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Exact left-right symmetry and spontaneous violation of parity*

G. Senjanovic and R. N. Mohapatra
Department of Physics, The City College of the City University of New York, New York, New York 10031
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Left-right–symmetric gauge models suggested previously to describe all interactions of elementary particles within a single unified framework are further shown to have the desirable property that maximal parity violation in low-energy weak processes arises purely as a result of spontaneous breakdown of the local gauge symmetry. We also discuss the question of dynamical origin of parity violation.

I. INTRODUCTION

It has recently been suggested[1-3] that the observed $V-A$ structure of weak interactions may only be a low-energy phenomenon which ought to disappear when one reaches energies of the order of 10^3 GeV. In such a picture, all interactions above these energies are supposed to be parity conserving and describable by a single gauge coupling constant g (where $g^2/4\pi \sim \alpha$). A left-right–symmetric model has been devised[2,3] within the framework of unified gauge theories, which not only exhibits the above properties, but also provides a natural basis for the CP-violating interactions[2] observed in K decays. An outstanding feature of this model is the close link between the magnitude of CP violation and the departure from exact left-right symmetry observed in nature. (For the first time, we seem to have a model for CP violation where its magnitude is not entirely arbitrary.)

A renormalizable gauge model based on the gauge group $SU(2)_L \times SU(2)_R \times SU(4')$ has been constructed which translates these ideas into a realistic model of weak and electromagnetic interactions. The unification of the left and right gauge couplings in this model was achieved by demanding that the entire Lagrangian except for Higgs-boson mass terms be invariant under a discrete symmetry[3] that transforms $SU(2)_L$ to $SU(2)_R$. The requirement that $\mu_L{}^2 \neq \mu_R{}^2$ ($\mu_{L,R}$ represents the Higgs-boson masses) served two purposes: (a) It guaranteed that the right-handed gauge bosons are heavier than the left-handed gauge bosons, as required by the nonobservation of the right-handed current interactions at present energies; (b) Secondly, since the mass terms are of dimension 2, the equality of the bare gauge couplings $g_L^{(0)} = g_R^{(0)}$ implies that infinite renormalizations to these couplings are the same, thus giving $g_L^{(\text{ren})} = g_R^{(\text{ren})}$ + finite, computable terms. Parity violation, however, is not spontaneous, since, prior to spontaneous breakdown, one did have a left-right asymmetry (though a mild one) due to $\mu_L{}^2 \neq \mu_R{}^2$. Therefore, a calculation in the symmetric vacuum limit would exhibit this asymmetry in the form of a mild parity violation (of order $\mu_L{}^2/\mu_R{}^2 \approx M_{W_L}{}^2/M_{W_R}{}^2$).

In the present article we would like to report that it is possible to obtain genuine spontaneous breakdown of left-right discrete symmetry (i.e., without requiring from the beginning that $\mu_L \neq \mu_R$) in the class of theories described in Refs. 1 and 2. The distinguishing feature in this case is that when we pass to the symmetric vacuum limit, there does not remain any trace of parity violation in this theory. This could, in principle, be tested by subjecting weak interactions to acute environmental conditions (such as high magnetic fields or high intensity laser fields), where one attains the symmetric vacuum limit. We further study the question of parity violation arising as a quantum effect in the sense of Coleman and Weinberg[4,5] for the simple case of $U(1)_L \times U(1)_R$ and find that there exists a range of values of the Higgs boson couplings, where this is possible.

