Spectral-function sum rules, asymptotic SU_4, and vector-meson leptonic decays

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The spectral-function sum rules corresponding to asymptotic SU_4 symmetry are used to study the decays of the vector mesons ρ, ω, ϕ, and $\psi(3105)$ into electron-positron pairs. The Glashow-Iliopoulos-Maiani model gives reasonable predictions.

Employing the spectral-function sum rules,[1] several authors[2] have studied the leptonic decays of the vector mesons to investigate the structure of the electromagnetic current of the hadrons in SU_3. In this note we generalize these sum rules in the framework of SU_4 and use them to study the vector-meson decays with a view to learning something about the SU_3 nature of the electric current. In doing so it may be possible to distinguish among the various models proposed to explain the hadron spectrum because the SU_3 structure of the electromagnetic current is expected to be different in different models.

Similarly to asymptotic SU_3 symmetry,[3] we assume asymptotic SU_4 to obtain the first spectral-function sum rule for the sixteen vector current components $V_\mu^a(x)$. The second spectral-function sum rule is assumed to have been modified[4] with the broken SU_4 model[5] quite similarly to the one in the SU_3 case. Therefore, we have

$$\int_0^\alpha \frac{\rho_{ab}(m^2)}{m^2}dm^2 = A\delta_{ab} + B\delta_{a0}\delta_{b0}\,, \tag{1}$$

$$\int_0^\alpha \rho_{ab}(m^2)dm^2 = C\delta_{ab} + Dd_{gab} + Ed_{15ab} + F\delta_{a0}\delta_{b0}\,, \tag{2}$$

where $a, b = 0, 1, 2, \ldots, 15$. Following Ref. 4 we will take $B = F = 0$ in what follows. Equations (1) and (2), with the hypothesis of vector dominance for the spectral functions, give the following relations for the various vacuum to single-particle matrix elements:

$$\frac{G_\rho^2}{m_\rho^2} = \frac{G_\omega^2}{m_\omega^2} + \frac{G_\phi^2}{m_\phi^2} + \frac{G_\psi^2}{m_\psi^2}\,, \tag{3a}$$

$$\frac{G_\rho^2}{m_\rho^2} = \frac{\sigma_\omega^2}{m_\omega^2} + \frac{\sigma_\phi^2}{m_\phi^2} + \frac{\sigma_\psi^2}{m_\psi^2} = \frac{h_\omega^2}{m_\omega^2} + \frac{h_\phi^2}{m_\phi^2} + \frac{h_\psi^2}{m_\psi^2}\,, \tag{3b}$$

$$\frac{G_\omega\sigma_\omega}{m_\omega^2} + \frac{G_\phi\sigma_\phi}{m_\phi^2} + \frac{G_\psi\sigma_\psi}{m_\psi^2} = 0\,, \tag{3c}$$

$$\frac{G_\omega h_\omega}{m_\omega^2} + \frac{G_\phi h_\phi}{m_\phi^2} + \frac{G_\psi h_\psi}{m_\psi^2} = 0\,, \tag{3d}$$

$$\frac{\sigma_\omega h_\omega}{m_\omega^2} + \frac{\sigma_\phi h_\phi}{m_\phi^2} + \frac{\sigma_\psi h_\psi}{m_\psi^2} = 0\,, \tag{3e}$$

$$4G_{K^*}^2 - G_\rho^2 - 3(G_\omega^2 + G_\phi^2 + G_\psi^2) = 0\,, \tag{4a}$$

$$G_\rho^2 - (G_\omega^2 + G_\phi^2 + G_\psi^2) = 2(\tfrac{2}{3})^{1/2}(G_\omega h_\omega + G_\phi h_\phi + G_\psi h_\psi)\,, \tag{4b}$$

$$G_\rho^2 + (G_\omega^2 + G_\phi^2 + G_\psi^2) = 2(h_\omega^2 + h_\phi^2 + h_\psi^2) + \frac{2}{\sqrt{3}}(\sigma_\omega h_\omega + \sigma_\phi h_\phi + \sigma_\psi h_\psi)\,, \tag{4c}$$

$$(\sigma_\omega^2 + \sigma_\phi^2 + \sigma_\psi^2) - (h_\omega^2 + h_\phi^2 + h_\psi^2) = -\frac{2}{\sqrt{3}}(h_\omega\sigma_\omega + h_\phi\sigma_\phi + h_\psi\sigma_\psi)\,, \tag{4d}$$

and

$$\sqrt{3}\,(G_\omega\sigma_\omega + G_\phi\sigma_\phi + G_\psi\sigma_\psi) = (G_\omega h_\omega + G_\phi h_\phi + G_\psi h_\psi)\,. \tag{4e}$$

The coupling constants are defined through

$$\begin{aligned} &\langle 0 | V_\mu^i | V\rangle = G_V \epsilon_\mu^V(k)(2k_0V)^{-1/2} \text{ for } i = 1, 2, \ldots, 8,\\ &\langle 0 | V_\mu^{15} | V\rangle = \sigma_V \epsilon_\mu^V(k)(2k_0V)^{-1/2}\,,\\ &\text{and}\\ &\langle 0 | V_\mu^0 | V\rangle = h_V \epsilon_\mu^V(k)(2k_0V)^{-1/2}. \end{aligned} \tag{5}$$

In terms of the quark field $q(x)$ the vector current components are

$$V_\mu^a(x) = \tfrac{1}{2}\bar{q}(x)\Lambda^a\gamma_\mu q(x)\,, \tag{6}$$

where Λ^a are 4×4 matrices with

$$\begin{aligned} &[\Lambda^a, \Lambda^b] = 2if_{abc}\Lambda^c\,,\\ &\{\Lambda^a, \Lambda^b\}_+ = 2d_{abc}\Lambda^c\,. \end{aligned} \tag{7}$$

For our purposes ψ is a vector meson which along with ρ, κ^*, ω, ϕ and their charmed relatives may belong to $15\oplus 1$ representation of SU_4. We shall later take it to be $\psi(3105)$, a narrow resonance observed[6] recently in e^+e^- annihilation.

