PHYSICAL REVIEW D VOLUME 12, NUMBER 1 1 JULY 1975

Hadron masses in a gauge theory*

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(Received 24 February 1975)

We explore the implications for hadron spectroscopy of the "standard" gauge model of weak, electromagnetic, and strong interactions. The model involves four types of fractionally charged quarks, each in three colors, coupling to massless gauge gluons. The quarks are confined within colorless hadrons by a long-range spin-independent force realizing infrared slavery. We use the asymptotic freedom of the model to argue that for the calculation of hadron masses, the short-range quark-quark interaction may be taken to be Coulomb-like. We rederive many successful quark-model mass relations for the low-lying hadrons. Because a specific interaction and symmetry-breaking mechanism are forced on us by the underlying renormalizable gauge field theory, we also obtain new mass relations. They are well satisfied. We develop a qualitative understanding of many features of the hadron mass spectrum, such as the origin and sign of the Σ-Λ mass splitting. Interpreting the newly discovered narrow boson resonances as states of charmonium, we use the model to predict the masses of charmed mesons and baryons.

I. INTRODUCTION

Once upon a time, there was a controversy in particle physics. There were some physicists[1] who denied the existence of structures more elementary than hadrons, and searched for a self-consistent interpretation wherein all hadron states, stable or resonant, were equally elementary. Others,[2] appalled by the teeming democracy of hadrons, insisted on the existence of a small number of fundamental constituents and a simple underlying force law. In terms of these more fundamental things, hadron spectroscopy should be qualitatively described and essentially understood just as are atomic and nuclear physics.

Many recent theoretical and experimental developments seem to confirm the latter philosophy, and lead towards a unique, unified, and remarkably simple and successful view of particle physics. All hadrons are built up of a quark and an antiquark or of three quarks properly chosen from among the twelve quark species, and all interactions (but not yet gravity) arise from (unified?) renormalizable gauge couplings.

This point of view is very tightly constrained if it is both to agree with experiment and to be theoretically consistent. In this paper, we explore the origins of the hadron mass spectrum from this standpoint. Not only do we find that the observed hadron mass splittings can be understood, but we uncover correct new relations among them. We also consider hadron states containing one or more charmed quarks in order to describe the newly observed[3-5] narrow boson resonances and the necessarily (we believe) soon-to-be-discovered charmed hadrons.

Let us review briefly those diverse strands of thought we now recognize to be converging.

Nonrelativistic SU(6). Hadron spectroscopists have long known[6-8] that almost all the meson and baryon resonances can be accommodated in a few representations of $SU(6)_q \times SU(6)_{\bar{q}} \times O(3)$—most baryons in S- and D-wave $\underline{56}$'s and a P-wave $\underline{70}$, and most mesons in S- and P-wave $\underline{36}$'s. To the extent that the members of these representations have the same mass, the quark binding forces are spin-independent and the $\mathcal{P}$, $\mathfrak{N}$, λ quarks are degenerate. Since nature makes use only of wave functions totally symmetric in all conventional dynamical variables, an artifice like color is needed for this picture to be compatible with the Fermi statistics of the quarks.

Deep-inelastic lepton scattering. Aside from the classification of hadron states, the most compelling success of the quark-parton picture[9] is in its surprisingly adequate description of inclusive electron and neutrino scattering in the deep-inelastic region.[10] At short shutter times, the nucleon behaves as if it consists of three noninteracting quarks.

Color is the notion that each type of quark ($\mathcal{P}$, $\mathfrak{N}$, etc.) comes in three versions, transforming as a triplet under color SU(3).[11] This group is assumed to be an exact symmetry of nature, and all physical states are assumed to be color singlets. This solves the statistics problem of spectroscopy, for the color-singlet state of three quarks is completely antisymmetric in the color variable. It also explains why the only observed hadrons are qqq = baryon, $\bar{q}\bar{q}\bar{q}$ = antibaryon, and $\bar{q}q$ = meson. Color SU(3) has just three invariant tensors, so that *all* color singlets may be decomposed into systems of mesons and baryons. Regrettably this explanation is not logically complete. Since it depends on the assertion that physical states are color singlets, its justification awaits a rigorous understanding of the quark binding mechanism. Nonetheless, the color hypothesis does have indirect experimental support. Away from resonances and below charm threshold, the experimental value of $R = \sigma(\text{hadrons})/\sigma(\mu^+\mu^-)$ in e^+e^- annihilation[12] is in far better agreement with $R = 2$ (the result with color quarks) than with $R = \frac{2}{3}$. And, the Adler calculation[13] of the π^0 decay rate agrees well with experiment when color is included, but disagrees by an order of magnitude without it.

Strong interactions as an exact SU(3) *gauge theory*. We shall assume that color SU(3) is not just an exact global symmetry, but an exact local symmetry, and that there are eight massless, neutral, yet unobservable color gluons.[14,15] We stress that this point of view excludes the possibility of integer charge (Han-Nambu[16]) quarks. For local SU(3) symmetry to be exact, the gluons must be electrically neutral. The observed behavior of electroproduction cross sections agrees with this notion: Although as much as half the nucleon's momentum resides on gluons, there is evidence that they do not directly participate in deep-inelastic scattering, for the longitudinal form factors are quite small.[17]

The viability of this idea depends on the fact that this model of strong interactions displays *asymptotic freedom*[15,18,19] and *infrared slavery*. At high momentum or short distance, the effective gauge coupling constant α_s is small and decreases with energy—a behavior indicated by the scaling of deep-inelastic lepton scattering, where the phenomena are well described in terms of scattering from free pointlike quarks.[19-21] Asymptotic freedom also justifies the interpretation of the new resonances as states of orthocharmonium, and is the key to our understanding of their narrow widths.[22,23]

Infrared slavery is the converse notion, wherein the gauge coupling becomes strong at small momentum or large distance. This property may explain why the quarks are confined and why colored states may not be produced. Should the force between quarks remain constant at large distances it is evident that the production of $q\bar{q}$ pairs will be energetically favored over the macroscopic isolation of a single quark. This happens in lattice gauge theories.[24]

Asymptotic freedom and infrared slavery will be kept in mind as we construct the Hamiltonian responsible for mass splittings within hadron multiplets. We assume that the principal binding forces of hadrons are the long-range quark-confining forces. These should be independent of quark spins and quark masses, depending just on the spatial separations of the constituent quarks.[24] If the quarks were degenerate in mass, these forces would produce hadrons in supermultiplets of very high multiplicity.

Many of the observed hadron mass splittings are produced by the splittings among the quark masses. We have nothing to say about quark mass splittings: Quark and lepton masses comprise the next level of spectroscopy.

In addition, there exist short-range forces depending on the quark spins and masses. We will argue that for the calculation of hadron masses it is as if these forces arose from one-gluon exchange (Sec. II). They should have the same form as the two-body Fermi-Breit interaction between charged Dirac particles.

Much research[6,7] has been devoted to describing hadron masses in terms of two-body quark-quark forces, often with considerable success. On the other hand, there is a wide variety of conceivable quark-quark forces. Our approach is unique because it is partially dynamical insofar as the form of the force is precisely prescribed by the underlying theoretical structure: There is only SU(3)-singlet exchange, the Fermi term varies with the inverse product of the quark masses, the coefficients of the spin-orbit and tensor forces are related, etc.

As a brief preview to our analysis, consider the origins of some characteristic mass splittings. The Fermi term (hyperfine splitting) is responsible for the Δ-N (Sec. III) and ρ-π (Sec. V) mass splittings. This term is not operative for the even-parity P-wave mesons, but it is the Breit interaction (spin-orbit, etc.) which produces the splittings among A_1, A_2, B, and δ (Sec. VII). In the case of P-wave baryons, both the Fermi and Breit terms contribute, and the situation is more complicated. The fact that the λ quark is heavier than its nonstrange siblings reveals itself most clearly in the equal spacing of $J^P = \frac{3}{2}^+$ baryons. But, what explains the Σ-Λ mass splitting? Here we see directly the quark-mass dependence of the Fermi interaction: The essential difference between Σ and Λ is in the spin of the lighter quark constituents. The model explains both the sign and magnitude of this mass splitting.

We shall also explain the anomalous behavior of the pseudoscalar-meson mass spectrum: Why it is that of all neutral mesons, only the η is almost purely octet, others being made up either of strange or of nonstrange quarks almost exclusively (Sec. VI). Where there is the possibility of virtual annihilation of a quark-antiquark pair into gluons, we must depart from the gluon-exchange model.

