## Determination of coupling constants and helicity amplitudes in decay processes $*$

Chien-er Lee and Gan-shu Lee

Physics Department, National Cheng Kung University, Tainan, Taiwan, Republic of China (Received 2 April 1975)

The two coupling constants and the helicity amplitudes of the decay processes  $(1)$  a spin-J baryon decaying into a pseudoscalar meson and a spin- $\frac{3}{2}$  baryon, and (2) a spin-J boson decaying into a pseudoscalar meson and a spin-1 boson with normality opposite to that of the decaying boson] are expressed in terms of the partial-wave amplitudes. The expressions are exact, without involving any kinematical and dynamical assumptions except Lorentz invariance. The values of the coupling constants and the helicity amplitudes are then calculated for baryons of spin  $\frac{3}{2}$ ,  $\frac{5}{2}$ , and  $\frac{7}{2}$  and for bosons of spin 1 by ignoring the high-partial-wave contribution. Under the above approximation, simple relations between the coupling constants and between the helicity amplitudes are obtained. The results are found to be exactly the same as those predicted by  $SU(2)_\mathbf{W}$  as long as  $SU(2)_\mathbf{W}$  does not forbid any helicity amplitude.

## I. INTRODUCTION

The decay of the high-spin resonance into a pseudoscalar meson and another high-spin resonance involves two or more independent coupling constants. In the case of strong decays, the number of independent coupling constants is equal to the number of partial waves allowed. Therefore, when the resonances are baryons, the number of the independent coupling constants is equal to  $J+\frac{1}{2}$ , where J is the lesser spin of the two baryons. When the resonances are bosons, there are  $J+1$ or  $J$  independent coupling constants, depending upon whether the normality of the resonances changes or not, where  $J$  is the lesser spin of the two boson resonances, and the normality of the boson is defined as  $P(-)^J$ , with P and J as the intrinsic parity and spin of the boson. The above conclusions are consequences of the conservations of parity and angular momentum.

The usual way to define the coupling constants is to associate them with the increasing number of momenta, since it gives the simplest kinematical factors. Many authors' calculated the coupling constant, which is associated with the minimum number of momenta, from the experimental decay width by ignoring other coupling constants. However, it is possible to calculate all the above coupling constants from the experimental decay widths. Our method is described in Ref. 2.

In the present work, we considered the following decay processes:  $(1)$  a spin-J baryon decays into a pseudoscalar meson and a spin- $\frac{3}{2}$  baryon, (2) a spin-J boson decays into a pseudoscalar meson and a spin-1 boson with normality opposite to that of the decaying boson. The above decay processes contain two partial waves. We first express the coupling constants and the helicity amplitudes in

terms of the partial-wave amplitudes, by assuming Lorentz invariance only, without involving any dynamical and kinematical assumptions. Since the high partial wave contributes little as compared with the low partial wave, in general, owing to the high centrifugal barrier, we calculate the above coupling constants and the helicity amplitudes from the known experimental decay widths by ignoring the high-partial-wave contributions. Under the above approximation, we find simple relations between the two coupling constants and between the helicity amplitudes. From the available data, we calculate the values of the above decay amplitudes for the baryons of spin  $\frac{3}{2}$ ,  $\frac{5}{2}$ , and  $\frac{7}{2}$ , and for the boson of spin 1 we also calculate the  $SU(3)$  coupling constants and find that  $SU(3)$ symmetry is badly broken for the baryons of spin  $\frac{7}{2}$ . The calculated helicity amplitudes are also compared with those predicted by  $SU(2)_w$ . We find that the same results are obtained, as long as  $SU(2)_w$  does not forbid any helicity amplitude.

#### II. BARYON DECAY AMPLITUDES

We define the two independent coupling constants for the decay of a spin- $J$  baryon into a pseudoscalar meson and a spin- $\frac{3}{2}$  baryon by associating them with increasing number of momenta. The express ion is

$$
\left(\frac{p_0 q_0}{Mm}\right)^{1/2} \langle \frac{3}{2}(q, \lambda') | j_{\pi}(0) | J(p, \lambda) \rangle
$$
  

$$
= \overline{U}_{\mu}(q, \lambda') \Gamma' i \gamma_s \left(\delta_{\mu \nu_1} F + \frac{p_{\mu} q_{\nu_1}}{m^2} G\right) q_{\nu_2} \cdots q_{\nu_n}
$$
  

$$
\times \Gamma U_{\nu_1} \cdots \nu_n (\rho, \lambda) M_0^{1-n} . \tag{1}
$$

In expression (1), the  $\lambda$ 's are the helicities, M and *m* are the masses of the spin- $J$  and spin- $\frac{3}{2}$ .

