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Semilocal duality in π^0 photoproduction

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It is found that a simple effective Regge parameterization of high-energy differential-cross-section data for π^0 photoproduction when extrapolated to energies below 3 GeV satisfactorily explains the small-$|t|$ data, except for the first resonance, in the sense of semilocal duality.

I. INTRODUCTION

The problem of determining the structure of multipoles, phase shifts, or amplitudes in low-energy pion photoproduction processes is formidable. Compared with pion-nucleon scattering, there are twice as many invariant amplitudes and hence twice as many independent partial-wave amplitudes to determine. Furthermore, the cross-section data are more sparse in the low-energy region and only a small number of single-particle polarization measurements are available.[1] Without any further constraints, it is impossible to determine low-energy multipoles. By imposing smoothness criteria,[2] dispersion relations,[3] Watson's theorem,[4] or consistency with πN resonance spectra,[5] various authors have obtained solutions for multipoles or partial-wave projections in the low-energy regions.

In what may be called the dual problem, in the last few years a great deal of effort has been devoted to the determination of the structure of the amplitudes in the high-energy realm. This work has included fitting available data with Reggeized absorption models,[6] dual absorption models,[7] and most recently, requiring the consistency of effective Regge parameterizations with fixed-t dispersion relations (FtDR's) and finite-energy sum rules (FESR's).[8] This last approach has relied on the existing fits to the low-energy data in order to evaluate the FtDR's and FESR's. The reliability of the resulting high-energy structure is thus dependent on the reliability of the low-energy multipoles. This interdependence is most striking in the recent high-energy analysis of π^0 photoproduction of Barker, Donnachie, and Storrow,[9] who find that the FESR for the unnatural-parity nucleon nonflip helicity amplitude is sizeable, and requires such an amplitude in the high-energy region, although the high-energy data alone can be explained without such a term.[6]

Whether or not it is possible to fit both low- and high-energy data simultaneously, with the constraints that FtDR's and FESR's be satisfied exactly, is of crucial importance in settling detailed questions of amplitude structure. We have taken the first steps by asking the simpler question: How useful are the fits to high-energy data in constraining the low-energy multipoles through FtDR's and FESR's? There are several reasons why such an inverted program may be fruitful. The energy dependences of differential cross section[10] and polarized photon asymmetry data[10,11] are very simple at photon energies above 3 GeV. Natural-parity-exchange amplitudes are dominant away from the forward direction. So a reasonable—but by no means unique—smooth effective Regge parameterization can be obtained. Then by requiring that the low-energy amplitudes satisfy the FESR's in a *semilocal*[12] sense, stringent constraints are imposed. To see the plausibility of such a procedure, we have considered the particularly simple reaction $\gamma P \to \pi^0 P$, for which most high-energy analyses[6–8] suggest the marked dominance of a single amplitude—the single-flip natural-parity-exchange amplitude W_0^+ to which the ω Regge trajectory contributes. We will present some findings here, that corroborate the utility of this program for π^0 photoproduction.

II. THE AMPLITUDE A_4

In agreement with high-energy models, we assume that the CGLN (Chew-Goldberger-Low-Nambu) amplitude A_4 (Ref. 13) (which is proportional to the W_0^+ amplitude at high energies) dominates the other three complex amplitudes at lab energies above 5 GeV for near forward scattering, $-0.1 \geq t \geq -0.4$ and $-0.6 \geq t$ (i.e., excluding the dip region). With this simple assumption, global duality (FESR's) then requires that the integral or average of $\mathrm{Im}\, A_4$ over the low-energy region should dominate over the integrated imaginary parts of $A_{1,2,3}$. Semilocal duality[12] requires, furthermore, that over the low-energy region $\mathrm{Im}\, A_4$ oscillates about the values of $\mathrm{Im}\, A_i$ extrapolated from high-energy data. (The oscillations, of course, are due to resonances, and since the resonances contribute to all of the CGLN amplitudes, the "amplitudes of the oscillations" of all $\mathrm{Im}\, A_i$'s are of roughly the same magnitude; the "period of the oscillations" is roughly the typical resonance spacing, which decreases as the energy increases). Qualitatively, then, we expect $\mathrm{Im}A_4$ to be the dominant imaginary part in the low-energy region as well, and this provides a simple test of the semilocal duality notion. With one further observation, however, we can test these ideas by looking at the data directly. That observation concerns the real parts of the amplitudes.

