Covariant harmonic oscillators and diffractive excitations

Y. S. Kim*
Center for Theoretical Physics, Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742

Marilyn E. Noz†
Department of Physics, Indiana University of Pennsylvania, Indiana, Pennsylvania 15701
(Received 18 March 1974)

We use convariant harmonic-oscillator wave functions to describe quark-model hadrons in Glauber's model of diffractive scattering. It is shown that the Glauber model can be constructed in the center-of-mass system in terms of fully covariant quantities. For elastic scattering, the covariant model gives the same result as that using nonrelativistic harmonic-oscillator wave functions. For the transition from the $n = 0$ to $n = 2$ states, which includes the diffractive excitations to the $N(1470)$ and $N(1690)$ resonances, the relativistic effect is simply a multiplication of the nonrelativistic amplitude by the factor $(1-\alpha^2)$, α being the velocity difference between the incoming nucleon and the final-state resonance. We discuss the effects of these results on the existing nonrelativistic calculations.

I. INTRODUCTION

In our previous paper[1] we discussed a covariant harmonic-oscillator wave function which contains all the required properties of the nonrelativistic bound-state wave function and which contains also the relativistic retardation effect that is consistent with the observed behavior of the nucleon elastic form factor. This wave function satisfies a Lorentz-invariant differential equation and is a normal-coordinate solution of that differential equation. This differential equation allows us to add a gauge-invariant electromagnetic interaction while preserving all the desirable properties.

Our wave function, however, differs from all the wave functions used extensively in the literature for calculating decay rates,[2,3] form factors,[4,5] and high-energy production cross sections.[6,7] It is certainly different from the nonrelativistic wave function. Its ground-state wave function is the same as that used by Fujimura *et al.*[8] and by Lipes.[9] The unique feature of our harmonic-oscillator wave function is that the space-like independent variables in which the harmonic oscillator is excited are unambiguously defined. They satisfy the $O(3)$ requirement.[10] Also the excited-state wave functions in these variables satisfy the above-mentioned Lorentz-invariant differential equation.

While the measurement-theoretic aspect of the covariant bound-state wave function is still an unsolved problem, we take the view that the present high-energy experiments will play an important role in a possible future reexamination of the quantum superposition principle.[11] The purpose of the present investigation is to look for experimental evidence for the existence of harmonic-oscillator excitations in the covariant variables introduced earlier.[1]

Our covariant wave function grew out of a study of the baryonic mass spectrum and of the nucleon elastic form factor. Our next experimental concern should be with decay rates, production processes, and other high-energy reactions where the spatial wave function plays an important role. The baryonic spectrum indicates the existence of static oscillator-like excitations.[12] The form-factor behavior of the nucleon confirms the Lorentz-contraction property of the ground-state oscillator. Our next theoretical concern should naturally be to combine the effects of Lorentz contraction and radial excitation.

In order to study covariant radial excitations, we have to look into production processes where excited-state resonances are produced. We can divide all the observed production processes into two groups. The first group consists of production processes induced by γ rays. The second group includes all the processes caused by pion and K-meson beams. We can treat the first group by using the gauge-invariant electromagnetic interaction, which can be added to the covariant oscillator differential equation.[9] For the meson-induced reactions, however, we have to resort to various models. There are many relativistic models where nonrelativistic wave functions are used to describe hadrons in the quark model.[6,7] The covariant bound-state wave functions may allow us to make these models completely relativistic.

In this paper we study the role of our covariant wave functions in diffractive excitations. We are particularly interested in the Glauber model of diffractive scattering by composite hadrons.[13] In the Glauber model, single-particle scattering can be represented by the two-dimensional impact-parameter space. The compositeness of the particle can be described by the transverse probability function.[14] One of the major difficulties in the past has been the lack of wave functions for fast-moving hadrons. The use of our covariant harmonic-oscillator wave function will completely eliminate this difficulty.

