Implications of broken SU(4)⊗SU(4) and scale symmetry for the masses of charmed particles

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Making use of chiral SU(4)⊗SU(4) and scale-symmetry breaking together with ϵ saturation we obtain limits for the charmed-pseudoscalar-meson masses. These limits only allow charmed ($C=\pm 1$) pseudoscalar states with masses below 2.5 GeV. The existence of such states with masses above 1.5 GeV would imply strong restrictions on the theory: δ has at least dimension $l_\delta = 1$, u has at least dimension $l_u = 2$, and the ϵ mass is higher than the upper limit noted in the Particle Data Group table. If the masses of these states are above 2 GeV, it furthermore follows than $l_u = 3$ and $l_\delta = 2$. Assuming the Gaillard-Lee-Rosner relation between the masses of the vector mesons and the quarks, identification of the $\psi(3.1)$ with the ϕ_c implies $l_u = 3$, $l_\delta = 2$, $m_\epsilon \approx 0.92$ GeV, $\Gamma_\epsilon \approx 0.42$ GeV, and $m_D = 2.2$ GeV. We have assumed integer dimensions. We have furthermore assumed that the effective m_ϵ is not much above 0.9 GeV.

A simple extension of the Weinberg-Salam model to include hadrons requires the existence of charm as a new quantum number of the strong interactions.[1] One is thus led to SU(4) rather than SU(3) as the fundamental symmetry of the strong interactions. Since the masses of the charmed and noncharmed particles are considerably different, it is clear from the outset that this symmetry is broken in the real world. Assuming that the SU(4) symmetry-breaking Hamiltonian density transforms as the sum of a singlet and the eighth and 15th components of an SU(4) 15-plet, SU(4) mass formulas then follow in analogy to the Gell-Mann–Okubo mass formula for SU(3). [These SU(4) mass formulas have already been noted in Ref. 2.] In case of the pseudoscalar mesons we will recover these mass formulas in the context of broken SU(4) ⊗ SU(4) symmetry and turn our attention to the breaking of this larger symmetry.

In analogy to broken chiral SU(3)⊗SU(3), it is appealing to consider the situation in which all the pseudoscalar mesons are Goldstone bosons in the symmetry limit. In this way one is able to formulate and deal with broken SU(4)⊗SU(4) in precisely the same way as chiral SU(3)⊗SU(3) has been dealt with in the past. Since there are at present several experimental indications[3] for a new class of particles the enlarged multiplets as required by SU(4) might actually exist. We will take this point of view in the present paper and start by writing the chiral-symmetry-breaking Hamiltonian density u as

$$u(x)=\sigma_0(x)+c_8\sigma_8(x)+c_{15}\sigma_{15}(x)\,, \tag{1}$$

where σ_0 through σ_{15} denote the 16 scalar densities contained in the $(4,\bar{4})\oplus(\bar{4},4)$ representation of SU(4)⊗SU(4). [As is well known, such a symmetry breaking follows from assuming that SU(4)⊗SU(4) is perfect except for a quark mass term.] Furthermore, we denote by ϕ_0 through ϕ_{15} the 16 pseudoscalar densities in the representation considered. Previous treatments of the assumption in Eq. (1) will be discussed after establishing our results. It might be obvious that within the present scheme additional assumptions are required to obtain information on the masses of the charmed pseudoscalar mesons. For example, one might arbitrarily require a certain transformation property of the symmetry-breaking Hamiltonian density under subgroups of SU(4)⊗SU(4). However, we will not take this attitude in what follows. Rather, we investigate the possibility that along with broken SU(4)⊗SU(4), a theory of broken dilation symmetry may be formulated. In this way the conventional ideas[4,5] already contained in the literature concerning scale symmetry breaking may be taken over to the present situation. In other words, we assume that there is a Goldstone ϵ meson state which is responsible for the breaking of scale symmetry. Furthermore, we continue to assume that scale symmetry is broken by u as well as by a possible q-number δ, both of which are assumed to have a dimension. These assumptions then yield restrictions on the masses of the charmed pseudoscalar mesons.

