General space-time structure of the neutral-current interactions*

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We present a general analysis of the neutral-current inclusive cross section taking into account all the covariants (scalar, pseudoscalar, tensor, vector, and axial-vector) in the interaction. We derive the model-independent formula for the inclusive cross section and we also discuss the consequences of various models. We deduce bounds on the ratio $R \equiv \sigma(\bar{\nu}N)/\sigma(\nu N)$ in the case in which scaling is true, i.e., the inclusive cross section at high energies is proportional to the incident neutrino energy E. The scaling violations arising from spin $\neq \frac{1}{2}$ partons and the bounds of R in that case are also discussed.

I. INTRODUCTION

Ever since the neutral-current weak interactions were experimentally discovered,[1] it has usually been assumed that the interaction is made up of vector and axial-vector currents. It is obviously of great importance to establish whether this is really so.

If the neutrino were a two-component field satisfying

$$\gamma_5 \nu = \nu, \quad \bar{\nu}\gamma_5 = -\bar{\nu}\,,$$

then the neutral-current weak interaction of the neutrinos would have to be through vector and axial-vector currents. But how do we know that neutrinos are two-component objects? The fact that neutrinos behave like two-component objects in charged-current interactions does not prove them to be intrinsically so. It is possible that in neutral-current interactions the missing two components of the neutrinos manifest themselves.[2] In any case, this is purely an experimental question.

Hence, the question of the space-time structure of the neutral-current weak interaction is completely open. We have yet to determine which of the covariants, S, P, V, A, or T, occurs in neutral-current weak interactions.[3]

This paper is devoted to a general analysis of the inclusive processes

$$\nu(\bar{\nu}) + N \rightarrow \nu(\bar{\nu}) + \text{anything}$$

taking into account all the covariants S, P, V, A, T in the interaction. These are certainly not simple reactions to study. But, unfortunately, these are our most copious supply of neutral-current events so far and so we consider it useful to know the general structure of the inclusive cross sections.

What we present is a general framework. The questions we ask are of the following type: How many invariant structure functions does the nucleon have, if all covariants S, P, V, A, T are allowed in the interaction? What are the properties of these structure functions? What kind of scaling hypothesis can be made? What do the parton models lead to?, etc.

In Sec. II, the inelastic structure functions of the nucleon are defined and the double-differential cross section for the neutral-current inclusive process is calculated. Section III deals with the scaling model and its consequences. Sections IV and V are devoted to spin $=\frac{1}{2}$ and spin $\neq\frac{1}{2}$ parton models, respectively. In Sec. VI we present the summary. In the Appendix the positivity properties of the structure functions are derived.

II. CALCULATION OF THE CROSS SECTION FOR THE INCLUSIVE PROCESSES

A. The form of the neutral-current interactions

The most general local (i.e., nonderivative) neutral-current interaction of the neutrinos with the hadrons can be written as[4]

$$\begin{aligned}\mathcal{L}_{\text{int}} = \frac{G}{\sqrt{2}}\big[&\bar{\nu}\gamma_\alpha(a_V + b_V\gamma_5)\,\nu V^\alpha + \bar{\nu}\gamma_\alpha\gamma_5(a_A + b_A\gamma_5)\,\nu A^\alpha \\ &+ \bar{\nu}(a_S + ib_S\gamma_5)\,\nu S + i\bar{\nu}\gamma_5(a_P + ib_P\gamma_5)\,\nu P \\ &+ \bar{\nu}\sigma_{\alpha\beta}(a_T + ib_T\gamma_5)\,\nu T^{\alpha\beta}\big]\,. \qquad (2.1)\end{aligned}$$

Here V^α, A^α, S, P, and $T^{\alpha\beta}$ are the vector, axial-vector, scalar, pseudoscalar, and tensor hadronic currents, which are all Hermitian.

We shall assume invariance under time reversal for the rest of this paper. The transformation properties of the leptonic currents under time reversal are the following:

$$T(\bar{\nu}\nu)\,T^{-1} = \bar{\nu}\nu\,,$$

$$T(i\bar{\nu}\gamma_5\nu)\,T^{-1} = -i\bar{\nu}\gamma_5\nu\,,$$

$$\left.\begin{aligned}T(\bar{\nu}\sigma_{\alpha\beta}\nu)\,T^{-1} &= -\eta\bar{\nu}\sigma_{\alpha\beta}\nu \\ T(i\bar{\nu}\sigma_{\alpha\beta}\gamma_5\nu)\,T^{-1} &= \eta i\bar{\nu}\sigma_{\alpha\beta}\gamma_5\nu\end{aligned}\right\}\eta = \begin{cases}+1 \text{ for } \alpha\beta = ij \\ -1 \text{ for } \alpha\beta = 0i \text{ or } i0,\end{cases}$$

$$\left.\begin{aligned}T(\bar{\nu}\gamma_\alpha\nu)\,T^{-1} &= -\eta\bar{\nu}\gamma_\alpha\nu \\ T(\bar{\nu}\gamma_\alpha\gamma_5\nu)\,T^{-1} &= -\eta\bar{\nu}\gamma_\alpha\gamma_5\nu\end{aligned}\right\}\eta = \begin{cases}+1 \text{ for } \alpha = i \\ -1 \text{ for } \alpha = 0\,.\end{cases}$$

