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New mesons and broken SU(3)′×SU(3)″ (color)

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An attempt is made to classify the newly discovered heavy mesons into broken SU(3)′×SU(3)″ (color) multiplets. Mass formulas and decay properties are discussed in connection with a suggestive scheme of symmetry breaking. The model leads to numerous predictions which will soon allow it to be substantiated or rejected on the basis of experiment. One of them is the existence of a broad resonance at ≃ 4.8 GeV in the photon channel. From presently known experimental data or theoretical considerations no stringent objections arise; on the contrary, several facts do favor this approach.

I. INTRODUCTION AND DESCRIPTION OF BROKEN SU(3)′× SU(3)″

The recent discovery of heavy mesons at Brookhaven[1] and SLAC[2] is of great importance to elementary particle physics since new degrees of freedom for hadrons seem to have shown up in these experiments. The new mesons may provide decisive information on the internal symmetries of elementary particles. The new degrees of freedom may be connected with "charm," i.e., an extension of SU(3) to SU(4).[3] Several indications speak in favor of this possibility as outlined in recent interesting publications.[4] However, it would be even more exciting if the new states would be manifestations of a still richer internal structure, as implied by an SU(3)′×SU(3)″ symmetry group. Such a structure is required in order to have integer parton charges and Fermi statistics for partons,[5,6] to obtain the correct $\pi^0 \to 2\gamma$ rate,[7] or to have parton fields which form an SU(3) nonet with baryon number one together with the above properties.[8] The corresponding new degree of freedom is now usually called "color." Different authors, however, use the color SU(3) group in very different ways. In the present paper we would like to take the point of view that the quantum numbers of the internal symmetry group are all observable,[8-10] so that color excitations can occur. We assume that the new heavy mesons, usually named ψ mesons, are such excited states and we will try to show that presently known experimental results are consistent with this interesting possibility.

Several proposals to classify the ψ resonances in color-octet multiplets have already been made.[11] In a recent excellent review paper Greenberg[12] discusses in particular those models which use color as an exact symmetry of strong interactions. We intend to show, however, that the heavy mesons fit naturally into a broken color symmetry scheme where the color symmetry breaking occurs at the level of strong interaction.[13]

The internal symmetry we are going to use is broken SU(3)′×SU(3)″ (color). Within this group we take the usual hadronic SU(3) to be the diagonal subgroup. With this choice the currents of SU(3)′ and those of SU(3)″ carry SU(3) octet quantum numbers and thus can both be used to construct the weak hadronic currents. The parton field is supposed to be in the Han-Nambu[6] (3, 3*) representation of the group and therefore forms a nonet (singlet + octet) under conventional SU(3) with integer charges. The partons may be quarks (Han-Nambu quarks) or gnomes.[8] The use of the broken SU(3)′×SU(3)″ internal symmetry group in connection with strong and weak currents was advocated by one of us in recent reports.[14] After the discovery of the new mesons the possible significance of this internal symmetry was evident and specific assignments which we also follow here could be made.[13]

The SU(3)′×SU(3)″ symmetry and the parton-antiparton constituent picture for mesons predict the existence of many new particles with different charge, isospin, and strangeness quantum numbers. There should exist 72 new heavy mesons of a given spin, parity, and internal structure and with masses not much different from the masses of the already observed new particles. Future qualitative experiments may already by sufficient to show whether or not such a rich particle spectrum occurs. The detailed assignments of the new mesons and even the qualitative predictions for their decay rates are not trivial. In a broken symmetry with two broken SU(3) groups, mixing between the states occurs and masks the simple selection rules.

The breaking of SU(3) symmetry is well known. It is due to a term in the effective Hamiltonian which transforms like the hypercharge operator Y. Unfortunately, there is no unique way to generalize this symmetry breaking to the full group.

However, if we assume the group SU(3)′×SU(3)″ to be spontaneously broken by mass terms transforming like the group generators, the specific direction of symmetry breaking can simply be used to define the Y' and Y'' axes in the representation space: Then, the effective Hamiltonian contains a $(Y,1)$ and a $(1,Y)$ symmetry-breaking term only. This is the generalization of the well-known SU(3) symmetry breaking we will assume. Obviously, it does not destroy the isospin invariance of strong interaction ($\vec{I}=\vec{I}'+\vec{I}''$). In fact, at this stage, the U(1) and SU(2) subgroups of SU(3)′ and SU(3)″ are still unbroken and the operators $Y', \vec{I}'$ and $Y'', \vec{I}''$ describing "hypercharge" and "isospin" of SU(3)′ and SU(3)″ are separately conserved.

One further expects also that symmetry-breaking terms which violate the two isospins occur in the effective Hamiltonian, but with a much smaller strength. We call them tadpole[15] terms but do not imply that they are necessarily due to the electromagnetic self interaction. We assume them to be of the form $(I_3,1)$ and $(1,I_3)$. The $(I_3,1)$ term is well known and supposed to be responsible for the $|\Delta\vec{I}|=1$ part of electromagnetic mass splitting,[15,16] for η decay and for the decay $\omega\to 2\pi$. The latter process has the remarkably large width of 130 keV.[17] It is known that this tadpole dominates second-order electromagnetic transitions. Thus, we have again chosen a simple generalization of what is already known. *A priori*, however, a further (I_3,I_3) tadpole cannot be excluded.[18] As we will see later, the decay properties of the $\psi(3,1)$ meson require us to assume that this (8,8) term is absent or small.

