Covariant harmonic oscillators and excited meson decays

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(Received 31 October 1974)

We use the covariant harmonic-oscillator wave functions to formulate the vacuum quark-pair-creation model of three-meson vertices and meson-decay matrix elements. It is pointed out first that the harmonic-oscillator wave functions can be given a covariant probability interpretation. We then use this probability concept to construct the three-meson vertex function. Using a relativistic L-S coupling scheme, we calculate the decay amplitudes for the $B \rightarrow \omega\pi$ and $A_1 \rightarrow \rho\pi$ decays. For the B decay, the calculated polarization ratio turns out be in excellent agreement with the experimental value. For the A_1 decay, the calculated ratio is consistent with the present experimental value. We have also calculated the decay ratio for the well-established $\rho \rightarrow \pi\pi$ and $A_2 \rightarrow \rho\pi$ decays. It is shown that the harmonic-oscillator wave functions produce a number which is very close to the experimental value. The limitations of the L-S coupling scheme are discussed.

I. INTRODUCTION

In our previous papers[1,2] we discussed covariant harmonic-oscillator wave functions which contain all desirable features for giving a Lorentz-contracted probability interpretation.[3] We discussed also the areas of high-energy physics to which our oscillator formalism can be applied.[2] The purpose of this paper is to use our covariant harmonic-oscillator wave functions for calculating the decay rates of mesonic resonances.

In discussing resonance decays, there are at present three different approaches. They are the standard quark model,[4] l-broken $SU(6)_W$ model,[5] and the models based on the Melosh transformation.[6] The successes and limitations of these approaches are well known. The strength of the standard quark model is that, while it has many serious limitations, it does enable us to think in terms of a complete theory in which everything can be calculated from a basic input. In this model quarks move in a potential well, and their wave functions are determined by the interaction. It is this bound-state wave function which is the basic ingredient for calculating dynamical quantities. One promising line along which this model has been developed is that of the relativistic quark model based on harmonic-oscillator wave functions.

In this paper we work within the harmonic-oscillator framework of Feynman *et al.*[4] We make the following changes, if not innovations, from their original work. Feynman *et al.* do not use normalizable wave functions. In this paper we start from the same harmonic-oscillator differential equation as theirs, but we use different solutions. Our wave functions are normalizable and can be given a covariant probability interpretation. While Feynman *et al.* resort to partial conservation of axial-vector current (PCAC) to treat mesonic decays, we use the probability approach and treat all participating mesons equally. Feynman *et al.* ignore recoil effects on spinors, but we use our covariant wave functions to calculate the spin recoil effects. Feynman *et al.* attempt to calculate a large number of decay rates in order to establish the general validity of the harmonic-oscillator approach. In this paper we shall concentrate only on the decays of the A and B mesons which are regarded as a crucial testing ground for the quark model. As in the case of Feynman *et al.*, we avoid the difficult question of treating the effect of symmetry breaking and simply use, to determine the parameters of the covariant wave functions, the observed masses and momenta which have been affected by the symmetry-breaking interaction.

The probability approach to the decay problem is not new. In fact, most of the quark-model calculations resort to one or another form of the quark distribution. The model which completely by-passes the question of quarks interacting directly with the external meson is the one proposed by Le Yaouanc *et al.*,[7] who formulated their program based on the duality diagram approaches adopted previously by Micu,[5] and by Carlitz and Kislinger.[8] They construct their three meson vertex function by taking a simple probability overlap integral of the three participating mesons. However, the model of Le Yaouanc *et al.* is nonrelativistic, lacks crossing symmetry, and requires a phenomenological addition of the Mitra-Ross recoil term.[9]

The purpose of using the covariant bound-state wave function is to eliminate the above-mentioned weaknesses of Le Yaouanc *et al.*[7] Our model is fully relativistic, is crossing-symmetric, and contains all the recoil terms including the Mitra-Ross term.[9]

In this paper we first present the formalism of the covariant harmonic oscillator without spin. Next, we adopt the covariant L-S coupling scheme using covariant Dirac spinors, as in the case of Feynman *et al.*[4,10] While this coupling is an incomplete theory, we note that one seldom goes beyond the L-S coupling even in nonrelativistic quantum mechanics, and that this approximation usually gives satisfactory numerical results. This does not necessarily imply that L-S coupling will work in the relativistic region. While we use this scheme to calculate the polarization parameters in the A_1 and B decays, we shall critically examine its validity in the relativistic region using the decay rates of the A_2 and ρ mesons.

Using the covariant procedures outlined above, we calculate in this paper the polarization ratio of the $B\to\omega\pi$ decay. Our result agrees quite well with the existing experimental data. For the $A_1\to\rho\pi$ decay, we obtain a qualitative agreement with the experimental numbers, which numbers are not yet firmly established. We attempted to use the same L-S coupling scheme to calculate the $A_2\to\rho\pi$ and $\rho\to\pi\pi$ decay rates. Because of the large symmetry breaking and relativistic effects, the $\rho\to\pi\pi$ decay amplitude becomes one order of magnitude larger than the A_2 amplitude. In order to trace the source of this discrepancy, we have calculated the decay rates without spins, so that the amplitudes are purely from the harmonic-oscillator overlap integrals. These overlap integrals give a correct ratio between the A_2 and ρ decay rates.

Our conclusion from this work is that the covariant harmonic oscillator which we introduced in Ref. 1 could serve useful purposes in relativistic quark-model calculations.