We shall assume that the hadronic currents also have unique transformation properties and that they transform in the same way as the leptonic currents, i.e.,

$$TST^{-1}=S\,,$$
$$TPT^{-1}=-P\,,$$
$$TT_{\alpha\beta}T^{-1}=-\eta T_{\alpha\beta}\,,$$
$$TV_\alpha T^{-1}=-\eta V_\alpha\,,$$
$$TA_\alpha T^{-1}=-\eta A_\alpha\,.$$

Hence, for invariance of the interaction under time reversal[5] we have to set $b_S=b_P=b_T=0$. Thus, for S, P, T interactions, time-reversal invariance implies parity conservation, which is reminiscent of the Feinberg-Gupta-Soloviev theorem for Yukawa couplings.[6]

We next note that, for collisions initiated by ν_L or $\bar{\nu}_R$ which alone are produced in the well-known π or K decays, we can replace $(a_V+b_V\gamma_5)$ and $\gamma_5(a_A+b_A\gamma_5)$ in Eq. (2.1) by (a_V+b_V) and (a_A+b_A), respectively. Hence, absorbing the constants into the hadronic currents, we get the simpler form for the effective interaction

$$\mathcal{L}_{\rm int}=\frac{G}{\sqrt{2}}\,[\bar{\nu}\gamma_\alpha\nu(V^\alpha+A^\alpha)+\bar{\nu}\nu S+i\bar{\nu}\gamma_5\nu P+\bar{\nu}\sigma_{\alpha\beta}\nu T^{\alpha\beta}]\,. \tag{2.2}$$

This is the form of the interaction we shall use in our calculation.

The V,A interactions do not flip the helicity of the neutrinos, whereas S, P, T interactions always flip the helicity. Hence the cross sections can be separated into two noninterfering parts, one arising from V,A interactions and the other from S, P, T interactions, and we may consider these two classes of interactions separately. The V,A interactions have already been studied in detail, and so we shall concentrate on the S, P, T interactions.

B. The inelastic structure functions of the nucleon

For the V,A interactions, there are generally six invariant structure functions, $W_1,\ldots,W_6$, one of which arises from time-reversal violation. For the S, P, T case, we introduce the structure functions $W_7,\ldots,W_{15}$ through the following definitions (the structure functions arising from time-reversal violation will be ignored here):

$$\begin{aligned}
W_{S,S}&=\tfrac{1}{2}\sum_i\langle p|S|i\rangle\langle i|S|p\rangle(2\pi)^3\delta^4(p-p_i-q)\\
&=W_7\,,\\
W_{P,P}&=\tfrac{1}{2}\sum_i\langle p|P|i\rangle\langle i|P|p\rangle(2\pi)^3\delta^4(p-p_i-q)\\
&=W_8\,,\\
W_{S,\alpha\beta}&=\tfrac{1}{2}\sum_i\langle p|S|i\rangle\langle i|T_{\alpha\beta}|p\rangle(2\pi)^3\delta^4(p-p_i-q)\\
&=\frac{i}{m_N^2}(p_\alpha q_\beta-q_\alpha p_\beta)W_9\,,\\
W_{P,\alpha\beta}&=\tfrac{1}{2}\sum_i\langle p|P|i\rangle\langle i|T_{\alpha\beta}|p\rangle(2\pi)^3\delta^4(p-p_i-q)\\
&=\frac{i}{m_N^2}\epsilon_{\alpha\beta\mu\nu}(p_\mu q_\nu-q_\mu p_\nu)W_{10}\,,\\
W_{\mu\nu,\alpha\beta}&=\tfrac{1}{2}\sum_i\langle p|T_{\mu\nu}|i\rangle\langle i|T_{\alpha\beta}|p\rangle(2\pi)^3\delta^4(p-p_i-q)\\
&=(g_{\mu\alpha}g_{\nu\beta}-g_{\nu\alpha}g_{\mu\beta})W_{11}+\frac{1}{m_N^4}(p_\mu q_\nu-q_\mu p_\nu)(p_\alpha q_\beta-q_\alpha p_\beta)W_{12}\\
&\quad-\frac{1}{m_N^2}(g_{\mu\alpha}p_\nu p_\beta-g_{\nu\alpha}p_\mu p_\beta-g_{\mu\beta}p_\nu p_\alpha+g_{\nu\beta}p_\mu p_\alpha)W_{13}\\
&\quad-\frac{1}{m_N^2}(g_{\mu\alpha}q_\nu q_\beta-g_{\nu\alpha}q_\mu q_\beta-g_{\mu\beta}q_\nu q_\alpha+g_{\nu\beta}q_\mu q_\alpha)W_{14}\\
&\quad+\frac{1}{m_N^2}[(g_{\mu\alpha}p_\nu q_\beta-g_{\nu\alpha}p_\mu q_\beta-g_{\mu\beta}p_\nu q_\alpha+g_{\nu\beta}p_\mu q_\alpha)+(p\leftrightarrow q)]W_{15}\,.
\end{aligned} \tag{2.3}$$

In these equations $|p\rangle$ denotes a spin-averaged nucleon state with four-momentum p and $W_7, \ldots, W_{15}$ are the functions of the two invariant variables $\nu = -(p\cdot q)/m_N$ and q^2. The mass of the nucleon is denoted by m_N. Hermiticity of S, P, and $T_{\mu\nu}$ implies that W_7, W_8, W_{11}, W_{12}, W_{13}, W_{14}, and W_{15} are real, whereas the transformation properties under time reversal imply that W_9 and W_{10} are real.