By simply replacing the nonrelativistic wave functions by the covariant wave functions in the center-of-mass system, we derive a result which differs from the nonrelativistic amplitude by the factor $(1-\alpha^2)^{1/2}$, α being the velocity difference between the incoming nucleon and the final-state resonance, for the transitions from the $n=0$ to $n=2$ states. The transitions to the $n=4$ or higher states may exhibit nontrivial Lorentz-contraction effects.

In Sec. II we discuss those properties of our covariant wave functions which are relevant to the present work. In Sec. III we study the kinematics in the center-of-mass system. It is pointed out that the longitudinal and time-like momentum transfers vanish in the high-energy limit. In Sec. IV the Glauber diffractive scattering amplitude is written in terms of purely covariant quantities. We carry out explicit calculations to derive the above-mentioned results. In Sec. V we discuss the implications of our results on the existing calculations.

II. PROPERTIES OF THE COVARIANT WAVE FUNCTION

The present work is an application of the covariant harmonic-oscillator wave function which we discussed in Ref. 1. In this section we discuss in detail the properties of the wave function which are particularly relevant to production reactions and diffractive scattering.

Our covariant harmonic-oscillator wave functions satisfy the following conditions:

(1) The oscillator must produce the nonrelativistic mass spectrum in its rest system, and the mass operator should be Lorentz-invariant and should produce Lorentz-invariant eigenvalues when it is boosted.

(2) When the wave function is boosted, it should have an acceptable Lorentz-contraction property, preferably of the type exhibited by the Bethe-Salpeter wave function. Furthermore, there must be experimental evidence to support this property.

(3) The wave function should satisfy a Lorentz-invariant differential equation, and this should allow us to add a gauge-invariant electromagnetic interaction.

(4) The wave function, under appropriate conditions, should accept the probability interpretation of nonrelativistic quantum mechanics. Since this is a relativistic wave function, it contains more information than is available from Schrödinger quantum mechanics. The properties of the relativistic wave function which cannot be explained in conventional quantum mechanics should be clearly separated from those that admit a nonrelativistic interpretation. The former may be subjected to a possible new measurement theory.

There is at present enough experimental data to confirm the Lorentz-contraction property[8,9] and the existence of harmonic-oscillator-like radial excitations.[12] Our covariant harmonic oscillator can simultaneously explain these two important high-energy phenomena. As is illustrated in Fig. 1, we can now use this wave function to calculate

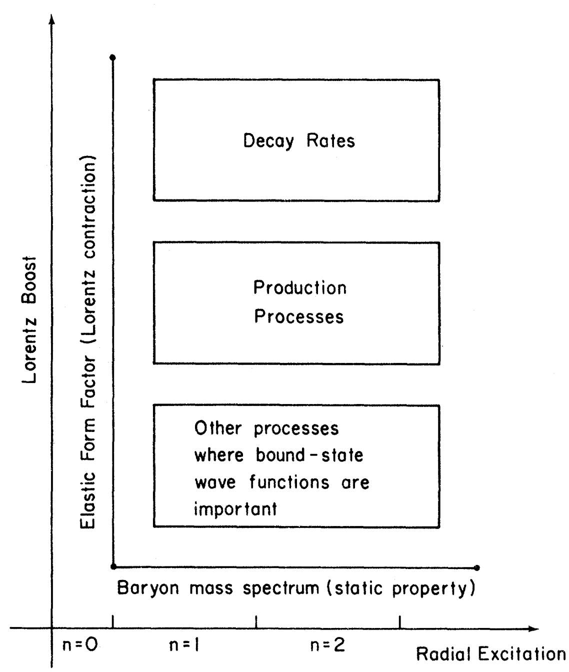

FIG. 1. Two-dimensional plot indicating the applicability of the covariant harmonic-oscillator wave function. The horizontal axis represents bound-state quantum mechanics, and the vertical axis corresponds to relativity. The covariant harmonic-oscillator wave function can thus serve as a simple model in tackling the old and persisting problem of combining the bound-state physics with relativity (see Refs. 11 and 27). This wave function accommodates simultaneously the Lorentz contraction and the radial excitation which is responsible for the discreteness of the mass spectrum. This covariant wave function can be used for calculating decay rates, production processes, and other high-energy processes where bound-state wave functions play important roles.

decay rates, production cross sections, and other high-energy quantities.