The still weaker, nontadpole electromagnetic self interaction allows for transitions with the simultaneous change of SU(3)′ and SU(3)″ representations and has $|\Delta\vec{I}|=0,1,2$ pieces. Finally, the separate conservation of I_3', Y', I_3'', Y'' is destroyed by weak interactions. A weak hadronic current which might be relevant for the case of broken color symmetry has been discussed in Refs. 9 and 13.

II. MESON STATES AND ASSIGNMENTS

The SU(3)′×SU(3)″ representations for the mesons are obtained from the product $(3,3^*)\times(3^*,3)$ which leads to (1,1), (8,1), (1,8), and (8,8) states. We suppose that the well-known vector mesons ω, ϕ, ρ, and K^* are SU(3)″ singlets to a good approximation and write

$$\omega=(\omega,1),\quad \phi=(\phi,1),\quad \rho=(\rho,1),\quad K^*=(K^*,1)\ . \tag{1}$$

The particle names in parentheses refer to SU(3) wave functions. ω and ϕ denote the well-known mixed states with nearly ideal mixing angle. This mixing is one of the consequences of having a broken symmetry. The relevant symmetry-breaking term is the $(Y,1)$ term in the effective Hamiltonian.

Color excited states which will be used to describe the heavy mesons are obtained by replacing the SU(3)″ singlet in (1) by SU(3)″ octet states. One obtains the 27 states

$$(\omega,\rho),\quad (\phi,\rho),\quad (\rho,\rho),\quad (K^*,\rho)\ , \tag{2}$$

the 9 states

$$(\omega,\phi_8),\quad (\phi,\phi_8),\quad (\rho,\phi_8),\quad (K^*,\phi_8)\ , \tag{3}$$

and the 36 states

$$(\omega,K^*),\quad (\phi,K^*),\quad (\rho,K^*),\quad (K^*,K^*)\ . \tag{4}$$

Again, the names in parentheses characterize the SU(3)′×SU(3)″ transformation properties. ϕ_8 is the $I''=0$ member of the SU(3)″ octet. In the wave functions of the new states we have taken ideal ω, ϕ mixing with respect to the SU(3)′ group as a plausible generalization of the singlet-octet degeneracy observed for the usual vector mesons.[11,13] In view of our effective Hamiltonian this is a natural assumption. It would be fortuitous if ideal mixing only existed for the lowest states.

Because of the perturbation Hamiltonian $H(1,Y)$ there will also be some singlet-octet mixing with respect to the color degree of freedom. The states (a,ϕ_8), $(a=\omega,\phi,\rho,K^*)$ will mix with the color-singlet wave packet contained in

$$H(1,Y)|(a,\phi_8)\rangle\ . \tag{5}$$

Spontaneous symmetry breaking and the parton picture suggest that the operator $H(1,Y)$ is a mass term which leaves the basic internal properties of the state (a,ϕ_8), e.g., the parton-antiparton sea and the spatial properties, essentially unchanged. One obtains from (5) an "analog state" formed out of continuum states from a reasonably large energy range around the resonance mass. The corresponding $(a,1)$ wave packet is not supposed to have a good overlap with the $(a,1)$ wave function of a low-energy meson. Their different energy content can induce differences in their internal structure. We will denote the perturbed (a,ϕ_8) meson state by (a,ϕ) where now $\phi(\mathrm{SU}(3)'')$ is no longer a pure octet.

To see which of the color excited states can possibly be identified with the new vector particles found at Brookhaven and SLAC we have to consider the electromagnetic current. In our model, it is a member of the SU(3) current octet which is a current of the diagonal subgroup of SU(3)′×SU(3)″:

$$J(\mathrm{SU}(3)) = J(8,1) + J(1,8)\ . \tag{6}$$

We decompose the electromagnetic current into components which transform like the vector meson states and find

$$J^{\text{e.m.}} = \frac{1}{\sqrt{6}}\,(\omega,1) - \frac{1}{\sqrt{3}}\,(\phi,1) + \sqrt{\tfrac{3}{2}}\,(\rho^0,1) - (\omega,\rho^0) - \frac{1}{\sqrt{2}}\,(\phi,\rho^0) + \frac{1}{\sqrt{3}}\,(\omega,\phi_8) + \frac{1}{\sqrt{6}}\,(\phi,\phi_8)\ . \tag{7}$$

Thus, four of the color-octet states are directly coupled to the photon. A fifth meson, namely (ρ^0,ϕ), can also be excited because of its mixing with ρ-mesonlike continuum states. Its coupling relative to the couplings occurring in Eq. (7) is $\sqrt{\frac{3}{2}}\sin\theta''$ where θ'' denotes the corresponding mixing angle.