The structure functions are not completely arbitrary, but have to satisfy certain positivity properties. Let us introduce the Hermitian matrix $\underline{W}$ through

$$W_{i,j} = \begin{pmatrix} W_{S,S} & W_{S,\alpha\beta} & 0 \\ W^*_{S,\alpha\beta} & W_{\mu\nu,\alpha\beta} & W^*_{P,\alpha\beta} \\ 0 & W_{P,\alpha\beta} & W_{P,P} \end{pmatrix}, \tag{2.4}$$

where the index i or j stands for S, $\alpha\beta$, or P and the elements $W_{S,S}$, etc. are already defined in Eq. (2.3). From the definition of these matrix elements, it is clear that $\underline{W}$ is a semipositive matrix:

$$\sum_{i,j} V_i^* W_{i,j} V_j \geq 0 ,$$

where V is an arbitrary vector. This leads to the following positivity properties of the structure functions (for details, see the Appendix):

(a) $W_7 \geq 0,\ W_8 \geq 0,\ W_{11} \geq 0,$

(b) $X \geq 0,\ Z \geq 0,\ U \geq 0$,

(c) $ZU \geq Y^2$,

(d) $W_7 X \geq \dfrac{1}{m_N^2}(\nu^2 - q^2) W_9^2$,

(e) $W_8 W_{11} \geq \dfrac{1}{m_N^2}(\nu^2 - q^2) W_{10}^2$,

where (2.5)

$$X \equiv -W_{11} + \frac{1}{m_N^2}(\nu^2 - q^2) W_{12} + W_{13} + \frac{q^2}{m_N^2} W_{14} + \frac{2\nu}{m_N} W_{15} ,$$

$$Z \equiv W_{11} - \frac{\nu^2}{q^2} W_{13} - \frac{q^2}{m_N^2} W_{14} - \frac{2\nu}{m_N} W_{15} ,$$

$$U \equiv -W_{11} - \frac{1}{q^2}(\nu^2 - q^2) W_{13} ,$$

$$Y \equiv -\frac{\nu}{q^2}(\nu^2 - q^2)^{1/2}\left(W_{13} + \frac{q^2}{m_N \nu} W_{15}\right) .$$

Conditions (b) and (c) refer to the tensor structure functions alone. Condition (c) is the Schwarz inequality relating the off-diagonal elements of the tensor matrix $W_{\mu\nu,\alpha\beta}$ to the diagonal elements. Inequalities (d) and (e) are the Schwarz inequalities relating the S-T and P-T interference terms to the diagonal terms.

C. The cross section

We want to calculate the double-differential cross section for the neutral-current inclusive process

$$\nu^{\pm} + N \to \nu^{\pm} + \text{anything} ,$$

where we denote neutrino and antineutrino by ν^+ and ν^-, respectively. In the laboratory system where the nucleon N is at rest, the incident neutrino energy is E, the final neutrino energy is E', and the scattering angle of the neutrino is θ. In this system,

$$\nu = E - E' \qquad q^2 = -4EE' \sin^2(\tfrac{1}{2}\theta) . \tag{2.6}$$

The double-differential cross section can be written as

$$\frac{d^2\sigma(\nu^{\pm}N)}{dE'\, d\cos\theta} = \frac{G^2}{16\pi m_N}\frac{E'}{E}\sum_{ij} w_{i,j} W_{i,j} , \tag{2.7}$$

where $w_{i,j}$ is the leptonic analog of the hadronic $W_{i,j}$. The final result is

$$\frac{d^2\sigma(\nu^{\pm}N)}{dE'\, d\cos\theta} = \frac{G^2}{8\pi m_N^3} E'^2 \{2 m_N^2 \sin^2(\tfrac{1}{2}\theta)(W_7 + W_8) + 8\sin^2(\tfrac{1}{2}\theta)[2EE'\sin^2(\tfrac{1}{2}\theta) + (E+E')^2] W_{12}$$
$$+ 4 m_N^2 [4 - \sin^2(\tfrac{1}{2}\theta)] W_{13} - 32 EE' \sin^4(\tfrac{1}{2}\theta) W_{14} + 16 m_N (E - E')\sin^2(\tfrac{1}{2}\theta) W_{15}$$
$$\pm 8 m_N (E + E')\sin^2(\tfrac{1}{2}\theta)(W_9 + W_{10})\} . \tag{2.8}$$

The function W_{11} does not occur in the above cross section since its contribution is proportional to the mass of the lepton. It is important to note that the S-T and P-T interference terms change sign as we go from the neutrino to the antineutrino.