In spite of the above-mentioned far-reaching requirements and consequences, the mathematics of our harmonic oscillator is amazingly simple. It is a matter of solving a harmonic-oscillator differential equation using a normal-coordinate method. We start with the invariant differential equation

$$\frac{1}{2}\left\{-\nabla^2+\frac{\partial^2}{\partial t^2}+\omega^2[(\vec{x})^2-t^2]\right\}\psi(x,p)=\lambda\psi(x,p)\,, \qquad (1)$$

where $\vec{x}$ and t are the relative space and time separations between the two bound quarks. This equation was studied first by Yukawa[15] in connection with Born's reciprocity relation, but extensive use of it was made only after the appearance of the quark model. The equation was a starting point for infinite-component theories,[16] relativistic quark models,[3,9] dual resonance and string models.[17] Since this equation serves this many useful purposes, it is worthwhile, in our opinion, to look into a possible deeper physical meaning of the wave function which satisfies Eq. (1).

The physics of the $\vec{x}$ separation is well known in nonrelativistic quantum mechanics. The time separation t cannot be explained in the conventional quantum theory, and there is at present no measurement theory to give proper quantum-mechanical interpretation to this quantity. We believe that this "interpretation," if meaningful, should come from laboratory observations.

In an attempt to find the physical meaning of the wave function, the present authors compared the properties of this wave function with those of the bound-state Bethe-Salpeter wave function.[1,18] The basic advantage the harmonic oscillator offers is its mathematical simplicity. The differential equation is soluble. In solving the differential equation, we note that Eq. (1) is completely separable in the $\vec{x}$, t variables. It is also separable in the homogeneous Lorentz transforms of the variables

$$\begin{aligned} &y_1=x_1,\quad y_2=x_2\,,\\ &y_3=(1-\beta^2)^{-1/2}(x_3-\beta t)\,,\\ &y_0=(1-\beta^2)^{-1/2}(t-\beta x_3)\,. \end{aligned} \qquad (2)$$

where β is the speed variable. The velocity of the hadron is assumed to be in the z direction. If we solve the differential equation by using the y variables, we obtain the Lorentz-contraction factor

$$\exp\left[-\frac{\omega}{2}({y_1}^2+{y_2}^2+{y_3}^2+{y_0}^2)\right] = \exp\left\{-\frac{\omega}{2}\left[-x^\mu x_\mu+2\left(\frac{p\cdot x}{m}\right)^2\right]\right\}\,, \qquad (3)$$

which was extensively discussed in the literature.[1,8,9]

We can construct excited-state wave functions by multiplying the above factor by Hermite polynomials in the y variables. The wave functions so constructed satisfy the differential equation of Eq. (1) with harmonic-oscillator eigenvalues.

It was noted in Ref. 1 that the parameter β defines a space-like hyperplane, and that this hyperplane can be regarded as the Lorentz transform of the rest system. We saw there that the orthonormality relation of the harmonic-oscillator wave function is preserved within a given hyperplane. The inner product of two wave functions belonging to different hyperplanes is expected to show a probability reduction whose concept does not exist in nonrelativistic quantum mechanics.

Thus far, the mathematics of the oscillator was quite simple. The first difficulty we have to face is, however, the fact that the observed mass of the hadron is quite different from the harmonic-oscillator eigenvalue λ of Eq. (1). The mass differences are caused by the SU(6) symmetry-breaking effects. By observing the fact that the mass value in Eq. (3) need not be related to the eigenvalue and also the fact that β is purely a velocity parameter, we can circumvent (do not necessarily solve) this difficulty. Without changing the value of λ, we can adjust the velocity parameter to conserve energy and momentum. Since this velocity is determined from the mass-energy relation, the wave function will contain, through the observed velocity, the effect of the SU(6) symmetry-breaking interactions.

