Comments on strongly interacting W-meson theories*

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(Received 19 May 1975)

We discuss the phenomenology and possible neutral-current tests of theories in which the W mesons have strong as well as semiweak and electromagnetic interactions. We then show that, at least in one such model, divergence difficulties are still present in the usual way for amplitudes such as $K_L \to \mu^+\mu^-$.

INTRODUCTION

If W mesons mediate the weak interactions, it is known that they must have a fairly large mass, probably greater than 10 GeV.[1] Though the conventional pictures (in this category we shall include gauge models) suppose that the W mesons have only weak and electromagnetic interactions, it is possible that they also have strong interactions as long as these are such that the relevant quantum numbers and symmetries are preserved. Models of this type have been considered over the past ten years,[2-6] most recently by Marshak and Mohapatra.[7] It has been hoped that allowing the W mesons to have strong interactions would introduce a natural damping mechanism to eliminate divergence difficulties. We will present in Sec. II arguments which strongly suggest that this is not the case. In Sec. I we will comment on the phenomenology of models of strongly interacting W mesons and in particular discuss what tests in neutral currents there are for the model of Ref. 7 and how it might be altered if it fails these tests.

I. PHENOMENOLOGY

In Ref. 7 a model is displayed in which the W mesons transform like a triplet under an internal U(3) symmetry group. They are integrally charged and consist of an isotopic spin doublet (W^+, W^0) with $Y=1$ and an isotopic spin singlet $(W^{0\prime})$ with $Y=0$. The interaction Hamiltonian is

$$H = fW^{a}_{\lambda}W^{\lambda}_{b}S^{b}_{a}(h) + g\,\bar{l}_{\lambda}W^{\lambda}_{1} + g[\cos\theta\, j^{a}_{\lambda 2} + \sin\theta\, j^{a}_{\lambda 3}]W^{\lambda}_{a} \qquad (1)$$

where $a = 1, 2, 3$ or $+, 0, 0'$, f is a dimensionless coupling constant of order 1, $S^{b}_{a}(h)$ is an octet function of the hadron fields, g is the semiweak coupling constant, l_λ is the lepton current (charged), θ is the Cabibbo angle, and $j^{a}_{\lambda b}$ is the octet hadron current.

Note that there are no neutral lepton currents present in the interaction Hamiltonian. The muonless neutrino events[8] are presumed to be caused, to order g^2, by the diagram of Fig. 1. This has several interesting consequences. Assuming that we are below threshold for W production, these are the following:

(a) $\nu_\mu(\bar{\nu}_\mu) + e^- \to \nu_\mu(\bar{\nu}_\mu) + e^-$ is forbidden. As discussed in Ref. 7, this is only allowed by a combination of weak and electromagnetic interactions, and naively the amplitude is of order $G_F\alpha$, so we expect this process to be strongly suppressed.

(b) The cross sections for $\nu + N \to \nu + X$ and $\bar{\nu} + N \to \bar{\nu} + X$ are equal, where X is any final hadronic state.

(c) The effective neutral hadronic current has only $I = 0, 2$ components. This is because $W^{+}_{\lambda}W^{\lambda -}$ couples in (1) to $S^1_1 + S^2_2$ which has $I = 0, 2$. This implies, e.g., that $\nu + N \to \nu + \Delta$ is forbidden.

We assume that these observations are known to the authors of Ref. 7. There is one additional one which is more obscure, however, owing to crossing symmetry:

(d) Consider elastic scattering of neutrinos off a target α, $\nu(k_1) + \alpha(q_1) \to \nu(k_2) + \alpha(q_2)$. The amplitude has the form

$$M = G_F\bar{u}(k_1)(\not{q}_1 + \not{q}_2)(1-\gamma_5)u(k_2) \times B(k_1\cdot q_1, \Delta^2 = (q_1 - q_2)^2). \qquad (2)$$

Crossing symmetry tells us that

$$B^*(k_1\cdot q_1, \Delta^2) = -B(-k_1\cdot q_1, \Delta^2). \qquad (3)$$

This is just like pion-nucleon scattering[9] except that the A amplitudes are absent because of helicity and the B^- is absent because we only have even

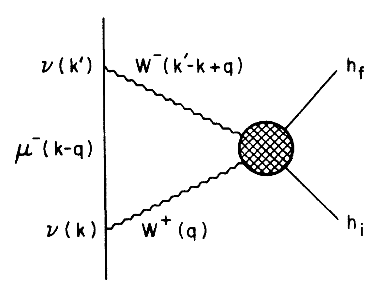

FIG. 1. Neutrino-hadron scattering diagram.

isospin. Since we are below threshold for W production B and B adjoint are equal, so for $\Delta^2=0$ we have $B(k_1q_1,0)=-B(-k_1q_1,0)$ or B is proportional to $k_1\cdot q_1$. This implies that, e.g., the ν-N invariant scattering amplitude is proportional to the incoming neutrino energy.

These four predictions are quite striking. If, e.g., $\nu_\mu e^-$ scattering is of the same order as $\nu_e e^-$ scattering, one will probably have to reject the model as it stands. A modified model, which would incorporate directly a neutral current

$$l^0_\lambda=\bar{e}\gamma_\lambda(1-\gamma_5)e+\bar{\nu}_e\gamma_\lambda(1-\gamma_5)\nu_e+(e\to\mu), \tag{4}$$

could then be considered. As an example of a modified interaction let H be the interaction Hamiltonian of Eq. (1), and let

$$H\to H+g\bar{l}^0_\lambda W^\lambda_3+gj^3_{\lambda 3}W^\lambda_3. \tag{5}$$

Of the four predictions (a)–(d), only (c) continues to hold, and even that is of course flexible since j^3_3 could be replaced by a combination of, e.g., j^1_1, j^2_2, and j^3_3. This would also not change the octet nature of the effective strangeness-changing nonleptonic weak Hamiltonian. At this point, though, several free parameters have been introduced, and the model does not seem to be interesting unless it has other desirable properties, in particular a gentler asymptotic behavior and the accompanying curing of divergence difficulties. We shall turn our attention to this point in the next section.