In Sec. II we outline the procedure for constructing three-meson vertex functions in terms of the probability overlap integral. In Sec. III quark spins are introduced and the decay amplitudes for the A and B mesons are constructed. In Sec. IV we compute these decay amplitudes and discuss in detail their physical implications.

II. CONSTRUCTION OF COVARIANT VERTEX FUNCTIONS

We consider a pair of quarks (or one quark and one antiquark) bound by a harmonic-oscillator force and write the equation of motion

$$\left\{2[\Box_1+\Box_2]-\tfrac{1}{16}\omega^2(x_1-x_2)^2+m_0^{\,2}\right\}\psi(x_1,x_2)=0\,, \quad (1)$$

where x_1 and x_2 are spatial coordinates of the quarks. Following Feynman *et al.*,[4] we make the coordinate transformation

$$X=\tfrac{1}{2}(x_1+x_2),\quad x=\frac{1}{2\sqrt{2}}(x_1-x_2)\,.$$

Then the above equation of motion becomes

$$\left[\frac{\partial^2}{\partial X_\mu^{\,2}}+m_0^{\,2}+\frac{1}{2}\left(\frac{\partial^2}{\partial x_\mu^{\,2}}-\omega^2 x_\mu^{\,2}\right)\right]\psi(X,x)=0\,. \quad (2)$$

This equation is separable in the X and x variables, and the wave function $\psi(X,x)$ can be written in the form

$$\psi(X,x)=e^{-iP\cdot X}\phi(P,x)\,, \quad (3)$$

where $P^2=m_0^{\,2}+\lambda$, and $\phi(P,x)$ satisfies the covariant harmonic-oscillator equation

$$\frac{1}{2}\left(\frac{\partial^2}{\partial x_\mu^{\,2}}-\omega^2x_\mu^{\,2}\right)\phi(P,x)=\lambda\phi(P,x)\,, \quad (4)$$

with the subsidiary condition

$$P^\mu\left(\omega x_\mu-\frac{\partial}{\partial x^\mu}\right)\phi(P,x)=0\,, \quad (5)$$

which eliminates time-like oscillations. We can now construct the ghost-free normalizable solutions of the above equations

$$\phi_n(P,x)=NH_{n_1}(y_1)H_{n_2}(y_2)H_{n_3}(y_3)\times\exp\left[\frac{-\omega}{2}(y_1^{\,2}+y_2^{\,2}+y_3^{\,2}+y_0^{\,2})\right]\,, \quad (6)$$

where the y variables are given in Eq. (2) of Ref. 2. The eigenvalue λ is like that of the nonrelativistic harmonic oscillator, and

$$P^2=m^2=m_0^{\,2}+\omega(n_1+n_2+n_3)\,. \quad (7)$$

The Gaussian factor in Eq. (6) may also be written in the covariant form given in Eq. (3) of Ref. 2.

The above covariant wave function has the following orthogonality relation[3]:

$$\int\phi_n(P,x)\phi_m(P',x)d^4x=\delta_{n_1m_1}\delta_{n_2m_2}\delta_{n_3m_3}\left[1-\left(\frac{\beta-\beta'}{1-\beta\beta'}\right)^2\right]^{(n_3+1)/2}\,. \quad (8)$$

This relation tells us that the wave functions $\phi_n(P,x)$ behave like those of nonrelativistic quantum mechanics if they belong to the same Lorentz frame. This relation further enables us to define a Lorentz-contracted probability for the wave functions belonging to different Lorentz frames. Since the concept of covariant probability could contain deeper physical implications, we review its recent development in Appendix A. We shall use this covariant probability in constructing three-point functions for hadrons.

We next consider a hadron vertex function where the three hadrons a, b, and c come in with their respective four-momenta p_a, p_b, and p_c, as is described in Fig. 1. In Fig. 1 hadron a consists of quark 1 and antiquark 2. Similar explanations can be given to hadrons b and c. We can now consider the vertex function defined by the following probability overlap integral[11]:

$$F = G\int d^4x_1 d^4x_2 d^4x_3 \psi_a(x_1, x_2)\psi_b(x_2, x_3)\psi_c(x_3, x_1)\,. \tag{9}$$

While we do not yet have a completely consistent dynamical scheme, the nonrelativistic version of this probability integral has been used in the literature for calculating decay rates in a vacuum pair-creation model.[7] Our model is of course completely covariant.

If we use the decomposition of Eq. (3) for each ψ, then

$$F = G\int d^4x_1 d^4x_2 d^4x_3 \exp[-i(p_a\cdot X_a + p_b\cdot X_b + p_c\cdot X_c)] \times \phi_a(x_a)\phi_b(x_b)\phi_c(x_c)\,, \tag{10}$$

where

$$X_a = \tfrac{1}{2}(x_1 + x_2),\quad x_a = \frac{1}{2\sqrt{2}}(x_1 - x_2),\ \text{and cyclic}\,.$$

The constant G characterizes the strength of the vacuum pair creation and is to be determined experimentally. In the following discussion, this constant will absorb all trivial numerical factors.