A renormalizable unified gauge theory of weak interactions and electromagnetism is another part of the new dogma. Most naturally implemented, such a synthesis predicts the existence of neutral-current weak interactions.[25] Such interactions, conserving strangeness, were recently observed.[26] But, there are no neutral-current interactions with $\Delta Y = \pm 1$. The simplest way to avoid these unwanted interactions[27] is through the introduction of a fourth quark type $\mathcal{P}'$ with charm.[28] Thus, we are led to our belief in twelve fundamental hadronic constituents. The fourth quark type may not be too heavy, lest $\Delta Y = \pm 1$ neutral-current effects become too large even at second order in weak interactions: Charmed hadrons simply must exist and be found with masses no greater than several GeV.

Two additional constraints must be satisfied if all is to be theoretically consistent. The local gauge invariance of weak and electromagnetic interactions must be compatible with the local gauge invariance of strong interactions. This is certainly true in the model we are led to, where the two groups commute with each other. Finally, the gauge theories must be free of triangle anomalies.[29] It is the introduction of charm and of color that leads miraculously to the cancellation of all such anomalies[30]—surely a most favorable augury for this picture.

II. HADRON MASSES WITHOUT CHARM

In order to understand the hadron mass spectrum from the underlying gauge field theory, we must know something about the long-range force responsible for quark trapping in mesons and baryons. At present, there is no completely satisfactory description of this long-range force in a non-Abelian gauge theory, but the lattice gauge theory of Wilson and Kogut and Susskind[24] may give us some useful clues. In this approach, the quark fields are defined only at the sites of a cubic lattice, and the "gauge fields" are associated with the links between neighboring sites. The gauge symmetries of the model consist of independent SU(3) rotations at each lattice site. The interaction energy between two distant static quarks can be written as an expansion in inverse powers of the quark-gluon coupling constant α_s, which is conjectured to be large for large lattice spacing. The leading term in this expansion is proportional to N, the number of excited lattice links, connecting the quarks. Thus, the force between distant quarks is constant. On the other hand, the interaction energy associated with spin-spin coupling decreases with distance as α_s^{-N}. Thus, the spin-spin interaction is short range: It is exponentially damped relative to the leading spin-independent force. The above arguments can be made for the simplest quark-link configuration in which the quarks are separated along a lattice axis, with the excited links forming a straight "string" between the two quarks. Suppose instead that the quarks are separated along a diagonal, so that the excited links must take a zig-zag path between them. Such a state has a larger interaction energy for a given quark separation than the straight string. Since this breakdown of rotation invariance cannot persist in the real world, we must decide which is the "right" expression for the energy.

Let us make it plausible that the right state is the one with the straight string. We can regard the lattice gauge theory as a non-Abelian gauge theory that has been cut off by putting it on a lattice, with a momentum cutoff of the order of the inverse lattice spacing. If we restrict our attention to processes in which all momenta are very small compared to the inverse lattice spacing, we should be able reliably to calculate in the cutoff theory. But we certainly cannot use the cutoff theory to analyze processes involving momenta of the order of the inverse lattice spacing. We might trust the lattice-model estimate of the energy of the system of two quarks connected by a string of excited lattice links if the string is long on the scale of the lattice spacing and straight. We certainly do not believe the calculation for a string which zigs or zags. We can also trust the assertion that the spin-spin interaction falls exponentially at large distances even though we cannot use the lattice theory to find the form of the short-range force.

The lattice theories also apparently give rise to long-range forces resembling spin-orbit couplings. These arise on the lattice because one end of the string can move while most of the string is stationary. But this calculation crucially depends on strings with bends, and is no more to be trusted than the calculation of energy of a diagonal string. The spin-orbit effects involved in rotating a long string rigidly are again suppressed exponentially.[31]

Thus we argue that the color gauge couplings produce a long-range spin-independent force. This force leads to the appearance of $\mathrm{SU}(6)_q \times \mathrm{SU}(6)_{\bar{q}} \times \mathrm{O}(3)$ supermultiplets of hadrons. [For brevity, we refer to these as SU(6) supermultiplets.]

In order to take into account the breaking of SU(3), we allow the various quark types ($\mathcal{P}$, $\mathfrak{N}$, λ) to have different masses. No other mechanism for SU(3) breaking is admitted, for none is consistent with the notion that all interactions are renormalizable gauge interactions.

It is only the short-range force between quarks that is spin dependent. In our calculation of hadron masses, we argue that the effective short-range force arises from one-gluon exchange.

Hadron masses can be calculated by looking for poles in two-point functions of color-gauge invariant operators. The only momentum in the calculation is the *total* hadron momentum. If at a typical hadronic mass the effective coupling is fairly small, the short-range interaction in our calculation is governed by a small coupling and is Coulomb-like. Thus, we simply use the Fermi-Breit interaction, generalized to describe fermions of arbitrary mass, with the fine-structure constant α replaced by $-\frac{4}{3}\alpha_s$ for $\bar{q}q$ pairs in a meson and by $-\frac{2}{3}\alpha_s$ for qq pairs in a baryon. These factors are the only pale residue of the non-Abelian nature of the gauge couplings: qq attract in an antisymmetric color state just half as much as $\bar{q}q$ do in a color singlet state.

We conclude that the strong and electromagnetic part of the Hamiltonian for a color neutral three-quark of quark-antiquark hadron state has the following form:

$$H = L(\vec{r}_1, \vec{r}_2, \ldots) + \sum_i \left(m_i + \frac{\vec{p}_i^{\,2}}{2m_i} + \cdots\right) + \sum_{i>j} (\alpha Q_i Q_j + k\alpha_s) S_{ij}\,. \tag{1}$$

In (1), L describes the universal interaction responsible for quark binding; r_i, p_i, m_i, and Q_i are the position, momentum, mass, and charge of the ith quark; and k is $-\frac{4}{3}$ for mesons and $-\frac{2}{3}$ for baryons. The two-body Coulombic interaction is S_{ij}, which has the form

$$S_{ij} = \frac{1}{|\vec{r}|} - \frac{1}{2m_i m_j}\left(\frac{\vec{p}_i\cdot\vec{p}_j}{|\vec{r}|} + \frac{\vec{r}\cdot(\vec{r}\cdot\vec{p}_i)\vec{p}_j}{|\vec{r}|^3}\right) - \frac{\pi}{2}\delta^3(\vec{r})\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{16\vec{s}_i\cdot\vec{s}_j}{3m_i m_j}\right)$$
$$- \frac{1}{2|\vec{r}|^3}\left\{\frac{1}{m_i^2}\vec{r}\times\vec{p}_i\cdot\vec{s}_i - \frac{1}{m_j^2}\vec{r}\times\vec{p}_j\cdot\vec{s}_j + \frac{1}{m_i m_j}\left[2\vec{r}\times\vec{p}_i\cdot\vec{s}_j - 2\vec{r}\times\vec{p}_j\cdot\vec{s}_i - 2\vec{s}_i\cdot\vec{s}_j + 6\frac{(\vec{s}_i\cdot\vec{r})(\vec{s}_j\cdot\vec{r})}{|\vec{r}|^2}\right]\right\} + \cdots, \tag{2}$$

where $\vec{r} = \vec{r}_i - \vec{r}_j$ and $\vec{s}_i$ is the spin of the ith quark, and in (1) and (2) $\cdots$ denotes neglected relativistic corrections.

Typical mass differences between low-lying SU(6) supermultiplets of the same parity are of the order of one GeV [e.g., $N(1780) - N(940)$ or $L(1770) - K(498)$ (see Ref. 32)], while characteristic splittings within supermultiplets due to quark mass differences and spin-dependent Coulombic interactions are a few hundred MeV [e.g., $K^*(892) - \rho(770)$ or $\Sigma^*(1385) - \Sigma(1190)$]. This suggests that a perturbative approach can give a satisfactory description of hadron mass splittings within a supermultiplet. We write $H = H_0 + V$, where

$$H_0 = \sum L(r_1, r_2, \ldots) + \sum_i (m_{\mathcal{P}} + \vec{p}_i^{\,2}/2m_{\mathcal{P}})$$

and V is everything else. The eigenstates of H_0 are the degenerate SU(6) supermultiplets. First-order perturbation theory in V introduces splittings within the supermultiplets. Lacking detailed information about the zeroth-order eigenstates, we can only parametrize the expectation value of V and fit to observed particle masses. There are fewer parameters than there are masses, so that we deduce a number of mass formulas which should be satisfied to first order in V. Although they are modified in higher order, we expect the first-order relations among hadron mass splittings within supermultiplets to be satisfied to within 20%, because the splittings within supermultiplets are typically about 20% of the splitting between supermultiplets.

In one special circumstance, we must recognize the analysis so far to be incomplete. For the S-wave quark-antiquark system, there is an important additional short-range interaction. In a neutral meson (i.e., $T = Y = 0$) the quarks may annihilate into gluons and reappear as a different quark-antiquark pair. This interaction acts to break $SU(6)_q \times SU(6)_{\bar{q}}$ down to diagonal SU(3) and to decompose a nonet of mesons into a singlet plus an octet. This interaction is crucial to our understanding of the η.

