,\theta)+\cdots]_F\}=0\,, \quad (30)$$

where we used the invariance of the integration measure. The dots in this equation represent the contributions of the additional source terms. This result must be valid for all possible values of $\tilde{\Lambda}_\pm(x,\theta)$.

The next step is to write this equation in a form that no longer implicitly depends on $\Lambda_\pm(x,\theta)$. Before doing that we prove a useful identity. Consider the expression

$$\int [dV][dS^1][dS^2][d\Phi][d\Phi']F[V,S^1,S^2]\Phi_\pm(x,\theta)\int d^4y[\Phi'_+(y,\theta)\tilde{\Lambda}_+(y,\theta)-\Phi'_-(y,\theta)\tilde{\Lambda}_-(y,\theta)]_F$$
$$\times\exp\left(iS_{\rm eff}+i\int d^4x'(J_VV)_D+\cdots\right)\,, \quad (31)$$

where F is an arbitrary functional of the fields V, S^1, and S^2. Using Eq. (29) this is equal to

$$\int [dV][dS^1][dS^2][d\Phi][d\Phi']F[V,S^1,S^2]\Phi_\pm(x,\theta)\int d^4y\left[\left(\Phi'_+\frac{\delta C_+}{\delta\Lambda_+}-\Phi'_-\frac{\delta C_-}{\delta\Lambda_+}\right)\Lambda_+ + \left(\Phi'_+\frac{\delta C_+}{\delta\Lambda_-}-\Phi'_-\frac{\delta C_-}{\delta\Lambda_-}\right)\Lambda_-\right]_F$$
$$\times\exp\left(iS_{\rm eff}+i\int d^4x'(J_VV)_D+\cdots\right)\,.$$

The quantities $\Phi'_+(\delta C_+/\delta\Lambda_\pm)-\Phi'_-(\delta C_-/\delta\Lambda_\pm)$ are, of course, related to the derivatives of the functional expression $\exp(iS_{\rm eff})$ with respect to the components of $\Phi_\pm$. Making use of this we find that after integration by parts (which is also valid for integrals over anticommuting parameters), Eq. (31) is simply given by

$$\int [dV][dS^1][dS^2][d\Phi][d\Phi']F[V,S^1,S^2]\Lambda_\pm(x,\theta)\exp\left(iS_{\rm eff}+i\int d^4x'(J_VV)_D+\cdots\right)\,.$$

If we now use this in order to replace the implicit $\Lambda_\pm$ dependence in Eq. (30) we find the generalized Ward-Takahashi identity for the generating functional:

$$\int [dV][dS^1][dS^2][d\Phi][d\Phi']\exp\left(iS_{\rm eff}+i\int d^4x'(J_VV)_D+\cdots\right)$$
$$\times\int d^4x\left\{\tfrac{1}{4}\rho[C_+(x,\theta)\tilde{\Lambda}_-(x,\theta)+C_-(x,\theta)\tilde{\Lambda}_+(x,\theta)]_D\right.$$
$$+\int d^4y[[J_V(x,\theta)(i\Phi_-(x,\theta)-i\Phi_+(x,\theta))]_D+[J^1_{S_+}(x,\theta)ig\Phi_+(x,\theta)S^1_+(x,\theta)+\cdots]_F]$$
$$\left.\times[\Phi'_+(y,\theta)\tilde{\Lambda}_+(y,\theta)-\Phi'_-(y,\theta)\tilde{\Lambda}_-(y,\theta)]_F\right\}=0\,. \quad (32)$$

This identity is supposed to be valid for all values of $\tilde{\Lambda}_\pm$. One easily recognizes that the basic structure of Eq. (32) is completely analogous to that of the generalized Ward-Takahashi identities for conventional gauge theories. There is, however, one important difference concerning the proof of these identities. In the conventional theories one has a very powerful regularization procedure for Feynman integrals, the dimensional regularization, and it is this aspect that makes the proof of the generalized Ward-Takahashi identities rigorous. For supersymmetric gauge theories such a powerful method is lacking,[17] and the identity (32) should be looked upon as a formal result.