It is not difficult to show that the above probability overlap integral can be reduced to

$$F = G\delta(p_a + p_b + p_c) \times \int d^4x_a d^4x_b \exp[i\sqrt{2}(p_b\cdot x_a - p_a\cdot x_b)] \times \phi_a(x_a)\phi_b(x_b)\phi_c(-x_a - x_b)\,. \tag{11}$$

We can simplify this expression by introducing the momentum wave function $\varphi(q)$ defined as

$$\varphi(q) = \int \phi(x)\exp(i\sqrt{2}\,q\cdot x)d^4x\,. \tag{12}$$

Then the decay amplitude takes the form

$$F = G\delta(p_a + p_b + p_c) \times \int d^4q\, \varphi_a(q)\varphi_b(q + p_c)\varphi_c(q - p_b)\,. \tag{13}$$

The argument of the φ function corresponds to the internal momentum of the oscillator system. In other words, q in φ_a is the momentum difference between quark 1 and antiquark 2. If we insist on four-momentum conservation at each hadronic point, we can assign four-momenta $(q + p_a)/2$, $(q - p_a)/2$, and $(q - p_b + p_c)/2$ to the quarks 1, 2, and 3, respectively, as is indicated in Fig. 2.

Equation (13) is a probability overlap integral in momentum space. The momentum of each quark varies as q goes over all possible values. This variation is consistent with the energy-momentum conservation law. The momentum distribution is of course determined by the wave functions in the integrand. Equation (13) is manifestly covariant and crossing-symmetric. In the following section we shall take into account quark spins.

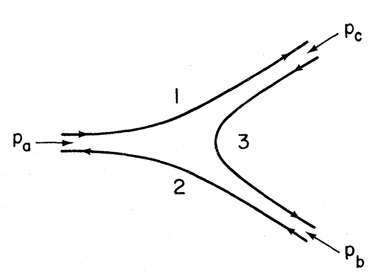

FIG. 1. Three-meson vertex in the quark-pair-creation model. This figure indicates the continuity of internal quantum numbers.

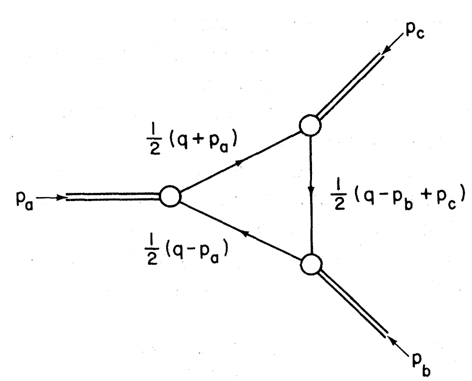

FIG. 2. Three-meson vertex in the quark-pair-creation model. This figure shows the internal momenta of the participating quarks.

III. CONSTRUCTION OF MESONIC DECAY AMPLITUDES

As we stated in Sec. I, we use the L-S coupling scheme for calculating the total angular momentum of the bound system. We first add up the spin angular momenta of the quarks and then couple the total spin to the orbital angular momentum. For the quark-antiquark system, the total spin can be either 0 or 1. We denote the spin-0 system by P and the spin-1 system by V_μ with the condition $p_\mu V^\mu = 0$. We can then write the spin wave function as

$$M_i(p_i) = \left(\frac{m_i + \gamma\cdot p_i}{2m_i}\right)(\gamma_5 P_i + \gamma_\mu V_i^\mu), \tag{14}$$

where $i = a$, b, or c. This $M(p)$ is to be sandwiched between two spinors with their respective spin states. γ_5 projects out only singlet states while $\gamma_\mu V^\mu$ projects out triplet states. Considering all possible spin states for each quark with given four-momentum as is indicated in Fig. 2, the spin structure in the probability integrand becomes

$$S(q) = \mathrm{Tr}\left\{M_a(p_a)\left(\frac{\gamma\cdot(q+p_a)+m}{2m}\right)M_c(p_c)\left(\frac{\gamma\cdot(q-p_b+p_c)+m}{2m}\right)M_b(p_b)\left(\frac{\gamma\cdot(q-p_a)+m}{2m}\right)\right\}, \tag{15}$$

where m is twice the quark mass. The spin amplitude $S(q)$ depends also on the momenta and spins of the hadrons a, b, and c. These spins are eventually to be coupled to the orbital angular momenta specified by the momentum wave functions φ_a, φ_b, and φ_c which we discussed in Sec. II.

We are familiar with the construction of singlets and triplets with nonrelativistic Pauli spinors. However, replacing Pauli spinors by momentum-dependent Dirac spinors is a new procedure whose physical meaning has yet to be clarified. Though it is an incomplete theory, this practice not only generates encouraging numerical results, but also gives physical insights into the quark spin structure. In constructing the above spin amplitude, we have adopted this widely accepted procedure and have not made any attempt to improve the existing situation.

In the decay process where the initial particle decays into two final-state particles, we can work in the Lorentz frame where the initial particle is at rest, and the momenta of the final-state particles are collinear. Since the internal momentum q can take all possible values and directions, the integration over this variable will give departures from the $\mathrm{SU}(6)_W \otimes \mathrm{O}(2)_{Lz}$ symmetry, which is known to be badly broken.

In our previous paper we discussed covariant orbital angular momenta as it is used in the harmonic-oscillator system. We can now couple the orbital angular momentum to the total spin of each hadron. With this understanding, we can write the transition amplitude as

$$T = G\int d^4q\,\varphi_a(q)\varphi_b(q+p_c)\varphi_c(q-p_b)S(q). \tag{16}$$

The decay rate in the center-of mass system is then

$$\Gamma = \frac{b}{8\pi m_a^2}\,|T|^2, \tag{17}$$

where b is the magnitude of the momenta of the final-state particles. T is to be averaged over initial polarizations and is to be summed over final-state polarizations.