From the five mesons coupled to the photon only two, namely the states (ω,ρ^0) and (ϕ,ρ^0), are stable with respect to strong decays into normal hadrons. Their SU(3)$''$ isospin which is a conserved quantity is one. The remaining states (ω,ϕ_8), (ρ^0,ϕ_8), (ϕ,ϕ_8) are unstable since they are connected with the color-singlet continuum by the $H(1,Y)$ perturbation in the effective Hamiltonian. Thus, it is very tempting to identify the (ω,ρ^0) and the (ϕ,ρ^0) states with the sharp resonances found at 3.1 and 3.7 GeV. Since the (ϕ,ρ^0) is expected to be heavier and, according to Eq. (7), has only half the e^+e^- decay width of the (ω,ρ^0) (neglecting mass-dependent effects) our assignment is

$$\psi(3.1) = (\omega,\rho^0),\quad \psi'(3.7) = (\phi,\rho^0)\ . \tag{8}$$

This identification gives an explanation for the remarkable stability of the $\psi(3.1)$, provided, of course, that other states with the same SU(3)$''$ quantum number $I''=1$ do not lie much below 3 GeV. This remark concerns in particular the color-octet pseudoscalar mesons which, in an obvious notation, may be denoted by (π,π^0), (η,π^0), etc. Then, the $\psi(3.1)$ can only decay via the $(1,I_3)$ tadpole term, via the smaller nontadpole electromagnetic self interaction, and by radiative transitions. The former leads to hadron final states with G parity -1. In purely hadronic decays mainly $G=-1$ states have been observed.[19] Our suggestion is, therefore, that the $(1,I_3)$ tadpole term is dominant as is the case with the $(I_3,1)$ tadpole in the transitions $\omega\to 2\pi$, $\eta\to 3\pi$ and that a sizeable (I_3,I_3) tadpole does not exist. The first-order radiative transition of $\psi(3.1)=(\omega,\rho^0)$ leads to hadronic compounds with the quantum numbers of an ω but with G parity $+1$. One expects the radiative decay mode of the ψ and ψ' to be rather important. Some aspects of these decays are discussed in Sec. V.

The $\psi'(3.7)=(\phi,\rho^0)$ particle can decay "strongly" into (ω,ρ^0) and other states with $I''=1$. These decays are suppressed by the Okubo-Zweig-Iizuka rule,[20] hereafter simply called "Zweig's rule," which may provide the reason for the small width $0.2\ \text{MeV} < \Gamma < 0.8\ \text{MeV}$ of the ψ' particle.[19]

The broad structure in the cross section which has been found at 4.15 GeV[21] allows a further particle identification. In fact, one of the advantages of the broken SU(3)$'\times$SU(3)$''$ model is the possibility of describing this state as a member of the color-octet family of particles which decays strongly into $I''=0$ hadrons. We expect it to be the (ω,ϕ) since the (ϕ,ϕ) should lie higher in mass. Also, according to Eq. (7), the (ω,ϕ_8) state has a stronger photon coupling than the (ϕ,ϕ_8) state. We take, therefore,

$$\psi''(4.15) = (\omega,\phi),\quad \phi(\mathrm{SU}(3)'') \approx \phi_8. \tag{9}$$

With this assignment we know the mass difference between the (ω,ϕ) and (ω,ρ) mesons and thus the strength of the $(1,Y)$ symmetry-breaking interaction which determines the total width of the (ω,ϕ). We will show in Sec. III that a rough calculation of this width gives in fact the observed value of ≈ 300 MeV.

The (ϕ,ϕ) state should be heavier than the (ω,ϕ) by about the same amount the $\psi'=(\phi,\rho)$ is heavier than the $\psi=(\omega,\rho)$. Thus, we expect the (ϕ,ϕ) state at $\simeq 4.8$ GeV (see Sec. III). An inspection of the data points[21] indicates that there could indeed be a broad resonance at this energy. Very tentatively we therefore write

$$\psi'''(4.8\,?) = (\phi,\phi),\quad \phi(\mathrm{SU}(3)'') \approx \phi_8\,. \tag{10}$$

This particle must decay strongly to $I''=0$, $I=0$ final states. The remaining state (ρ^0,ϕ), which is coupled to the photon by the perturbation of color symmetry, may to some extent contribute to the bump at 4.15. The mass formula given in Sec. III suggests this degeneracy. The particle assignments together with rough estimates of the widths to be discussed below are displayed in Table I.

To complete this section we briefly comment on the stability of the remaining color excited states. The particles with quantum numbers I_3'', Y'' different from zero cannot decay by strong or electromagnetic interactions. Thus, to each octet quantum number with $I_3''\neq 0$ there exists a meson which can decay by weak interaction only. The predictions for weak decays depend, of course, on the form of the weak current. We prefer the weak current given in Refs. 9 and 13 since it is constructed from (8, 1) and (1, 8) currents only, apart from a unitary rotation conserving charge and strangeness. It is important to note that this current can excite or deexcite all color-octet states irrespective of their SU(3)″ quantum number. Thus, all heavy mesons can decay weakly.

The stability of the color-octet mesons also implies that the mesons with $I_3'' \neq 0$ can only be produced by pair production or in neutrino interactions. Particles with isospin I' and I'' both different from zero have further interesting properties which we will not discuss here, however.