We start by noting that from Eq. (1) it follows that

$$c_8=2(\tfrac{2}{3})^{1/2}\,\frac{m_\pi^{\,2}-m_K^{\,2}}{m_K^{\,2}+m_D^{\,2}}\,, \tag{2a}$$

$$c_{15}=\frac{1}{\sqrt{3}}\,\frac{2m_\pi{}^2+m_K{}^2-3m_D{}^2}{m_K{}^2+m_D{}^2}\,, \tag{2b}$$

$$4m_K{}^2-3m_8{}^2-m_\pi{}^2=0\,, \tag{2c}$$

$$m_D{}^2-m_F{}^2=m_\pi{}^2-m_K{}^2\,, \tag{2d}$$

and

$$m_{15}{}^2=-\tfrac{2}{3}m_\pi{}^2+\tfrac{1}{6}m_K{}^2+\tfrac{3}{2}m_D{}^2\,. \tag{2e}$$

In the above m_D and m_F denote the masses of the $I=\frac{1}{2}$, $S=0$ and $I=0$, $S=\pm1$ charmed mesons, respectively, while m_{15} denotes the mass of the state corresponding to the $I=0$, $Y=0$, $S=0$ 15th component of $\partial^\mu A_\mu$. No assumptions concerning its mixing with $\eta'(958)$ are made here. Equations (2c)–(2e) have already been derived in Ref. 2. The method of that reference is, however, not appropriate in the context of broken scale symmetry.

A simple derivation[6] starts by assuming SU(4) in the form

$$\langle\Omega|A_\mu^\alpha(x)|P^\beta\rangle=if\delta_{\alpha\beta}p_\mu e^{-ip\cdot x} \tag{3}$$

and

$$\langle\Omega|\phi^\alpha(x)|P^\beta\rangle=\delta_{\alpha\beta}re^{-ip\cdot x}\,, \tag{4}$$

where P^α denotes the SU(4) 15-plet of pseudoscalar mesons. Upon multiplying Eq. (3) by ∂^μ and computing $\partial^\mu A_\mu^\alpha$ in terms of the ϕ^α by use of the usual formula $i[Q,u]=\partial^\mu J_\mu$ and comparing to Eq. (4), the results in Eqs. (2) then follow. Another transparent way of arriving at Eqs. (2) would be to start from Eq. (3) together with the saturation assumption

$$f^2m_\alpha{}^2=-i\langle[Q_A^\alpha,\partial^\mu A_\mu^\alpha]\rangle_0 \tag{5}$$

and the SU(4) assumption

$$\langle\sigma_8\rangle_0=\langle\sigma_{15}\rangle_0\approx0\quad, \tag{6}$$

which hold to lowest order in SU(4)$\otimes$SU(4).

We also note[7] that the symmetry-breaking Hamiltonian density u may be expressed in terms of the pseudoscalar meson σ terms by means of the formula

$$u=\frac{i}{C_R}\sum_{\alpha=1}^{15}([Q_V^\alpha,\partial^\mu V_\mu^\alpha]+[Q_A^\alpha,\partial^\mu A_\mu^\alpha]), \tag{7}$$

with C_R the value of the SU(4)$\otimes$SU(4) Casimir operator $C_2\equiv\sum_\alpha\{Q_V^\alpha Q_V^\alpha+Q_A^\alpha Q_A^\alpha\}$ in the irreducible representation of SU(4)$\otimes$SU(4) and parity. Equation (7) follows by first noting that $\partial^\mu J_\mu=i[Q,u]$, so that the expression in Eq. (7) represents the action of the Casimir operator C_2 on u. The result then follows by noting that C_2 has the same value in the (m,n) as in the (n,m). Since C_2 is a positive operator it thus follows that C_R is also positive. In the case at hand a straightforward computation yields $C_R=\frac{15}{2}$ for the $(4,\bar{4})\otimes(\bar{4},4)$ model. Note that, of course, there is also a completely pedestrian method to check Eq. (7) in our model: Compute the σ terms and add them. For our purpose it suffices to check the vacuum expectation value of Eq. (7) under the assumptions in Eq. (6).