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baryons, respectively,  $n = J - \frac{1}{2}$ ,  $M_0 = 1$  MeV, and the  $\Gamma$ 's are +1 or  $i\gamma_5$ , according as the normality of the baryon is  $+1$  or  $-1$  [the normality of the baryon is defined as  $P(-)^n$ , with P as the intrinsic parity]. We note that expression (1) satisfies parity conservation, and the two independent coupling constants  $F$  and  $G$  are dimensionless.

If we require a real coupling constant  $g$  in the interaction

$$
(-i)^n g \overline{\psi}_{\nu_1} \dots \nu_n(x) Q \psi_N(x) \partial_{\nu_1} \cdots \partial_{\nu_n} \phi_{\pi}(x),
$$

with  $Q = i \gamma_5$  or 1, depending upon whether the normality of the spin-J baryon is  $+1$  or  $-1$ , then the spin-J baryon transforms, under time reversal, as follows:

$$
T\psi_{\nu_1} \dots \psi_n(x) T^+ = P\gamma_1 \gamma_3 \psi_{\nu_1} \dots \psi_n(\bar{x}, -t),
$$
  
\n
$$
T|J(\bar{p}, \lambda)\rangle = P\bar{\eta} * |J(-\bar{p}, \lambda)\rangle,
$$
\n(2)

where P is the intrinsic parity and  $\overline{\eta}$  is determined by

$$
\overline{\eta} U_{\nu_1}^* \dots \nu_n (-\overline{\mathfrak{p}}, \lambda) = \gamma_1 \gamma_3 U_{\nu_1} \dots \nu_n (\overline{\mathfrak{p}}, \lambda)
$$

By using the above convention for time reversal, we can show that the two independent coupling constants defined in expression (1) are real.

By using the Carruthers decomposition<sup>3</sup> for the high-spin spinor, we can express the decay matrix element (1) in terms of the coupling constants and explicit kinematic factors. The expression is

$$
\left(\frac{p_0 q_0}{Mm}\right)^{1/2} \left\langle \frac{3}{2}(q, \lambda')|\, j_\pi(0)|\, J(p, \lambda)\right\rangle = (i \operatorname{sign}\lambda')^{1+\delta} \eta^{-1} \left(\frac{(J+\frac{1}{2})!}{(2J)!!}\right)^{1/2} \left(\frac{q_0-\eta m}{2m}\right)^{1/2} q^{J-3/2} d_{\lambda\lambda'}^{(J)}(\theta)
$$

$$
\times M_0^{-1-n} \left\{ \delta_{\lambda'\pm 3/2} \left(\frac{J+\frac{3}{2}}{2J-1}\right)^{1/2} F + \delta_{\lambda'\pm 1/2} \frac{1}{\sqrt{6}} \left[ \left(\frac{2q_0}{m}-\eta\right) F - \frac{2q^2 M}{m^3} G \right] \right\},\tag{3}
$$

where  $\eta$  is the product of the normalities of the spin-J and spin- $\frac{3}{2}$  baryons. In the above expression, we choose the  $XZ$  plane as the plane formed by q and the spin quantization axis of the spin- $J$  baryon. The helicity amplitude, denoted by  $F_{\lambda}$ , is then defined by the expression

$$
\left(\frac{p_0q_0}{Mm}\right)^{1/2} \left\langle \frac{3}{2}(q,\lambda')\right|j_\pi(0)|J(p,\lambda)\rangle = (i)^{1+\delta\eta-1} \left(\frac{(J+\frac{1}{2})!}{(2J)!!}\right)^{1/2} \left(\frac{q_0-\eta m}{2m}\right)^{1/2} q^{J-3/2} M_0^{-1-n} d_{\lambda\lambda'}^{(J)}(\theta) F_{\lambda'}.
$$
\n(4)

Then by comparing with expression (8) we can express the helicity amplitudes in terms of the coupling constants as follows:

$$
F_{\pm 3/2} = (\pm)^{\delta} \eta_1 \left( \frac{J + \frac{3}{2}}{2J - 1} \right)^{1/2} F,
$$
  
\n
$$
F_{\pm 1/2} = (\pm)^{\delta} \eta_1 \frac{1}{\sqrt{6}} \left[ \left( \frac{2q_0}{m} - \eta \right) F - \frac{2q^2 M}{m^3} G \right].
$$
\n(5)