III. S-WAVE BARYONS

The zeroth-order wave functions for the 56-plet of S-wave baryons consist of SU(6) wave functions multiplying a single completely symmetric function of the positions of the quarks $\Psi_0(r_1, r_2, r_3)$ and an antisymmetric color wave function. Our first-order mass formula is

$$M = M_0 + \sum_i\left[\Delta m_i + a\left(\frac{1}{m_i} - \frac{1}{m_{\mathcal{P}}}\right)\right] + \sum_{i>j}(\alpha Q_i Q_j - \tfrac{2}{3}\alpha_s)\left[b - \frac{c}{m_i m_j} - d\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{16\vec{s}_i\cdot\vec{s}_j}{3m_i m_j}\right)\right], \tag{3}$$

where $\Delta m_i = m_i - m_{\mathcal{P}}$, $H_0|\Psi_0\rangle = M_0|\Psi_0\rangle$, and

$$a = \tfrac{1}{2}\langle\Psi_0|p_i^2|\Psi_0\rangle\,, \tag{4a}$$

$$b = \left\langle\Psi_0\left|\frac{1}{|\vec{r}_{12}|}\right|\Psi_0\right\rangle\,, \tag{4b}$$

$$c = \tfrac{1}{2}\left\langle\Psi_0\left|\frac{r_{12}^2\vec{p}_1\cdot\vec{p}_2 + \vec{r}_{12}\cdot(\vec{r}_{12}\cdot\vec{p}_1)\vec{p}_2}{|\vec{r}_{12}|^3}\right|\Psi_0\right\rangle\,, \tag{4c}$$

$$d = \frac{\pi}{2}\langle\Psi_0|\delta^3(\vec{r}_{12})|\Psi_0\rangle\,. \tag{4d}$$

The expectation values of the $\vec{L}\cdot\vec{s}$ and tensor-force pieces of S_{ij} have not been included because they vanish when the relative orbital angular momentum of the two-quark subsystem is zero. We expect that the ground-state wave function Ψ_0 is predominantly S wave in any pair of quark positions, so that these higher angular momentum contributions should be negligible.

Let us use (3) first to obtain an estimate of the ratio of mass of strange and nonstrange quarks:

$$\frac{m_{\mathcal{P}}}{m_\lambda} = \frac{2(\Sigma^* - \Sigma)}{2\Sigma^* + \Sigma - 3\Lambda} = 0.622\,, \tag{5}$$

where particle names stand for particle masses. This ratio differs from unity by ~40%, so that an expansion to first order in SU(3) breaking incurs a ~15% error. Since higher-order corrections in our perturbative approach are of a similar size, we may as well expand (3) to first order in Δm. Then, the mass formula (neglecting electromagnetism) describes the eight masses of the 56-plet (N, Λ, Σ, Ξ, Δ, Σ^*, Ξ^*, Ω) in terms of four parameters, A, B, C, and $\Delta m_\lambda/m_{\mathcal{P}}$:

$$M = A + B\sum_i \frac{\Delta m_i}{m_{\mathcal{P}}} + C\sum_{i>j}\vec{s}_i\cdot\vec{s}_j\left(1 - \frac{\Delta m_i + \Delta m_j}{m_{\mathcal{P}}}\right), \tag{6}$$

where

$$A = M_0 - 2\alpha_s[b - (c+2d)/m_{\mathcal{P}}^2]\,, \tag{7a}$$

$$B = m_{\mathcal{P}} - a/m_{\mathcal{P}} - 4\alpha_s(c+2d)/3m_{\mathcal{P}}^2\,, \tag{7b}$$

$$C = 32\alpha_s d/9m_{\mathcal{P}}^2\,. \tag{7c}$$

Four mass formulas are implied by (6). These are the Gell-Mann–Okubo formula[33] for the baryon octet,

$$2N + 2\Xi = 3\Lambda + \Sigma\,, \tag{8}$$

the equal-spacing rules for the decuplet,

$$\Delta - \Sigma^* = \Sigma^* - \Xi^* = \Xi^* - \Omega^-\,, \tag{9}$$

and the SU(6) relation[34]

$$\Sigma^* - \Sigma = \Xi^* - \Xi\,. \tag{10}$$

More important is the fact that the forces arising from the exchange of color SU(3) gluons provide a natural qualitative explanation of the baryon mass splittings. The decuplet is heavier than the octet because, in an attractive Coulomb potential, two Dirac particles in an $L=0$ state have a higher energy when their spins are aligned than when their spins are opposite. The Σ-Λ mass difference arises because of the difference in mass between λ and $\mathcal{P}$ quarks, as follows.

In Σ^0, the $\mathcal{P}$ and $\mathcal{N}$ quarks are in an isotopic-triplet state which is symmetric in internal symmetry space. This pair must also be symmetric in spin, so that $(\vec{s}_{\mathcal{P}} + \vec{s}_{\mathcal{N}})^2 = 2$ and $\vec{s}_{\mathcal{P}}\cdot\vec{s}_{\mathcal{N}} = \frac{1}{4}$. The spins are aligned, so the Fermi interaction of the $\mathcal{P}$-$\mathcal{N}$ pair gives a positive contribution to the energy proportional to $(2m_{\mathcal{P}})^{-2}$. Since the total spin is $\frac{1}{2}$, $\vec{s}_{\mathcal{P}}\cdot\vec{s}_{\mathcal{N}} + \vec{s}_{\mathcal{P}}\cdot\vec{s}_\lambda + \vec{s}_{\mathcal{N}}\cdot\vec{s}_\lambda = -\frac{3}{4}$, and the Fermi interaction of the $\mathcal{P}$-λ and $\mathcal{N}$-λ pairs gives a negative contribution proportional to $-(m_{\mathcal{P}}m_\lambda)^{-1}$. For Λ, the $\mathcal{P}$-$\mathcal{N}$ pair has isospin zero, therefore spin zero, so that $\vec{s}_{\mathcal{P}}\cdot\vec{s}_{\mathcal{N}} = -\frac{3}{4}$. The entire Fermi contribution comes from this quark pair, and is proportional to $-3(2m_{\mathcal{P}})^{-2}$. All other interactions contribute equally to Σ^0 and Λ masses, so that the Σ^0-Λ mass difference is proportional to $m_{\mathcal{P}}^{-1}(m_{\mathcal{P}}^{-1} - m_\lambda^{-1})$. The λ quark is heavier than the $\mathcal{P}$ quark, so that Σ^0 is heavier than Λ.

In other words, both the decuplet-octet mass splitting and the Σ-Λ mass difference are "hyperfine" splittings. They are related by

$$\Sigma - \Lambda = \tfrac{2}{3}(1 - m_{\mathcal{P}}/m_\lambda)(\Delta - N)\,. \tag{11}$$

The value of $m_{\mathcal{P}}/m_\lambda$ given by (5) or (11) does not coincide with the value obtained from the pseudoscalar-meson masses via current algebra. Ours are effective masses of quarks bound in hadrons, not the masses appearing in the phenomenological Lagrangians describing the breaking of SU(3) × SU(3).

IV. ELECTROMAGNETIC PROPERTIES OF BARYONS

We may use (3) to examine the electromagnetic mass differences among the baryons. Neglecting terms of order $\alpha(\Delta m_\lambda/m_{\mathcal{P}})$, we find that the octet electromagnetic mass differences satisfy the Coleman-Glashow relation,[35]

$$\Sigma^+ - \Sigma^- = p - n + \Xi^0 - \Xi^-\,, \tag{12}$$

and the decuplet electromagnetic splittings are predicted to be[36]

$$\Delta^+ - \Delta^{++} = (n-p) - (\Sigma^+ + \Sigma^- - 2\Sigma^0)\,, \tag{13a}$$

$$\Delta^0 - \Delta^+ = \Sigma^{*0} - \Sigma^{*+} = (n-p)\,, \tag{13b}$$

$$\Delta^- - \Delta^0 = \Sigma^{*-} - \Sigma^{*0} = \Xi^{*-} - \Xi^{*0} = (n-p) + (\Sigma^+ + \Sigma^- - 2\Sigma^0) . \quad (13c)$$

We also consider the magnetic moments of the baryons. The successful nonrelativistic quark model relation [37] $\mu(n) = -\frac{2}{3}\mu(p)$, obtained by adding the magnetic moments of the constituent quarks with weights dictated by the SU(6) wave function, survives in the present framework. This prediction presupposes that quark magnetic moments are proportional to their charge to mass ratio, which is true in our model to all orders in the gluon coupling constant for $m_{\wp} = m_\lambda$, and to second order in α_s for $m_{\wp} \neq m_\lambda$.