As an illustration let us calculate the Ward-Takahashi identities for the propagators in lowest order, with a gauge condition that was defined in Eq. (24). The identities for the propagators are found by taking the derivative with respect to the sources and subsequently taking them equal to zero. In lowest order the pertinent part of Eq. (32) is represented by

$$\begin{aligned}\int [dV][dS^1][dS^2][d\Phi][d\Phi'] \exp(iS_{\text{eff}}) & \\ \times \int d^4x\, d^4y \{ [[J_V(x,\theta)V(x,\theta)]_D + [J^1_{S_+}(x,\theta)S^1_+(x,\theta) + \cdots]_F] & \\ \times \tfrac{1}{4}\rho[C_+(y,\theta)\tilde{\Lambda}_-(y,\theta) + C_-(y,\theta)\tilde{\Lambda}_+(y,\theta)]_D & \\ + [J_V(x)(\Phi_-(x,\theta) - \Phi_+(x,\theta))]_D[\Phi'_+(y,\theta)\tilde{\Lambda}_+(y,\theta) - \Phi'_-(y,\theta)\tilde{\Lambda}\ (y,\theta)]_F\} &= 0\,.\end{aligned}$$

Let us first consider the expression $\frac{1}{4}\rho(C_+\tilde{\Lambda}_- + C_-\tilde{\Lambda}_+)_D$ and $(\Phi'_+\tilde{\Lambda}_+ - \Phi'_-\tilde{\Lambda}_-)_F$. The various terms in these expressions that are proportional to the same independent component of $\tilde{\Lambda}_\pm$, carefully taking into account the commuting or anticommuting character of the various fields, are given, respectively, by

$$\begin{array}{lll} \rho\partial^2 F + \frac{1}{2}\rho\rho'\partial^2(H' + H'^\dagger) & \text{and} & -\frac{1}{2}(\xi' - \xi'^\dagger)\,, \\ \rho\partial^2 G + \frac{1}{2}i\rho\rho'\partial^2(H' - H'^\dagger) & \text{and} & -\frac{1}{2}i(\xi' + \xi'^\dagger)\,, \\ \rho D + \rho\partial^2 A + \frac{1}{2}\rho\rho'(M' + M'^\dagger) & \text{and} & -\frac{1}{2}(\phi' - \phi'^\dagger)\,, \\ \rho\partial_\mu B_\mu - \frac{1}{2}i\rho\rho'(M' - M'^\dagger) & \text{and} & -\frac{1}{2}i(\phi' + \phi'^\dagger)\,, \\ -\rho\bar{\chi}\overleftarrow{\not\partial} + \rho\partial^2\bar{\psi} - \rho\rho'\bar{\Psi}'\overleftarrow{\not\partial} & \text{and} & \bar{\zeta}' \quad . \end{array}$$

If we denote the corresponding elements of the first and second column by X^i and Y^i, respectively, $(i = 1, \ldots, 5)$, we find the following identities for the two-point Green's functions:

$$\begin{aligned} &\langle A(x)X^i(y)\rangle - \tfrac{1}{2}\langle(\phi - \phi^\dagger)(x)Y^i(y)\rangle = 0\,, \\ &\langle\psi(x)X^i(y)\rangle - \langle\zeta(x)Y^i(y)\rangle = 0\,, \\ &\langle F(x)X^i(y)\rangle - \tfrac{1}{2}\langle(\xi - \xi^\dagger)(x)Y^i(y)\rangle = 0\,, \qquad \text{(33a)} \\ &\langle G(x)X^i(y)\rangle + \frac{i}{2}\langle(\xi + \xi^\dagger)(x)Y^i(y)\rangle = 0\,, \\ &\langle B_\mu(x)X^i(y)\rangle - \tfrac{1}{2}\langle\partial_\mu(\phi + \phi^\dagger)(x)Y^i(y)\rangle = 0\,. \end{aligned}$$