III. KINEMATICS

We are considering the near-forward scattering of two hadrons in which a projectile of mass μ is scattered by a target hadron of mass m. We restrict ourselves to the case where the mass of the projectile is not changed while the target mass is increased from m to M. Since most experimental studies of diffractive scattering are done in terms of the familiar s, t variables, and since these variables take their most convenient form in the center-of-mass system,[19] we shall formulate our problem in the center-of-mass frame.

We are studying the process where the projectile and target particles come in with their respective four-momenta q and p, and then go out with q' and p'. In the center-of-mass system,

$$\vec{p}+\vec{q}=\vec{p}\,'+\vec{q}\,'=0\,,$$

and the variables s and t are

$$s=(p+q)^2=(p'+q')^2\,,\quad t=-(p-p')^2=-(q-q')^2\,. \tag{4}$$

The target particle initially comes in with the energy and momentum,

$$p_0=\frac{s+m^2-\mu^2}{2\sqrt{s}}$$

and

$$|\vec{p}|=\left[\frac{(s-m^2+\mu^2)^2-4s\mu^2}{4s}\right]^{1/2}. \tag{5}$$

The energy of the incoming projectile is

$$q_0=\frac{s-m^2+\mu^2}{2\sqrt{s}}\,. \tag{6}$$

Its momentum is of course equal in magnitude and opposite in sign to that of the incoming target. We can obtain the final-state energies and momenta by simply replacing m^2 by M^2 in the above formulas.

In the center-of-mass frame, we can define a Cartesian coordinate in which the projectile moves along the z direction. Because of the m-M mass difference, the four-momentum transfer

$$k=q-q' \tag{7}$$

has nonzero longitudinal and time-like components,

$$k_0=\frac{M^2-m^2}{2\sqrt{s}}\,,\quad k_3=\frac{M^2-m^2}{2\sqrt{s}}+O\left(\frac{1}{s}\right)\,, \tag{8}$$

for large s. Both k_3 and k_0 vanish in the high-energy limit. Since we are interested only in the high-energy diffractive scattering, we can ignore these quantities and assume that the momentum transfer is purely transverse.

In the following sections we shall assume that the projectile is a point particle and that the target is a bound state of two quarks. Generalizations to more complicated and realistic cases are trivial. The target particle comes in along the z direction with its velocity parameter

$$\beta=-|\vec{p}|/p_0\,, \tag{9}$$

and goes out along the z direction (in the small-angle approximation) with the outgoing velocity

$$\beta'=-|\vec{p}'|/p_0'\,. \tag{10}$$

The incoming target is in the ground state, and its wave function takes the form

$$\psi_{in}(x)=\exp\left[-\frac{\omega}{2}(y_1^2+y_2^2+y_3^2+y_0^2)\right]\,, \tag{11}$$

with proper normalization constant. ω is the well-known spring constant in the quark model. The y variables are defined in Eq. (2). The out-going target is in an excited state, and

$$\psi_{out}=H_{n_1}(y_1')H_{n_2}(y_2')H_{n_3}(y_3')\times\exp\left[-\frac{\omega}{2}(y_1'^2+y_2'^2+y_3'^2+y_0'^2)\right]\,, \tag{12}$$

where

$$y_1'=x_1\,,\ y_2'=x_2\,,\quad y_3'=(1-\beta'^2)^{-1/2}(x_3-\beta'x_0)\,,$$

and

$$y_0=(1-\beta'^2)^{-1/2}(x_0-\beta'x_3)\,.$$

The $H_n(y)$ are Hermite polynomials. Naturally

$$n_1+n_2+n_3=n\,, \tag{13}$$

where n determines the energy level. In Sec. IV we shall study the cases where $n=0$, $n=2$, and $n=4$ or higher.