The SU(3) prediction[35] $\mu(\Lambda) = -\frac{1}{3}\mu(p) = -0.93$ is modified by the inequality of strange and nonstrange quark masses to read

$$\begin{aligned}\mu(\Lambda) &= -\tfrac{1}{3}(m_{\wp}/m_\lambda)\mu(p)\\ &= -\frac{2(\Sigma^* - \Sigma)}{3(2\Sigma^* + \Sigma - 3\Lambda)}\mu(p)\\ &= -0.6 ,\end{aligned} \quad (14)$$

bringing it into closer agreement with experiment. Our predictions, and the measured values, of all octet magnetic moments are shown in Table I, with all numbers in nuclear magnetons.

The data on radiative decays $N^* \to N\gamma$ and $\omega \to \pi^0\gamma$ suggest[7] that the anomalous magnetic moments of the quarks are small. In our framework, the masses of the quarks have a realistic meaning, in spite of the fact that the long-range forces forbid the macroscopic isolation of a quark. Their masses may be estimated from observed magnetic moments in the nonrelativistic approximation and with the assumption that quark anomalous magnetic moments are negligible. We obtain

$$m_{\wp} = p/\mu(p) = 336 \text{ MeV} . \quad (15)$$

The mass of the λ quark obtained from (15) and (5) is

$$m_\lambda = 540 \text{ MeV} . \quad (16)$$

V. S-WAVE MESONS ($I \neq 0$)

The zeroth-order wave function for the 36-plet of S-wave mesons consists of the $SU(6)_q \times SU(6)_{\bar{q}}$ wave function multiplied by a spatial wave function $\Psi_0'(|\vec{r}_1 - \vec{r}_2|)$. Our first-order mass formula is

$$M = M_0' + \Delta m_1 + \Delta m_2 + a'(m_1^{-1} + m_2^{-1} - 2m_{\wp}^{-1}) + (\alpha Q_1 Q_2 - \tfrac{4}{3}\alpha_s)\left[b' - \frac{c'}{m_1 m_2} - d'\left(\frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{16\vec{s}_1\cdot\vec{s}_2}{3m_1 m_2}\right)\right] + X . \quad (17)$$

The parameters are defined as in Sec. III with Ψ_0' replacing Ψ_0. X stands for the annihilation term, relevant only to $I = 0$ states, which are separately discussed in Sec. VI.

Expanding to first order in SU(3) breaking, and neglecting electromagnetic effects, we obtain the mass formula (for $I \neq 0$ states)

$$M = A' + B'\left(\frac{\Delta m_1 + \Delta m_2}{m_{\wp}}\right) + C'\vec{s}_1\cdot\vec{s}_2\left(1 - \frac{\Delta m_1 + \Delta m_2}{m_{\wp}}\right) . \quad (18)$$

We obtain the four masses π, K, ρ, and K^* in terms of three new parameters and $(\Delta m_\lambda / m_{\wp})$, which is determined from baryon masses. Thus we obtain one relation, which may be written

$$\frac{K^* - K}{\rho - \pi} = \frac{2(\Sigma^* - \Sigma)}{2\Sigma^* + \Sigma - 3\Lambda} = \frac{m_{\wp}}{m_\lambda} . \quad (19)$$

This relation is well satisfied by observed masses.

Again, we can understand this relation qualitatively: The K^*-K and ρ-π mass differences are hyperfine splittings, and they are inversely proportional to the product of the masses of the constituent quarks.

For the electromagnetic mass differences among

TABLE I. Baryon magnetic moments, in nuclear magnetons.

Particle	p	n	Λ	Σ^+	Σ^-	Ξ^-	Ξ^0
Theory	input	−1.86	−0.60	2.67	−1.05	−0.46	−1.39
Experiment	2.793	−1.91	−0.67 ± 0.06	2.62 ± 0.41	−0.4 ± 1.2	−1.93 ± 0.75	?

mesons, we obtain two relations:

$$\tfrac{1}{4}(K^0 - K^+) + \tfrac{3}{4}(K^{*0} - K^{*+}) \leq \xi\,, \tag{20a}$$

$$(K^0 - K^+) + \tfrac{2}{3}(\pi^+ - \pi^0) = \xi\,, \tag{20b}$$

where

$$\xi = [(n-p) + \tfrac{1}{3}(\Sigma^+ + \Sigma^- - 2\Sigma^0)]\frac{K - \pi + 3(K^* - \rho)}{2(3\Lambda - 2N - \Sigma)}$$
$$= 2.4\ \text{MeV}\,.$$

Neither relation is satisfied by experimental data: The left-hand side of (20a) is 5.6 ± 1.4 MeV and the left-hand side of (20b) is 7.03 ± 0.13 MeV. We interpret the failure of the first relation to mean that the experimental determination of the K^* electromagnetic mass splitting[38] is in error. The failure of the second formula may result from the neglect of virtual annihilation of π^0 into a state containing a photon and gluons.

VI. THE PROBLEM OF NEUTRAL ISOSCALAR MESONS

Our formula (18) for the masses of $L = 0$ mesons predicts the following values for the isoscalar members of the two nonets:

$$\eta(548) = \pi(138)\,,$$

$$\eta'(958) = 2K - \pi = 854\ \text{MeV}\,,$$

$$\omega(784) = \rho(770)\,,$$

$$\varphi(1019) = 2K^* - \rho = 1014\ \text{MeV}\,.$$

While these results are acceptable for the 1^- states, they are no sensible approximation for the 0^- states. The reason for this apparent failure of our approach, the mechanism that splits the η and π masses, can be discovered in our previous analysis[23] of the newly discovered[3,4] $\Psi(3105)$ or J particle. This state was interpreted as a bound state of a charmed quark and its antiquark. The problem was to understand its narrow hadronic width. To lowest order in α_s, the decay of J into hadrons involves the annihilation of the charmed quarks into three colored gluons, and is proportional to the imaginary part of the diagram in Fig. 1(a). The ratio of hadronic to leptonic widths, in the nonrelativistic limit for the bound state, is[22]

$$\frac{\Gamma(J \to \text{hadrons})}{\Gamma(J \to e^+e^-)} = \frac{5(\pi^2 - 9)\alpha_s^{\,3}}{18\pi\alpha^2}\,. \tag{21}$$

Using 80 keV and 4 keV for the observed hadronic and leptonic widths, we obtain[39] $\alpha_s(J) = 0.23$ from (21). A very similar expression can be written[23] for the much larger ratio $\Gamma(\varphi \to 3\pi)/\Gamma(\varphi \to e^+e^-)$. We obtain $\alpha_s(\varphi) = 0.5$. These values of α_s correspond to the effective coupling constant at 3.1 GeV and 1 GeV in the sense of renormalization-group improved perturbation theory: To this order the three-gluon cut is the only contribution to the imaginary part if the particles are lighter than twice the mass of the constituent quarks. On the other hand, $\alpha_s(\varphi)$ can be related to $\alpha_s(J)$ by the familiar asymptotic-freedom argument[15,18]

$$\alpha_s(J) = \left[1 - \frac{25}{12\pi}\alpha_s(\varphi)\ln(\varphi/J)\right]^{-1}\alpha_s(\varphi) \sim 0.28\,, \tag{22}$$

with a result in agreement with the preceding estimates. Below the φ mass, α_s becomes large (infrared slavery) and we no longer expect (22) to continue to be a good approximation.

For 0^- mesons, the annihilation diagram [Fig. 1(b)] has a two-gluon intermediate state and is expected to be more important: It contains fewer powers of α_s.

The mechanism annihilating a quark-antiquark pair $\bar{q}_i q_i$ into gluons and back to a pair $\bar{q}_j q_j$ contributes to the mass matrix of isoscalar mesons. The real parts of the diagrams in Figs. 1(a) and 1(b) contribute to the mass matrix for 1^- and 0^- states, respectively. For masses below 1 GeV, higher-order diagrams are apt to be important as well.