This paper is organized as follows: In Sec. II we study the Higgs sector for a simple model to demonstrate these ideas; in Sec. III we construct the potential for the realistic model choosing the gauge group to be $SU(2)_L \times SU(2)_R \times U(1)$; in Sec. IV we discuss the question of dynamical breakdown of parity using a simple $U(1)_L \times U(1)_R$ model. Section IV summarizes the results and gives a comparison with another model of spontaneous violation suggested by Fayet.[6]

II. THE $SU(2)_L \times SU(2)_R \times U(1)$ GAUGE GROUP AND A SIMPLE MODEL FOR SPONTANEOUS PARITY VIOLATION

In this section we will illustrate our idea with a simple example. We choose the gauge group for unifying weak and electromagnetic interactions as $SU(2)_L \times SU(2)_R \times U(1)$ for this purpose. Strong interactions can be included either using the "color" $SU(3')$ degree of freedom, or using the $SU(4')$ group unifying baryons and leptons.[1] The fermion sector[7] and the corresponding weak phenomenology for such models has previously been extensively studied in Refs. 1 and 2. So we will focus our attention here on the Higgs sector. We take two Higgs multiplets χ_L and χ_R, where $\chi_L \equiv (\frac{1}{2}, 0, 1)$ and $\chi_R \equiv (0, \frac{1}{2}, 1)$. We note that the charge formula (which exhibits the left-right symmetry explicitly) in such a theory is

$$Q = T_{3L} + T_{3R} + \tfrac{1}{2}Y. \tag{1}$$

So we have

$$\chi_{L,R} = \begin{pmatrix} \chi^{+}_{L,R} \\ \chi^{0}_{L,R} \end{pmatrix}$$

and $\chi_L \leftrightarrow \chi_R$ under the discrete symmetry. We note that the most general potential (constructed out of χ_L and χ_R) consistent with renormalizability, gauge invariance, and discrete left-right symmetry is the following:

$$\begin{aligned} V(\chi_L, \chi_R) = &-\mu^2(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R) \\ &+\lambda_1[(\chi_L^\dagger\chi_L)^2 + (\chi_R^\dagger\chi_R)^2] \\ &+\lambda_2(\chi_L^\dagger\chi_L)(\chi_R^\dagger\chi_R). \end{aligned} \tag{2}$$

We would like to demonstrate that for a range of the parameters μ^2, λ_1, and λ_2 we can choose $\langle\chi_L\rangle = 0$ and $\langle\chi_R\rangle = \binom{0}{U_R}$. (We call this the asymmetric solution.) The equations for the minima are $\partial V/\partial\chi_L^\dagger = 0$ and $\partial V/\partial\chi_R^\dagger = 0$:

$$\begin{aligned} &(-\mu^2 + 2\lambda_1\chi_L^\dagger\chi_L + \lambda_2\chi_R^\dagger\chi_R)\chi_L = 0, \\ &(-\mu^2 + 2\lambda_1\chi_R^\dagger\chi_R + \lambda_2\chi_L^\dagger\chi_L)\chi_R = 0. \end{aligned} \tag{2'}$$

The only constraint that must be satisfied by the parameters is

$$-\mu^2 + 2\lambda_1 U_R^2 = 0,$$

which yields

$$U_R = \left(\frac{\mu^2}{2\lambda_1}\right)^{1/2}. \tag{3}$$

The first obvious requirement is therefore that $\mu^2 > 0$ and $\lambda_1 > 0$. Now, we would like to show that it is indeed a minimum, i.e., the matrices $\partial^2 V/\partial\chi_L^\dagger\partial\chi_L$ and $\partial^2 V/\partial\chi_R^\dagger\partial\chi_R$ are positive-semidefinite. It is easy to show that χ_R^+ and $\mathrm{Im}\chi_R^0$ remain massless and $\mathrm{Re}\chi_R^0$ becomes massive with mass $2\mu^2$. For $M_{\chi_L} > 0$, we find that $\lambda_2 > 2\lambda_1$ and the same constraint also guarantees that $V_{\min}$ corresponding to the asymmetric solution is less than one corresponding to the symmetric solution of Eq. (2).