We are considering here the decay of an $L=1$ meson into $L=0$ mesons. The initial-state meson is at rest, and the wave function takes the form

$$\varphi_a^i(q) = q_i \exp\left[-\frac{1}{3W^2}(\vec{q}^2 + q_0^2)\right], \tag{18}$$

where $3W^2 = \omega$. ω is the spring constant defined in Eq. (2). The index i runs from 1 to 3. This index is to be coupled with the spin index to produce the total angular momentum. The final-state wave functions have the form

$$\varphi_b(q) = \exp\left\{-\frac{1}{3W^2}\left[-q^\mu q_\mu + 2\left(\frac{q\cdot p_b}{m_b}\right)^2\right]\right\}$$

and (19)

$$\varphi_c(q) = \exp\left\{-\frac{1}{3W^2}\left[-q^\mu q_\mu + 2\left(\frac{q\cdot p_c}{m_c}\right)^2\right]\right\}.$$

As was noted in Ref. 1, the normalization constants are momentum-independent and thus can be swept into the constant G in Eq. (16).

In the decays of the A and B mesons, the initial-state spins are 1 and 0, respectively. One of the final-state particles, say particle b, has spin 1, and the other has spin 0 in both the A and B meson cases. For the $A_1 \to \rho\pi$ decay, we have

$$S_A(q)=\mathrm{Tr}\left[\left(\frac{1+\gamma_0}{2}\right)\gamma\cdot\epsilon\left(\frac{\gamma\cdot(p_a+q)+m}{2m}\right)\left(\frac{m_\pi+\gamma\cdot p_c}{2m_\pi}\right)\gamma_5\left(\frac{\gamma\cdot(q-p_b+p_c)+m}{2m}\right)\left(\frac{m_\rho+\gamma\cdot p_b}{2m_\rho}\right)\gamma\cdot\eta\left(\frac{\gamma\cdot(q-p_a)+m}{2m}\right)\right], \tag{20}$$

where ϵ and η are the spin polarization vectors of the A and ρ mesons respectively.

For the $B\to\omega\pi$ decay,

$$S_B(q)=\mathrm{Tr}\left[\left(\frac{1+\gamma_0}{2}\right)\gamma_5\left(\frac{\gamma\cdot(p_a+q)+m}{2m}\right)\left(\frac{m_\pi+\gamma\cdot p_c}{2m_\pi}\right)\gamma_5\left(\frac{\gamma\cdot(q-p_b+p_c)+m}{2m}\right)\left(\frac{m_\omega+\gamma\cdot p_b}{2m_\omega}\right)\gamma\cdot\eta\left(\frac{\gamma\cdot(q-p_a)+m}{2m}\right)\right]. \tag{21}$$

Here η denotes the spin polarization vector of the ω meson.

The trace calculations are tedious but straightforward. The traces become polynomials in q. We then have to couple the orbital angular momentum represented by the momentum wave function with the spin angular momentum represented by the polarization vector ϵ. This calculation is also a well-known procedure. We then have to evaluate the integrals in q. Since the exponent of the momentum wave function is quadratic in q, the integral becomes trivial if the exponent is brought to diagonal form. Since this procedure is not widely known, we outline the evaluation of the Gaussian integral in Appendix B.

IV. CALCULATION OF DECAY AMPLITUDES

In this section we present the calculational details of the decay amplitudes and discuss their physical implications.

Let us start with the $B\to\omega\pi$ decay. The B meson is regarded as an $S=0$, $n=L=1$ state of the quark-antiquark system. We are calculating the amplitude in the Lorentz frame where this initial-state meson is at rest. Thus the momentum wave function takes the form

$$\varphi_a(q)\propto q_i\exp\left[-\frac{1}{3W^2}(\vec{q}^2+q_0^2)\right], \tag{22}$$

where the index i runs from 1 to 3 and represents the orientation of the orbital angular momentum. As far as the spin is concerned, the B meson has the same spin structure as the π meson, and this fact has been taken into account in Eq. (21). The total angular momentum of the B meson is represented entirely and only by the orbital angular momentum.

We have to evaluate the trace of Eq. (21) for both the transverse and the longitudinal polarizations of the ρ meson, which polarization is denoted by η. When η is longitudinal, its timelike component should also be taken into account. The trace calculation shows that when $\vec{\eta}=\vec{e}_1$, only the q_1 component of the wave function of Eq. (22) gives a nonzero contribution in the q integral. We obtain a similar result for $\vec{\eta}=\vec{e}_2$. When η is longitudinal and timelike, only the q_3 component of the momentum wave function gives a nonzero contribution.

In order to simplify the trace calculations, we set the quark mass equal to one half that of the initial-state meson mass. This is of course a very crude approximation. However, we note that there is no reason at present to expect the quark mass to be another universal constant. In fact, the prevailing idea is quite the opposite. For instance, the quark mass is assumed to be one half of the meson mass when we deal with mesons, and to be one third of the baryon mass when we deal with baryons. The quark mass is a locally adjustable parameter. It is adjusted in such a way that the quark velocity would be the same as that of the hadron which the quarks constitute. The approximation we are using is crude but is not against the prevailing idea.