The reader should notice the important result[7] implied by Eq. (7): Independent of the model of chiral-symmetry breaking, $\langle-u\rangle_0$ is positive. This has interesting consequences for broken scale symmetry [Eq. (10)]. We may now get an expression for $\langle u\rangle_0=\langle\sigma_0\rangle_0$ by using the fact that to lowest order in SU(4)$\otimes$SU(4) the Goldstone pion saturates $\langle[Q_A^1,\partial^\mu A_\mu^1]\rangle_0$, so that

$$\begin{aligned}f^2m_\pi{}^2&=-i\langle[Q_A^1,\partial^\mu A_\mu^1]\rangle_0\\&=\frac{-1}{2\sqrt{3}}(\sqrt{3}+\sqrt{2}\,c_8+c_{15})\langle\sigma_0\rangle_0.\end{aligned} \tag{8}$$

This has the consequence

$$-\langle u\rangle_0=\frac{2\sqrt{3}\,f^2m_\pi{}^2}{\sqrt{3}+\sqrt{2}\,c_8+c_{15}}\quad. \tag{9}$$

Equation (9) is also easily derived independently of the result in Eq. (7) by explicitly computing $\langle[Q_A^1,\partial^\mu A_\mu^1]\rangle_0$ and using Eq. (6). Note that upon use of Eqs. (2) and (9), knowledge of $\langle u\rangle_0$ is thus equivalent to knowledge of the masses of the charmed pseudoscalar particles.

In the following we next use broken scale symmetry in order to obtain a prediction for $\langle u\rangle_0$. For this purpose we start by noting that if the strong-interaction Hamiltonian density H_{strong} is written as $u(x)+\delta(x)+$(a chiral-invariant part with dimension 4), it then follows that[5]

$$\begin{aligned}-\langle u_0\rangle&=\frac{3}{32\pi\Gamma_\epsilon}\,\frac{[m_\epsilon{}^2+(l_u-2)m_\pi{}^2]^2}{(4-l_u)(l_u-l_\delta)}\\&\quad\times(m_\epsilon{}^2-4m_\pi{}^2)^{1/2},\end{aligned} \tag{10}$$

where l_u and l_δ are the dimensions of u and δ, respectively, and where m_ϵ and Γ_ϵ denote the mass and width of the ϵ meson. As is well known, $l_u<4$ so that from Eqs. (7) and (10) it thus follows that $l_\delta<l_u<4$. We will assume integer values for these dimensions in what follows.

Solving Eqs. (2) and (9) we obtain for m_D the expression

$$m_D=m_\pi\left[\frac{\langle-u\rangle_0}{f^2m_\pi{}^2}-\left(\frac{m_K}{m_\pi}\right)^2\right]^{1/2}\,. \tag{11}$$

The physical values of l_u under our assumption of integer dimensions are $l_u=1,2,3$. In the quark model, the σ_i are given by $\bar{q}\lambda_i q$ such that $l_u=3$.

(The λ_i, d_{ijk}, f_{ijk}, etc., can be found e.g. in Ref. 8). We shall do the calculation for $l_u = 3$ in what follows. For $l_u = 1, 2$ similar predictions result. It is well known that $l_u = 3$ implies violation of multiplet symmetries, such as SU(4) for boson matrix elements of σ_i and/or δ. We refer to the literature for this (e.g., Ref. 9).