II. DIVERGENCE PROBLEMS

Consider a slightly different version of the strongly interacting W model for which the total Lagrangian is

$$\mathcal{L}=-\tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_b(\delta^b_a+fS^b_a(h))-\tfrac{1}{2}M_W{}^2W_\lambda W^\lambda$$
$$+g\mathcal{J}^a_\lambda W^\lambda_a+\mathcal{L}_h+\mathcal{L}_l+\text{h.c.}, \tag{6}$$

with

$$\mathcal{J}^a_\lambda=l_\lambda\delta^a_1+\cos\theta\, j^a_{\lambda 2}+\sin\theta\, j^a_{\lambda 3}. \tag{7}$$

The equation of motion for $W_\lambda{}^2$ implies

$$W^a_\lambda=\frac{g}{M_W{}^2}\{\mathcal{J}^a_\lambda+\partial^\nu[F_{\lambda\nu b}(\delta^a_b+fS^a_b(h))]\}, \tag{8}$$

and calling π^a_i the momentum conjugate to W^a_i, the above implies that

$$W^a_0=\frac{g}{M_W{}^2}(\mathcal{J}^a_0+\partial^i\pi^a_i). \tag{9}$$

With these notions before us, let us turn to the divergence problem. Consider the diagram of Fig. 1. The amplitude, to all orders in the strong interactions, is given by (neglecting lepton masses)

$$M_{\nu+h_i\to\nu+h_f}=g^2\bar{u}(k')\left[\int\frac{d^4q}{(2\pi)^4}\gamma^\beta\frac{\not{k}-\not{q}}{(k-q)^2}\gamma^\alpha(1-\gamma_5)T_{\beta\alpha}\right], \tag{10}$$

$$T_{\beta\alpha}(q,p_i)=\int d^4z\,e^{iq\cdot z}\langle h_f|T^*\{W_{\beta 1}(z)W^+_{\alpha 1}(0)\}|h_i\rangle .$$

Note that by writing the time-ordered product of W fields, instead of sources, we have included all W-propagator corrections as well as the full W-hadron scattering amplitude.[10]

The large-q behavior of the matrix element in (10) can be analyzed by using the Bjorken-Johnson-Low limit,[11,12] i.e., letting $q_0\to\infty$ with $\vec{q}$ fixed.

$$\lim_{\substack{q_0\to\infty\\ |\vec{q}|\ \text{fixed}}} T_{\beta\alpha}=-\frac{1}{q_0}\int d\vec{z}\,e^{-i\vec{q}\cdot\vec{z}}\langle h_f|[W_{\beta 1}(\vec{z},0),W^+_{\alpha 1}(0)]|h_i\rangle+\text{polynomials}. \tag{11}$$

Using (9) we see that, e.g.,

$$\lim_{\substack{q_0\to\infty\\ |\vec{q}|\ \text{fixed}}} T_{00}=-\frac{1}{q_0}\frac{g^2}{M_W{}^4}\langle h_f|J_0(0)|h_i\rangle , \tag{12}$$

with $J_\mu(x)$ being an octet $V-A$ current with transformation properties

$$J_\mu=(V_\mu-A_\mu)_3\cos^2\theta+[\tfrac{1}{2}(V_\mu-A_\mu)_3+\tfrac{1}{2}\sqrt{3}\,(V_\mu-A_\mu)_8]\sin^2\theta-(V_\mu-A_\mu)_6\sin\theta\cos\theta. \tag{13}$$

The most divergent contribution of T_{00} to (10) is

$$\left(\frac{g^2}{M_W{}^2}\right)^2\bar{u}(k')\gamma_0(1-\gamma_5)u(k)\int\frac{d^4q}{(2\pi)^4}\frac{1}{(k-q)^2}\langle h_f|J_0(0)|h_i\rangle ,$$

which leads to the usual $G_F(G_F\Lambda^2)$ quadratic divergence present in an ordinary model in which only charged W's without strong interactions are present. In particular, since J_μ has strangeness-violating terms in it, we obtain a quadratically divergent contribution to $K_L \to \mu^+\mu^-$ and other forbidden decays in order G_F^2. The conclusion to draw seems to be that, at the very least, the model requires some sort of GIM[13] cancellation mechanism and hence the introduction of charm.

There are several remarks one can make at this point. First, one may not trust the employed limiting procedure of Eq. (12); it is known to be violated in perturbation theory,[14] but such violations are usually in the $1/q_0^2$ term when the $1/q_0$ term vanishes. The $1/q_0$ term is presumably accurate. Secondly, one may say that a perturbative calculation for a strongly interacting theory is meaningless, but we are only calculating perturbatively in g, not in f, the strong coupling constant. One would hope the theory gives sensible answers as a power-series expansion in g. Third, we have examined only one model[15] of strongly interacting W's, with the particular form given in Eq. (6), which is in fact different from the form of Eq. (1). For the latter we have not been able to show unambiguously the existence of a $G_F(G_F\Lambda^2)$ term in the matrix element of a $\Delta S = 1$, $\Delta Q = 0$ semileptonic decay, but we do not believe there are essential differences in this respect between the two models.

Note added in proof. Many of the topics contained in Sec. I have also been discussed by Marshak and Mohapatra in a recent Physical Review D article.[16]

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*Work supported in part by ERDA.

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