For all the remaining components of V, S^1, or S^2 we find

$$\langle P(x)X^i(y)\rangle = 0\,, \qquad \text{(33b)}$$

where P denotes the particular component. Using the explicit form of the propagators of the Faddeev-Popov ghost fields, as given in Eq. (28), we can express the identities (33) that are nontrivial in a matrix form:

$$S(\chi,\psi,\Psi';q)\begin{pmatrix} i\rho\not{q} \\ -\rho q^2 \\ \rho\rho' i\not{q}\end{pmatrix} = \begin{pmatrix} 0 \\ -i\not{q}q^{-2} \\ 0\end{pmatrix}\,,$$

$$D(B_\mu, M'_I, H'_I, G;q)\begin{bmatrix} -\rho i q_\nu \\ \rho\rho' \\ 0 \\ 0\end{bmatrix} = \begin{bmatrix} iq_\mu q^{-2} \\ 0 \\ 0 \\ 0\end{bmatrix}\,,$$

$$D(B_\mu, M'_I, H'_I, G;q)\begin{bmatrix} 0 \\ 0 \\ \rho\rho' q^2 \\ -\rho q^2\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1\end{bmatrix}\,,$$

$$D(D, A, M'_R, H'_R, F;q)\begin{bmatrix} \rho \\ -\rho q^2 \\ \rho\rho' \\ 0 \\ 0\end{bmatrix} = \begin{bmatrix} 0 \\ q^{-2} \\ 0 \\ 0 \\ 0\end{bmatrix}\,,$$

$$D(D, A, M'_R, H'_R, F;q)\begin{bmatrix} 0 \\ 0 \\ 0 \\ -\rho\rho' q^2 \\ -\rho q^2\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1\end{bmatrix}\,.$$

One can easily show that these identities are indeed valid for the propagators (28) that we calculated in Sec. IV.

VI. AN EXAMPLE OF A CLOSED-LOOP CALCULATION

From the Ward-Takahashi identities that were discussed in the previous section the gauge independence of the S matrix can be derived along the same lines as for the conventional gauge theories.[29,30] To demonstrate how in practice the cancellations among gauge-dependent parts occur, we will present the calculation of the propagator for the field Ψ as defined in Eq. (26) in the one-loop approximation. We will perform this calculation in the gauge that was defined in Eq. (24). Because the Faddeev-Popov ghost fields are interacting in this gauge, this offers us an opportunity to demonstrate the role that is played by the spinor ghost fields ζ in actual calculations.

Let us first recall the pertinent part of the interaction Lagrangian (14), which we have divided into three terms:

$$-\tfrac{1}{4}g^2[A^2\bar\Psi\not\partial\Psi-\tfrac{1}{4}(\bar\psi\gamma_\mu\gamma_5\psi)(\bar\Psi\gamma_\mu\gamma_5\Psi)]\,, \tag{34a}$$

$$-\tfrac{1}{2}g[A\bar\Psi\overleftrightarrow{\not\partial}\Psi'+\bar\Psi(H_R'-i\gamma_5H_I')(\chi+\not\partial\psi)+\bar\Psi\not\partial(H_R'+i\gamma_5H_I')\psi+\bar\Psi(M_R'-i\gamma_5M_I')\psi+iB_\mu\bar\Psi\gamma_\mu\gamma_5\Psi']\,, \tag{34b}$$

$$\tfrac{1}{2}\rho' g(-i\phi_I'\bar\Psi\zeta-\phi_R'\bar\Psi\gamma_5\zeta+i\bar\zeta'\Psi\phi_I+\bar\zeta'\gamma_5\Psi\phi_R)\,. \tag{34c}$$

These three terms give rise to the self-energy diagrams (a), (b), and (c), respectively, that are depicted in Fig. 1.