III. MASS FORMULAS AND DECAY WIDTHS

In our model SU(3)′ and SU(3)″ breaking is essential in order to accommodate the observed resonances in mass-split multiplets. The effective Hamiltonian of strong interaction consists of the three parts

$$H = H(1,1) + H(Y,1) + H(1,Y)\ . \tag{11}$$

The perturbation terms may be due, for instance, to spontaneous symmetry breaking. The SU(3)″ violating term accounts for the mass difference between the 3.1 and 4.15 GeV resonances. From the large masses of the new mesons it is immediately clear that at least the first term in Eq. (11) has reduced matrix elements which depend strongly on the special SU(3)′×SU(3)″ representation of the states. In other words, at least the (1, 1) "parton mass" is a dynamical quantity depending on the representation of the state. Since in the general case we obtain too many reduced matrix elements, additional input information is required to obtain predictive formulas. One suggestive possibility is to generalize the SU(3)′ relations with ideal ω-ϕ mixing, as (approximately) observed for the usual vector mesons, to the heavy mesons: As mentioned in Sec. II, we take singlet-octet degeneracy and ideal mixing with respect to the SU(3)′ group for all relevant states. (This property could be due, for example, to an SU(3)′ singlet gluon exchange interaction and a symmetry-breaking mass term which only depends on the SU(3)″ representation). We obtain immediately the first-order mass formulas (particle names stand for the masses):

$$(\rho, b) = (\omega, b)\ , \tag{12a}$$

$$(K^*, b) = \tfrac{1}{2}(\omega, b) + \tfrac{1}{2}(\phi, b)\ , \tag{12b}$$

$$(a, K^*) = \tfrac{3}{4}(a, \phi_8) + \tfrac{1}{4}(a, \rho)\ , \tag{12c}$$

and for $b, b' \neq 1$

$$(\phi, b) - (\omega, b) = (\phi, b') - (\omega, b')\ . \tag{12d}$$

TABLE I. Particle properties as measured by experiment and predicted by our model.

Experiment				Broken SU(3)′ × SU(3)″ (color) model						
Particle	Mass (GeV)	$\Gamma^{e^+e^-}$ (keV)	Γ^{tot} (MeV)	Name	Mass (GeV)	$J^P(I',I'')^G$	$\Gamma^{e^+e^-}$ (keV)	Decay mechanism	Γ (MeV)	$(I',I'')^G$ hadronic final states
ψ	3.1	5.2 ± 1.3	0.08	(ω,ρ^0)	3.1 (input)	$1^-(0,1)^+$	5.2 (input)	radiative tadpole second-order e.m.	≈0.03	$(0,0)^+$ (see footnote a) $(0,0)^-$ $(1,0)^+$; $(0,0)^-$
				(ρ,ρ^0)	≃3.1	$1^-(1,1)^-$	0	strong radiative tadpole		$(1,1)^-$ $(\pi,\pi)+\pi$; $(\eta,\pi)+2\pi$ $(1,0)^-$ $(1,0)^+$
ψ'	3.7	2.2 ± 0.5	0.2 to 0.8	(ϕ,ρ^0)	3.7 (input)	$1^-(0,1)^+$	≃2.6	strong (hindered by Zweig's rule)		$(0,1)^+$ $(\omega,\rho)+2\pi$ $(\eta,\pi)+3\pi$ $(\rho,\rho^0)+\pi$ (see footnote b) $(\omega,\rho^0)+\eta$ (see footnote b) $(\pi,\pi)+2\pi$ (see footnote b)
ψ''	4.15	≈4	≈300	(ω,ϕ) (ρ,ϕ)	4.15 (input) ≃4.15	$1^-(0,0)^-$ $1^-(1,0)^+$	≃2.2 ≈0.3 [c]	strong strong	≈280 ≈280	$(0,0)^-$ $(1,0)^+$ 12% of the 4.15 GeV resonance [c]
ψ'''	≈4.8 (?)			(ϕ,ϕ)	≃4.8	$1^-(0,0)^-$	≈0.4 [c]	strong	≈280	$(0,0)^-$

[a] In this entry $I''=1$ states are not considered. [b] p-wave decays suppressed by Zweig's rule. [c] Sensitive to mixing angle θ''.

In these equations the label a stands for any of the SU(3)′ wave functions ω, ϕ, ρ, K^* and the label b for the SU(3)″ wave functions $1, \phi_8, \rho, K^*$. Obviously, all masses are determined from the five input masses $(\omega,1)$, $(\phi,1)$, (ω,ρ), (ϕ,ρ), (ω,ϕ_8). Before evaluating the masses we should consider the corrections which are due to SU(3)″ singlet-octet mixing. This mixing arises from the term $H(1,Y)$ which couples the high-mass resonances with the color-singlet continuum. It is not easy to treat. However, the only mass formula which is affected is Eq. (12c): The color-octet mass (a,ϕ_8) will be different from the physical mass of the (a,ϕ) particle. No drastic effects are expected; the mass (a,ϕ_8) might be smaller than the mass (a,ϕ) by an amount less or roughly equal to the total width of this meson (see the discussion and formulas given below). Thus, the masses of all heavy vector mesons can be predicted qualitatively from the masses of ψ, ψ', and ψ''. Notably, we have the mass degeneracy

$$(\rho,\rho)=(\omega,\rho),\quad (\rho,\phi)=(\omega,\phi)\ . \tag{13}$$

The masses of the particles with a strange color and a nonstrange SU(3)′ index, i.e., of those states which should preferably be found in neutrino reactions, if the weak current of Refs. 9 and 13 is correct, are

$$(\omega,K^*)=(\rho,K^*)\simeq 3.8\text{ GeV};\quad (\phi,K^*)\simeq 4.4\text{ GeV}. \tag{14}$$

The mass of the (ϕ,ϕ) meson which we discussed already in Sec. II follows from Eq. (12d): $(\phi,\phi)\simeq 4.8$ GeV. Clearly, the values given should not be taken too seriously. A slight deviation from the ideal mixing angle, for example, has sizeable effects on the mass differences.