We conclude, therefore, that for the range of parameters $\mu^2 > 0$ and $\lambda_2 > 2\lambda_1$ the lowest-energy solution of the Hamiltonian violates parity. This gives us a model with spontaneous parity violation because prior to spontaneous breaking $L \leftrightarrow R$ symmetry demands $g_L = g_R$, and therefore there is no parity violation in any physical matrix element, even though we are working with left- and right-handed gauge groups. This model is, of course, not realistic, since here the $SU(2)_L \times U(1)$ local symmetry remains completely unbroken and therefore left-handed interactions are long range. However, in a realistic model, the only unbroken local symmetry we can tolerate will be U(1) and further, the $SU(2)_R$— local symmetry must be broken more strongly than the $SU(2)_L$ — local symmetry. Moreover, to include *CP* violation, some of the vacuum expectation values must have a nonzero phase. We discuss these questions in the next section.

III. A REALISTIC MODEL

In this section we construct the classical Higgs potential for a realistic model of weak interaction based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$. For the present, we will ignore the question of *CP* violation. The *CP* problem can be treated without much effort by doubling the $\sigma(\frac{1}{2}, \frac{1}{2}, 0)$ multiplet described below. We choose the following Higgs multiplets:

$$\begin{aligned} &\chi_L \equiv (\tfrac{1}{2}, 0, 1), \\ &\chi_R \equiv (0, \tfrac{1}{2}, 1), \\ &\sigma \equiv (\tfrac{1}{2}, \tfrac{1}{2}, 0) \equiv \phi_1, \\ &\phi_2 = \tau_2\phi_1^*\tau_2. \end{aligned} \tag{4}$$

The most general potential consistent with renormalizability and gauge and discrete symmetry is

$$\begin{aligned} V(\sigma, \chi_L, \chi_R) = &-\mu^2(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R) - \sum_{i,j=1,2}\mu_{ij}^2\mathrm{Tr}(\phi_i^\dagger\phi_j) + \lambda_1[(\chi_L^\dagger\chi_L)^2 + (\chi_R^\dagger\chi_R)^2] + \lambda_2(\chi_L^\dagger\chi_L)(\chi_R^\dagger\chi_R) \\ &+ \sum_{i,j,k,l=1,2} C_{ijkl}\mathrm{Tr}(\phi_i^\dagger\phi_j\phi_k^\dagger\phi_l) + \sum_{i,j,k,l=1,2} d_{ijkl}\mathrm{Tr}(\phi_i^\dagger\phi_j)\mathrm{Tr}(\phi_k^\dagger\phi_l) + \sum_{i,j} e_{ij}(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R)\mathrm{Tr}(\phi_i^\dagger\phi_j) \\ &+ \sum_{i,j=1,2} f_{ij}[\chi_L^\dagger\phi_i\phi_j^\dagger\chi_L + \chi_R^\dagger\phi_i^\dagger\phi_j\chi_R] + \mathrm{H.c.} \end{aligned} \tag{5}$$

The minimum conditions can be easily written down and again we find by explicit calculation, as in the previous section, that for domain of parameters

$$\mu_1^2 > 0, \quad \mu_{ij}^2 > 0,$$
and
$$\lambda_2 > 2\lambda_1 \tag{6}$$

we have a minimum of the potential as follows:

$$\langle \chi_L \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \chi_R \rangle = \begin{pmatrix} 0 \\ U_R \end{pmatrix}, \quad \langle \sigma \rangle = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}, \tag{7}$$

where U_R satisfies the relation

$$-\mu_1^2 + 2\lambda_1 U_R^2 + 2(e_{11} + e_{22})(a^2 + b^2) + 2ab(e_{12} + e_{21}) + 2f_{11}a^2 + 2f_{22}b^2 + 2ab(f_{12} + f_{21}) = 0. \tag{8}$$