Let $\beta(\mu)$ denote the contribution of the annihilation term to the mass matrix connecting $\bar{q}_i q_i$ to $\bar{q}_j q_j$. Evidently, β is independent of i and j since the colored gluons couple equally to different quark types: β contributes only to the SU(3) [or, SU(4)] singlet state. Gauge invariance requires $\beta(M) \sim M^2$ as $M \to 0$, and asymptotic freedom re-

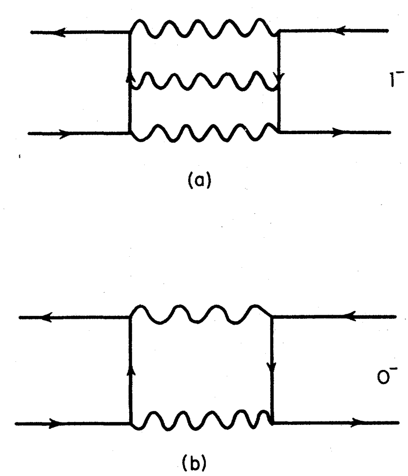

FIG. 1. Virtual annihilation of neutral mesons into gluons. (a) States with $J^P = 1^-$ are coupled to no fewer than three gluons. (b) States with $J^P = 0^-$ couple to two gluons.

quires $\beta \to 0$ as $(\log M)^{-p}$ (with $p=2$ or 3 for $J=0$ or 1) as $M \to \infty$. For the 3×3 mass matrix describing π^0, η, and η' we may write

$$M = \begin{pmatrix} \beta+\pi & \beta & \beta \\ \beta & \beta+\pi & \beta \\ \beta & \beta & \beta+2K-\pi \end{pmatrix}.$$

One eigenvector of M is π^0 with mass degenerate with $\pi^{\pm}$: The annihilation mechanism preserves isotopic-spin invariance. The other eigenvalues follow from the characteristic equation

$$\beta(M) = (2K+\pi-M)(M-\pi)(4K-\pi-3M)^{-1}. \qquad (23)$$

Identifying these eigenvalues with $\eta(549)$ and $\eta'(948)$ we obtain

$$\beta(549) = 630 \text{ MeV}, \quad \beta(948) = 83 \text{ MeV}. \qquad (24)$$

Indeed, β is large, and decreases with energy as we anticipated.

In Fig. 2 we display the right-hand side of (23) and a guess for $\beta(M)$ consistent with the constraints of gauge invariance, infrared slavery, and asymptotic freedom, and intersecting the former curve at the observed masses of η and η'. Observe that our formalism would break down if the π mass were too small: If one imagines reducing the pion mass, a point is reached where two additional roots appear somewhat above the pion mass and with opposite metric.

We apply the same analysis to the 1^- states. Here, we expect smaller values of β', since the annihilation term for 1^- requires three gluons rather than two. We obtain

$$\beta'(\omega) = 7.2 \text{ MeV}, \quad \beta'(\varphi) = 5.4 \text{ MeV}. \qquad (25)$$

As expected, β' is small and even seems to be decreasing.

The annihilation term is relevant only to the four states η, η', ω, and φ. No other mesons are S-wave, light, and neutral.

Before closing this section, let us mention the so-called η problem of current algebra. With the usual smoothness assumptions that the inverse meson propagators are linear in q^2, it is possible to deduce[40] that the light isoscalar 0^- meson is lighter than $\sqrt{3}\,\pi \sim 240$ MeV. We have not encountered this problem simply because the annihilation term does not satisfy the usual smoothness assumption. Most other current-algebra results depend on smoothness of the pion propagator and are unchanged, because the pion propagator receives no annihilation contribution. It remains, of course, a challenge to correctly compute the decay rates of η and η'.

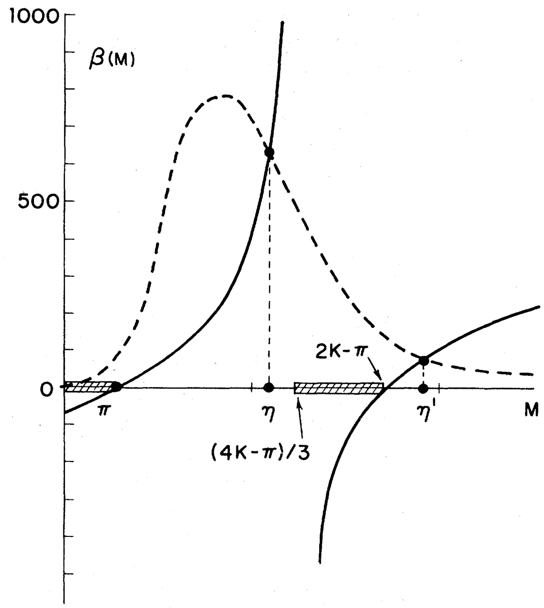

FIG. 2. Characteristic equation (solid line) for the masses of the neutral $J^P = 0^-$ mesons and a guess for $\beta(M)$ (dashed line).

Our solution to the η-π mass-splitting problem is somewhat *ad hoc*. It would be consistent with our view to treat the annihilation term perturbatively, as we treat the rest of the short-range interactions. This would lead to the mass relation $\beta(m_\eta) = \beta(m_{\eta'})$, which is in disagreement with experiment. Thus we are forced to assume that the annihilation term is more strongly mass dependent than the other interactions. The arguments given previously to support this assumption may not be completely convincing, but at least the mass dependence we need to explain the π-η-η' mass spectrum is in agreement with our theoretical expectations.

VII. P-WAVE HADRONS

The first-order mass formula for the P-wave mesons is

$$-\tfrac{4}{3}\alpha_s\Big(b'' - \frac{c''}{m_1 m_2} - \frac{d''}{m_1 m_2}[\vec{L}\cdot(\vec{s}_1+\vec{s}_2) + \tfrac{4}{5}\vec{s}_1\cdot\vec{s}_2 - \tfrac{3}{5}\{\vec{s}_1\cdot\vec{L}, \vec{s}_2\cdot\vec{L}\}]$$

$$- d''\frac{m_1^2+m_2^2}{4m_1^2 m_2^2}\vec{L}\cdot(\vec{s}_1+\vec{s}_2) + d''\frac{m_1^2-m_2^2}{4m_1^2 m_2^2}\vec{L}\cdot(\vec{s}_1-\vec{s}_2)\Big), \qquad (26)$$

where $d'' = \langle | \vec{r}_1 - \vec{r}_2 |^{-3} | \rangle$. There is no Fermi term because $\langle | \delta(r_1 - r_2) | \rangle = 0$ in a P-wave state. For a similar reason, we have omitted the annihilation term. The Breit interaction (the d'' term) involving spin-orbit and tensor interactions appears instead. Expanding to first order in SU(3) breaking, we find

$$M = A'' + B''\left(\frac{\Delta m_1 + \Delta m_2}{m_{\mathcal{P}}}\right) + C''\left[\left(1 - \frac{\Delta m_1 + \Delta m_2}{m_{\mathcal{P}}}\right)\left[\tfrac{3}{2}\vec{L}\cdot(\vec{s}_1 + \vec{s}_2) + \tfrac{4}{5}\vec{s}_1\cdot\vec{s}_2 - \tfrac{3}{5}\{\vec{s}_1\cdot\vec{L}, \vec{s}_2\cdot\vec{L}\}\right] + \frac{1}{2}\left(\frac{\Delta m_2 - \Delta m_1}{m_{\mathcal{P}}}\right)\vec{L}\cdot(\vec{s}_1 - \vec{s}_2)\right]. \tag{27}$$

We may determine the three new parameters A'', B'', and C'' by inputting the masses of $A_2(1310)$, $\delta(976)$, and $K^*(1420)$. Then, all other P-wave meson masses are predicted. The results of this exercise are compared with experiment in Table II. Note that the successful prediction of the B mass depends upon the explicit relation between the spin-orbit and tensor pieces of the Coulomb interaction: Our result is an improvement over the equal-spacing rules ($A_2 - B = B - A_1 = A_1 - \delta$) obtained with a spin-orbit term alone. The $D(1285)$ is not included in Table II. It could be the $I = 0$, $J^{PC} = 1^{++}$ state which we predict at 1161 MeV, but we are at a loss to explain the 124 MeV discrepancy in our prediction of its mass. Alternatively, it may be that D is a 0^- or 2^- excitation of η. The A_1 and κ, if they truly exist, could be anywhere near 1100 MeV and 1300 MeV, respectively: There is no evident disagreement here. Two strange states are observed in the so-called Q region, and indeed, two are predicted. There are three missing $\bar{\lambda}\lambda$ states which should have decay modes only into states containing a pair of strange mesons, or an η.

The P-wave baryon system is more complicated. We will make the simplifying assumption that the relative orbital angular momentum of each quark pair is a superposition of $L = 0$ and 1. Even with this restriction, it is difficult to parameterize $\langle S_{ij}\rangle$ in any model-independent way. However, the contribution of the Fermi interaction is expressed in terms of a single parameter. This term splits the nonstrange P-wave baryons as follows: The two Δ's with $J = \frac{3}{2}$ and $\frac{1}{2}$ and spin $\frac{1}{2}$ together with the three N's with $J = \frac{5}{2}$, $\frac{3}{2}$, and $\frac{1}{2}$ and spin $\frac{3}{2}$ are all left degenerate. They should be heavier than the remaining two states, the two N's with $J = \frac{3}{2}$ and $\frac{1}{2}$ and spin $\frac{1}{2}$. Indeed, this is consistent with what is observed. The well-established states $\Delta(1650, \frac{1}{2}^-)$, $\Delta(1670, \frac{3}{2}^-)$, $N(1670, \frac{5}{2}^-)$, and $N(1700, \frac{1}{2}^-)$, as well as a questionable $N(1700, \frac{3}{2}^-)$ state, are nearly degenerate. Significantly lower in mass lie the two approximately degenerate states $N(1520, \frac{3}{2}^-)$ and $N(1535, \frac{1}{2}^-)$. These are just the seven states required to fill the $L = 1$, 70-plet, and there is no evidence for the existence of any other low-lying odd-parity N or Δ state.

