Weak decays of charmed hadrons. II. Soft-meson theorems*

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Soft-meson theorems, inclusive and exclusive, are derived for weak nonleptonic decay of charmed hadrons in the context of the four-quark model of weak interactions.

In a previous paper[1] we discussed weak, nonleptonic decay of charmed hadrons[2] in the framework of the four-quark model of the weak interactions. In this model the charm-changing piece of the effective Hamiltonian has a simple SU(3) structure, and SU(3) considerations were accordingly exploited to relate various two-body decays of charmed mesons and baryons. These relations take on an especially striking form under the hypothesis that the SU(3) $\underline{6}\oplus\overline{\underline{6}}$ terms dominate in the effective Hamiltonian. It was shown that $\underline{6}\oplus\overline{\underline{6}}$ dominance for the charm-changing interactions is implied by the familiar and well-established octet dominance of charm-conserving reactions, insofar as the latter is a reflection of the short-distance properties of the weak interactions.[3]

In the present note we discuss the application of soft-pion ideas[4] to weak nonleptonic decay of charmed hadrons. It will be sufficient here to concentrate on the leading effects, obtained by setting to zero the Cabibbo angle θ_C. The effective charm-changing Hamiltonian then has the structure (up to an over-all constant factor)

$$H_{\text{int}} \sim \bar{\lambda}\gamma_\mu(1+\gamma_5)\mathcal{P}'\bar{\mathcal{P}}\gamma_\mu(1+\gamma_5)\mathcal{N}+\text{H.c.}\,, \qquad (1)$$

where $\mathcal{P},\mathcal{N},\lambda$ are the usual quarks and $\mathcal{P}'$ is the charmed quark. In the previous paper[1] we invoked the approximation of SU(3) symmetry for the strong interactions. In the case of two-body decays, where the available phase volume is ample, this approximation is probably reasonable, but since we will here be considering multiparticle decays we avoid any appeal to SU(3) symmetry. On the other hand, SU(2) symmetry considerations *will* be invoked. Notice that the effective Hamiltonian behaves like an isovector.

The soft-pion results follow in the standard way from the ideas of current algebra and PCAC (partially conserved axial-vector current). They concern the decay spectrum of a produced pion in the limit where the pion four-momentum q goes to zero. If one integrates over all final-state variables other than the pion energy E, then $q\to 0$ involves not only neglect of pion mass but also extrapolation in E to the unphysical point $E=0$. If the data for small, physical values of E display gentle and smooth behavior, the extrapolation may not be too hazardous; if the variations are rapid, it may be hazardous. The hazards of PCAC are known well enough and need not be labored here any further. We start with a discussion of charmed-meson decays. There is the technical simplification here that the most important decay channels are not likely to be those involving baryon pairs. Where baryons appear, soft-pion insertions on external baryon lines have to be taken into account. In the absence of baryons the soft-pion limits are determined solely by equal-time commutator terms. However, at the end we also discuss a special class of charmed-baryon decays; for example, those in which the outgoing baryon is a Λ particle. Since a pion cannot be inserted on the Λ-particle line, here again only the equal-time commutators come into play. Also in this class are states involving a proton and soft π^+, a neutron and soft π^-, or Σ^- and soft π^-.

Soft-pion limits in charmed meson decay. We deal here with the triplet of charmed mesons $F^+\equiv(\mathcal{P}',\bar{\lambda})$, $D^+\equiv(\mathcal{P}',\bar{\mathcal{N}})$, $D^0\equiv(\mathcal{P}',\bar{\mathcal{P}})$, where the symbols in parentheses characterize the quark content. For our present purposes the only features of this characterization that are relevant have to do with the SU(2) properties of these mesons: F^+ is a singlet; D^+, D^0 form a doublet. The soft-pion results can be worked out case by case for individual channels. We begin, however, with the inclusive situation, where one sums over all channels containing a soft pion of specified charge.

Let M denote any particular one of the charmed mesons and let Γ_M be the *total* nonleptonic decay rate. Let $\partial^3\Gamma_{M\to\pi}/\partial^3 q$ be the rate spectrum in the momentum $\vec{q}$ of the produced pion, summed over all accompanying particles. Define

$$\omega_{M\to\pi}(E)\equiv 16\pi^3 E\,\frac{\partial^3\Gamma_{M\to\pi}}{\partial^3 q}\,. \qquad (2)$$

Notice that

$$\frac{1}{\Gamma}\int\frac{\partial^3\Gamma}{\partial^3 q}\,d^3q$$

is equal to the mean multiplicity for pions of the specified charge. The soft-pion theorems concern the $E\to 0$ limit of $\omega(E)$ and are summarized as follows:

For $M=F^+, D^+, D^0$

$$\omega_{M\to\pi^0}\underset{E\to 0}{\longrightarrow}\frac{1}{f_\pi^{\,2}}\Gamma_M \qquad (3)$$

and

$$\omega_{M\to\pi^-}\underset{E\to 0}{\longrightarrow}0\;; \qquad (4)$$

also

$$\omega_{F^+\to\pi^+}\underset{E\to 0}{\longrightarrow}\frac{1}{f_\pi^{\,2}}\Gamma_{F^+}\;, \qquad (5)$$

$$\lim_{E\to 0}\omega_{D^0\to\pi^+}=\omega_{D^+\to\pi^+}=\frac{1}{2f_\pi^{\,2}}(\Gamma_{D^0}+\Gamma_{D^+})\;. \qquad (6)$$

In these expressions f_π, which has the dimension of mass, is the PCAC constant defined in the context of $\pi\to\mu+\nu$ decay by

$$\left\langle 0\left|\frac{\partial A_\mu^{1+i2}}{\partial x_\mu}\right|\pi^-\right\rangle=\sqrt{2}\,f_\pi m_\pi^{\,2}\;,$$

where A_μ^{1+i2} is the usual charge-raising axial-vector current; experimentally, $f_\pi\approx 95$ MeV.

The application of similar PCAC ideas for soft-kaon emission could be carried out, formally, in the same way as for pions. The validity of these ideas for kaons is of course considerably more dubious. Nevertheless, in passing, and as a very crude guide to the possibilities for charmed-particle decay, we may note the following results. For $M=F^+, D^+, D^0$ one finds

$$\omega_{M\to K^0}\underset{E\to 0}{\longrightarrow}0\;' \qquad (7)$$

and, if $\underline{6}\oplus\underline{\overline{6}}$ dominance holds,

$$\omega_{M\to K^\pm}\underset{E\to 0}{\longrightarrow}0\;. \qquad (8)$$

If the soft-kaon ideas were confirmed in connection with Eq. (7), then Eq. (8) would become interesting as a test of $\underline{6}\oplus\underline{\overline{6}}$ dominance.

The preceding results[5] for inclusive decays can easily be extended to cover semi-inclusive and exclusive processes. It is of course immediately evident that Eqs. (4), (7), and (8) hold for every exclusive channel. Similarly Eq. (3) holds for any exclusive decay, $M\to\pi^0+\beta_1+\beta_2+\cdots$, where Γ_M on the right-hand side of the equation is to be replaced by the partial rate for $M\to\beta_1+\beta_2+\cdots$. It is also evident that the soft-π^+ results of Eqs. (5) and (6) hold for semi-inclusive reactions, in the following sense. Consider a class of states $\{\beta\}$ in which the numbers of particles belonging to various isotopic multiplets are specified, e.g., the numbers of π's, of K's, of η's, etc. Then sum over all states differing in the charge assignments within each group but preserving these numbers. This leads again to Eqs. (5) and (6), with an obvious reinterpretation of the symbols, e.g.,

$$\omega(F^+\to \text{soft }\pi^+ + 3\text{ pions}+K+\overline{K})\underset{E\to 0}{\longrightarrow}\frac{1}{f_\pi^{\,2}}\Gamma(F^+\to 3\text{ pions}+K+\overline{K})\;,$$

where on each side a sum over all three pion and $K, \overline{K}$ charge states is understood.

Finally, for soft-π^+ emission we observe that the all of the results discussed above stem from the basic theorem on exclusive amplitudes

$$\langle\beta,\pi^+(q)|H^{1+i2}|M\rangle\underset{q\to 0}{\longrightarrow}\frac{2}{\sqrt{2}f_\pi}\langle\beta|H^3|M\rangle\;, \qquad (9)$$

where H^{1+i2} is the charm-lowering Hamiltonian of Eq. (1), the superscript being appended here to signify the isotopic spin character of this interaction; H^3 is the neutral member of the same isotopic triplet. This equation provides a relation between the soft-pion amplitude and an amplitude involving the same final state, with only the soft π^+ removed. The matrix element on the right-hand side of Eq. (9) can now be related to physical amplitudes of isotopically rotated analogs of the state β. The results are simplest for simple channels, and here we merely note some examples:

$$\lim_{E\to 0}\omega(F^+\to\pi^+ +[\overline{K}^0+K^0])=\omega(F^+\to\pi^+ +[K^+ +K^-])$$
$$=\frac{1}{2f_\pi^{\,2}}\Gamma(F^+\to K^+ +\overline{K}^0)\;, \qquad (10)$$

$$\omega(F^+\to\pi^+ +[2\pi^0])\underset{E\to 0}{\longrightarrow}0\;, \qquad (11)$$

$$\omega(F^+\to\pi^+ +[\pi^+ +\pi^-])\underset{E\to 0}{\longrightarrow}\frac{1}{f_\pi^{\,2}}\Gamma(F^+\to\pi^+ +\pi^0)\;, \qquad (12)$$

$$\omega(D^0\to\pi^+ +[\overline{K}^0+\pi^-])\underset{E\to 0}{\longrightarrow}\frac{1}{f_\pi^{\,2}}\Gamma(D^0\to\overline{K}^0+\pi^0)\;. \qquad (13)$$

We conclude this subsection with the obvious remark that for decay of the charge-conjugate mesons $F^-, D^-, \overline{D}^0$ all of the results obtain with the replacements $\pi^-\leftrightarrow\pi^+$, $\pi^0\leftrightarrow\pi^0$, $K^0\leftrightarrow\overline{K}^0$, etc.