The leptonic decay widths of the new mesons can be taken from the couplings (7) by normalizing to the width of the $\psi(3.1)$ ($\Gamma^{e^+e^-}(\psi)\simeq 5.2$ keV) and assuming the widths to be independent of the masses in the mass range considered. In order to be able to use Eq. (7) for the e^+e^- decays of (ω,ϕ), (ρ,ϕ), and (ϕ,ϕ) one needs to know to what extent these particles mix with color-singlet wave packets $(a,1)$ of the same internal structure as the unperturbed (a,ϕ_8) mesons. Denoting the unknown mixing angle by θ'' we find from Eq. (7)

$$\Gamma^{e^+e^-}(\phi,\rho)\simeq\tfrac{1}{2}\Gamma^{e^+e^-}(\omega,\rho)\ , \tag{15a}$$

$$\Gamma^{e^+e^-}(\omega,\phi)\simeq\left(\frac{1}{\sqrt{3}}\cos\theta''+\frac{1}{\sqrt{6}}\sin\theta''\right)^2\times\Gamma^{e^+e^-}(\omega,\rho)\ , \tag{15b}$$

$$\Gamma^{e^+e^-}(\rho,\phi)\simeq\tfrac{3}{2}\sin^2\theta''\,\Gamma^{e^+e^-}(\omega,\rho)\ , \tag{15c}$$

$$\Gamma^{e^+e^-}(\phi,\phi)\simeq\left(\frac{1}{\sqrt{6}}\cos\theta''-\frac{1}{\sqrt{3}}\sin\theta''\right)^2\times\Gamma^{e^+e^-}(\omega,\rho)\ . \tag{15d}$$

The angle θ'' could best be obtained by analyzing the fraction (15c) of $G=+1$, $I=1$ final states among the decay products of the 4.15 resonance. A theoretical estimate of it is difficult. The mixing angle is determined by the formula

$$\tan\theta''=\left\langle(a,1)\left|\frac{1}{M-H(1,1)-H(Y,1)}\,H(1,Y)\right|(a,\phi_8)\right\rangle. \tag{16}$$

We mentioned already in Sec. II that the wave packet $(a,1)$ which is not identical with the state of a low-mass meson may be taken from the color-singlet part of (5). Equation (16) can then be rewritten (P projects color-singlet states):

$$\tan\theta''\simeq[\langle(a,\phi_8)|H(1,Y)|(a,1)\rangle]^{-1}\left\langle(a,\phi_8)\left|H(1,Y)\,\frac{P}{M-H(1,1)-H(Y,1)}\,H(1,Y)\right|(a,\phi_8)\right\rangle . \tag{17}$$

The denominator can be expressed by the mass difference ψ''-ψ since the internal wave functions overlap:

$$\langle(a,\phi_8)|H(1,Y)|(a,1)\rangle=\frac{1}{\sqrt{2}}\,[(\omega,\phi)-(\omega,\rho)]\ . \tag{18}$$

The numerator in (17) gives the shift δM of the resonance position because of the resonance continuum interaction. Unfortunately, we can only guess this shift: Since the resonance occurs at high energy far above the color-singlet threshold the sign will be positive and the magnitude somewhat smaller than the total width of the resonance. If one takes $\delta M\approx\Gamma^{\text{tot}}/2\simeq 150$ MeV one gets

$$\tan\theta''\approx 0.2\ . \tag{19}$$

Although this value for θ'' is only a guess we used it in Table I for an orientation about the e^+e^- widths. The obtained total contribution of (ω,ϕ)

and (ρ, ϕ) to the e^+e^- width of $\psi(4.15)$ is 2.5 keV to be compared to the observed width[22] of ≈ 4 keV. The e^+e^- width of (ϕ, ϕ) is predicted to be small because of the destructive interference in (15d).

It is important to note that the angle θ'' obtained in (19) is not relevant for the color-octet admixture to the usual ω, ϕ, ρ, K^* states. There is, presumably, little overlap between wave functions describing quantum states of so different energies and there is no color-octet continuum at small mass to compensate for this effect. In fact, the e^+e^- decay ratios follow the rule $\omega{:}\phi{:}\rho = 1{:}2{:}9$ pretty well, which corresponds to no admixture.