If the quark model for the σ_i is assumed, one may define

$$R \equiv (m_c - m_u)/(m_s - m_u) \qquad (12)$$

in terms of the quark masses. If SU(4)⊗SU(4) is only broken by the quark mass terms,

$$\sigma_0 + c_8\sigma_8 + c_{15}\sigma_{15} \equiv \bar{q}(\lambda_0 + c_8\lambda_8 + c_{15}\lambda_{15})q,$$

one may then obtain from Eqs. (2) that

$$\frac{m_s}{m_u} = 2\left(\frac{m_K}{m_\pi}\right)^2 \qquad (13)$$

and

$$R = (m_D{}^2 - m_\pi{}^2)/(m_K{}^2 - m_\pi{}^2)\ . \qquad (14)$$

(The reader should compare Ref. 2 for a derivation of these results under assumptions different from ours.) Thus, we may also express our results in terms of m_c/m_u.

According to Ref. 2, the masses of the charmed vector mesons can also be computed from m_c/m_u. For the ϕ_c with hidden charm one has[2]

$$2R = (m_{\phi c}{}^2 - m_\rho{}^2)/(m_{K^*}{}^2 - m_\rho{}^2)\ . \qquad (15)$$

We assume that this result can be taken over from Ref. 2. In the analogous case of the pseudoscalars, the corresponding relation [Eq. (14)] has been derived above under our assumptions. It should be noticed that the derivation of Eqs. (14), (15), and the vector mesons mass formulas given in Ref. 2 is not obviously correct once broken scale invariance is taken into account. This is related to the fact that, as mentioned above, SU(4) violations are implied by u having a unique dimension (e.g., Ref. 9). We take our derivation of Eqs. (2c)–(2e) and (14) as an indication that also the analogous relations for the vector mesons will be correct.

We are now ready to list our estimates under various assumptions. The simplest version of the model has a c-number δ with $l_\delta = 0$. Table I shows that the extremely low results for m_D, m_F, and m_{15} exclude these possibilities if the limits on m_ϵ and Γ_ϵ (≤ 0.7 GeV and ≥ 0.6 GeV, respectively) given in the Particle Data Group table are correct. However,[10] the effective m_ϵ might also be as big as about 0.9 GeV. Table I also shows that the resulting charmed pseudoscalar masses under that assumption are somewhat low though not really excluded.[11] If Eq. (15) is correct, the recently discovered[3] $\psi(3.1)$ in this case has too large a mass to be the ϕ_c. (For $l_u = 1, 2$ even lower charmed masses are predicted.)

If l_δ is closer to 3, the value of $\langle -u\rangle_0$ increases, and so do the predictions for the masses of the charmed states. We conclude from Table I that for any reasonable set of the ϵ-meson parameters the present theoretical scheme implies m_D, m_F, $m_{15} \lesssim 2.5$ GeV.

A reasonably high mass (perhaps 1.5 GeV) of the charmed-pseudoscalar-meson masses implies $l_\delta \geq 1$ (such that δ cannot be a c number). It furthermore follows from this lower limit on the mass that $l_u \geq 2$. If the lower limit on the pseudoscalar charmed masses is 2 GeV, $l_\delta = 2$ and $l_u = 3$ follows. Results on l_u are easily obtained by observing that [Eq. (10)] $\langle -u\rangle_0$ depends on l_u and l_δ mainly via the denominator $(4 - l_u)(l_u - l_\delta)$. This has its minimum value, 1, only for $l_u = 3$ and $l_\delta = 2$. For any other allowed dimensions, $(4 - l_u)(l_u - l_\delta) \geq 2$. The value 2 implies $l_u = 3$ and $l_\delta = 1$ or $l_u = 2$ and $l_\delta = 1$.

Finally, if the $\psi(3.1)$ were the ϕ_c and if we could use Eq. (15), then it would follow that $l_u = 3$, $l_\delta = 2$, and m_ϵ is almost fixed at 0.9 GeV. In order to derive $l_u = 3$, $l_\delta = 2$ we note that (units GeV)

$$m_{\phi_c}[(4 - l_u)(l_u - l_\delta) \geq 2,\ m_\epsilon \leq 1,\quad \Gamma_\epsilon \geq 0.5] \leq 2.5. \qquad (16)$$

If $m_{\phi_c} = 3.1$ GeV and $l_u = 3$, $l_\delta = 2$ then m_ϵ and Γ_ϵ

TABLE I. Upper limits on the charmed masses under various assumptions. It is assumed in the table that $l_u = 3$. Units: GeV.