Since the coupling constants can be expressed in terms of the partial-wave amplitudes'

$$
F = F' + \frac{q^2}{m^2} G',
$$
  
\n
$$
G = \frac{m}{M} \left[ \frac{m}{q_0 + m} F' + \left( \frac{q_0}{m} + 1 + \frac{2(2 + \eta)}{2J - 1} \right) G' \right],
$$
\n(6)

where  $F'$  and  $G'$  are the low- and high-partialwave amplitudes, respectively. We can also express the helicity amplitudes in terms of the partial-wave amplitudes. The expressions are

$$
F_{13/2} = (\pm)^{\delta} n_1 \left( \frac{J + \frac{3}{2}}{2J - 1} \right)^{1/2} \left( F' + \frac{q^2}{m^2} G' \right),
$$
  
\n
$$
F_{11/2} = (\pm)^{\delta} n_1 \frac{1}{\sqrt{6}} \frac{1}{2J - 1} \left[ (2 - \eta)(2J - 1) F' - (2 + \eta)(2J + 3) \frac{q^2}{m^2} G' \right].
$$
  
\n(7)

We note that all the above decay amplitudes are real and dimensionless and that the above relations involve no kinematical and dynamical assumptions except Lorentz invariance for decay matrix elements. We also have the ratios

$$
\frac{F_{1/2}}{F_{-1/2}} = \frac{F_{3/2}}{F_{-3/2}} = -\eta,
$$
\n(8)

which are just the consequences of rotational invariance.

#### III. BOSON DECAY AMPLITUDES

By analogy to the baryon cases, the two independent coupling constants for the decay of a spin- $J$  boson into a pseudoscalar meson and a spin-1 boson with normality opposite to that of the decaying boson are defined by

$$
(4p_0q_0)^{1/2}\langle 1(q,\lambda')|j_\pi(0)|J(p,\lambda)\rangle
$$
  

$$
= \overline{\epsilon}_{\mu}(q,\lambda')\left(\delta_{\mu\nu_1}F+\frac{p_\mu q_{\nu_1}}{m^2}G\right)q_{\nu_2}\cdots q_{\nu_J}
$$
  

$$
\times \epsilon_{\nu_1}\cdots\epsilon_{\nu_J}(p,\lambda)M_0^{2-J},
$$
 (9)

where *m* is the mass of the spin-1 boson,  $\lambda$ 's are helicities, and  $M_0$  is equal to 1 MeV. The nor-

mality of the boson is defined as  $P(-)^{J}$ , with P as the intrinsic parity. We note that the introduction of the factor  $M_0^{2-J}$  makes the coupling constants dimensionless.

As in the baryon case, we adopt the following convention for the time-reversal operation on spin-J bosons:

$$
T\phi_{\mu_1}\ldots\mu_J\left(x\right)T^*=P\phi_{\mu_1}\ldots\mu_J\left(\bar{x},-t\right). \hspace{1cm} (10)
$$

This implies that

$$
T|J(\vec{p},\lambda)\rangle = P\overline{\eta} * |J(-\overline{p},\lambda)\rangle,
$$

with  $\overline{\eta}$  determined by

$$
\overline{\eta} \epsilon_{\mu_1 \ldots \mu_J}^* (-\overline{\mathfrak{p}}, \lambda) = \epsilon_{\mu_1 \ldots \mu_J} (\overline{\mathfrak{p}}, \lambda).
$$

Under the above convention, we can show that the coupling constants defined by expression (9) are real.

By using the Lee decomposition<sup>4</sup> for the polarization tensor, we can express the decay matrix element (9) as

$$
T\phi_{\mu_1} \dots \mu_J(x) T^+ = P\phi_{\mu_1} \dots \mu_J(\bar{x}, -t).
$$
 (10) By using the Lee decomposition<sup>4</sup> for the polarization tensor, we can express the decay matrix element (9) as  
\n
$$
(4p_0q_0)^{1/2} \langle 1(q, \lambda')|j_{\pi}(0)|J(p, \lambda)\rangle = \left(\frac{(J-1)!}{(2J-1)!!}\right)^{1/2} q^{J-1} d_{\lambda, \lambda'}^{(J)}(\theta) M_0^{2-J} \left[\delta_{\lambda'_{1}}\left(\frac{J+1/2}{2}\right)^{1/2} F + \delta_{\lambda',0} \sqrt{J}\left(\frac{q_0}{m}F - \frac{q^2M}{m^3}G\right)\right],
$$
\n(11)

where  $M$  and  $m$  are the masses of the spin- $J$  and spin-1 bosons, respectively. The helicity amplitudes are then defined by the expression

$$
(4p_0 q_0)^{1/2} \langle 1(q, \lambda') | j_\pi(0) | J(p, \lambda) \rangle
$$
  
=  $\left( \frac{(J-1)!}{(2J-1)!} \right)^{1/2} q^{J-1} d_{\lambda, \lambda'}^{(J)}(\theta) M_0^{2-J} F_{\lambda'}.$  (12)