The Fermi term also acts to depress the mass of SU(3)-singlet states relative to SU(3) octets with spin $\frac{1}{2}$. This could account for the anomalously small masses of the approximately SU(3) singlet states $\Lambda(1405, \frac{1}{2}^-)$ and $\Lambda(1520, \frac{3}{2}^-)$. However, this term does not explain the 115 MeV splitting between these two states. A complete description of the P-wave baryon system must await a more detailed model for the zeroth-order wave functions.

TABLE II. Predicted and observed $L = 1$ mesons. (Masses in MeV.)

J^{PC} \ Constituent quarks	$\bar{\mathcal{P}}\mathcal{P}, \bar{\mathcal{P}}\mathcal{N}$ $I = 1$	$\bar{\mathcal{P}}\mathcal{P}, \bar{\mathcal{N}}\mathcal{N}$ $I = 0$	$\lambda\mathcal{P}, \bar{\lambda}\mathcal{N}$	$\bar{\lambda}\lambda$
2^{++}	input $A_2(1310)$	1310 $f(1270)$	input $K^*(1420)$	1530 $f'(1516)$
1^{+-}	1223 $B(1237)$	1223 not seen	 1372 and 1321	1508 not seen
1^{++}	1161 $A_1(1100?)$	1161 not seen	$Q(1240\text{–}1400)$	1494 not seen
0^{++}	input $\delta(976)$	976 $S(993)$	1212 $\kappa(1300?)$	1448 not seen

VIII. CHARM

Our view of the current picture of particle physics, as sketched in the Introduction, leads to an adequate dynamical explanation of mass splittings within supermultiplets. Of course, much more could and should be done using explicit models (potentials, strings, bags, etc.) to compute more about the intermultiplet splittings, and even the identities of and splittings among observed supermultiplets. We have thus far restricted ourselves to a world without charm. But we know that charm must exist, and that charmed hadrons may not be too heavy, if this interpretation of particle physics is to make sense.

It is with great joy, therefore, that we interpret[23] the recently discovered resonance at 3.105 GeV as a vector meson, the lightest $L=0$, $J=1$ bound state of a charmed quark and antiquark. We expect the dominant contribution to its mass to be the rest mass of the constituent quarks. Thus, the charmed-quark mass is expected to be near 1.5 GeV. The difference in mass between $\mathcal{P}'$ and the uncharmed quarks is comparable to (in truth, larger than) the mass splitting between radial excitations [it is only 600 MeV between J or Ψ and its first excitation $\Psi(3.7)$]. Thus, we cannot expect our previous perturbative analysis reliably to extend to states containing charmed quarks. On the other hand, we do expect our approach to give a reasonably good description of the mass splittings of mesons or baryons with a fixed value of charm (e.g., strange and nonstrange, $J=0^-$ and 1^- mesons with charm one.) These should be well described in terms of the quark masses and the hyperfine interactions. Of course, we may not expand reciprocal quark masses to first order in $m_{\mathcal{P}'} - m_{\mathcal{P}}$, and consequently we do not anticipate the success of SU(4) formulas analogous to the Gell-Mann–Okubo formula.[41,42] Our results indicate which of the charmed hadrons decay weakly, and which have allowed electromagnetic or strong decay modes. These qualitative features should certainly survive in a more ambitious approach using explicit potentials and wave functions, while our estimates of absolute masses of charmed states are less trustworthy.

Before we proceed further, we note that our supermultiplets are representations of $SU(8)_q \times SU(8)_{\bar{q}} \times O(3)$. The $L=0$ mesons are $J=0$ and $J=1$ 16-plets consisting of an SU(4) singlet and 15-plet, containing an SU(3) $\bar{3}$ of charm-one states and a 3 of charm-minus-one states. The representations of SU(4) may be displayed as polyhedra in a space whose coordinates are T_3, Y, and charm. Corresponding to the meson multiplets is the Archimedean solid shown in Fig. 3. It is

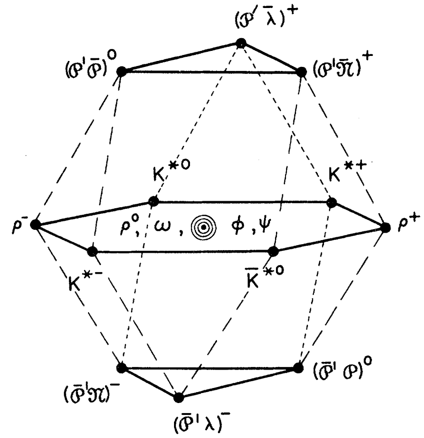

FIG. 3. The 16 vector mesons: The vertical axis is charm. The bull's eye in the center indicates four states.

known as a cuboctahedron. The SU(4) representation corresponding to the baryon octet is 20-dimensional, and it is recognized in Fig. 4 to describe another Archimedean solid, the truncated tetrahedron. As well as an uncharmed octet, it contains a $6+\bar{3}$ of singly charmed states and a 3 of doubly charmed states. The baryon decuplet also extends in SU(4) to another inequivalent isocuplet containing the uncharmed decuplet, a singly charmed 6, doubly charmed 3, and a doubly charged, triply charmed singlet (the $J=\frac{3}{2}$ $3\mathcal{P}'$ state taking over Ω^-'s role as the heaviest "stable" hadron). The regular tetrahedron corresponding to this representation is shown in Fig. 5.

Not until many of these predicted charmed states are discovered and measured can the subject of hadron spectroscopy join its distinguished colleagues, atomic and nuclear spectroscopy, as

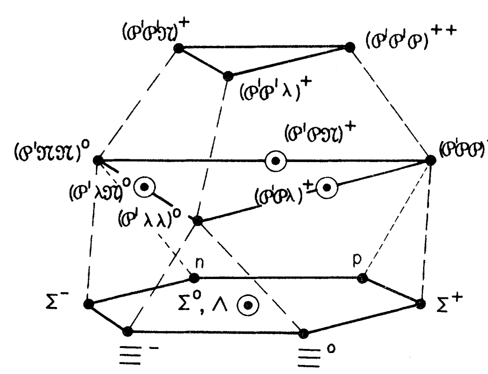

FIG. 4. The $J^P=\frac{1}{2}^+$ baryon octet and its charmed partners. Circled dots indicate positions where two states are located.

subjects certainly worthy of continued study, but understood (at some level) in principle.

IX. NEW HADRONS

By new hadrons, we mean states containing one or more charmed quarks. The first such state to be seen is the J or Ψ at 3.105 GeV. Recently published data[12] on the total cross section for $\sigma(e^+e^- \to \text{hadrons})$ show what could be interpreted as a threshold between center-of-mass energies of 3.5–4 GeV, temptingly associated with the production of pairs of charmed mesons. Recently published experiments in neutrino physics showing departures from charge symmetry,[43] the observation of dimuon events,[44] and the appearance of structure in hadron mass distributions,[45] and scaling-variable plots[46] suggest that the final hadron state sometimes contains a single charmed hadron. All these effects had been predicted.[47-49] Under these exciting circumstances, we are compelled to offer our predictions concerning the new hadrons, despite our qualms about their reliability.

The levels of charmonium. We apply (17) to the 3.105 GeV J or Ψ particle. The annihilation term, small for the φ, is certainly negligible here. The mass of charmonium determines the mass of the charmed quark, with a small uncertainty associated with the kinetic-energy term. We find

$$1690\text{ MeV} \geq m_{\mathcal{P}'} \geq 1630\text{ MeV}\,. \qquad (28)$$

This evaluation will be used in our calculation of charmed-hadron masses. For the pseudoscalar counterpart of the J or Ψ, so-called paracharmonium, the hyperfine splitting analogous to ρ-π splitting is predicted to be quite small

$$\text{ortho-para} = (\rho - \pi)(m_{\mathcal{P}}/m_{\mathcal{P}'})^2 \sim 27\text{ MeV}\,, \qquad (29)$$

where we used our estimate (15) for $m_{\mathcal{P}}$. We interpret $\Psi(3.695)$ as a radial excitation of the charmonium ground state analogous to $\rho'(1600)$ but still below, or just above, charm threshold. Its ortho-para splitting should be similar to that of the ground state. Neither pseudoscalar para state is formed by e^+e^- to order α, but they should both be more readily produced in hadron collisions than the vector states. This is for the same reason that their hadronic decay widths are expected to be larger than those of orthocharmonium: They are coupled to uncharmed quark pairs by the exchange of two gluons rather than three.