		$l_\delta = 0$	$l_\delta = 1$	$l_\delta = 2$
$m_\epsilon \leq 0.7$	m_D	0.27	0.48	0.85
$\Gamma_\epsilon \geq 0.6$	m_F	0.55	0.68	0.97
	m_{15}	0.37	0.62	1.1
	R	0.25	0.96	3.1
	m_c/m_u	6.9	24	75
	m_{ϕ_c}	0.83	1.0	1.4
$m_\epsilon \leq 0.9$	m_D	1.1	1.4	2.1
$\Gamma_\epsilon \geq 0.45$	m_F	1.2	1.5	2.1
	m_{15}	1.4	1.8	2.5
	R	5.5	8.9	19
	m_c/m_u	133	214	454
	m_{ϕ_c}	1.7	2.0	2.9

are also almost fixed. One has for example

$$m_{\phi_c}(l_u=3,\quad l_\delta=2,\quad m_\epsilon=0.92\text{ GeV},\quad \Gamma_\epsilon=0.42\text{ GeV}) =3.1\text{ GeV},$$

$$m_{\phi_c}(l_u=3,\quad l_\delta=2,\quad m_\epsilon=1.1\text{ GeV},\quad \Gamma_\epsilon=1.1\text{ GeV}) =3.1\text{ GeV}.$$

Thus values of m_ϵ larger than 1.1 GeV make no sense since they would require $\Gamma_\epsilon > m_\epsilon$. At the border of the possibilities for m_ϵ appears to be $m_\epsilon=0.92$ GeV such that $\Gamma_\epsilon=0.42$ GeV in this scheme. The resulting charmed masses are given by

$$m_D=2.2\text{ GeV},\quad m_F=2.3\text{ GeV},\quad m_{15}=2.8\text{ GeV},$$

$$R=22,\quad m_c/m_u=500,\quad \text{and } m_{\phi_c}=3.1\text{ GeV}.$$

The uncertainties of these results should be obvious from the above.

If the charmed-pseudoscalar-meson masses should turn out to be considerably larger than the masses noted in Table I, part of our theoretical scheme would have to be abandoned. A crucial assumption going into our derivation was the Goldstone nature of the possibly existing ϵ meson. Use of the ϵ might be replaced by direct use of the $I=0$ s-wave $\pi\pi$ phases. This already might bring considerable changes and is under present investigation. [Such changes have also been found in previous treatments of SU(3)⊗SU(3) symmetry breaking.[12]] An easy way to obtain arbitrary large predictions for the masses of the charmed psuedoscalars would be to give up the idea that u and δ have integer dimensions. It is seen from Eq. (10) that with l_δ arbitrarily close to l_u and/or l_u arbitrarily close to 4, arbitrarily large charmed pseudoscalar meson masses result.

In conclusion we would like to note that the assumption in Eq. (1) has already been considered by several authors.[8,13,14] Our treatment differs from each one of these references on several points. Firstly, in accord with expectation based on Refs. 2 and 3 we assume that SU(4) is an approximate multiplet symmetry and that just the pseudoscalar mesons are the Goldstone bosons of broken SU(4)⊗SU(4). In particular—unlike Ref. 14—we do not assume Goldstone-breaking of part of the vector SU(4); as can be seen from Ref. 14, making this assumption will only yield physical results if at the same time the *total* vector-SU(4) group is used to obtain equality of coupling constants.

Secondly, in obtaining our values of c_8 and c_{15} we have made no assumptions on the η' mixing with the $I=0$, $Y=0$, $S=0$ member of the 15 plet. We remind the reader that particular assumptions concerning that mixing were essential to obtain the clash of Eq. (1) with positivity in Refs. 13 and 14. Our values of c_8 and c_{15} are in agreement with positivity.

As our main point, we have introduced scale symmetry breaking in order to derive Eq. (10). It is this result which implies the physical consequences given in our paper.

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