By comparing expression  $(11)$  with expression (12), we can express the helicity amplitudes in terms of the coupling constants as

$$
F_{\pm 1} = \left(\frac{J+1}{2}\right)^{1/2} F,
$$
  
\n
$$
F_0 = \sqrt{J} \left(\frac{q_0}{m} F - \frac{q^2 M}{m^3} G\right).
$$
\n(13)

By means of a method similar to that for the baryon case, we can express the coupling constants in terms of the partial-wave amplitudes. The expressions are

$$
F = F' + \frac{q^{2}}{m^{2}} G',
$$
  
\n
$$
G = \frac{m}{M} \left[ \frac{m}{q_{0} + m} F' + \left( 1 + \frac{q_{0}}{m} + \frac{1}{J} \right) G' \right],
$$
\n(14)

where  $F'$  and  $G'$  are the low- and high-partialwave amplitudes, respectively.

The helicity amplitudes can then be expressed in terms of the partial-wave amplitudes as

$$
F_{\pm 1} = \left(\frac{J+1}{2}\right)^{1/2} \left(F' + \frac{q^2}{m^2} G'\right),
$$
  

$$
F_0 = \sqrt{J} \left[F' - \left(1 + \frac{1}{J}\right) \frac{q^2}{m^2} G'\right].
$$
 (15)

It is obvious that  $F_{+1} = F_{-1}$ , which is just the consequence of rotational invariance.

### IV. APPROXIMATION

It is generally granted that the high-partial-wave contribution can be ignored, in general, because of the high centrifugal barrier. Therefore, we may ignore the contributions from the high-partialwave amplitude  $G'$  in expressions (6), (7), (14), and (15}, and obtain relations which express the coupling constants  $F$  and  $G$  and the helicity amplitudes  $F_{\lambda}$  in terms of the low-partial-wave  $F'$ only.

Under the above approximation, we obtain the correlations between coupling constants and between helicity amplitudes. The relations are

$$
G = \frac{m^2}{M(q_0 + m)} F \tag{16}
$$

and

$$
\frac{F_{3/2}}{F_{1/2}} = \frac{\sqrt{6}}{2-\eta} \left(\frac{J+\frac{3}{2}}{2J-1}\right)^{1/2},
$$
\n(17)

for a spin-J baryon decaying into a pseudoscalar meson and a spin- $\frac{3}{2}$  baryon. For a spin-J boson decaying into a pseudoscalar meson and a spin-1 boson with normality opposite to that of the decaying boson, we obtain the relations

$$
G = \frac{m^2}{M(q_0 + m)} F,
$$
\n(18)

$$
\frac{F_{\pm 1}}{F_0} = \left(\frac{J+1}{2J}\right)^{1/2}.
$$
\n(19)

It is interesting to note that expressions  $(16)$  and (16) are identical in form.

The decay width formula, under the approximation which ignores the high-partial-wave contri-

bottom, becomes

\n
$$
\Gamma(J + \frac{3}{2}0^{-}) = \frac{1}{4\pi} \frac{(J - \frac{3}{2})!}{(2J)!} \frac{2J + 1 + \eta}{2 + \eta} \frac{q_0 - \eta m}{M}
$$
\n
$$
\times q^{2J - 2} M_0^{3 - 2J} F'^2 \tag{20}
$$



TABLE I. Baryon decay amplitudes.  $\alpha$  denotes the SU(3) multiplet. L and S are the total quark orbital angular momentum and spin, respectively. F and G are

<sup>a</sup> Octet coupling constants.<br><sup>h</sup> Singlet coupling constants without dividing by zero. The mixing angle is  $(-23 \pm 4)^\circ$ .<br><sup>c</sup>  $\Lambda$ \* is a  $\frac{3}{2}^-$  resonance with  $m = 1517.8 \pm 1.0$  MeV.