As energy increases through a resonance, the real part of the resonance contribution to an amplitude changes sign, as is well known. If the next resonance is roughly of the same width, spaced one full width higher in energy, and of the same sign (in its imaginary part), the real parts destructively interfere. Then in regions of many closely spaced resonances of roughly the same magnitude, but random relative signs in the imaginary parts, we expect the real part of the resulting amplitude to be more slowly varying, with smaller fluctuations about an average value than the imaginary part. Then, if the imaginary part satisfies semilocal duality, the more slowly varying real part would be given approximately by the Regge phase applied to the averaged imaginary part (since the extrapolated Regge amplitude satisfies a dispersion relation).

Under these fortuitous circumstances, then, the differential cross section at low energies will be interpolated by the extrapolated high-energy cross section in energy regions of densely packed resonances and momentum transfer regions in which the single amplitude dominates (at high energies).

III. THE FIT

In the high-energy region we follow Ref. 14 and take

$$A_4^R(\nu,t)=ie^{-i\pi\alpha(t)/2}\beta(t)(\nu^2-\nu_0^2)^{[\alpha(t)-1]/2}, \quad (1)$$

where $\alpha(t)$ is an effective trajectory (e.g., ω with an ω-P cut), and $\beta(t)$ being an effective residue obtained from the high-energy data,

$$\nu=\frac{s-u}{4M}$$

and

$$\nu_0=\frac{\mu^2-t}{4M}\left(1-\frac{4M^2}{t}\right)^{1/2},$$

where s, t, and u are the usual Mandelstam variables and μ and M the pion and proton masses, respectively. Then, assuming the other amplitudes are negligible, the differential cross section in the high-energy region is

$$\frac{d\sigma}{dt}\cong K(\nu,t)\,|A_4^R(\nu,t)|^2 = K(\nu,t)\beta^2(t)(\nu^2-\nu_0^2)^{\alpha(t)-1}, \quad (2)$$

where $K(\nu,t)$ is the kinematic factor

$$K(\nu,t)\equiv K(s,t) = \frac{1}{32\pi}\left[(s-M^2)\left(\frac{\mu^2-t}{s-M^2}\right)^3 - (2s+\mu^2)\left(\frac{\mu^2-t}{s-M^2}\right)^2 + 2(s-M^2+\mu^2)\left(\frac{\mu^2-t}{s-M^2}\right)-2\mu^2\right]. \quad (3)$$

Note that $K\to 0$ in the forward direction as $s\to\infty$.

The FESR for $\nu\,\mathrm{Im}A_4$, including the Born term, is

$$\int_{\nu_0}^{N} d\nu\,\nu\,\mathrm{Im}A_4+\frac{\pi e g}{16M^3}\chi_P(t-\mu^2) = \int_{\nu_0}^{N} d\nu\,\nu\,\mathrm{Im}A_4^R = [\cos(\tfrac{1}{2}\pi\,\alpha(t))]\beta(t)(N^2-\nu_0^2)^{[\alpha(t)+1]/2}/[\alpha(t)+1], \quad (4)$$

where g is the usual πN coupling constant, χ_P is the anomalous moment in units of Bohr magnetons. Now define the quantity

$$I(\nu,t)\equiv\nu\cos(\tfrac{1}{2}\pi\alpha(t))\left[\frac{d\sigma}{dt}(\nu,t)/K(\nu,t)\right]^{1/2}. \quad (5)$$