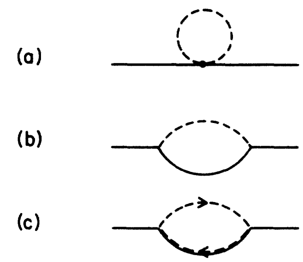

FIG. 1. Self-energy diagrams for the field Ψ. The solid lines correspond to fermions and the dashed lines to bosons. The oriented dashed and solid-dashed lines denote, respectively, the spinless and spinor ghost fields.

Using the lowest-order propagators that were listed in Eqs. (28) we can easily calculate the contributions of the various diagrams. The first part of the interaction Lagrangian (34a) whose second term gives no contribution gives rise to the following expression:

$$\tfrac{1}{2}g^2 i\not k\int d^4q\left(\frac{1+\rho}{\rho q^4}+\frac{\rho'^2}{q^2(q^2+m^2)}\right)\,. \tag{35}$$

The momentum of the external fermion is denoted by k_μ.

We distinguish three different contributions from the second part (34b). The first contribution involves at least one $A\Psi\Psi'$ vertex, the second one at least one $B_\mu\Psi\Psi'$ vertex, whereas the third one originates only from vertices that contain H' or M' fields. These three different contributions are given, respectively, by

$$-\tfrac{1}{4}ig^2\frac{1+\rho}{\rho q^4(p^2+m^2)}[(\not k-im)(3p^2-q^2+k^2)+(\not p+im)(p^2-k^2)]$$
$$-\tfrac{1}{4}g^2\frac{\rho'^2}{p^2q^2(p^2+m^2)(q^2+m^2)}[m(3p^4-q^4+k^2q^2+5k^2p^2+p^2q^2)$$
$$+i\not k(4p^4+3p^2q^2-4m^2p^2+2m^2q^2)-i\not q(2p^2q^2+2m^2k^2+p^4-k^2p^2)]\,, \tag{36}$$

$$\tfrac{1}{4}g^2\frac{1}{q^2(p^2+m^2)}\left(\frac{1+\rho}{\rho}(i\not k+m)+\frac{1+\rho}{\rho}\frac{i\not q}{q^2}(p^2-k^2)-4m-2i\not p\right)$$
$$+\tfrac{1}{4}g^2\frac{\rho'^2}{p^2q^2(p^2+m^2)(q^2+m^2)}[m(q^4-p^4-q^2k^2+p^2k^2+3p^2q^2)+i\not k q^2(p^2-2m^2)+i\not q(p^4-p^2k^2+2m^2k^2+2p^2q^2)]\,, \tag{36$'$}$$

$$\tfrac{1}{2}g^2\frac{1}{\rho p^4(q^2+m^2)}[-(i\not k+m)p^2+\rho(i\not q+m)p^2+(1+\rho)(i\not q+m)(q^2-k^2)]$$
$$+\tfrac{1}{2}g^2\frac{\rho'^2}{p^2(p^2+m^2)(q^2+m^2)}[-(i\not k+m)p^2+(i\not q+m)(q^2-k^2)]\,. \tag{36$''$}$$

The momenta of the internal fermion and boson lines are denoted by p_μ and q_μ, respectively, and an integration $\int d^4p\,d^4q\,\delta^4(p+q-k)$ is understood.

Using the same definitions, we finally find the following contribution from the interactions with the ghost fields (34c):

$$g^2\rho'^2\frac{i\not p}{p^2q^2}\,, \tag{37}$$

where p_μ and q_μ are the momenta of the internal spinor and spinless ghost-field lines.

We will now establish the gauge independence of the mass of the field Ψ in the one-loop approximation. The mass follows from Eqs. (35)–(37) by going to the mass shell: $\not{k}=im$ and $k^2=-m^2$. If we use symmetric integration, the ρ- and ρ'-dependent contributions of Eq. (36″) will simply vanish on the mass shell. The same is true for the ρ-dependent terms in Eq. (36′). Finally, the ρ-dependent part of Eq. (36) cancels exactly the ρ-dependent contribution from Eq. (35), after symmetrical integration and on the mass shell. The cancellation of the ρ'-dependent contributions is somewhat more complicated. The ρ'-dependent terms of Eqs. (36) and (36′) are given by