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$\alpha$ , <sup>2S+1</sup> L, $J^P$	Decay modes	Width (MeV)	F	G	$F_{1}$ $\boldsymbol{F}$	$F_{0}$ $\bm{F}$ .	$F_{\lambda}$ $F_{-1}$	$SU(2)_w$ prediction
$8,~^{1}P,~1^+$	$B(1237 \pm 10) \rightarrow \omega \pi$	$\pm 20$ 120	$(3.62 \pm 0.34) \times 10^{3}$	$(1.10 \pm 0.10) \times 10^3$				$F_{\pm 1} = 0$
$8, \, {}^3P, \, 1^*$	$A_1(1100) \rightarrow \rho \pi$ $E(1416\pm10)\rightarrow K\sqrt{kT}$ $K_4(1242 \pm 10) \rightarrow K^* \pi$ $\rightarrow \rho K$ $\rightarrow \omega K$	300 $6 \pm 2$ $31.8 \pm 14.2$ $95.3 \pm 32.7$ $1.27 \pm 0.68$	$(5.99) \times 10^3$ $(1.5 \pm 0.5) \times 10^3$ $(2.11 \pm 0.50) \times 10^3$ $(5.55 \pm 2.08) \times 10^3$ $(0.57 \pm 0.20) \times 10^3$	$(2.04) \times 10^3$ $(0.46 \pm 0.16) \times 10^3$ $(0.74 \pm 0.18) \times 10^3$ $(1.74 \pm 0.64) \times 10^{3}$ $(0.18 \pm 0.05) \times 10^3$				$F_0=0$ , $\frac{F_{+1}}{F_{-1}}$ = 1

TABLE II. Boson decay amplitudes.

for the baryon decays, and becomes

$$
\Gamma(J \to 10^{-}) = \frac{1}{8\pi} \frac{(J-1)!}{(2J-1)!!} \frac{q^{2J-1}}{M^2} M_0^{4-2J} F'^2 \qquad (21)
$$

for the boson decays with normality changes.

#### V. CALCULATIONS AND RESULTS

We first calculate the absolute values of the low-partial-wave amplitudes by setting the experimental partial widths of baryons and bosons equal to expressions (20) and (21), respectively. The coupling constants  $F$  and  $G$  and the helicity amplitudes  $\mathbf{F}_{\lambda}$  are then calculated by using expressions (6) and (7) for baryons and expressions (14) and (15) for bosons by neglecting the highpartial-wave contribution. The SU(3) coupling constants denoted by  $F^0$  and  $G^0$  are also calculated for baryons, by dividing the coupling constants with proper SU(3) isoscalar factors. We also calculate the  $SU(2)_w$  predictions on the helicity amplitudes. The results are given in Table I for the baryon decay amplitudes, and in Table II for the boson decay amplitudes.

In the above calculations, the input data are taken from Samios  $et al.^5$  and the Particle Data Group<sup>6</sup> except the  $A_1$  data, which are taken from Rinaudo et al.'

# VI. CONCLUDING REMARKS

The partial-wave expansions for the coupling constants and the helicity amplitudes given by expressions  $(6)$ ,  $(7)$ ,  $(14)$ , and  $(15)$  are exact without involving any kinematical and dynamical assumptions except Lorentz invariance. The approximation used in the calculations is reasonable because the higher partial wave has the higher contrifugal barrier, which lessens the decay probability. We find that under the above approximation the phase  $constraint<sup>8,9</sup>$ 

$$
2\left(\frac{F_1}{F_0}\right)_{A_1 \to \rho \pi} = \left(\frac{F_0}{F_1}\right)_{B \to \omega \pi} + 1
$$

is satisfied.

From the tables, we see that SU(3) symmetry is badly broken for the decay:  $10(\frac{7}{2}^+) - 10(\frac{3}{2}^+) + 8(0^-)$ . We also see that  $SU(2)_w$  symmetry allows only (1) the longitudinally polarized  $\omega$  meson in  $B(1237)$  $+\omega\pi$ , (2) the transversely polarized  $\rho$  meson in  $A_1(1100) \rightarrow \rho \pi$ , and (3) the helicity amplitudes with  $\lambda = \pm \frac{1}{2}$  in  $N(1520) \rightarrow \Delta \pi$  and  $N(1687) \rightarrow \Delta \pi$ . These predictions are different from the results obtained by neglecting the high partial wave. But  $SU(2)_\nu$ symmetry does not forbid any helicity amplitudes in  $N(1674) \rightarrow \Delta \pi$  and  $\Delta(1930) \rightarrow \Delta \pi$ , and its predicted ratios between the helicity amplitudes are exactly the same as those obtained by ignoring the high partial wave. That is,  $SU(2)_w$  symmetry predicts a vanishing high-partial-wave amplitude as long as it does not forbid any helicity amplitude. The above conclusion is also valid for  $\Sigma(1767) \rightarrow \Lambda^*$  $(1520)\pi$ , in which all helicity amplitudes are allowed by  $SU(2)_w$ . We note that the final  $\Lambda^*$  resonance has one unit of quark orbital angular momentum.

Rev. 159, 1456 (1967).

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