Let us now estimate the total width of the states (a, ϕ). We use the formula for the decay rate in perturbation theory

$$\Gamma((a, \phi)) = \frac{1}{2M} \int d^4x \langle (a, \phi_8)| \mathcal{H}_{(1,Y)}(x) \mathcal{H}_{(1,Y)}(0) | (a, \phi_8) \rangle , \tag{20}$$

where $\mathcal{H}_{(1,Y)}(x)$ is the Hamiltonian density of the perturbation. Replacing the effective time interval by $1/\Delta M$ where ΔM is a characteristic level spacing between old and new mesons, one gets

$$\Gamma((a, \phi)) = \frac{1}{2M} \frac{1}{\Delta M} \int d^3x \langle (a, \phi_8)| \mathcal{H}_{(1,Y)}(0, \vec{x}) \mathcal{H}_{(1,Y)}(0, \vec{0}) | (a, \phi_8) \rangle . \tag{21}$$

This equal-time matrix element can be expressed in terms of known mass differences if we use again the assumption that, at infinite momentum, $H(1, Y)$ acts essentially on the SU(3)′×SU(3)″ indices only and not on other internal variables. One obtains

$$\Gamma[(a, \phi)] \simeq \frac{3}{4\Delta M} [(a, \phi) - (a, \rho)]^2 . \tag{22}$$

Putting $\Delta M \approx 3$ GeV Eq. (22) gives for the total width of the (a, ϕ) mesons

$$\Gamma[(a, \phi)] \approx 280 \text{ MeV} . \tag{23}$$

It may, of course, for fortuitous that this estimate agrees with the observed width of the 4.15 GeV bump. In any case, the (ϕ, ϕ) meson predicted at $\simeq 4.8$ GeV should have the same width as the (ω, ϕ) resonance.

For the $\psi(3.1)$ resonance we can write down formulas analogous to Eqs. (21) and (22). In this case, we can turn the arguments around and estimate the $(1, I_3)$ tadpole contribution s to the $|\Delta \vec{I}''| = 1$ mass difference. One obtains

$$s \simeq [\Delta M \, \Gamma_{\text{tadpole}}(\psi)]^{1/2} , \tag{24}$$

where this time $\Delta M \approx 2.3$ GeV. To use this estimate one has to subtract from the total width of the $\psi(3.1)$ the nontadpole part, the leptonic decay width, and the width for radiative decays. Since the latter is unknown, we obtain (for $\Gamma_{\text{tadpole}}(\psi) \lesssim 55$ keV) the upper limit $s \lesssim 11$ MeV. A radiation width of 10–30 keV (see below) gives $s \approx 8$–10 MeV which is a reasonable value not too different from the strength of the well-known $(I_3, 1)$ tadpole.

IV. THE HADRONIC DECAYS

As we have seen in the preceding sections our broken SU(3)′×SU(3)″ model is supported by several facts: the spectrum of observed resonances in the e^+e^- channel, the e^+e^- branching ratios, and the total decay width of the 4.15 GeV bump. Also, the value of R, the ratio of hadron to μ-pair production in e^+e^- collisions, is at high energy not far from the expected value $R = 4$. The I_3'' part of the electromagnetic current is responsible for an increase of R by 1.5. The corresponding rise of R will set in when $I'' = 1$ continuum states can be produced. This should happen near 3.7 GeV since $\psi = (\omega, \rho^0)$ and to some extent also $\psi' = (\phi, \rho^0)$ are bound states below the $I'' = 1$ threshold. This expectation is in agreement with the data points, at least if one interprets them optimistically.

On the other hand, our model predicts unusual properties of hadrons and an enormous number of new states. Some of the new mesons should even have charge 2, and strangeness-2 states should also occur. The model is, therefore, very vulnerable and its confirmation or rejection should come from the study of the spectrum and the decays of the new particles. The analysis of the decay properties of the resonances in the e^+e^- channel may already provide some information in this respect. Let us therefore discuss the hadronic decay modes of the SU(3)″ octet resonances produced in e^+e^- annihilation.

The $1\,{}^3S_1$ state of highest mass should be the (ϕ, ϕ) expected at $\simeq 4.8$ GeV. It should decay strongly (with a width of about 280 MeV) into states with G parity -1, $I'' = 0$, and isospin zero. From the SU(3)′ wave function of this state we expect that many decay events contain a pair of strange particles. The strong decays to ψ' (3.7) and ψ (3.1) are forbidden. Decays to the unstable pseudoscalar meson states (a, η_8) are allowed.

The state (ω, ϕ) assigned to the bump at 4.15 GeV has the properties of an ω with high mass. This resonance is expected to be degenerate in mass with the (ρ, ϕ) which decays like a very heavy ρ meson. The admixture of (ρ^0, ϕ) to ψ'' (4.15) could be around 12%. After subtraction of the background, $\approx 12\%$ of the final hadrons near 4.15 GeV would then have $I=1$ and G parity +1. The strong decays to $\psi'(3.7)$ and $\psi(3.1)$ are not allowed. Decays to unstable (a, η_8) pseudoscalar mesons can occur, however.