$$\tfrac{1}{2}g^2\frac{\rho'^2}{p^2q^2(p^2+m^2)(q^2+m^2)}$$
$$\times[m(3m^2p^2+p^2q^2-p^4)-i\not{k}(2p^4+p^2q^2)-2i\not{q}(p^2q^2-m^4)]\,,$$

where we allowed ourselves to change the integration variables $p \leftrightarrow q$, and took $k^2=-m^2$. Using the same arguments once more, $\not{q}$ can be replaced by $\frac{1}{2}(\not{p}+\not{q})=\frac{1}{2}im$, so that we find on the mass shell

$$\tfrac{1}{2}g^2m\frac{\rho'^2}{p^2q^2(p^2+m^2)(q^2+m^2)}(p^4+3m^2p^2+p^2q^2+m^4)$$
$$=\tfrac{1}{2}g^2m\frac{\rho'^2}{q^2(q^2+m^2)}+\tfrac{1}{2}g^2m\frac{\rho'^2}{p^2q^2}\,.$$

This then cancels the ρ'-dependent parts of Eqs. (35) and (37), respectively, on the mass shell.

Hence we find that the mass of the field Ψ is gauge independent, and given by

$$M_\Psi=m\left(1-\tfrac{1}{2}\frac{ig^2}{(2\pi)^4}\int d^4q\frac{1}{q^2(q^2-2kq)}\right),$$

which is logarithmically divergent. The infinite part is in agreement with a result previously found by Wess and Zumino[8] in a different gauge. Of course, we should mention that the results of this section were sometimes obtained by manipulating linearly divergent integrals. However, under the assumption that this calculation can be made rigorous by using a regularization scheme, the gauge independence of a physical quantity has been demonstrated.

VII. CONCLUSIONS

We have formulated the quantization procedure for supersymmetric gauge theories, and we have found that one needs spinor ghost fields in addition to the usual spinless Faddeev-Popov ghost fields. All these ghost fields can be assigned to scalar chiral superfields that are anticommuting, i.e., the spinless components anticommute, whereas the spinors commute. We have discussed supersymmetric gauge conditions in some detail, and we have shown how they lead to a supersymmetric generating functional for the Green's functions. The Slavnov-Taylor identities for this generating functional have been formulated and to demonstrate the consistency of our approach we have presented several explicit examples.

Our discussion was mainly in the context of the Abelian gauge model, but the generalization to the non-Abelian models is obvious.

Clearly, the formulation of the quantization procedure is only a first step to the more intricate question of the possible renormalizability of the supersymmetric gauge theories. As we have pointed out earlier, one important ingredient in the study of renormalizability is the existence of a regularization procedure, preferably in a class of gauges that contains both supersymmetric gauges and the physical gauge.

It would also be of interest to give an independent formulation of the quantization of these theories in the context of the canonical formalism.

Note added in proof: There have been several recent publications[31] dealing with supersymmetric gauge theories quantized in a manifestly supersymmetric way. Some of them claim to have proven the renormalizability in such gauges.

ACKNOWLEDGMENT

We are grateful for valuable discussions with Dr. Y. Dothan, Dr. D. Z. Freedman, and Dr. B. Zumino.

APPENDIX

In this appendix we will list some formulas that are useful in dealing with superfields.

For anticommuting Majorana spinors θ_α one can derive the following identities:

$$\begin{aligned}\bar\theta_\alpha\theta_\beta&=\tfrac{1}{4}[(\bar\theta\theta)\delta_{\beta\alpha}+(\bar\theta\gamma_5\theta)(\gamma_5)_{\beta\alpha}\\&\quad+(\bar\theta i\gamma_\mu\gamma_5\theta)(i\gamma_\mu\gamma_5)_{\beta\alpha}]\,,\\ \bar\theta_\alpha\theta_\beta\theta_\rho&=\tfrac{1}{4}(\bar\theta\theta)[\delta_{\beta\alpha}\theta_\rho-(\gamma_5)_{\beta\alpha}(\gamma_5\theta)_\rho\\&\quad-(i\gamma_\mu\gamma_5)_{\beta\alpha}(i\gamma_\mu\gamma_5\theta)_\rho]\,,\\ \bar\theta_\alpha\theta_\beta\bar\theta_\rho\theta_\sigma&=\tfrac{1}{16}(\bar\theta\theta)^2[\delta_{\beta\alpha}\delta_{\sigma\rho}-(\gamma_5)_{\beta\alpha}(\gamma_5)_{\sigma\rho}\\&\quad-(i\gamma_\mu\gamma_5)_{\beta\alpha}(i\gamma_\mu\gamma_5)_{\sigma\rho}]\,.\end{aligned}\tag{A1}$$

Subsequently we list how the generators of the supersymmetry transformations G_α act on the components of a vector field and a commuting or anticommuting scalar chiral field, which were defined in Eqs. (4), (5), and (22), respectively. We consider the effect of an infinitesimal transformation given by $\bar\epsilon G$, where ϵ is an anticommuting constant Majorana spinor:

Real Vector Field V

$$\delta A=\bar\epsilon\psi\,,$$
$$\delta\psi=(\partial\!\!\!/ A+i\partial\!\!\!/ B\gamma_5+F+i\gamma_5 G)\epsilon\,,$$
$$\delta F=\bar\epsilon(\chi+\partial\!\!\!/\psi)\,,$$
$$\delta G=i\bar\epsilon\gamma_5(\chi+\partial\!\!\!/\psi)\,, \tag{A2}$$
$$\delta B_\mu=i\bar\epsilon\gamma_5(\gamma_\mu\chi+\partial_\mu\psi)\,,$$
$$\delta\chi=D\epsilon-\tfrac{1}{2}i(\partial_\mu B_\nu-\partial_\nu B_\mu)\gamma_\mu\gamma_\nu\gamma_5\epsilon\,,$$
$$\delta D=\bar\epsilon\partial\!\!\!/\chi\,;$$

Chiral Field S

$$\delta H=\bar\epsilon(1+\gamma_5)\Psi\,,$$
$$\delta\Psi=\tfrac{1}{2}\partial\!\!\!/[(H+H^\dagger)-\gamma_5(H-H^\dagger)]\epsilon$$
$$\qquad+\tfrac{1}{2}[(M+M^\dagger)+\gamma_5(M-M^\dagger)]\epsilon\,, \tag{A3}$$
$$\delta M=\bar\epsilon\partial\!\!\!/(1+\gamma_5)\Psi:$$

Anticommuting chiral field Φ

$$\delta\phi=\bar\epsilon(1+\gamma_5)\zeta\,,$$
$$\delta\zeta=-\tfrac{1}{2}\partial\!\!\!/[(\phi-\phi^\dagger)-\gamma_5(\phi+\phi^\dagger)]\epsilon$$
$$\qquad-\tfrac{1}{2}[(\xi-\xi^\dagger)+\gamma_5(\xi+\xi^\dagger)]\epsilon\,, \tag{A4}$$
$$\delta\xi=\bar\epsilon\partial\!\!\!/(1+\gamma_5)\zeta\,.$$

Here H and M and ϕ and ξ correspond to components of the left-handed fields, S_+ and Φ_+, respectively, and we have $S_+^\dagger=S_-$, $\Phi_+^\dagger=\Phi_-$.