To solve Eqs. (3) and (4) we suppose, as is popular, that ψ couples predominantly to the SU_3 singlet piece of the electromagnetic current, i.e., $G_\psi = 0$. We then obtain

$$h_\omega = \sqrt{3}\,\sigma_\omega, \quad h_\phi = \sqrt{3}\,\sigma_\phi, \quad h_\psi = -\frac{1}{\sqrt{3}}\sigma_\psi$$

and

$$\frac{G_\rho{}^2}{m_\rho{}^2} = \frac{3G_\omega{}^2}{m_\rho{}^2} = \frac{3}{2}\frac{G_\phi{}^2}{m_\phi{}^2} = 3\sqrt{2}\,\frac{G_\omega\sigma_\omega}{m_\omega{}^2} = -3\sqrt{2}\,\frac{G_\phi\sigma_\phi}{m_\phi{}^2} \tag{8}$$

and

$$\frac{G_\rho{}^2}{m_\rho{}^2} = 6\frac{\sigma_\omega{}^2}{m_\omega{}^2} = 12\frac{\sigma_\phi{}^2}{m_\phi{}^2} = \frac{4}{3}\frac{\sigma_\psi{}^2}{m_\psi{}^2}\cdots.$$

If Q_1, Q_2, Q_3, and Q_4 are the charges of the quarks, the hadronic electromagnetic (em) current can be written as

$$j_\mu^{\text{em}}(x) = aV_\mu^3(x) + bV_\mu^8(x) + cV_\mu^{15}(x) + dV_\mu^0(x),$$

with

$$\begin{aligned} a &= Q_1 - Q_2, \\ b &= \frac{1}{\sqrt{3}}(Q_1 + Q_2 - 2Q_3), \\ c &= \frac{1}{\sqrt{6}}(Q_1 + Q_2 + Q_3) - (\tfrac{3}{2})^{1/2}Q_4, \end{aligned} \tag{9}$$

and

$$d = \frac{1}{\sqrt{2}}(Q_1 + Q_2 + Q_3 + Q_4).$$

The couplings f_ω, f_ϕ, and f_ψ at the ω-γ, ϕ-γ, and ψ-γ junctions are then in general

$$\begin{aligned} f_\omega &= \frac{G_\omega}{\sqrt{3}} + c\sigma_\omega + dh_\omega, \\ f_\phi &= \frac{G_\phi}{\sqrt{3}} + c\sigma_\phi + dh_\phi, \\ f_\psi &= c\sigma_\psi + dh_\psi, \end{aligned} \tag{10}$$

and

$$a = 1 \text{ and } b = \frac{1}{\sqrt{3}} \text{ in most models.}$$

The decay rate for $V \to \bar{l}l$ is given by

$$\Gamma(V \to \bar{l}l) = \frac{4\pi}{3}\left(\frac{e^2}{4\pi}\right)^2 \frac{f_V{}^2}{m_V{}^3}[1 + O(m_l{}^2/m_V{}^2)]. \tag{11}$$

In the Glashow-Iliopoulos-Maiani model[7]

$$c = 2(\tfrac{2}{3})^{1/2}, \quad d = -\tfrac{2}{3}\sqrt{2},$$

so that we obtain

$$\begin{aligned} \frac{f_\omega{}^2}{m_\omega{}^2} &= \frac{1}{3}\frac{G_\omega{}^2}{m_\omega{}^2} = \frac{1}{9}\frac{G_\rho{}^2}{m_\rho{}^2}, \\ \frac{f_\phi{}^2}{m_\phi{}^2} &= \frac{1}{3}\frac{G_\phi{}^2}{m_\phi{}^2} = \frac{2}{9}\frac{G_\rho{}^2}{m_\rho{}^2}, \end{aligned} \tag{12}$$

and

$$\frac{f_\psi{}^2}{m_\psi{}^2} = \frac{32}{27}\frac{\sigma_\psi{}^2}{m_\psi{}^2} = \frac{8}{9}\frac{G_\rho{}^2}{m_\rho{}^2}.$$

If we now calculate G_ρ from the observed decay width $\Gamma(\rho \to \bar{l}l)$ we get

$$\begin{aligned} \Gamma(\omega \to \bar{l}l) &= \tfrac{1}{9}\Gamma(\rho \to \bar{l}l) \simeq 0.7 \text{ keV}, \\ \Gamma(\phi \to \bar{l}l) &= \tfrac{2}{9}(m_\rho/m_\phi)\Gamma(\rho \to \bar{l}l) \simeq 1.1 \text{ keV}, \end{aligned} \tag{13}$$

is reasonable agreement with experiment.[8]

Identifying ψ with $\psi(3105)$ we have $\Gamma(\psi \to \bar{l}l) \approx 1.5$ keV, which is a bit lower than the observed value (~5 keV).

It is interesting to note that for ω and ϕ the effective electromagnetic current seems to belong to the SU_3 octet. The effects of V_μ^{15} and V_μ^0 are canceling each other. It is possible that at the SU_3 level this cancellation may be taking place for all the hadrons built from the three charmless quarks. That may be the reason why $j_\mu = V_\mu^3 + (1/\sqrt{3})V_\mu^8$ works well for such hadrons. It may be noted that this observation does not depend on the identification of ψ with $\psi(3105)$. It will be valid so long as there is an SU_3-singlet vector meson ψ.

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