The decay modes of the 3.7 GeV particle,[23] our (ϕ, ρ^0), involve the violation of the Zweig rule which is not well understood. Thus, only qualitative remarks can be made. The "strong" hadronic decays should lead to new particle states with the same SU(3)″ quantum number as the 3.7 resonance. Since the $\epsilon(2\pi$–S-wave) resonance is close in energy to the mass difference between $\psi'(3.7)$ and $\psi(3.1)$, the decay $\psi'(3.7) \rightarrow \psi(3.1) + 2\pi$ (see Ref. 22) may be interpreted as an s-wave quasi-two-body decay

$$(\phi, \rho^0) \rightarrow (\omega, \rho^0) + \epsilon \ . \tag{25}$$

The process

$$(\phi, \rho^0) \rightarrow (\rho, \rho^0) + \pi \tag{26}$$

should also be present. Its detection would reveal the existence, mass, and width of the (ρ, ρ^0). It is a difficulty for our model that this p-wave decay has not been seen so far. Another p-wave decay is the process

$$\psi'(3.7) \rightarrow \psi(3.1) + \eta \ . \tag{27}$$

If both decays (26) and (27) proceed simply through deviations from ideal SU(3)′ mixing, the process (26) is expected to be stronger since phase space favors it while the decay amplitudes for the two processes are comparable. On the other hand, if the deviations from ideal mixing are unimportant the amplitudes for (26) and (27) could be quite different depending on the details of the Zweig suppression mechanism for p-wave decays and the strange parton content of η. The (ρ, ρ^0) state can be narrow or broad depending on whether or not it can further decay to new pseudoscalar mesons or—if heavy enough—to the $(\omega, \rho^0) = \psi(3.1)$ by π emission. As an upper limit for the (ρ, ρ) mass we suggest 3.5 GeV which is the calculated mass of the fictive SU(3)′ octet state (ϕ_8, ρ^0). A lower bound is obtained from the stability of the $\psi(3.1)$:

$$(\rho, \rho^0) \gtrsim (\omega, \rho^0) - \pi \ . \tag{28}$$

The $(\phi, \rho^0) = \psi'(3.7)$ may also directly decay into pseudoscalar meson states. We will denote the 0^- mesons by (a, b) where $a, b = \eta', \eta, \pi, K$ describe the SU(3)′ and SU(3)″ wave functions and quantum numbers. The masses of the new 0^- mesons are not much different from the corresponding vector meson masses if the generalization of the formula

$$\rho^2 - \pi^2 \simeq K^{*2} - K^2 \simeq 0.55 \text{ GeV}^2 \tag{29}$$

holds. We expect, for example,

$$(\rho, \rho)^2 - (\pi, \pi)^2 \simeq 0.55 \text{ GeV}^2 \ , \tag{30}$$

i.e., mass differences (of unmixed states) of the order of 100 MeV only. The lowest lying states are presumably the (π, π) and the (η, π). They are stable with respect to strong interactions. Thus, also the processes

$$\begin{aligned} (\phi, \rho) &\rightarrow (\pi, \pi) + 2\pi \\ &\rightarrow (\eta, \pi) + 3\pi \end{aligned} \tag{31}$$

could occur.

From the stability of the $\psi(3.1) = (\omega, \rho)$ we finally have the additional information

$$\begin{aligned} (\eta, \pi) &\geq (\omega, \rho) - \pi \ , \\ (\pi, \pi) &\geq (\omega, \rho) - 2\pi \ . \end{aligned} \tag{32}$$

V. OBJECTIONS AND CONCLUDING REMARKS

An objection which can be raised against "color-excitation" models is connected with the first-order radiative transitions.[24] In particular, the decays

$$\begin{aligned} \psi(3.1) &\rightarrow \eta'(\text{or } \eta) + \gamma \\ &\rightarrow 2\pi + \gamma \\ &\rightarrow 4\pi + \gamma \end{aligned} \tag{33}$$

are expected, but appear to be suppressed. For high-energy γ rays where $kR_0 \gg 1$ (R_0 is the radius of the decaying particle) an argument for this suppression can be given: The plane wave of the photon oscillates inside the charge distribution and averages the matrix element almost to zero. A quantitative estimate is not possible at present. In a parton picture and for a Gaussian form of the density distribution the γ width for η' or η decay is expected to go as

$$\Gamma^{\eta'\gamma} \propto kR_0 e^{-(kR_0)^2/n}, \qquad R_0^{\,2} = \langle r^2 \rangle_{\text{Gauss}} , \tag{34}$$

where n depends on the number of constituents: n is equal to 12 if the total momentum is carried by 2 partons only. It rapidly approaches the value $n=3$ for states containing more (charged or neutral) constituents. Since $kR_0 \simeq 7.1$ ($R_0 \simeq 1$ fermi) for the decay $\psi \rightarrow \eta' + \gamma$ while kR_0 is only $\simeq 1.3$ for the decay $\omega \rightarrow \pi^0 + \gamma$, it can be seen from formula (34) that a relative suppression factor of the order of 10^{-2} to 10^{-3} due to the short wavelength of the photon can easily be understood.[25] A further suppression occurs because of the presumably small overlap between the internal wave functions of the old and new mesons which we have mentioned before. On the other hand, γ rays of lower energy are not suppressed by these effects. Therefore, we have to estimate the total width for the radiative decay of the ψ to the continuum states below 3.1 GeV. The formula for first-order radiative decays reads

$$\Gamma^{\gamma}_{\text{tot}}(\psi) = \frac{e^2}{2M} \int \frac{d^3k}{(2\pi)^3 2k} \times \int d^4x\, e^{ikx} \langle (\omega, \rho) | J_\perp(x) J_\perp(0) | (\omega, \rho) \rangle . \tag{35}$$