In the high-energy region, this will be $\nu\,\mathrm{Im}A_4^R$ of Eq. (1). In the low-energy region, assuming our arguments about single amplitude dominance and density of resonances are correct, $I(\nu,t)$ should oscillate about $\nu\,\mathrm{Im}A_4^R$ as a function of ν, for fixed t in the relevant regions. That is, the average of $I(\nu,t)$, over an energy region covering a few resonances, should be approximately the same as $\nu\,\mathrm{Im}A_4(\nu,t)$ averaged over the same region. By semilocal duality, $\nu\,\mathrm{Im}A_4(\nu,t)$ should have the same average as $\nu\,\mathrm{Im}A_4^R(\nu,t)$ in such a region.

IV. RESULTS AND CONCLUSIONS

In Fig. 1 we have plotted $I(\nu,t)$ (extracted from the low-energy data[15] and $\nu\,\mathrm{Im}A_4^R(\nu,t)$ (extrapolated from a fit to data bove 5 GeV) in the low-energy region for several values of t. Although data above 2 GeV is scanty, we see that $I(\nu,t)$ does oscillate about $\nu\,\mathrm{Im}A_4^R(t)$ in the energy region above the first resonance, $\Delta(1236)$. [We have smoothed the data for $I(\nu,t)$ below $E_{\mathrm{lab}}=1.2$ GeV by using Walker's fit.[16] We take this to be strong evidence for our conjecture that semilocal duality imposes strong constraints on the low-energy differential cross section for this particular reaction in the appropriate regions of energy and momentum transfer. Near the Δ(1236) we do not expect the constraint to apply since this resonance is separated from the higher resonances by a large gap in energy and hence its real part is not altered. Furthermore, the Regge extrapolation in this lowest energy region is most sensitive to low-lying Regge singularities. We have ignored the detailed structure in the complex angular momentum plane by using a single effective trajectory which fits the high-energy cross-section data. Lower-lying trajectories and nondominant CGLN amplitudes will be important for the "fine-structure" (e.g., polarization asymmetries) at high energies; we have chosen to emphasize the over-all qualitative features over the full range of energies. Although