With these calculational features in mind, we have computed both the transverse and longitudinal amplitudes. With the spring constant $\omega=1\ \mathrm{GeV}^2$, the result is

$$\left|\frac{M(0)}{M(\pm1)}\right|_{B\to\omega\pi}=0.69\,. \tag{23}$$

This is in excellent agreement with the latest experimental value of Ascoli *et al.*,[12] which is

$$\left|\frac{M(0)}{M(\pm1)}\right|^{\exp}_{B\to\omega\pi}=0.68\pm0.12\,. \tag{24}$$

Let us next consider the $A_1\to\rho\pi$ decay. The A_1 meson is believed to have $L=1$, $S=1$, and $J=1$. Again in the Lorentz frame where the initial hadron is at rest, we can couple the spin and orbital angular momenta using the well-known nonrelativistic formula. The polarization of this $J=1$ meson is determined by the axial vector J_i defined as

$$J_i=\epsilon_{ijk}q_j\epsilon_k\,.$$

The spin polarization vector ϵ_k was introduced in

Eq. (20).

Here again, $\vec{J}$ has to be transverse when η is transverse, and longitudinal when η is longitudinal and timelike. The above spin-orbit coupling together with the evaluation of the traces will lead to the polynomials to be multiplied with the Gaussian factor in the integrand. These polynomials are written down in Appendix C. The evaluation of the integrals gives

$$\left(\frac{M(\pm 1)}{M(0)}\right)_{A_1\to\rho\pi} = 1.2\,. \tag{25}$$

This value is somewhat larger than the largest experimental value given by Colglazier and Rosner,[5] which is

$$\left(\frac{M(\pm 1)}{M(0)}\right)^{\mathrm{exp}}_{A_1\to\rho\pi} = 0.95\,. \tag{26}$$

There is a discrepancy between the above two numbers, but this should not alarm us. First of all, the difference is not big. Second, our treatment of the quark mass is only a crude approximation. Finally, the experimental situation is by no means ideal. There are big differences among the available experimental numbers.[7] The experimental difficulty primarily comes from the fact that the A_1 meson is a non-Breit-Wigner resonance.[13]

Considering all these factors, the above qualitative agreement is an encouraging result, and further efforts should be made to improve the relativistic L-S coupling scheme, while experimentalists improve their experiments.

Finally, let us compare the decay rate of the $\rho\to\pi\pi$ decay with that of the $A_2\to\rho\pi$ decay. The purpose of this comparison is to see whether the pair-creation constant of Eq. (20) can really be regarded as a constant for all harmonic-oscillator states. As is well known, the ρ meson is in the ground state while the A_2 meson is in the first excited state. We have computed the decay amplitudes using exactly the same procedure as above. Because the ρ-meson mass is much larger than the π-meson mass, and because the A_2-meson mass is much larger than the ρ-meson mass, the $\rho\to\pi\pi$ decay amplitude turns out to be one order of magnitude larger than the $A_2\to\rho\pi$ amplitude. It is possible to readjust the quark mass in order to get the desired number, but we are not interested in such a refinement in this paper. However, we should point out that this discrepancy comes from the L-S coupling scheme and not from the harmonic-oscillator wave functions. In order to support this assertion, we have calculated the decay rates without spins but with the proper statistical weights due to spins. The result is

$$\frac{\Gamma(A_2\to\rho\pi)}{\Gamma(\rho\to\pi\pi)} = 0.5\,. \tag{27}$$

If we take the experimental value of $\Gamma(\rho\to\pi\pi)$ to be 140 MeV, and $\Gamma(A_2\to\rho\pi) = 80$ MeV, the experimental ratio turns out to be 0.6, which is in good agreement with the above prediction. This result is a strong indication that our harmonic-oscillator wave functions are basically good wave functions.

At this point, we have to explain why we expect the spin-orbit coupling to be valid in one case and to be invalid in the other case. As was pointed out in Sec. I, there is no reason to expect that the spin-orbit coupling with Dirac spinors will give correct answers for all relativistic calculations. We note that in the $B\to\omega\pi$ and $A_1\to\rho\pi$ decays, the ω and ρ mesons, whose polarizations we have considered, are relatively slow, and the relativistic effect on the Dirac spinors was not severe. One way of measuring this effect is to estimate the magnitudes of the longitudinal and transverse components of the internal momenta of the quarks. For the A and B meson decays, $\langle q_z^2\rangle$, which measures the longitudinal component, turns out to be of the same order of magnitude as $\langle q_x^2\rangle$ which represents the transverse internal momentum. For the $\rho\to\pi\pi$ decay, the longitudinal component turns out to be one order of magnitude smaller than the transverse internal momentum. This is of course due to the severe relativistic effect. In this case, we cannot expect the L-S coupling in the present form to be a good approximation.

V. CONCLUDING REMARKS

In this paper we have attempted to use the covariant harmonic-oscillator wave functions to calculate the decay amplitudes of the orbitally excited meson resonances. Unlike other relativistic wave functions, our harmonic-oscillator wave function can be given a probability interpretation. We constructed the decay amplitudes by taking probability overlap integrals. The present work suggests that the covariant probability in terms of the harmonic-oscillator wave function is a useful concept in relativistic particle physics.

Our calculation requires extensive spin calculations. We used a simple-minded relativistic L-S coupling scheme where static Pauli spinors are replaced by Dirac spinors, and we also used a crude approximation for the quark mass. We emphasize here that this is only an approximation and has a limited range of validity. This point has also been discussed.

We worked in the framework of the naive relativistic quark model. This model has still many limitations, but it has the basic advantage that it enables us to interpret everything in terms of the well accepted traditional dynamical variables, such as momentum distribution and probability overlap integrals.