It will take a long time before we have enough experimental data to exploit this completely general formula. So, at this stage it is worthwhile to construct simple models that may be relevant at high energies.

Before we go to the models, let us write down the cross section in the deep-inelastic limit which is obtained by taking E, ν, and $|q^2|$ large such that $x \equiv |q^2|/2m_N\nu$ and $y \equiv \nu/E$ are finite. In this limit, we have

$$\frac{d^2\sigma(\nu^\pm N)}{dxdy} = \frac{G^2 m_N E}{8\pi}\Big\{xy^2(W_7+W_8)+4\Big[\frac{E^2}{m_N{}^2}x(2-y)^2y^2W_{12}+4\frac{Ey}{m_N}(1-y)W_{13}-2\frac{E}{m_N}x^2y^3W_{14}+2\frac{E}{m_N}xy^3W_{15}\Big] \pm 4\frac{E}{m_N}xy^2(2-y)(W_9+W_{10})\Big\} . \tag{2.9}$$

This is still a model-independent formula.

III. SCALING MODEL

The first model we consider is the scaling model based on the following scaling hypothesis. For $\nu\to\infty$ and $|q^2|\to\infty$ such that x is finite, the following limiting functions are supposed to exist:

$$W_7\to F_7(x),\quad W_8\to F_8(x),\quad \frac{\nu}{m_N}W_9\to F_9(x),\quad \frac{\nu}{m_N}W_{10}\to F_{10}(x),\quad W_{11}\to F_{11}(x)\,,$$
$$\frac{\nu^2}{m_N{}^2}W_{12}\to F_{12}(x),\quad \frac{\nu}{m_N}W_{13}\to F_{13}(x),\quad \frac{\nu}{m_N}W_{14}\to F_{14}(x),\quad \frac{\nu}{m_N}W_{15}\to F_{15}(x)\,. \tag{3.1}$$

This hypothesis is motivated purely by analogy with Bjorken's scaling hypothesis[7] for the $V-A$ case. Under this hypothesis, the cross section is asymptotically proportional to E, and dropping the terms which are of lower order in E, we get

$$\frac{d^2\sigma(\nu^\pm N)}{dxdy} = \frac{G^2 m_N E}{8\pi}\{xy^2(F_7+F_8)+4[x(2-y)^2F_{12}+4(1-y)F_{13}-2x^2y^2F_{14}+2xy^2F_{15}]\pm 4xy(2-y)(F_9+F_{10})\}\,. \tag{3.2}$$

We see that the S, P interactions give a y^2 term, and the S-T and P-T interferences lead to a $y(2-y)$ term whereas the T interaction gives a more complicated quadratic in y. Integrating over x and y,

$$\sigma(\nu^\pm N) = \frac{G^2 m_N E}{24\pi}\int_0^1 dx[x(F_7+F_8)+4(7xF_{12}+6F_{13}-2x^2F_{14}+2xF_{15})\pm 8x(F_9+F_{10})]\,. \tag{3.3}$$

By making use of the positivity properties of the structure functions, we can derive lower and upper bounds for the ratio $R\equiv\sigma(\bar\nu N)/\sigma(\nu N)$. For this purpose, let us first write down the scaled versions of the positivity conditions (2.5):

$$
\begin{aligned}
&(\mathrm{a})\ F_7,\ F_8,\ F_{11} \geq 0\ ,\\
&(\mathrm{b})\ -F_{11}+F_{12}-2xF_{14}+2F_{15} \geq 0\ ,\\
&(\mathrm{c})\ F_{11}+\frac{1}{2x}F_{13}+2xF_{14}-2F_{15} \geq 0\ ,\\
&(\mathrm{d})\ -F_{11}+\frac{1}{2x}F_{13} \geq 0\ ,\\
&(\mathrm{e})\ \left(F_{11}+\frac{1}{2x}F_{13}+2xF_{14}-2F_{15}\right)\left(-F_{11}+\frac{1}{2x}F_{13}\right) \geq \frac{1}{4x^2}(F_{13}-2xF_{15})^2\ ,\\
&(\mathrm{f})\ F_7(-F_{11}+F_{12}-2xF_{14}+2F_{15}) \geq |F_9|^2\ ,\\
&(\mathrm{g})\ F_8F_{11} \geq |F_{10}|^2\ .
\end{aligned}
\tag{3.4}
$$

A simple manipulation of these inequalities leads to the following bound on the sum of S-T and P-T interference terms (see the Appendix for a derivation):

$$4\sqrt{7}x(|F_9|+|F_{10}|) \leq x(F_7+F_8)+4(7xF_{12}+6F_{13}-2x^2F_{14}+2xF_{15})\ . \tag{3.5}$$

By using (3.5) in (3.3), we get the minimum and maximum values of R:

$$
\begin{aligned}
R_{\min} &= \frac{\sqrt{7}-2}{\sqrt{7}+2} = 0.1389\ ,\\
R_{\max} &= \frac{\sqrt{7}+2}{\sqrt{7}-2} = 7.195\ .
\end{aligned}
\tag{3.6}
$$

This is the S, P, T analog of the well-known V,A result:

$$R_{\min}=\tfrac{1}{3},\quad R_{\max}=3\ .$$

Since S, P, T do not interfere with V, A the bounds in (3.6) are valid in the most general case of S, P, T, with V and A present. Also, in case of pure T or S, P only R is fixed to be 1.

