Is the relation $R \cong 16\pi^2/f_\rho^2$ of physical significance?*

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(Received 23 January 1975)

It is shown that the present remarkable agreement between the previously predicted asymptotic limit $R \cong 16\pi^2/f_\rho^2 (= 5.7 \pm 0.9)$ of $\sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ and the recently reported SLAC-LBL e^+-e^- colliding-beam data with $R = 5.1 \pm 0.4$ at 5 GeV center-of-mass energy is of physical significance if R stays approximately constant for higher energies.

In the previous papers[1] I have predicted the asymptotic limit R of $R(s) = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ to be approximately $16\pi^2/f_\rho^2 = 5.7 \pm 0.9$, where f_ρ is the ρ-dominance coupling constant ($f_\rho^2/4\pi = 2.2 \pm 0.3$). I have also predicted that the scaling[2] of $\sigma(e^+e^- \to \text{hadrons})$ will be seen in the near future since the CEA[3] and SLAC-LBL preliminary[4] data of e^+e^- colliding-beam experiments have shown $R(s)$ reaching 4.5–7.5 at the total center-of-mass energy squared $s \cong 25\ \text{GeV}^2$. The original derivation of the relation $R \cong 16\pi^2/f_\rho^2$, however, strongly depends on the assumption that the function $\Delta(q^2)$ defined by

$$(q_\mu q_\nu - q_{\mu\nu} q^2)\Delta(q^2) = \int dx\, dy\, e^{iqx} \langle 0 | T(J_\mu(x) J_\nu(0) \theta_\lambda^\lambda(y)) | 0 \rangle , \quad (1)$$

where J_μ and $\theta_{\mu\nu}$ are the electromagnetic current and the stress-energy tensor of hadrons, respectively, can be smoothly extrapolated from $q^2 = m_\rho^2$ to $q^2 = 0$. It has, therefore, been highly desirable to derive the same relation from different assumptions[5] or to study the extrapolation of $\Delta(q^2)$ more carefully.[6]

Very recently the SLAC-LBL group[7] has reported the detailed data on the total cross section for $e^+e^- \to$ hadrons, showing that $R(s)$ is approximately constant from $s^{1/2} = 2.4$ GeV to 3.8 GeV aside from the very narrow resonances $\psi(3105)$ and $\psi(3695)$, rises between $s^{1/2} = 3.8$ and 4.1 GeV, and at $s^{1/2} = 5$ GeV has a value of 5.1 ± 0.4 in remarkable agreement with the predicted value of $R \cong 5.7 \pm 0.9$. Although the constancy of $R(s)$ for higher s is still subject to future experiments, it seems worthwhile to find whether this agreement is real or a mere coincidence. In this short note I shall show that *it is of physical significance and not a mere coincidence if R(s) stays approximately constant for higher s.*

In deriving the relation $R \cong 16\pi^2/f_\rho^2$, I have started with the low-energy theorem by Crewther[8] and by Chanowitz and Ellis[9]:

$$\Delta(0) = -\frac{R}{6\pi^2} . \quad (2)$$

The theorem is an immediate consequence of the canonical trace Ward identity[9] which can be written in the form of

$$\Delta(q^2) = -\frac{q^2}{6\pi^2} \int ds \frac{R(s)}{(s-q^2)^2} - \frac{R}{6\pi^2} . \quad (3)$$

What I have proved rigorously is

$$E(m_\rho^2) = 1 , \quad (4)$$

with the definition of $E(q^2)$ given by

$$\Delta(q^2) = -\frac{2}{f_\rho^2} \left(\frac{m_\rho^2}{m_\rho^2 - q^2} \right)^2 E(q^2) . \quad (5)$$

Then, what I have assumed is that the extrapolation function $E(q^2)$ does not change much between $q^2 = m_\rho^2$ and $q^2 = 0$ so that

$$E(0) \cong \tfrac{4}{3} , \quad (6)$$

where the isoscalar contribution is included by assuming SU(3) symmetry. In general, $E(0) \cong 1 + r$ if the ratio of the isoscalar contribution to that of the isovector is r. It follows the general relation $R \cong 12\pi^2(1+r)/f_\rho^2$. Notice that the nonvanishing $E(0)$, which is a consequence of the ρ-double-pole dominance of $\Delta(q^2)$, is consistent with the nonvanishing R, which is an assumption leading to the low-energy theorem (2).

Now that fairly precise information on $R(s)$ has become available[3,4,10] for $s^{1/2}$ below 5 GeV, one can explicitly calculate $E(q^2)$ from Eqs. (3) and (5) to see whether the above-mentioned assumption is working if $R(s)$ is approximately constant for $s^{1/2}$ above 5 GeV. The result of such calculation of $E(q^2)$ as a function of q^2 for q^2 between 0 and m_ρ^2 is shown in Fig. 1. The shaded area indicates where $E(q^2)$ should be found with approximately 60% probabilities. For comparison, the three broken curves are presented to illustrate how $E(q^2)$ would behave if the right-hand side of Eq. (3) is calculated from the ρ resonance in $R(s)$ only, from the R-term only, or from their sum. The contri-

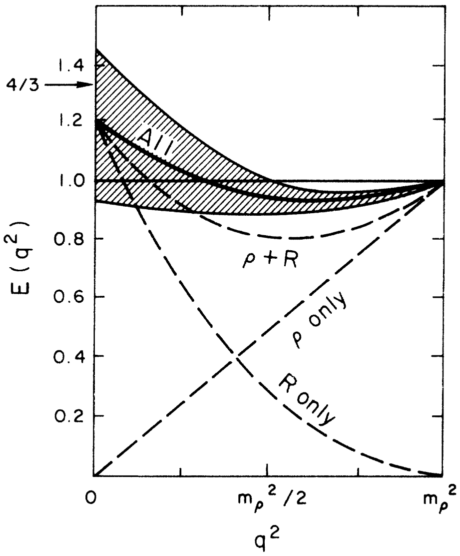

FIG. 1. The calculated extrapolation function $E(q^2)$ defined by Eqs. (3) and (5) for q^2 between 0 and m_ρ^2 (the shaded area). Also shown are the same functions calculated from the ρ resonance in $R(s)$ only, from the R term only, and from their sum (the dashed curves).

butions of ω and ϕ resonances and the continuum in $R(s)$, including ρ' (1250) and ρ'' (1600), to $E(q^2)$ for q^2 between 0 and m_ρ^2 are, at their maximum, 7%, 2%, and 9% of the total, respectively, and those of ψ(3105) and ψ(3695) are negligible (less than 1%). Although the statistics in the data[10] for $1 \lesssim s \lesssim 4$ GeV^2 are rather poor, the result would not be changed by more than 3%. This result clearly shows that $E(q^2)$ varies very little (less than 20%) from $q^2 = m_\rho^2$ to $q^2 = 0$ and that $\Delta(q^2)$ is always dominated by the ρ-double pole in this region,[11] which confirms the assumption mentioned above. Also it is remarkable that none of the single contributions of ρ, ω, ϕ, and the continuum in $R(q^2)$, or the R term in the right-hand side of Eq. (3) can make $E(q^2)$ flat in the extrapolation region. They are cooperating in such a way that only the total sum of them all can make this occur. The remaining question of whether $R(s)$ stays approximately constant for s larger than 25 GeV^2 will soon be answered in the coming SLAC (SPEAR) and DESY (DORIS) experiments at higher energies.

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*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-2232.

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[11]It cannot be stressed too strongly that the ρ-double-pole dominance of $\Delta(q^2)$ is *consistent with the scaling of $R(q^2)$ and inconsistent with the ρ-pole dominance of $R(q^2)$*.