IV. DIFFRACTIVE EXCITATIONS

According to the Glauber model, the amplitude for high-energy small-angle scattering takes the form

$$f(\vec{q},\vec{q}')=\frac{|\vec{q}|}{2\pi i}\int d^2b\, e^{i(\vec{q}-\vec{q}')\cdot\vec{b}}\{e^{i\chi(\vec{b})}-1\}\,, \tag{14}$$

where $\vec{b}$ is a two-component vector perpendicular to $\vec{q}$, and the "eikonal" $\chi(\vec{b})$ is defined by

$$\chi(\vec{b})=\frac{-\mu}{|\vec{q}|}\int_{-\infty}^{\infty}V(\vec{b}+\hat{q}\xi)d\xi\,. \tag{15}$$

$V(\vec{b}+\hat{q}\xi)$ is the Schrödinger potential.

By summing up a set of Feynman diagrams and by making suitable approximations, Lévy and Sucher derived the covariant form for the scattering amplitude,[20]

$$f(s,t)=\frac{-g^2}{8\pi\sqrt{s}}\int d^4x\, e^{-ik\cdot x}\Delta_F(x)\left(\frac{e^{i\eta}-1}{\eta}\right)\,, \tag{16}$$

where η is a purely field-theoretic quantity. The above covariant form can be made like the impact-parameter representation of Eq. (14) in the kinematical frame where the longitudinal and time-like components of the four-vector k vanish.

As in the case of all field-theoretic models, the Lévy-Sucher formula serves only limited purposes. It was not designed to produce experimentally observed numbers. It is still unknown whether their approximation preserves causality. Their formula, however, is manifestly covariant.

Although there are good reasons to believe that the forces between the quarks inside the hadron are like that of the harmonic oscillator,[12] there are no theories governing the scattering of a quark by a quark. For this reason, the quark-quark scattering amplitude has to be a purely phenomenological quantity. What we have in mind is to replace the field-theoretic quantities in the Lévy-Sucher formula by a covariant but phenomenological formula, which will produce the observed data in high-energy diffractive reactions.

The most commonly used nonrelativistic quark-quark scattering amplitude for small angle is of the form[5,6]

$$\exp\left\{-\frac{a^2}{2}(\vec{q}-\vec{q}\,')^2\right\} \tag{17}$$

in the region where the amplitude is not sensitive to the total energy. It is not difficult to find a covariant quantity which can reproduce the above form when it replaces the integrand of Eq. (16). Let us consider the covariant form

$$G(x,K)=A\exp\left\{-\frac{1}{2a^2}\left[-x^\mu x_\mu+2(x\cdot K)^2/K^2\right]\right\}, \tag{18}$$

where K is a time-like four-vector. We can choose K to be $(p+q)$ or $(q+q')$ in order to arrive at the desired result. The latter choice for K will exhibit a Lorentz contraction of the interaction range.

If we replace the integrand of Eq. (16) by the above phenomenological quantity,

$$f(s,t)=\int d^4x\, e^{-ik\cdot x}G(x,K), \tag{19}$$

the integration can be performed trivially, and this will produce the phenomenological amplitude of Eq. (17). In the kinematical frame which we discussed in Sec. III, the above form can be put into an impact-parameter representation,

$$f(s,t)=\left(\frac{iq}{2\pi}\right)A\int d^2b\, e^{i\vec{k}\cdot\vec{b}}e^{-b^2/2a^2}. \tag{20}$$

The constant A absorbs the trivial kinematical factors.

This impact-parameter representation together with the transverse probability distribution of the quarks in the target hadron makes it possible to construct diffractive scattering by composite particles.[14]

We now construct the transverse probability function $\rho(\vec{b})$ using the covariant harmonic-oscillator wave functions. We can calculate this quantity by integrating the probability density over the longitudinal and time-like directions

$$\rho(\vec{b})=\int dt\, dz\, \psi_f^*(x)\psi_i(x). \tag{21}$$

Then the Glauber single scattering amplitude takes the form

$$f_1(s,t)=\left(\frac{iq}{2\pi}\right)A\int d^2b\int d^2s\, e^{i\vec{k}\cdot\vec{b}}e^{-(1/2a^2)(\vec{b}-\vec{s})^2}\rho(\vec{s}), \tag{22}$$

and the double scattering term can be written as

$$f_2(s,t)=\left(\frac{iq}{2\pi}\right)A^2\int d^2b\int d^2s\, e^{i\vec{k}\cdot\vec{b}}e^{-(1/2a^2)(\vec{b}-\vec{s})^2}\times e^{-(1/2a^2)(\vec{b}+\vec{s})^2}\rho(\vec{s}). \tag{23}$$