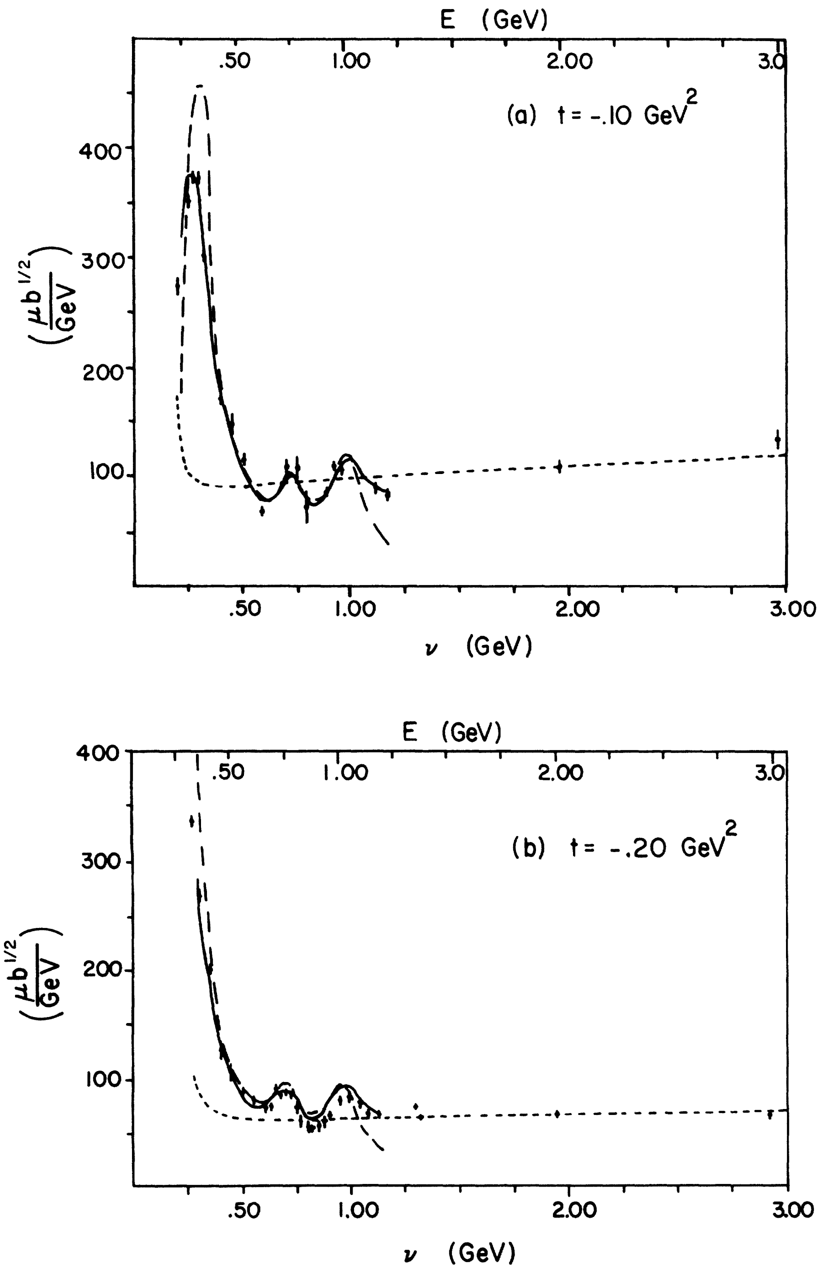

FIG. 1 *(Continued on following page)*

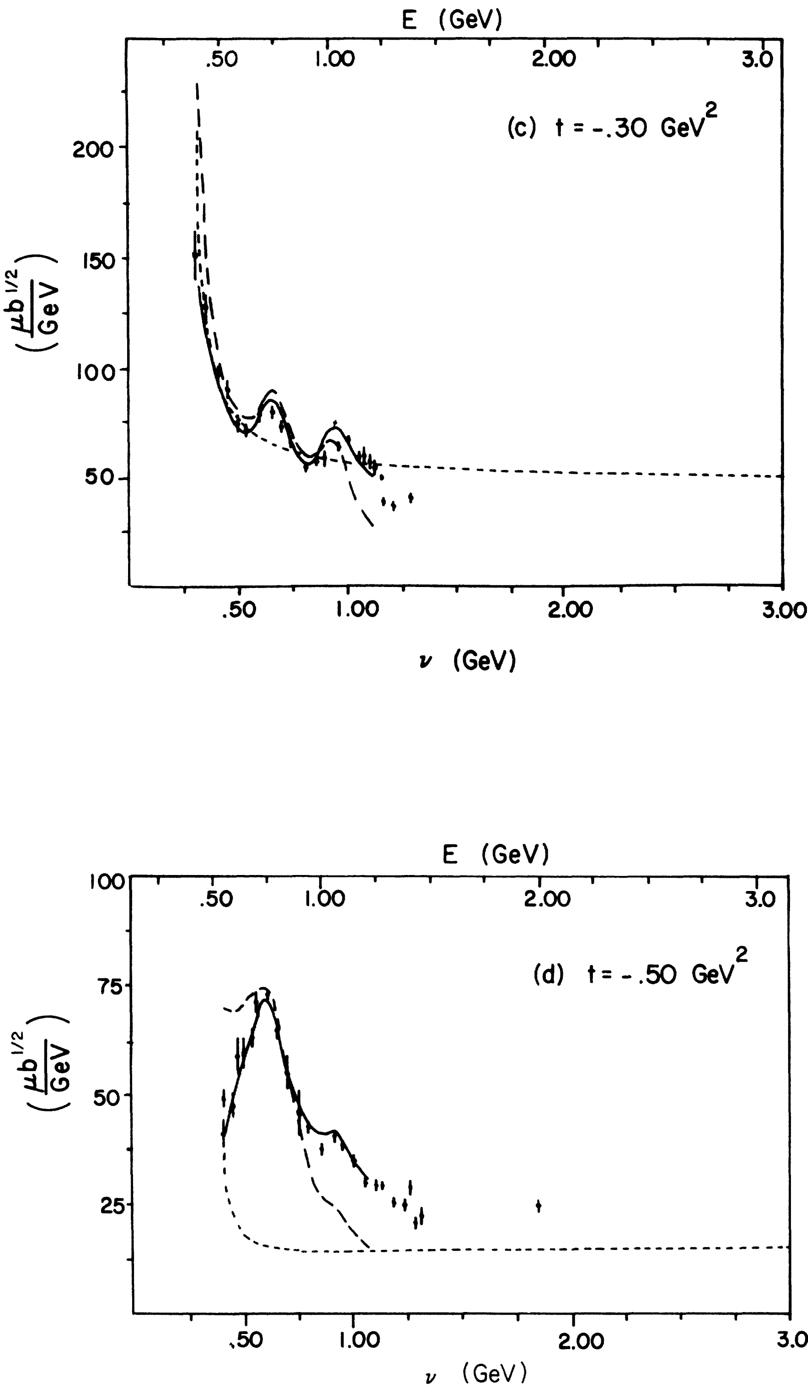

FIG. 1. (a)–(d) $I(\nu, t)$ versus ν for four values of t. The solid line interpolates the data and is given by $\nu\cos(\frac{1}{2}\pi\alpha)[(d\sigma/dt)/K(s,t)]^{1/2}$, where $d\sigma/dt$ is given by the fit to the data of Ref. 16. Representative data points are displayed to give a feeling for the precision involved. The short-dashed line is the Regge extrapolation parameterized as $\nu\cos(\frac{1}{2}\pi\alpha)\beta(\nu^2-\nu_0^{\,2})^{(\alpha-1)/2}$, where α and β are functions of t and have been determined by fits to the high-energy data. The long-dashed line is $\nu\,\mathrm{Im}\,A_4$, determined by the multipole fit of Ref. 16. E is the photon lab energy.

FESR's have been used extensively to constrain the high-energy parameterization, the simple qualitative realization of semilocal duality by the cross sections alone has not been noticed before, and is quite remarkable.

We have also tested Walker's 1968 fit[16] to see if it reflects the semilocal duality constraints indicated by the data. Using six resonances plus smooth backgrounds, Walker has fitted cross-section and polarization data up to $E_{1ab}=1.2$ GeV. We have plotted $\nu \mathrm{Im} A_4(\nu, t)$ obtained from his partial-wave helicity amplitudes in Fig. 1, and compared it with $I(\nu,t)$ and $\nu \mathrm{Im} A_4^R(\nu,t)$. Walker's $\nu \mathrm{Im} A_4(\nu, t)$ has almost the same magnitude and oscillations as $I(\nu, t)$ (except near the "dip" region, $t \sim -0.5$) and is always positive in the region under consideration ($|t| < 0.7$ GeV^2). The difference between I and $\nu \mathrm{Im} A_4$ is of course made up by the other amplitudes and their interference with $\mathrm{Im} A_4$. Although the other amplitudes may be as sizeable as $\mathrm{Im} A_4$, their rapid variations in magnitude and phase combine to give a smaller, more slowly varying contribution to $I(\nu, t)$. This fact and the positivity of $\mathrm{Im} A_4(\nu, t)$ suggest that Walker's fit does support semilocal duality even though his other amplitudes are sometimes larger than we would anticipate. Hence, the possibility of fitting the data with a dominant semilocally constrained $\mathrm{Im} A_4$ and smaller, slowly varying additional contributions is suggested by Walker's fit.

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