The P-wave ground states of charmonium belong to the same $SU(8) \times SU(8) \times O(3)$ supermultiplet as A_2, B, etc., and we may boldly use (26), together with our estimates of quark masses, to predict their masses. We obtain

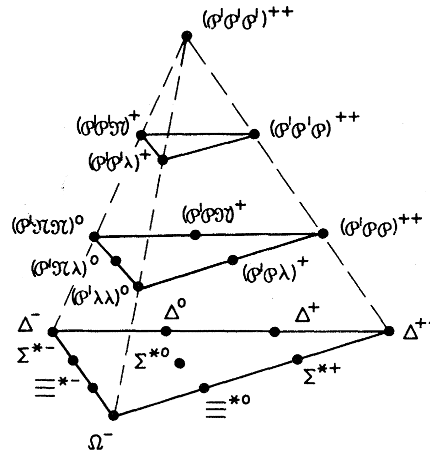

FIG. 5. The $J^P = \frac{3}{2}^+$ baryon decuplet and its charmed partners.

$$P(1^{+-}) \sim 3650\text{ MeV}\,,$$

$$P(2^{++}) - P(1^{+-}) \sim 3.6\text{ MeV}\,,$$

$$P(1^{++}) - P(1^{+-}) \sim -2.6\text{ MeV}\,,$$

$$P(0^{++}) - P(1^{+-}) \sim -10.4\text{ MeV}\,.$$

Their over-all mass is perhaps our least reliable prediction; probably more believable are various dynamical calculations.[50,51] The rather small predicted intermultiplet splittings follow directly from the large mass of the charmed quark, and are more likely to be true. Even more plausible are the weaker relations

$$\frac{P(2^{++}) - P(0^{++})}{P(2^{++}) - P(1^{++})} = \frac{27}{17}\,,$$

$$4P(1^{++}) = 3P(1^{+-}) + P(0^{++})\,.$$

Charmed mesons. The uncertainty of the charmed-quark mass drops out of one relation deduced from (17). We obtain a relatively firm prediction for the mass of the strange charmed vector meson which is numerically coincident with the equal-spacing rule,

$$M(\mathcal{P}'\bar{\lambda}, J^P = 1^-) = \tfrac{1}{2}(J + \varphi) = 2061\text{ MeV}\,.$$

Our predictions for the masses of the remaining charmed S-wave mesons are given in Table III.

TABLE III. The masses of the charmed mesons (in MeV).

S \ J^P	0^-	1^-
0	1800 to 1860	1930 to 1990
1	1975	2061

The lightest charmed meson is the nonstrange pseudoscalar at ~1.83 GeV. Thus charm threshold is ~3.7 GeV, consistent with the observed behavior of the cross section $\sigma(e^+e^- \to \text{hadrons})$.[12] In Fig. 6 we display the predicted spectrum of these states, the allowed strong and electromagnetic transitions among them, and some characteristic weak two-body decay modes. We predict that the 0^- charmed mesons decay weakly, and the 1^- charmed mesons largely if not exclusively decay radiatively. The main weak decays of the pseudoscalars are

$$(\mathcal{P}'\mathcal{P}),\ (\mathcal{P}'\mathcal{N}) \to \bar{K} + \text{pions}\,,$$

$$(\mathcal{P}'\bar{\lambda}) \to K\bar{K} + \text{pions}\,.$$

We are assuming that the dominant decay modes of charmed hadrons arise from the part of the weak Hamiltonian proportional to $\cos^2\theta$. It is possible that the part proportional to $\sin\theta\cos\theta$, which allows the transition $\mathcal{P}' \to \mathcal{P}$, is enhanced by the same mechanism that produces the $\Delta I = \frac{1}{2}$ law. Thus, charmed mesons may decay significantly into final states without kaons.[52]

Charmed baryons. As they were for charmed mesons, our estimates of the over-all masses of these states are not so reliable as our estimates of the splittings among them. We compute the charmed-baryon masses from (3) and from our estimates of quark masses (15), (16), and (28). Our predictions are shown in Table IV, where N stands for $J=\frac{1}{2}^+$ baryons and Δ for $J=\frac{3}{2}^+$ baryons. The masses of charm $=c$ baryons may be underestimated by as much as $c \times 100$ MeV owing to the uncertainty in $m_{\mathcal{P}'}$.

The predicted spectrum of singly charmed baryons is given in Fig. 7. Strong decay modes between states of different strangeness are kinematically forbidden. For each of the three values of strangeness, $S = 0, -1, -2$, the lightest $(J=\frac{1}{2}^+)$ is stable against all but weak decay.

A more detailed view of the singly charmed nonstrange baryons is shown in Fig. 8. In contrast to the Σ-Λ system, the strong (pionic) decay of the isovector $J=\frac{1}{2}$ state into the isoscalar $J=\frac{1}{2}$ state is allowed. This follows, in our analysis, from the fact that $\mathcal{P}'$ is much heavier than the uncharmed quarks.

An enlarged view of the singly charmed $S=-1$ baryons is shown in Fig. 9. Some interesting allowed weak two-body decays are

$$(\mathcal{P}'\lambda\mathcal{N};\ J=\tfrac{1}{2})^0 \to \begin{cases} \Xi^-\pi^+ \\ \Sigma^+K^- \end{cases},$$

$$(\mathcal{P}'\lambda\mathcal{P};\ J=\tfrac{1}{2})^+ \to \begin{cases} \Xi^0\pi^+ \\ \Sigma^+K^0 \end{cases}.$$

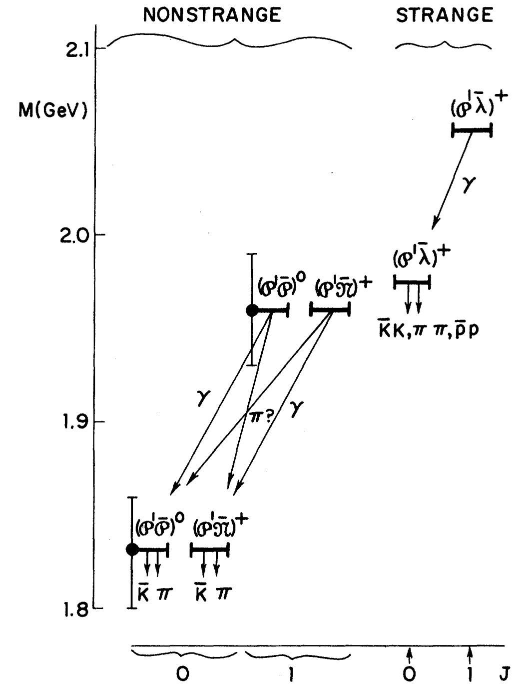

FIG. 6. Spectrum of charmed mesons with their allowed decays.

The $S=-2$, spin-$\frac{1}{2}$, charmed baryon is also stable; some of its weak decays are

$$(\mathcal{P}'\lambda\lambda;\ J=\tfrac{1}{2})^0 \to \begin{cases} \Omega^-\pi^+ \\ \Xi^0\bar{K}^0 \\ \Sigma\bar{K}\bar{K} \end{cases}.$$

For completeness, the spectrum of doubly charmed baryons is shown in Fig. 10. We do not expect these states to be discovered in the immediate future. The heaviest stable baryons would be $(\mathcal{P}'\mathcal{P}'\mathcal{P}')$, with spin $\frac{3}{2}$.

TABLE IV. The masses of the charmed baryons (in MeV). (N stands for $J^P=\frac{1}{2}^+$ states; Δ stands for $J^P=\frac{3}{2}^+$ states.)