There are other approaches to the decay problems. One promising line has been and still is the Melosh transformation.[6] We would like to point out that there has been a recent attempt to interpret the procedures of this transformation in terms of the traditional dynamical variables.[14]

ACKNOWLEDGMENT

We would like to thank Professor S. Oneda for helpful discussions.

APPENDIX A

The covariant harmonic-oscillator equation of Eq. (4) occupies an important place in present-day physics. It serves as a basis for relativistic dual and string models.[15] It is regarded as a possible generalization of nonrelativistic bound-state (standing wave) quantum mechanics. This equation was first written down by Yukawa in connection with Born's reciprocity hypothesis.[16] Though the equation is simple and separable, it contains all the known inconveniences of the relativistic wave equation. It represents a noncompact group which will lead either to nonunitary representation or to infinite-component wave functions. This sometimes appears as negative norms or negative energy eigenvalues.[4] These features prevented us, in the past, from attempting a probability interpretation for the solutions of this equation. Indeed, this lack of probability interpretation led us to alternative approaches to the problem of relativistic extended particles. The dual and string models are good examples along this line.[15] The bag model follows more conventional lines,[17] but it is not clear whether the model, while being a field theory within a confined region, will accommodate the probability concept.

The study of Eq. (4) in connection with a possible relativistic probability amplitude was revived by the form factor calculation of Fujimura *et al.*,[18] who related the asymptotic behavior of the nucleon form factor to the Lorentz-contraction properties of the relativistic wave function. This interest was further enhanced by Feynman *et al.*,[4] who proposed the use of Eq. (4) for relativistic quark models. While their work does not provide significant technical innovations for treating covariant wave functions, it contains a remark which may prove to be a turning point in modern physics. Feynman *et al.* state that it would be difficult to expect dynamical regularities among resonances from the conventional field theory, and that it is worth considering a new relativistic theory which is naive and obviously wrong in its simplicity, but which is definite and enables us to calculate as many things as possible. The model would be clearly "wrong" or incomplete if its wave functions did not carry a probability interpretation.

The relativistic wave functions which Feynman *et al.* used in their paper are not normalizable and do not give correct form factors. Lipes attempted to reformulate their work using normalizable wave functions.[19] However, his excited-state wave functions do not satisfy the harmonic-oscillator wave equation except in the rest frame. Kim and Noz noted this point and constructed ghost-free excited-state wave functions which are completely covariant and which satisfy the harmonic-oscillator equation in all Lorentz frames.[1] Kim and Noz went further to define a covariant inner product which is consistent both with nonrelativistic quantum mechanics and with the Lorentz-contraction property observed in the nucleon form factor. It was later shown by Ruiz[3] that these wave functions have the orthogonality and contraction properties which are summarized in Fig. 3.

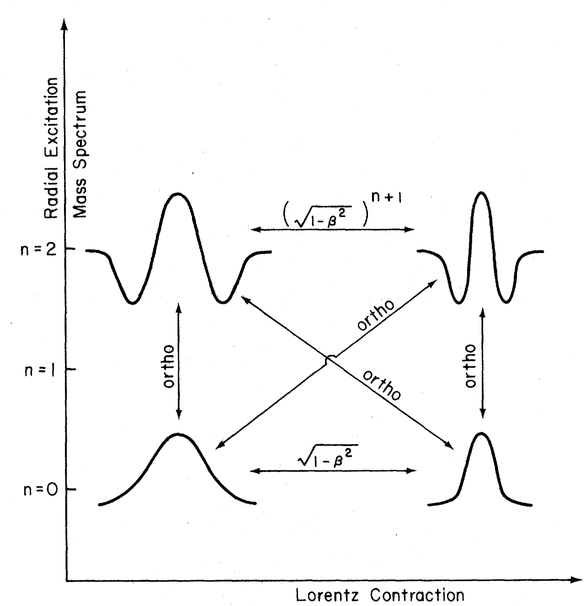

FIG. 3. Orthogonality relations and Lorentz-contraction properties of the covariant harmonic oscillators. These relations enable us to give a Lorentz-contracted probability interpretation to the covariant harmonic-oscillator wave functions.

In the following discussion we restrict ourselves to longitudinal excitations. According to the result summarized in Fig. 3, the orthogonality relation is preserved under Lorentz transformations. Furthermore, the ground-state wave function with one half wave and with no node is contracted like a rigid rod. The nth excited-state wave function with $n+1$ half waves and n nodes contains a polynomial of the nth degree. For this reason, the wave function should be contracted like a multiplication of $n+1$ rigid rods. In fact, our excited-state wave functions behave exactly like that.

While it still seems remote to attempt a completely relativistic measurement theory,[20] the above properties do not discourage us from attaching a probability interpretation to our harmonic-oscillator wave functions.

Our result does not contradict the inconveniences of relativistic wave functions mentioned at the beginning of this appendix. The point is that the hyperbolic partial differential equation of Eq. (4) has many forms of solutions with different boundary conditions. While other solutions have inconvenient features, our solution happens to have the attractive properties summarized in Fig. 3.