In the above expressions the relativistic effects come from the $\rho(\vec{s})$ function. Let us go back to Eq. (21). The initial wave function $\psi_i(x)$ is that of the ground state. The final-state wave function $\psi_f(x)$ is in either a ground or an excited state. It was noted in Ref. 1 that the normalization constants for these harmonic-oscillator wave functions do not depend on the parameter β. This will allow us to ignore the normalization constants in the following discussions. We shall first consider the transition from the $n=0$ to $n=0$ state. Now the $\rho(\vec{b})$ function takes the form

$$\rho_0(\vec{b})=e^{-b^2/2a^2}\int dz\, dt\exp\left\{\frac{-\omega}{2}\left[y_3^{\,2}+y_1'^2+y_0^{\,2}+y_0'^2\right]\right\}. \tag{24}$$

We can perform this integral easily if we use the new variables

$$\xi=\frac{z+t}{\sqrt{2}} \text{ and } \eta=\frac{z-t}{\sqrt{2}},$$

and the net effect is

$$\rho_0(\vec{b})=(1-\alpha^2)^{1/2}e^{-b^2/2a^2}\int dz\, dt\exp[-\omega(t^2+z^2)], \tag{25}$$

where α is the velocity difference between β and β':[21]

$$\alpha=\frac{\beta-\beta'}{1-\beta\beta'}. \tag{26}$$

In the high-energy small-angle scattering,

$$\alpha^2=\left(\frac{M^2-m^2}{M^2+m^2}\right)^2. \tag{27}$$

Equation (22) indicates that the over-all relativistic effect is simply a multiplication of the nonrelativistic calculation by the factor $(1-\alpha^2)^{1/2}$. This simple result will also hold in the transitions from the $n=0$ to $n=1$ states.

The most interesting case is the transition to the $n=2$ states which include the $N(1470)$ and $N(1690)$ resonances. In this case, the z-t integral will contain Hermite polynomials of order 1 or 2. The nonzero contribution to $\rho(\vec{b})$ will come from the integral of the form

$$\rho_2(\vec{b}) = e^{-\omega \vec{b}^2} \int dz\, dt [F(x,y) + z^2] \exp\left(\frac{-\omega}{2} \{z^2 + t^2 + (1-\alpha^2)^{-1}[(z-\alpha t)^2 + (t-\alpha z)^2]\}\right) , \quad (28)$$

where $F(x,y)$ does not depend on z. We can again use the variables ξ and η to evaluate the above integral, and

$$\rho_2(\vec{b}) = (1-\alpha^2)^{1/2} e^{-\omega \vec{b}^2} \int dz\, dt [F(x,y) + z^2] \times \exp[-\omega(z^2 + t^2)] . \quad (29)$$

Here again the relativistic effect is the over-all $(1-\alpha^2)^{1/2}$ factor. This simple result comes from the following cancellation. If we change the variables x and y to ξ and η, the exponent becomes completely separable. The z^2 term in the integrand becomes in effect $(\xi^2 + \eta^2)/2$. In addition to the over-all $(1-\alpha^2)^{1/2}$ factor, the ξ^2 integral receives the $(1-\alpha)$ correction, and the η^2 integral receives $(1+\alpha)$. These corrections cancel out, and the net result is the over-all multiplication factor $(1-\alpha^2)^{1/2}$. We do not expect this type of cancellation for the transitions to the $n=4$ or higher states.

We shall discuss the physical implications of the above calculations in the following section.