Next, let us be even more specific and go to the spin-$\frac{1}{2}$ parton model.

IV. SPIN-$\frac{1}{2}$ PARTON MODEL

If only the spin-$\frac{1}{2}$ constituents of the nucleon participate in the neutral-current coupling, then we have

$$
\begin{aligned}
&F_{12}=F_{14}=0\ ,\\
&F_{13}=2F_{12}x=2F_{11}x\ .
\end{aligned}
\tag{4.1}
$$

These are the tensor analogs of the Callan-Gross relations[8] originally derived for the V,A case. Using Eqs. (4.1) in (3.2) one can see that the y distribution for the tensor simplifies to $(2-y)^2$.

Furthermore, within the parton-model framework we can explicitly calculate all the scaled structure functions $F_7, \ldots, F_{15}$. We shall now do this for the quark-parton model in which the nucleon is assumed to be made up of an isodoublet of quarks u and d. We shall ignore the antiquark contribution and the strange-quark contribution, for simplicity.

Let us start with the explicit form of the interaction (including the V,A case):

$$
\begin{aligned}
\mathcal{L}_{\text{int}} = \frac{G}{\sqrt{2}}\,[&\bar{\nu}\gamma_\alpha\nu(h_V^u\bar{u}\gamma^\alpha u+h_A^u\bar{u}\gamma^\alpha\gamma_5 u)+h_S^u\bar{\nu}\nu\bar{u}u\\
&+h_P^u\bar{\nu}\gamma_5\nu\bar{u}\gamma_5 u+h_T^u\bar{\nu}\sigma_{\alpha\beta}\nu\bar{u}\sigma^{\alpha\beta}u+(u\leftrightarrow d)]\ .
\end{aligned}
\tag{4.2}
$$

Thus, the general interaction contains ten coupling constants, $h_V^{u,d}$, $h_A^{u,d}$, $h_S^{u,d}$, $h_P^{u,d}$, and $h_T^{u,d}$. The standard parton model leads to the following double-differential cross section for proton and neutron targets:

$$\frac{d^2\sigma(\nu^\pm p)}{dxdy}=\frac{G^2m_NE}{16\pi}[\phi_\pm^u(y)\,xU(x)+\phi_\pm^d(y)\,xD(x)]\ , \tag{4.3}$$

$$\frac{d^2\sigma(\nu^\pm n)}{dxdy}=\frac{G^2m_NE}{16\pi}[\phi_\pm^u(y)\,xD(x)+\phi_\pm^d(y)\,xU(x)]\ , \tag{4.4}$$

where $U(x)$ is the probability of finding the u quark with the fractional momentum x in the proton, $D(x)$ is the same for the d quark in the proton,

$$
\begin{aligned}
\phi_\pm^u(y) = [&2(h_V^u\pm h_A^u)^2+2(h_V^u\mp h_A^u)^2(1-y)^2\\
&+8h_T^{u2}(2-y)^2+(h_S^{u2}+h_P^{u2})\,y^2\\
&\mp 4h_T^u(h_S^u-h_P^u)\,y(2-y)]\ ,
\end{aligned}
\tag{4.5}
$$

and $\phi_\pm^d(y)$ is defined in a similar manner. For the isospin-averaged nucleon N, Eqs. (4.3) and (4.4) lead to

$$\frac{d^2\sigma(\nu^\pm N)}{dxdy}=\frac{G^2 m_N E}{16\pi}\left[\phi_\pm^u(y)+\phi_\pm^d(y)\right]xQ(x)\ , \quad (4.6)$$

where

$$Q(x)=\frac{U(x)+D(x)}{2}\ .$$

So, the y and x distributions factor out for the isospin-averaged target.

The question one may ask at this stage is whether it is possible to determine the space-time structure of the neutral-current interaction from the y distribution. To answer this, let us parametrize the y distributions in the following manner:

$$\begin{aligned}\frac{d\sigma(\nu N)}{dy}&=(a+by+cy^2)\int\frac{G^2 m_N E}{16\pi}\,xQ(x)dx\ ,\\ \frac{d\sigma(\overline{\nu}N)}{dy}&=(\overline{a}+\overline{b}y+\overline{c}y^2)\int\frac{G^2 m_N E}{16\pi}\,xQ(x)dx\ .\end{aligned}\quad (4.7)$$

By comparison with Eq. (4.6), we get

$$\begin{aligned}a&=\overline{a}=2\left[(h_V+h_A)^2+(h_V-h_A)^2\right]+32h_T^2\ ,\\ b&=-4(h_V-h_A)^2-32h_T^2-8h_T(h_S-h_P)\ ,\\ \overline{b}&=-4(h_V+h_A)^2-32h_T^2+8h_T(h_S-h_P)\ ,\\ c&=2(h_V-h_A)^2+8h_T^2+(h_S^2+h_P^2)+4h_T(h_S-h_P)\ ,\\ \overline{c}&=2(h_V+h_A)^2+8h_T^2+(h_S^2+h_P^2)-4h_T(h_S-h_P)\ .\end{aligned}\quad (4.8)$$

Here and hereafter, all quadratic expressions are understood to be summed over u and d; for instance, $(h_V+h_A)^2$ stands for $(h_V^u+h_A^u)^2+(h_V^d+h_A^d)^2$ and $h_T h_S$ stands for $h_T^u h_S^u+h_T^d h_S^d$.