APPENDIX B

In this appendix we outline the procedure for evaluating the overlap integrals corresponding to Eq. (16). Let us consider a particle at rest decaying into two particles which have four-momenta p and k, respectively. If we identify these momenta with the quantities in Fig. 2, then

$$p=-p_b \text{ and } k=-p_c .$$

The initial-state meson, particle a, is at rest. Hence, the Gaussian factor factor corresponding to $\varphi_a(q)$ is

$$\varphi_a(q) \propto \exp\left[-\frac{1}{3W^2}(\vec{q}^2+q_0^2)\right]. \tag{B1}$$

The Gaussian factors corresponding to $\varphi_b(q+p_c)$ and $\varphi_c(q-p_b)$ are

$$\varphi_b(q+p_c)=\varphi_b(q-k) \propto \exp\left\{-\frac{1}{3W^2}\left[-(q-k)^2+2\left(\frac{(q-k)\cdot p}{m_b}\right)^2\right]\right\}$$

and (B2)

$$\varphi_c(q-p_b)=\varphi_c(q+p) \propto \exp\left\{-\frac{1}{3W^2}\left[-(q+p)^2+2\left(\frac{(q+p)\cdot k}{m_c}\right)^2\right]\right\}.$$

We then have to add up the above three exponential terms and diagonalize so that we have a quadratic form in q. For this purpose, we use the following transformations of the q_3 and q_0 variables:

$$q_+=\frac{1}{\sqrt{2}}(q_0+q_3), \quad q_-=\frac{1}{\sqrt{2}}(q_0-q_3). \tag{B3}$$

Then the integral of Eq. (16) becomes

$$\int dq_1 dq_2 dq_+ dq_- P(q)\exp\left[-\frac{1}{W^2}(q_1^2+q_2^2)\right]\exp\left[-\left(\frac{q_+-r_+}{s_+}\right)^2-C_+\right]\exp\left[-\left(\frac{q_--r_-}{s_-}\right)^2-C_-\right], \tag{B4}$$

where

$$s_+^2=\frac{3W^2(1-\beta)(1-\alpha)}{4-(1+\alpha)(1+\beta)},$$

$$r_+=\frac{b}{\sqrt{2}}\,\frac{(1+\beta)(1+\alpha)}{4-(1+\alpha)(1+\beta)}\left(2-\frac{1}{\alpha}-\frac{1}{\beta}\right),$$

$$C_+=\frac{b^2}{6W^2}\,\frac{(1+\alpha)(1+\beta)}{(1-\alpha)(1-\beta)}\left\{\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}-2\right)-\frac{(1+\alpha)(1+\beta)}{4-(1+\alpha)(1+\beta)}\left(\frac{1}{\alpha}+\frac{1}{\beta}-2\right)^2\right\},$$

with

$$\beta=b/p_0, \quad \alpha-b/k_0 .$$

We can obtain s_-, r_-, and C_- from the above expression by changing the signs of b, α, and β.

The exponentials in Eq. (B4) are diagonal, and the integral is now in a manageable form.

APPENDIX C

In this appendix we list the polynomials which are to be multiplied with the Gaussian factor in the q integral for the decay amplitudes. We obtain these polynomials from the trace calculation and from the spin-orbit coupling of the initial-state meson.

Let us start with the polynomials in the $B\to\omega\pi$ decay. For this process, there are one longitudinal and two transverse amplitudes. Since the two transverse amplitudes are equal in magnitude, we list only one transverse component. For simplicity, we use the combinations of the energy and mass

$$f = E + m, \quad g = m - E\,, \tag{C1}$$

for each hadron. Then the polynomial for the transverse polarization is

$$\begin{aligned} P(\vec{\eta} = \vec{e}_x) = &- q_x{}^2(q_x{}^2 + q_y{}^2 + q_z{}^2)(b^2 + g_\pi g_\omega) + 4g_x{}^2 q_z q_0 b\, m_\omega \\ &+ q_x{}^2\{ q_0{}^2[f_\pi f_\omega + b^2 + f_\pi g_\omega - g_\pi f_\omega] \\ &\quad + q_0[(E_\omega - E_\pi)(f_\pi g_\omega - g_\pi f_\omega) + m_B(f_\pi g_\omega + g_\pi f_\omega) + 2b^2(2m_\pi + 2m_\omega - m_B)]\}\,. \end{aligned} \tag{C2}$$