V. PHYSICAL IMPLICATIONS

In this paper, we started with the covariant harmonic-oscillator wave function and discussed its applicability in the high-energy diffractive scattering. We noted in Eq. (25) that the Lorentz contraction of the ground-state wave function results in the reduction of the probability by the factor $(1-\alpha^2)^{1/2}$.

This covariant wave function was used to compute the transverse probability distribution which is needed in diffractive scattering by the composite target particle. The most interesting diffractive processes are of course nucleon elastic scattering and production of the $N(1470)$ and $N(1690)$ resonances. These resonances are assumed to have the same SU(6) structure as the nucleon. They have different spatial wave functions. The $N(1470)$ resonance is regarded as the $L=0$ member of the $n=2$ state while the $N(1690)$ resonance is assumed to have $L=2$ in the same n state. In studying these resonances, we can ignore the SU(6) structure and concentrate on the spatial wave functions.

For these production processes, there are numerous calculations in the literature[5,7,19,22]. However, none of them can be regarded as satisfactory because wave functions used are not covariant. In this paper we used completely covariant wave functions. Our calculation for the transitions to the $n=2$ states shows that the use of the relativistic wave functions leads to the over-all contraction factor $(1-\alpha^2)$ which multiplies the nonrelativistic amplitude.[23]

Although the values of α are 0.41 and 0.52 for the $N(1470)$ and $N(1690)$ resonances, respectively, the $(1-\alpha^2)$ factor is still very close to unity. For this reason, this effect cannot be detected from the present experimental data. Therefore, the use of nonrelativistic wave functions was a sound approach from the numerical point of view, and our work removes the well-known internal inconsistencies in the nonrelativistic calculation.

Because of the orthogonality between the $n=0$ and $n=2$ wave functions, the nonrelativistic calculation gives an unwanted forward dip for the $N(1690)$ amplitude.[7,22] Since our result reproduces this orthogonality, the Lorentz contraction alone does not remove this dip.[24] The experimental absence of this dip may indicate that there is a configuration mixing between these excited states and the ground state. However, the present experimental situation does not compel us to calculate this configuration mixing.[25]

In Sec. II we stated that the covariant harmonic oscillator plays important underlying roles in constructing many theoretical models. Of equal importance is its role in understanding experimental data. We noted in Sec. I that it produces the observed mass spectra and elastic form factors. In addition, the use of the covariant wave functions gives quite encouraging results for electroproduction[9] and large-angle nucleon scattering where the form factor plays the decisive role.[26] We note further that the relativistic approach to decay rate calculation, using a harmonic-oscillator model, has been successful.[3] In this paper we observed first that the Gaussian contraction factor gives the $(1-\alpha^2)^{1/2}$ reduction in probability, which is quite compatible with our concept of space contraction. We then showed that the relativistic treatment of diffractive scattering gives *numerically* the same result as that of the nonrelativistic treatment, which is quite consistent with the present experimental data. The covariant harmonic oscillator may not be a perfect model, but it certainly provides a framework for many important aspects of both the conceptual and phenomenological approaches to a possible new physics.[27]

Note added in proof. The result of this paper strongly indicates an orthogonality relation for the wave functions having different velocity parameters. After this work was submitted for publication, Ruiz[28] proved the orthogonality relation for the harmonic-oscillator wave functions which we introduced in Ref. 1 and which we use in this paper and also in the following paper.

---

*Work supported in part by the University of Maryland General Research Board and in part by the National Science Foundation under Grant No. NSF GP20709.

†Present address: Department of Radiology, New York University Medical Center, New York, New York 10016.

[1] Y. S. Kim and M. E. Noz, Phys. Rev. D 8, 3521 (1973).

[2] D. L. Katyal and A. N. Mitra, Phys. Rev. D 1, 338 (1970); D. K. Choudhury and A. N. Mitra, *ibid.* 1, 351 (1970).

[3] R. P. Feynman, M. Kisslinger, and F. Ravndal, Phys. Rev. D 3, 2706 (1971).