One can see from Eq. (4.8) that there are three relations among the six parameters a, b, c, $\overline{a}$, $\overline{b}$, and $\overline{c}$:

$$a=\overline{a}, \quad (4.9)$$

$$2a+b+\overline{b}=0\ , \quad (4.10)$$

$$(\overline{b}-b)+2(\overline{c}-c)=0\ . \quad (4.11)$$

Thus, only three independent equations are available for the determination of the coupling constants h_V, h_A, h_S, h_P, and h_T. Hence, the coupling constants for the five different interactions S, P, V, A, T cannot be determined from the y distribution.[9]

The three independent equations are

$$a=4(h_V^2+h_A^2)+32h_T^2\ , \quad (4.12)$$

$$c+\overline{c}-a=2(h_S^2+h_P^2)-32h_T^2\ , \quad (4.13)$$

$$c-\overline{c}=8h_V h_A-8h_T(h_S-h_P)\ . \quad (4.14)$$

From Eq. (4.13), we see that *a nonzero value for* $c+\overline{c}-a$ *is a sure indication of the presence of* S, P, *and/or* T *interactions*. One may further note that, if S, P, T interactions alone are present, then the three equations (4.12), (4.13), and (4.14) allow a determination of the amounts of each of these three interactions. The same statement is, in fact, valid if any three of the five interactions are present.

It may also be pointed out that if data on a proton target are available in addition, then by using Eqs. (4.3) and (4.4) a separate determination of h^u and h^d is possible.

Integrating (4.6) over y, one finds that the ratio $R\equiv\sigma(\overline{\nu}N)/\sigma(\nu N)$ has precisely the same maximum and minimum values given in Eq. (3.6). In other words, the quark-parton model does not restrict the possible range of R any further than the scaling model.

V. PARTONS WITH SPIN $\neq\frac{1}{2}$

A. General remarks

Finally, let us consider the possibility of spin $\neq\frac{1}{2}$ partons in the nucleon. Inelastic electron scattering and inelastic neutrino scattering have so far revealed the presence of only spin-$\frac{1}{2}$ *charged* constituents in the nucelon. However, there are also *neutral* constituents in the nucleon, which seem to carry nearly half the momentum of the nucleon. It is generally thought that these objects have integer spin. These neutral objects might be the ones that participate in neutral-current weak interaction. Hence, it is necessary to analyze the neutral-current interaction more generally, taking account of possible spin $\neq\frac{1}{2}$ constituents.

Once the possibility of spin $\neq\frac{1}{2}$ partons is granted, it should be noted that the cross sections do not scale any longer. In other words, *scaling is violated*. This is in sharp contrast to the situation in the case of V, A interactions, where Bjorken scaling can be maintained for spin $=\frac{1}{2}$ as well as for spin $\neq\frac{1}{2}$ partons.

This difference in scaling behavior between V, A and S, P, T cases arises from the fact that V and A currents have a unique dimension, namely 3, whatever may be the fields out of which the currents are made, whereas the dimensions of S, P, T currents depend on the dimensions of the constituent fields. For instance, the dimension of the currents $\overline{\psi}\psi$, $\overline{\psi}\gamma_5\psi$, $\overline{\psi}\sigma_{\alpha\beta}\psi$ made of spin-$\frac{1}{2}$ field ψ is 3; the dimensions of $\phi^\dagger\phi$ and $(\partial_\mu\phi^\dagger\partial_\nu\phi-\partial_\nu\phi^\dagger\partial_\mu\phi)$ made of spin-0 field ϕ are 2 and 4 respectively; and the dimension of $V_\mu^\dagger V_\mu$ and $V_\mu^\dagger V_\nu-V_\nu^\dagger V_\mu$ made of vector field V_μ is 2. (Note that to construct an antisymmetric tensor invariant from scalar fields, the fields have to carry a new quantum number. This is also true of vector fields if one wants to avoid derivatives. Could this be color?)

Hence, if we allow S, P, T interactions, we should be prepared to see scaling violations also. The scaling violations manifest themselves in (a) the energy dependence of the inclusive cross sections being different from the conventional linear E dependence and (b) the y distribution of the inclusive cross sections also being different from what is expected in the scaling model.

To illustrate these scaling violations, we have worked out some examples of spin-0 and spin-1 partons.