C \ S	0	−1	−2
1	N^+(2200) $N^{0,+,++}$(2360) $\Delta^{0,+,++}$(2420)	$N^{0,+}$(2420) $N^{0,+}$(2510) $N^{0,+}$(2560)	N^0(2680) Δ^0(2720)
2	$N^{+,++}$(3550) $\Delta^{+,++}$(3610)	N^+(3730) Δ^+(3770)	
3	Δ^{++}(4810)		

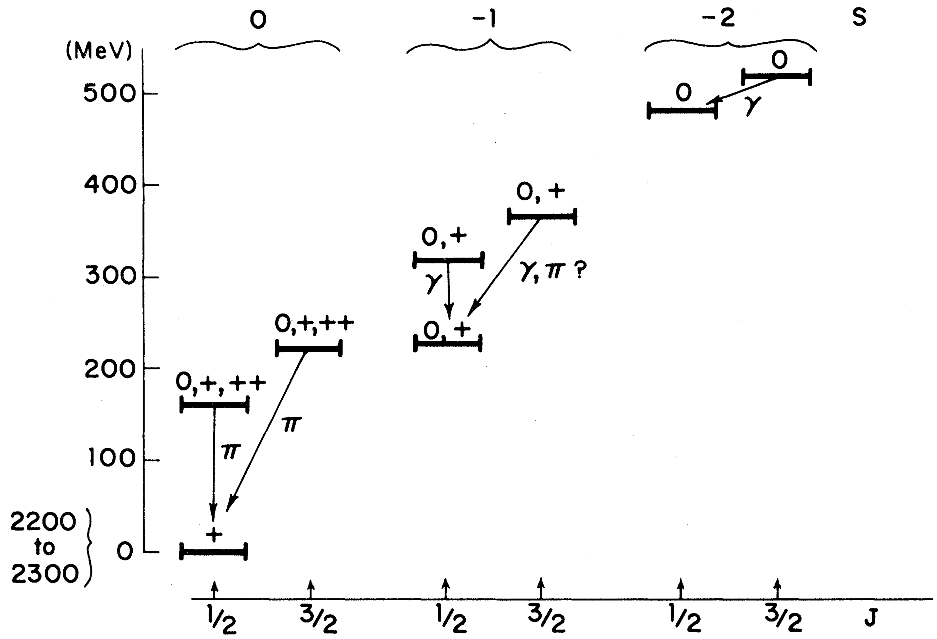

FIG. 7. Spectrum of singly charmed baryons with their strong and radiative decays.

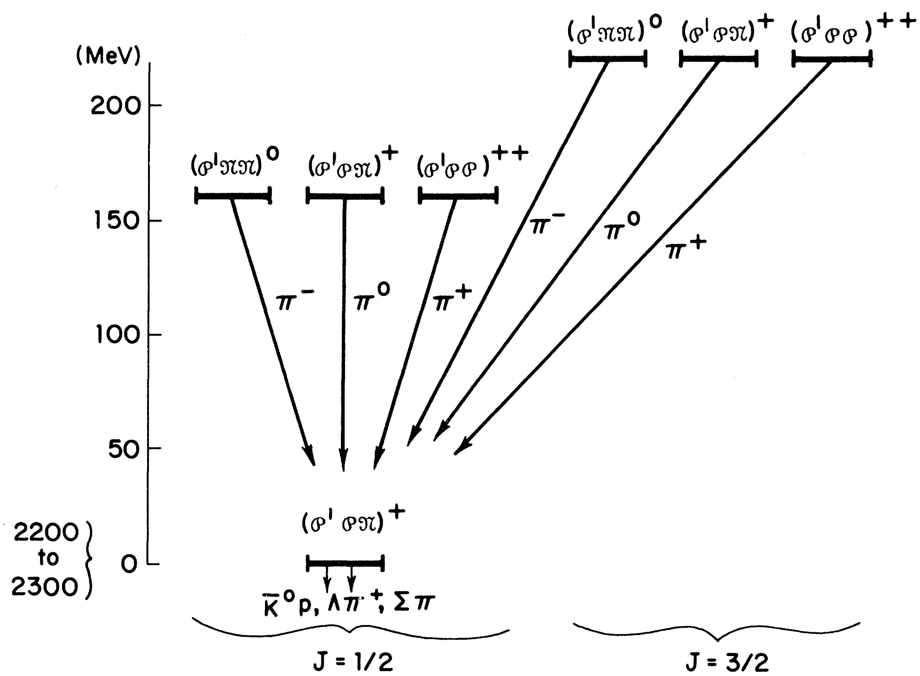

FIG. 8. Detail of the singly charmed nonstrange baryons.

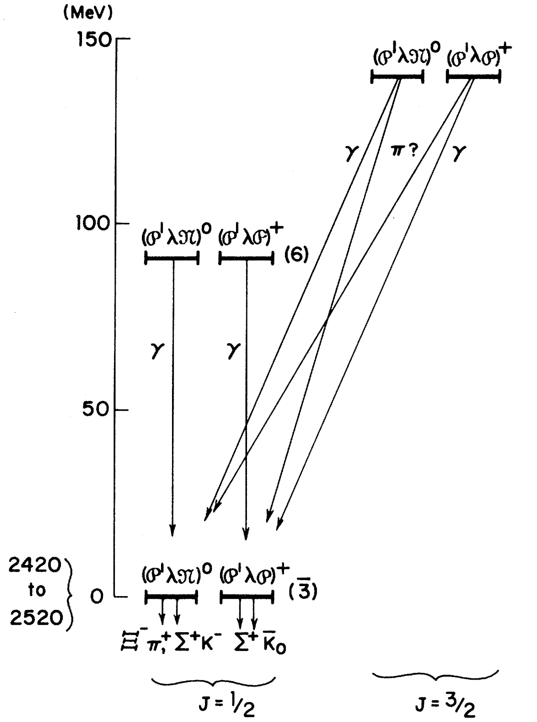

FIG. 9. Detail of the singly charmed $S = -1$ baryons.

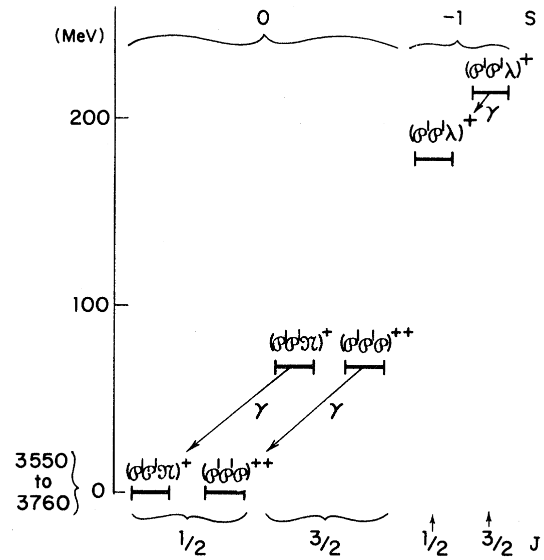

FIG. 10. Spectrum of doubly charmed baryons and their radiative decays.

X. AFTERWORD

There are several purposes to this perhaps too ambitious work: We are not merely interested in explaining hadron masses. We feel that many of the wilder theoretical oats sown in recent years, and nourished by new and exciting experimental discoveries, are yielding a truly bountiful harvest. The naive quark model, supplemented by color gauge theory, asymptotic freedom, and infrared slavery, is turning out to be not so naive, and more than just a model. The demand for charm coming from abstract arguments about selection rules and triangle anomalies in unified models of weak and electromagnetic interactions may soon be met by nature. We can see coming a time when the subject of hadron spectroscopy as it is now known will be generally recognized to be interesting, but no longer truly fundamental. Hadron masses, widths, and cross sections may soon be "understood" if not precisely calculable. This optimistic view may yet be mere illusion, for it depends crucially on the discovery of charmed hadrons, and on the continued development of our theoretical tools. Remember, for example, that arguments for quark confinement, perhaps plausible, are certainly not yet rigorous.

But, if the time does come that hadron physics becomes mere spectroscopy, what are the remaining fundamental questions?

Why are the masses of quarks and leptons what they are? What is the significance of the Cabibbo angle, and can it be computed? Who violates *CP*? And, most profoundly, what is the unified system containing the weak, electromagnetic, and strong interactions? We presume that this will be a gauge theory based on a large but simple local symmetry group. We have seen[53] how such a picture necessarily involves particles with masses comparable to the Planck mass. It has been suggested[54] that it is gravitational attraction that supplies the missing force leading to the binding of Goldstone bosons necessary for the spontaneous breaking of the gauge group, and it is at the Planck mass that this force becomes relevant.

A disquieting aspect of current particle theory is the appearance of exact global symmetries which are not local and not associated (as electric charge is) with massless gauge fields. Corresponding to these symmetries are exact conservation laws for baryon number, electron number, and muon number. These conservation laws are familiarly deduced by imagining the effects of infinitesimal operations performed over all of space-time–not only in the laboratory today, but behind the moon next week. We find such a theoretical construct to be *a priori* absurd, and are therefore relieved that conservation laws of this kind, in truly unified theories, are only approximate.[53]

The conjectured existence of black holes presents another philosophical argument against the existence of exact global symmetries: If a particle bearing electric charge should fall into a black hole, the memory of its charge is preserved by its electric flux, so that conservation of electric charge in a punctured space-time from which the black hole has been omitted remains a sensible construct. On the other hand, a fallen baryon leaves no trace at all, so that a conservation law for baryon number is logically inconsistent.

An obstacle to major progress in particle physics is the difficulty in accommodating our microscopic theory of Lorentz-invariant quantum mechanics with macroscopic gravitational theory. We have mentioned what we suspect are harbingers of an eventual rapprochement.

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*Work supported in part by the National Science Foundation under Grant No. MPS73-05038-A01.

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