[4] S. D. Drell, A. Finn, and M. Goldhaber, Phys. Rev. 157, 1402 (1967).

[5] A. L. Licht and A. Pagnamenta, Phys. Rev. D 2, 1150 (1970); 2, 1156 (1970).

[6] N. S. Thornber, Phys. Rev. 169, 1096 (1968).

[7] A. W. Hendry and J. S. Trefil, Phys. Rev. 184, 1680 (1969). We would like to thank J. S. Trefil for helpful discussions on this paper.

[8] K. Fujimura, T. Kobayashi, and M. Namiki, Prog. Theor. Phys. 43, 73 (1970).

[9] R. Lipes, Phys. Rev. D 5, 2849 (1972). Lipes calculates his transition amplitudes in the Lorentz frame where the excited-state resonance is at rest, and his wave functions coincide with our covariant wave functions in this particular Lorentz frame. For this reason, the use of our spatial wave function will lead to Lipes's calculation.

[10] O. W. Greenberg, Phys. Rev. Lett. 13, 598 (1964).

[11] G. F. Chew, Phys. Rev. D 4, 2330 (1971). In this paper Chew states that the construction of a theoretical model which is demonstrably compatible with both the quantum superposition principle and relativistic space-time is one of the most pressing problems of our time. We agree with him. See also Y. S. Kim and K. V. Vasavada, Phys. Rev. D 5, 1002 (1972).

[12] Y. S. Kim and M. E. Noz, Nuovo Cimento 11A, 513 (1972); T. De, Y. S. Kim, and M. E. Noz, *ibid.* 13A, 1089 (1973); Y. S. Kim and M. E. Noz, Nuovo Cimento 19A, 657 (1974).

[13] R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Brittin and G. Dunham (Interscience, New York, 1959), Vol. 1, p. 315.

[14] W. Czyż and L. C. Maximon, Ann. Phys. (N. Y.) 52, 59 (1969).

[15] H. Yukawa, Phys. Rev. 91, 416 (1953).

[16] A. Bohm, in *Lectures in Theoretical Physics: High Energy Physics and Fundamental Particles*, edited by W. E. Brittin *et al.* (Gordon and Breach, New York, 1967), Vol. XB.

[17] L. Susskind, Phys. Rev. Lett. 25, 545 (1969); Y. Nambu, in *Symmetries and Quark Models*, edited by R. Chand (Gordon and Breach, New York, 1970).

[18] For a detailed discussion on Lorentz contraction properties of the Bethe-Salpeter wave functions, see Y. S. Kim and R. Zaoui, Phys. Rev. D 4, 1764 (1971).

[19] Some authors in the past used different Lorentz frames on the grounds that the quark-model hadrons are relativistic and that nonrelativistic wave functions cannot be used in the center-of-mass system. See, for instance, A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal, Nucl. Phys. B29, 204 (1971).

[20] M. Lévy and J. Sucher, Phys. Rev. 186, 1656 (1969). We would like to thank J. Sucher for explaining the main point of this paper.

[21] A similar result was obtained for $\beta_2 = 0$ by M. Markov. See M. Markov, Nuovo Cimento Suppl. 3, 760 (1956). We would like to thank Dr. Markov for bringing this point to our attention.

[22] D. E. Parry, Nuovo Cimento Lett. 4, 267 (1972).

[23] We note that these resonances belong to the $n = 2$ state, and that each resonance contains two independent modes of oscillation, each giving the $(1-\alpha^2)^{1/2}$ factor.

[24] J. S. Bell derived a similar result using the infinite-momentum frame. See J. S. Bell, Nucl. Phys. B66, 293 (1973).

[25] F. Ravndal, Phys. Rev. D 8, 847 (1973).

[26] J. F. Gunion, S. J. Brodsky, and R. Blankenbecler, Phys. Rev. D 6, 2652 (1972).

[27] P. A. M. Dirac, Rev. Mod. Phys. 21, 392 (1949).

[28] M. J. Ruiz, Phys. Rev. D 10, 4306 (1974).