B. Spin-0 partons

Consider the interaction

$$\mathcal{L}_{\text{int}} = g_S \bar{\nu}\nu\phi^\dagger\phi + ig_T\bar{\nu}\sigma^{\alpha\beta}\nu(\partial_\alpha\phi^\dagger\partial_\beta\phi - \partial_\beta\phi^\dagger\partial_\alpha\phi) . \tag{5.1}$$

The contributions to the structure functions arising from this interaction can be calculated in parton model; these should be substituted in Eq. (2.9), which is the general formula for the inclusive cross section in the deep-inelastic limit. The tensor interaction in Eq. (5.1) contributes only to W_{12}, and the final result for the spin-0 contribution is

$$\frac{d^2\sigma(\nu^\pm N)}{dxdy} = [g_S^2 y + 4g_T^2 m_N^2 E^2(2-y)^2 yx^2 \pm 4g_S g_T m_N E(2-y)yx]\frac{S(x)}{\pi} , \tag{5.2}$$

where $S(x)$ is the probability of finding the spin-0 parton ϕ with fractional momentum x in the nucleon. This contribution should be added to the spin-$\frac{1}{2}$ contribution given in Eq. (4.6).

The scalar, tensor, and S-T interference terms in Eq. (5.2) are respectively proportional to E^0, E^2, and E. This can in fact be derived from dimensional arguments. For large E, the E^2 term will dominate over all other terms (including the spin $=\frac{1}{2}$ contributions).

The y distributions also are different from those arising from spin-$\frac{1}{2}$ partons. Actually, with every additional power of E (as compared to linear E) one can associate an additional power of y. *Note especially the presence of y^3 terms. The y distribution is no longer a quadratic.* Finally, in the high energy limit, since only the E^2 term survives, the ratio $R=\sigma(\bar{\nu}N)/\sigma(\nu N)$ goes to 1.

C. Spin-1 partons

We consider the interaction

$$\mathcal{L}_{\text{int}} = f_S\bar{\nu}\nu V_\mu^\dagger V_\mu + if_T\bar{\nu}\sigma^{\alpha\beta}\nu(V_\alpha^\dagger V_\beta - V_\beta^\dagger V_\alpha) . \tag{5.3}$$

The naive dimensional argument does not work in this case since the spin summation for finite-mass spin-1 partons leads to $g_{\mu\nu} - p_\mu p_\nu/m_V^2$, where m_V is the mass of the spin-1 parton. It is the $p_\mu p_\nu/m_V^2$ term that dominates for high energies, and the dominant contribution to the inclusive cross section is proportional to E^2. Keeping only this dominant contribution, we get[10]

$$\frac{d^2\sigma(\nu^\pm N)}{dxdy} = \frac{1}{48\pi}\left(\frac{m_N E}{m_V^2}\right)^2 [f_S^2 y^3 + 4f_T^2(2-y)^2 yx^2 \pm 4f_S f_T(2-y)y^2 x]x^2 V(x) , \tag{5.4}$$

where $V(x)$ is the probability function for spin-1 partons. Note again that the y distributions from all the terms are one power higher than in the spin-$\frac{1}{2}$ case, i.e., y^3 is present in each term.

Integrating Eq. (5.4) over y, we find

$$R \equiv \frac{\sigma(\bar{\nu}N)}{\sigma(\nu N)} = \frac{3f_S^2 + 44f_T^2 - 20f_S f_T}{3f_S^2 + 44f_T^2 + 20f_S f_T} . \tag{5.5}$$

The upper and lower bounds of this expression are

$$R_{\min} = \frac{\sqrt{33}-5}{\sqrt{33}+5} = 0.0697 , \quad R_{\max} = \frac{\sqrt{33}+5}{\sqrt{33}-5} = 14.33 . \tag{5.6}$$

Hence in this case the bounds on R given in Eq. (3.6) are not satisfied.

VI. SUMMARY AND DISCUSSION

If time-reversal invariance is valid, there are nine inelastic structure functions for the nucleon. We have obtained all the positivity conditions satisfied by these functions.

The double-differential cross section for the inclusive process has been written down in terms of these structure functions. This formula [Eq. (2.8)]—especially the characteristic $(E+E')$ dependence contained in it—may be made use of in the future when sufficient experimental data become available.

We have discussed the question of scaling in some detail. Although it is possible to construct a scaling model such that the inclusive cross section is proportional to the incident neutrino energy E, is is by no means clear that the S, P, T interactions would obey this scaling. *In fact, the S, P, T interactions treated in the spin $\neq \frac{1}{2}$ parton models violate scaling. It will be most interesting to look for these scaling violations, i.e., cross sections rising faster than E and y^3 terms in the y distributions, in the experimental data on neutral-current inclusive processes.*

If scaling holds such that the inclusive cross sections at high energies are proportional to E, then we have shown that the ratio $R \equiv \sigma(\bar{\nu}N)/\sigma(\nu N)$ has to lie between 0.1389 and 7.195. The scale-violating contributions from spin-1 partons [given in Eq. (5.4)] evaluated in the high energy limit violate this bound on R and satisfy a wider bound $0.0697 < R < 14.33$.