Finally, we give the superfield expressions of the quantities $\frac{1}{4}\bar D\gamma_\mu(1+\gamma_5)DV(x,\theta)$, $-\frac{1}{4}\bar D(1\pm\gamma_5)DV(x,\theta)$, and $-\frac{1}{4}\bar D(1+\gamma_5)DS_+(x,\theta)$:

$$\tfrac{1}{4}\bar D\gamma_\mu(1+\gamma_5)DV(x,\theta)=i(B_\mu+i\partial_\mu A)+\bar\theta[\gamma_\mu\gamma_5\chi-(1+\gamma_5)\partial_\mu\psi]$$
$$-\tfrac{1}{2}\bar\theta(1+\gamma_5)\theta\partial_\mu(F+iG)-\tfrac{1}{2}\bar\theta i\gamma_\rho\gamma_5\theta(i\delta_{\mu\rho}D+i\partial_\mu\partial_\rho A+\partial_\mu B_\rho+\epsilon_{\rho\mu\nu\sigma}\partial_\nu B_\sigma)$$
$$-(\bar\theta\theta)\bar\theta\left(\partial_\mu\chi-\tfrac{1}{2}\gamma_5\gamma_\mu\partial\!\!\!/\chi+\frac{1-\gamma_5}{2}\partial_\mu\partial\!\!\!/\psi\right)-\tfrac{1}{8}(\bar\theta\theta)^2[2\partial_\mu D+\partial_\mu\partial^2 A+i(\partial^2 B_\mu-2\partial_\mu\partial_\nu B_\nu)]\,, \tag{A5}$$

$$-\tfrac{1}{4}\bar D(1\pm\gamma_5)DV(x,\theta)=\exp(\pm\tfrac{1}{2}\bar\theta\partial\!\!\!/\gamma_5\theta)[(F\pm iG)+\bar\theta(1\mp\gamma_5)(\chi+\partial\!\!\!/\psi)+\tfrac{1}{2}\bar\theta(1\mp\gamma_5)\theta(D+\partial^2A\mp i\partial_\mu B_\mu)]\,, \tag{A6}$$

$$-\tfrac{1}{4}\bar D(1+\gamma_5)DS_+(x,\theta)=\exp(\tfrac{1}{2}\bar\theta\partial\!\!\!/\gamma_5\theta)[M+\bar\theta(1-\gamma_5)\partial\!\!\!/\Psi+\tfrac{1}{2}\bar\theta(1-\gamma_5)\theta\partial^2 H]\,. \tag{A7}$$

---

*Work supported in part by the National Science Foundation Grant No. GP-32998X.

†Present address: Institute for Theoretical Physics, University of Utrecht, Utrecht, The Netherlands.

[1]J. Wess and B. Zumino, Nucl. Phys. B70, 39 (1974).

[2]J. Wess and B. Zumino, Phys. Lett. 49B, 52 (1974); H. S. Tsao, *ibid.* 53B, 381 (1974).

[3]J. Iliopoulos and B. Zumino, Nucl. Phys. B76, 310 (1974).

[4]S. Ferrara and E. Remiddi, Phys. Lett. 53B, 347 (1974); B. Zumino, Nucl. Phys. B89, 535 (1975).

[5]A. Salam and J. Strathdee, Phys. Lett. 49B, 465 (1974); P. Fayet and J. Iliopoulos, *ibid.* 51B, 461 (1974).

[6]V. P. Akulov and D. V. Volkov, Teor. Mat. Fiz. 18, 39 (1974) [Theor. Math. Phys. (USSR) 18, 28 (1974)]; Phys. Lett. 46B, 109 (1973).

[7]B. de Wit, Phys. Rev. D 11, 898 (1975). See also B. Zumino, Ref. 4.

[8]J. Wess and B. Zumino, Nucl. Phys. B78, 1 (1974).

[9]A. Salam and J. Strathdee, Phys. Lett. 51B, 353 (1974); R. Delbourgo, A. Salam, and J. Strathdee, *ibid.* 51B, 475 (1974).

[10]A. Salam and J. Strathdee, Phys. Rev. D 11, 1521 (1975).

[11]S. Ferrara and B. Zumino, Nucl. Phys. B79, 413 (1974).

[12]P. Fayet, Nucl. Phys. B90, 104 (1975).

[13]References concerning the quantization and renormalization of gauge theories can be found in E. S. Abers and B. W. Lee, Phys. Rep. 9C, 1 (1973); M. Veltman, in *Proceedings of the Sixth International Symposium on Electron and Photon Interactions at High Energy, Bonn, Germany, 1973,* edited by H. Rollnik and W. Pfeil (North-Holland, Amsterdam, 1974).