We first notice that Eq. (7) reduces the local symmetry from $SU(2)_L \times SU(2)_R \times U(1)$ to U(1) leaving only the photon massless. The right-handed gauge mesons can be made massive by choosing

$$\mu_1^2 \gg \mu_{ij}^2. \tag{9}$$

This guarantees that $U_R \gg a, b$. This satisfies the only phenomenological constraint on our model, namely, that the right-handed current interactions be suppressed. In fact, in terms of the symmetry-breaking parameters, we find that

$$m_{W_L^+}{}^2 \simeq \tfrac{1}{4} g^2 (U_R^2 + a^2 + b^2),$$
$$m_{W_L^+}{}^2 \simeq \tfrac{1}{4} g^2 (a^2 + b^2). \tag{10}$$

As has been remarked before,[2] the mixing betwee W_L^+ and W_R^+ is proportional to ab and can be made arbitrarily small by choosing $b \ll a$. We further find that the masses of the eigenstates in the neutral gauge boson sector are

$$m_{A_\mu} = 0,$$
$$m_{U_\mu}{}^2 = \frac{g^2}{4}(a^2 + b^2)\left(\frac{g^2 + 2g'^2}{g^2 + g'^2}\right), \tag{11}$$
$$m_{Z_\mu}{}^2 = \tfrac{1}{4}(g^2 + g'^2)V_R^2 + \frac{g^4}{g^2 + g'^2}\frac{(a^2 + b^2)}{4}.$$

IV. DYNAMICAL BREAKDOWN OF PARITY

In this section we ask ourselves the question as to whether the spontaneous breakdown of parity discussed in the present paper could arise as a higher-order effect in the sense of Coleman and Weinberg.[4] To simplify discussions, we look at the $U(1)_L \times U(1)_R$ gauge group and take only two multiplets φ_L and φ_R with respective $U(1)_L \times U(1)_R$ quantum numbers as (1, 0) and (0, 1). Following Coleman and Weinberg,[4] we find that the effective potential in the one-loop approximation is given by

$$V(\varphi_L, \varphi_R) = \lambda_1[(\varphi_L^*\varphi_L)^2 + (\varphi_R^*\varphi_R)^2] + \lambda_2(\varphi_L^*\varphi_L)(\varphi_R^*\varphi_R) + \frac{3g^4}{64\pi^2}(\varphi_L^*\varphi_L)^2\left[\ln\frac{\varphi_L^*\varphi_L}{M^2} - \frac{25}{6}\right] + \frac{3g^4}{64\pi^2}(\varphi_R^*\varphi_R)^2\left[\ln\left(\frac{\varphi_R^*\varphi_R}{M^2}\right) - \frac{25}{6}\right] + O(\lambda_1^2, \lambda_1 g^2, \ldots), \tag{12}$$

where we have assumed that $\lambda_1 \sim g^4$ and we have worked in the Landau gauge.[8] It is then easy to check that for the range of the renormalized couplings $\lambda_2 > 3g^4/64\pi^2$ the minimum of the potential corresponds to $\langle \varphi_L \rangle = 0$ and $\langle \varphi_R \rangle^2 = V_R^2$, where V_R satisfies the following equation:

$$\ln\left(\frac{V_R^2}{M^2}\right) - \frac{25}{6} = -\frac{1}{2} - \frac{64\pi^2}{3g^4}\lambda_1. \tag{13}$$

We have checked that this minimum is lower than the stationary point[9] corresponding to $\langle \varphi_L \rangle^2 = \langle \varphi_R \rangle^2 = V^2$. In fact, this later point is not a minimum but corresponds to a saddle point in the φ_L, φ_R plane. This procedure can be easily extended to the realistic case of $SU(2)_L \times SU(2)_R \times U(1)$.