Added notes. (a) The y distributions for S, P, T for the spin-$\frac{1}{2}$ parton model were written down by Kayser *et al.*[9] and by Kingsley, Wilczek, and Zee.[11] More tests of S, P, T are also discussed by Kingsley *et al.*[12] (b) In the V, A case, if the neutrinos couple to spin-1 objects in the nucleon, then scaling is not satisfied because of the $q_\mu q_\nu/m_V{}^2$ term in the spin sum. This is discussed in detail by Rajasekaran and Roy.[13]

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APPENDIX: POSITIVITY PROPERTIES OF THE STRUCTURE FUNCTIONS

The positivity conditions are simply the statement that the eigenvalues of the matrix $\underline{W}$ defined in Eq. (2.4) are positive-semidefinite.

Instead of the full matrix $\underline{W}$ it is enough to consider the reduced matrix $\underline{W}_R$ obtained by considering the tensor indices $\alpha\beta = 03,\ 02,\ 01,\ 23,\ 31$, and 12 and ignoring 30, 20, 10, 32, 13, and 21. One can show this does not affect the positivity conditions because of the following theorem: If an $n\times n$ Hermitian matrix $\underline{M}$ has eigenvalues $\lambda_1, \ldots, \lambda_n$, then the $2n\times 2n$ Hermitian matrix

$$\begin{pmatrix} \underline{M} & -\underline{M} \\ -\underline{M} & \underline{M} \end{pmatrix}$$

has eigenvalues $2\lambda_1, \ldots, 2\lambda_n$ and 0 repeated n times.

It is convenient to go to the Lorentz frame in which q is purely spatial:

$$\begin{aligned} q &= (0, 0, 0, (-q^2)^{1/2}) , \\ p &= (m(1-\nu^2/q^2)^{1/2}, 0, 0, \nu m/(-q^2)^{1/2}) . \end{aligned} \tag{A1}$$

In this frame, $\underline{W}_R$ is the following matrix:

$$\begin{array}{c} \\ S \\ 03 \\ 02 \\ 01 \\ 23 \\ 31 \\ 12 \\ P \end{array}
\begin{array}{c} \begin{array}{cccccccc} S & 03 & 02 & 01 & 23 & 31 & 12 & P \end{array} \\
\begin{bmatrix} W_7 & W_9' & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
W_9' & X & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & U & \cdot & -Y & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & U & \cdot & Y & \cdot & \cdot \\
\cdot & \cdot & -Y & \cdot & Z & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & Y & \cdot & Z & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & W_{11} & W_{10}' \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & W_{10}' & W_8 \end{bmatrix} \end{array} , \tag{A2}$$

where the dots refer to zeros, X, Y, Z, and U are defined in Eq. (2.5), and

$$\begin{aligned} W_9' &= \frac{1}{m_N}(\nu^2 - q^2)^{1/2} W_9 , \\ W_{10}' &= \frac{1}{m_N}(\nu^2 - q^2)^{1/2} W_{10} . \end{aligned} \tag{A3}$$

The eigenvalues of the matrix in (A2) are

$$\begin{aligned} &\tfrac{1}{2}(Z+U) \pm \tfrac{1}{2}[(Z+U)^2 - 4(ZU - Y^2)]^{1/2} , \\ &\tfrac{1}{2}(W_7+X) \pm \tfrac{1}{2}[(W_7+X)^2 - 4(W_7X - W_9'^2)]^{1/2} , \\ &\tfrac{1}{2}(W_8+W_{11}) \pm \tfrac{1}{2}[(W_8+W_{11})^2 - 4(W_8W_{11} - W_{10}'^2)]^{1/2} , \end{aligned} \tag{A4}$$

where the eigenvalues given in the first line occur twice. The positivity of these eigenvalues is expressed by the conditions given in (2.5).

The scaled versions of the positivity conditions are given in (3.4). Let us derive the inequality (3.5) from these. First multiply the Schwarz inequalities (3.4f) and (3.4g) by an arbitrary α^2:

$$\begin{aligned} &F_7\alpha^2(-F_{11} + F_{12} - 2xF_{14} + 2F_{15}) \geq |\alpha F_9|^2 , \\ &F_8\alpha^2 F_{11} \geq |\alpha F_{10}|^2 . \end{aligned}$$

From these, we can derive the following linearized inequalities:

$$F_7 + \alpha^2(-F_{11} + F_{12} - 2xF_{14} + 2F_{15}) \geq 2\alpha|F_9| , \tag{A5}$$

$$F_8 + \alpha^2 F_{11} \geq 2\alpha|F_{10}| . \tag{A6}$$

From (3.4c) and (3.4d), we get

$$\beta^2\left(\frac{F_{13}}{x} + 2xF_{14} - 2F_{15}\right) \geq 0 , \tag{A7}$$

where β^2 is another arbitrary constant. Combining (A5), (A6), and (A7), we have

$$(F_7 + F_8) + \alpha^2 F_{12} + \beta^2\frac{F_{13}}{x} + (\alpha^2 - \beta^2)(-2xF_{14} + 2F_{15}) \geq 2\alpha(|F_9| + |F_{10}|) . \tag{A8}$$

The choice $\alpha = \sqrt{28}$ and $\beta = \sqrt{24}$ leads to (3.5).

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