[14]By "renormalizable by power counting" we mean that the theory can be rendered finite by the addition of counterterms with dimension less than or equal to four. In order to prove the full renormalizability of the theory one must show that the structure of those counterterms is compatible with the requirements from the supersymmetry and the local gauge symmetry.

[15]L. D. Faddeev and V. N. Popov, Phys. Lett. 25B, 29 (1967).

[16]G. 't Hooft, Nucl. Phys. B33, 173 (1971).

[17]In conventional gauge theories one has the dimensional regularization method: G. 't Hooft and M. Veltman, Nucl. Phys. B44, 189 (1972); C. G. Bollini and J. J. Giambiagi, Nuovo Cimento 12B, 20 (1972). However, for supersymmetric theories, one can show

that it is not possible to analytically continue Feynman integrals in the space-time dimension in a way that preserves supersymmetry [S.-Y. Pi and B. de Wit (unpublished)]. Recently, R. Delbourgo and V. B. Prasad have proposed to modify the Lagrangian for dimensions different from four by introducing additional bosonic and fermionic degrees of freedom. They claim that their prescription leads to a supersymmetric regularization scheme: J. Phys. G 1, 377 (1975).

[18]A. Salam and J. Strathdee, Nucl. Phys. B76, 477 (1974).

[19]S. Ferrara, J. Wess, and B. Zumino, Phys. Lett 51B, 239 (1974).

[20]We use the Pauli metric. The four γ matrices are Hermitian and satisty $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$ ($\mu, \nu = 1, 2, 3\ 4$). $\bar{\psi}$ is defined by $\psi^\dagger\gamma_4$, and $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$.

[21]Anticommuting quantities are discussed in the mathematical literature in the context of a Grassmann algebra. One can define derivatives on the elements of a Grassmann algebra, as well as integrals over the algebra. In the context of the quantization of fermions in the functional approach the main techniques in dealing with anticommuting quantities are discussed in F. A. Berezin, *The Method of Second Quantization*, Pure and Applies Physics, vol. 24, (Academic, New York, 1966). Lie groups with commuting and anticommuting parameters are studied in F. A. Berezin and G. I. Kac, Mat. Sb. 82, 343 (1970) [Math. USSR—Sbornik 11, 311 (1970)].

[22]R. P. Feynman, Rev. Mod. Phys. 20, 267 (1948).

[23]This result has been found independently in a different context by R. Arnowitt, P. Nath, and B. Zumino, Phys. Lett. 56B, 81 (1975).

[24]This is usually expressed by taking a and b complex and writing a' and b' as complex conjugates of a and b. It is only for notational reasons that we have not followed this convention.

[25]S. Ferrara and B. Zumino have been performing calculations in the generalized Landau gauge (private communication). Gauges defined by Eq. (24) lead, however, to certain complications when the supersymmetry is realized in a spontaneously broken way. This has been discussed more generally by B. de Wit, this issue, Phys. Rev. D 12, 1843 (1975).

[26]E. S. Fradkin and I. V. Tyutin, Phys. Rev. D 2, 2841 (1970).

[27]A. A. Slavnov, Teor. Mat. Fiz. 10, 153 (1972) [Theor. Math. Phys. (USSR) 10, 99 (1972)].

[28]J. C. Taylor, Nucl. Phys. B33, 436 (1971).

[29]B. W. Lee and J. Zinn-Justin, Phys. Rev. D 7, 1049 (1973).

[30]G. 't Hooft and M. Veltman, Nucl. Phys. B50, 318 (1972).

[31]A. A. Slavnov, Theor. Math. Phys. (USSR) 23, 3 (1975); Nucl. Phys. B97, 155 (1975); J. Honerkamp, F. Krause, M. Scheunert, and M. Schlindwein, *ibid.* B95, 397 (1975); S. Ferrara and O. Piguet, *ibid.* B93, 261 (1975).