For the longitudinal polarization, the polynomial takes the form

$$\begin{aligned} P(\vec{\eta}^* = \vec{e}_z) = &(q_z{}^4 + 2q_z{}^2 q_x{}^2)\left[\left(\frac{E_\omega}{m_\omega}\right)(b^2 - g_\pi g_\omega) - \left(\frac{b}{m_\omega}\right) b(f_\omega + g_\pi)\right] \\ &+ 2q_x{}^2\left(\frac{E_\omega}{m_\omega}\right) b\{ q_z q_0[g_\omega - g_\pi - 2(m_\omega - m_\pi)] + q_z[m_B(g_\omega + g_\pi) + (E_\omega - E_\pi)(g_\omega - g_\pi) + 2(b^2 - g_\pi g_\omega)]\} \\ &+ 2q_x{}^2\left(\frac{b}{m_\omega}\right)\{ q_z q_0(g_{\bar{u}} f_\omega + b^2 + g_\pi g_\omega - f_\pi f_\omega) + q_z[m(b^2 - g_\pi f_\omega) + (E_\omega - E_\pi)(g_\pi f_\omega + b^2) - 2b^2(f_\omega + g_\pi)]\} \\ &+ \left(\frac{bE_\omega}{m_\omega}\right)\{ q_0 q_z{}^3(g_\omega - g_\pi + 2m_\pi) + q_z{}^3[m_B(g_\omega + g_\pi) + (E_\omega - E_\pi)(g_\omega - g_\pi) + 2(b^2 - g_\pi g_\omega)]\} \\ &+ \left(\frac{b}{m_\omega}\right)\{ q_0 q_z{}^3(g_\pi f_\omega + b^2 + g_\pi g_\omega - f_\pi f_\omega) + q_z{}^3[m_B(b^2 - g_\pi f_\omega) + (E_\omega - E_\pi)(g_\pi f_\omega + b^2) - 2b^2(f_\omega + g_\pi)]\} \\ &+ \left(\frac{E_\omega}{m_\omega}\right)\{ q_z{}^2 q_0{}^2(f_\omega f_\pi - b^2 + f_\pi g_\omega - g_\pi f_\omega) \\ &\quad + q_0 q_z{}^2[(E_\omega - E_\pi)(f_\pi g_\omega - g_\pi f_\omega) + m(f_\pi g_\omega + g_\pi f_\omega) + 2b^2(2m_\pi + 2m_\omega - m_B)]\} \\ &+ \frac{b^2}{m_\omega}\{- q_z{}^2 q_0{}^2(f_\pi + g_\omega + 2m_\omega - 2m_\pi) + 2q_0 q_z{}^2[-(E_\omega - E_\pi)(m_\omega - m_\pi) + m_B(E_\omega + E_\pi) - (f_\pi f_\omega - g_\pi g_\omega)]\} \\ &+ \left(\frac{bE_\omega}{m_\omega}\right)\{ q_z q_0{}^3(f_\omega - f_\pi) + q_z q_0{}^2[(E_\omega - E_\pi)(f_\omega - f_\pi) + 2(f_\pi f_\omega - b^2) - m_B(f_\omega + f_\pi)]\} \\ &- \frac{b}{m_\omega}\{ q_z q_0{}^3(f_\pi g_\omega + b^2) + q_z q_0{}^2[(E_\omega - E_\pi)(f_\pi g_\omega + b^2) + 2b^2(f_\pi + g_\omega) + m_B(f_\pi g_\omega - b^2)]\}\,. \end{aligned} \tag{C3}$$

Next, we list the polynomials corresponding to the $A_1 \to \rho\pi$ decay. Here also there are one longitudinal and two transverse amplitudes. The transverse component takes the form

$$\begin{aligned} P(\vec{\eta} = \vec{e}_x) = &-(q_y{}^2 + q_z{}^2)(q_x{}^2 + q_y{}^2 + q_z{}^2)(b^2 + g_\pi g_\rho) + 2q_z{}^2(q_x{}^2 + q_y{}^2 + q_z{}^2)b^2 - 4(q_y{}^2 + q_z{}^2) b q_z q_0 E_\rho + 2q_z{}^2 q_0{}^2 b^2 \\ &- q_z(2q_x{}^2 + q_z{}^2) b\{ q_0(g_\pi - f_\rho - 2E_\pi) - m_A(g_\rho + g_\pi) + (E_\rho - E_\pi)(g_\pi - g_\rho) - 2(b^2 - g_\pi g_\rho)\} \\ &-(q_y{}^2 + q_z{}^2)\{ q_0{}^2(2E_\pi f_\rho - b^2 - g_\rho f_\pi) + q_0[m_A(g_\pi f_\rho - f_\pi g_\rho) + (E_\rho - E_\pi)(2b^2 - g_\pi f_\rho - f_\pi g_\rho)]\} \\ &- q_z q_0{}^2 b\{ q_0(f_\rho - f_\pi) + 2(f_\rho f_\pi - b^2) - m_A(f_\pi + f_\rho) + (E_\rho - E_\pi)(f_\rho - f_\pi)\}\,. \end{aligned} \tag{C4}$$

If the polarization is longitudinal,

$$\begin{aligned} P(\vec{\eta}^* = \vec{e}_z) = &-\left(\frac{E_\rho}{m_\rho}\right)(q_x{}^2 + q_y{}^2)(q_x{}^2 + q_y{}^2 + q_z{}^2)(b^2 + g_\pi g_\rho) \\ &+ \left(\frac{b^2}{m_\rho}\right)(q_x{}^2 + q_y{}^2)(q_x{}^2 + q_y{}^2 + q_z{}^2)(f_\rho - g_\pi) - 4\left(\frac{E_\rho}{m_\rho}\right)(q_x{}^2 + q_y{}^2) q_z q_0 b E_\rho \\ &- \left(\frac{E_\rho}{m_\rho}\right)(q_x{}^2 + q_y{}^2)\{ q_0{}^2(2E_\pi f_\rho - b^2 - g_\rho f_\pi) + q_0[m_A(g_\pi f_\rho - f_\pi g_\rho) + (E_\rho - E_\pi)(2b^2 - g_\pi f_\rho - f_\pi g_\rho)]\} \\ &+ \left(\frac{b}{m_\rho}\right)(q_x{}^2 + q_y{}^2) b\{ q_0{}^2(2E_\pi - 2E_\rho + f_\pi - g_\rho) + 2q_0[m_A(m_\pi + m_\rho) - (E_\rho - E_\pi)^2 + 2b^2 - (f_\pi f_\rho + g_\pi g_\rho)]\}\,. \end{aligned} \tag{C5}$$

We can derive similar polynomials for the $A_2 \to \rho\pi$ decay. In this case there are only two transverse polarizations of equal magnitude. Since we did not use this polynomial in deriving the result of Eq. (27), we shall not list it here.

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*Work supported in part by the National Science Foundation under Grant No. GP 20709.

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