V. SUMMARY AND DISCUSSIONS

To summarize, we have succeeded in demonstrating that the left-right–symmetric gauge model advocated in Refs. 1 and 2 has the additional attractive feature that parity violation may arise as a result of spontaneous breakdown of local symmetry which in turn may have dynamical origin. All the features of the model outlined in Refs. 1 and 2 remain unaffected. Furthermore, if it turns out that the $SU(2)_L \otimes SU(2)_R \otimes U(1) \otimes SU(3')$ or $SU(2)_L \otimes SU(2)_R \otimes SU(4')$ model we are considering is itself a part of a larger unified group like the $SU(4)_L \otimes SU(4)_R \otimes SU(4')_L \otimes SU(4')_R$ group, or the $SU(16)_L \otimes SU(16)_R$ group, the techniques of the present paper may be used to unify the various *a priori* unrelated gauge couplings. In the first case, one may impose $L \leftrightarrow R$ as well as "valency" ↔ "color" discrete symmetry to begin with for the purpose of unifying the coupling constants (imposing just $L \leftrightarrow R$ symmetry achieves the purpose in the second case). Thus, all distinctions between left- and right-handed sectors as well as that between "valency" and "color" sector could arise purely as a result of spontaneous breakdown of the local symmetry along the lines advocated in the present paper. An important result of this paper (apart from presenting a realistic model with spontaneous parity violation) is that it takes one more step in the direction of reducing the number of arbitrary parameters of unified gauge theories (more precisely, instead of having two mass parameters μ_L and μ_R, the techniques of the present paper enable us to fulfill the requirements of weak phenomenology with only one parameter, i.e., $\mu_L = \mu_R$).

Lastly, we would like to compare our model with the only other model[6] in literature where spontaneous breakdown of parity occurs. The model of Ref. 6 does not have left-right symmetry and is therefore less elegant in our opinion. Secondly, extra neutral heavy leptons are crucial to Ref. 6 and the neutrino is necessarily massless. In our model,[2] the neutrino is not massless; however, its mass could be arbitrarily small. We, of course, do not need any extra leptons and work within the very elegant 4×4 quark-lepton structure.[1] Another important difference between Ref. 6 and our model lies in the hadron sector. In Ref. 6, one requires six quarks and more importantly, the n and λ quarks must remain massless. This implies that SU(3) symmetry and therefore $U(3)_L \times U(3)_R$ symmetry must be exact. This is in conflict with our knowledge of the hadronic world. In our model, the quarks get their masses through their Yukawa couplings to σ; therefore, their masses are arbitrary and there is no conflict with observed broken symmetries of the hadron spectrum.

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[1]J. C. Pati and A. Salam, Phys. Rev. D 10, 275 (1974).

[2]R. N. Mohapatra and J. C. Pati, Phys. Rev. D 11, 566 (1975).

[3]R. N. Mohapatra and J. C. Pati, Phys. Rev. D 11, 2558 (1975).

[4]S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).

[5]Dynamical breakdown of other discrete symmetries such as *CP* has been discussed in the literature: See A. Zee, Phys. Rev. D 9, 1772 (1974); R. N. Mohapatra, *ibid*. 9, 3461 (1974); H. Georgi and A. Pais, *ibid*. 10, 1246 (1974).

[6]P. Fayet, Nucl. Phys. B78, 14 (1974).

[7]The assignments of fermions to this group (see Ref. 2) are as follows:

$$Q_{1L,R} = \begin{pmatrix} P \\ n^{(0)} \end{pmatrix}_{L,R}, \quad Q_{2L,R} = \begin{pmatrix} P' \\ \lambda^{(0)} \end{pmatrix}_{L,R}$$

$$Q_{1,2L} = (\tfrac{1}{2}, 0, 1), \quad Q_{1,2R} \equiv (0, \tfrac{1}{2}, 1),$$

where the superscripts stand for the fact that they are unrotated fields and are therefore not eigenstates of the fermion mass matrix.

[8]It has been shown that the gauge dependence is only in higher-order terms, i.e., $O(\lambda_1 g^2)$. See R. Jackiw, Phys. Rev. D 9, 1686 (1974).

[9]We have checked that there are no other solutions of the equations $\partial V/\partial\varphi_R^\dagger = \partial V/\partial\varphi_L^\dagger = 0$ except the ones mentioned, along with others obtained from them by the application of gauge and discrete symmetry.