We replace again the time integration by a characteristic level spacing $1/\Delta M$. Then, a parton-model calculation can be performed. Using a Gaussian distribution to describe the space correlation of the two currents at equal times inside the ψ, one finds

$$\Gamma^{\gamma,\, I''=0\, \text{hadrons}}(\psi) \simeq \frac{4}{3\pi} \frac{e^2}{4\pi} \frac{3}{2R_c^2} \frac{1}{\Delta M} . \tag{36}$$

In this formula, the square of the equal-time correlation radius R_c appears. To express it in terms of the mean square radius $R_0^2 = \langle r^2 \rangle_{\text{Gauss}}$ we use the known result for nucleons[26]

$$R_c^2 \simeq 3R_0^2 . \tag{37}$$

Choosing $\Delta M \approx 2.3$ GeV and $R_0 \simeq 1$ F our estimate for the above width is

$$\Gamma^{\gamma,\, I''=0\ \text{hadrons}}(\psi) \approx 26\ \text{keV}. \tag{38}$$

A slightly larger particle radius or a larger value for ΔM which includes the effect of the time oscillation in (35) would even give a smaller width. On the other hand, ψ decays into colored states could have a larger width since the relevant level spacing, in the neighborhood of the ψ, is much smaller. Thus, a difficulty of the model only arises if important high-mass resonances lie below 3.1 GeV.

These considerations suggest that the radiative decay of the $\psi'(3.7)$ could be sizeable. Here, the most pronounced photon transition should lead to the (η, π^0) pseudoscalar meson state if this is energetically possible. The corresponding width depends, of course, very much on the decay energy and on the content of strange partons [with respect to SU(3)′] in (η, π) which we do not know.

A further objection against the use of "color" concerns deep-inelastic scattering.[24,27] If color-excited particles can be produced in the final state, a change of the scaling functions with increasing energy is expected. We have no good explanation to offer to explain why such effects are not pronounced in present data. A few remarks can nevertheless be made: Because of the $(8,1)+(1,8)$ form of the current the effect of color can only be noticed in the SU(3)′ and SU(3)″ singlet part of the bilocal current which is suppressed at values of the scaling variable x away from zero; color threshold effects manifest themselves mainly in the magnitude of the structure functions; the ratio of the electron-nucleon and neutrino-nucleon structure functions changes only slightly; the final hadron state produced in the reaction has some color-octet admixture already at lower energy; and finally, the color symmetry classification for baryons might be more complicated than the quark model suggests.

To conclude, we believe to have shown that there are no stringent objections against the broken SU(3)′×SU(3)″ model neither from presently known experimental data nor from theoretical considerations, while several facts do support it. The truth may, of course, be different. In our discussion we left out possible SU(3)′×SU(3)″ (color) classifications of baryons, their spectrum, and the production of the new mesons by baryon-baryon and meson-baryon interactions. Also, we did not comment on the decay of the new mesons into baryon-antibaryon states or resonances. The answer to the corresponding questions will depend on the nature of the partons (quarks or gnomes) and on the spectrum of colored baryons. The possibility of parton pair production remains an open problem. Our interpretation of the observed resonances as stable bound states in the limit of exact color symmetry suggests that the region in which parton pairs can be produced abundantly lies beyond the presently available e^+e^- energies.

Added note. Feldman and Matthews[28] independently of us used the same strong symmetry-breaking term $(1, Y)$ and also predict a resonance at $\simeq 4.8$ GeV.

Note added in proof. According to recent measurements[29] the structure at 4.1 GeV looks like the superposition of two partly overlapping resonances of different heights. These could be the states (ρ^0, ϕ) and (ω, ϕ), as was suggested in the paper. More pronounced in the data is a new non-narrow resonance at 4.4 GeV. This $\psi'''(4.4)$ particle could be the (ϕ, ϕ) state (originally expected near 4.8 GeV) if one were to allow for a different ω-ϕ splitting of $I''=0$ and $I''=1$ states. With these assignments the leptonic decay widths predicted by (15) and (19) agree well with the new experimental values.

Also to be noted is the smallness or absence of cascade transitions $\psi'', \psi''' \to \psi, \psi'$ as well as the reported ratio[30] $\Gamma(\psi \to \rho\pi)/\Gamma(\psi \to K^*K) \approx 3$, which is just the value expected from the SU(3)′ transformation property of the $\psi(3.1)$. Further points in favor of the model are the indications for a heavy lepton[31] and the observation of a jet structure[29] at the highest SPEAR energies. Two charged heavy leptons (which would bring the effective R up to 6 at high energy) and a jet structure have been suggested in the framework of broken color symmetry.[13,14] A negative point is the fact that the charged heavy mesons required by the model have so far not been discovered. The decay constants for the $\psi'(3.7)$ decays to such states must be surprisingly small. The eventual detection of charged heavy mesons (and of doubly charged mesons in neutrino reactions) remains the decisive test of the model.

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