Soft-pion limits in charmed-baryon decay. The low-lying charmed baryons are expected to belong to the $\underline{\bar{3}}$ or $\underline{6}$ representations of SU(3). It is not at all clear that even the lowest-lying multiplet, whether $\underline{\bar{3}}$ or $\underline{6}$, will be stable against strong decay to a charmed meson and an ordinary hadron, but on the chance that this may be the case we consider here the soft-pion limit in weak decays. As remarked earlier, there is the complication that one has to allow in general for pole terms, corresponding to soft-pion emission from external baryons (for the $q\to 0$ limit only the outgoing baryon line is relevant; away from $q=0$ this pole term can become important). This of course can be handled for each individual channel, but it is simpler to avoid the effect altogether by specializing to channels where the pole term does not enter, namely, for soft pions of any charge if the outgoing baryon is a Λ particle, for soft-π^- emission if the outgoing baryon is a neutron or a Σ^- particle, and for soft-π^+ emission if the outgoing baryon is a proton or a Σ^+ particle. Although the pole term corresponding to soft-pion emission from an external baryon line can be ignored in the limit $q\to 0$, for processes involving a Λ particle in the final state it must be recognized that this term can be important for small, nonvanishing values of q. This is so because of the smallness of the Σ, Λ mass difference; that is, pole-term effects can produce rapid variations at small values of q en route to the extrapolation point $q=0$. Nevertheless, we state our results for this extrapolation point, and the restrictions noted above on the baryon content of final states is to be understood in the following discussion.

(i) For the SU(3) triplet of charmed baryons it is clear that all of the previous results on charmed-meson decay carry over without change. Thus, in Eqs. (3)–(6) and (10)–(13), one need only replace F^+, D^+, D^0 by the corresponding members of the baryon triplet, $(\mathcal{P}'\mathcal{P}\mathcal{N})$, $(\mathcal{P}'\mathcal{P}\lambda)$, $(\mathcal{P}'\mathcal{N}\lambda)$—call them C_0^+, A^+, A^0. Thus Γ_{F^+} becomes $\Gamma_{C_0^+}$, etc, but the ω's and Γ's are always restricted to states, or sums over states, containing the baryons specified above.

(ii) For the charmed baryons belonging to the SU(3) sextet we deal with an isotopic spin singlet $(\mathcal{P}'\lambda\lambda)$, a doublet $(\mathcal{P}'\mathcal{N}\lambda)$, $(\mathcal{P}'\mathcal{P}\lambda)$, and an isotriplet $(\mathcal{P}'\mathcal{P}\mathcal{P})$, $(\mathcal{P}'\mathcal{P}\mathcal{N})$, $(\mathcal{P}'\mathcal{N}\mathcal{N})$. Since our soft-pion results rely only on SU(2) considerations it is again clear that the isosinglet and the two members of the isodoublet enter into all the previous formulas in the same way, respectively, as F^+ and D^+, D^0. Also for the isotriplet states one again has the analogs of Eqs. (3) and (4). It remains therefore only to consider soft-π^+ emission in decay of the charmed isovector baryons, which we denote by C^{++}, C^+, C^0. For "inclusive" decays (subject to the final states containing Λ, p, or Σ^+) one finds

$$\lim_{E\to 0} \omega_{C^{++}\to\pi^+} = \omega_{C^0\to\pi^+} = \frac{1}{f_\pi^{\,2}}\Gamma_{C^+}\,, \tag{14}$$

$$\omega_{C^+\to\pi^+} \xrightarrow[E\to 0]{} \frac{1}{f_\pi^{\,2}}\left[\Gamma_{C^{++}} + \Gamma_{C^0} - \Gamma_{C^+}\right]. \tag{15}$$

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[1]R. L. Kingsley, S. B. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D 11, 1919 (1975).

[2]For a review of the conjectured properties of charmed hadrons, see M. K. Gaillard, B. W. Lee, and J. L. Rosner, Rev. Mod. Phys. 47, 277 (1975).

[3]This point was independently discussed by G. Altarelli, N. Cabibbo, and L. Maiani, PTENS Report No. 74/5 (unpublished).

[4]S. L. Adler and R. F. Dashen, *Current Algebras* (Benjamin, New York, 1968).

[5]We note that of course these results depend on the assumed structure of the weak Hamiltonian. By contrast, in an alternative scheme of A. Pais and V. Rittenberg [Phys. Rev. Lett. 34, 707 (1975)] in which the weak Hamiltonian is dominated by the $\underline{15}$ representation of SU(4), one would obtain an extra factor of $\frac{1}{4}$ on the right-hand side of Eq. (3) and an extra factor of $\frac{1}{2}$ on the right-hand side of (5), and (6) would be replaced by $\omega_{D^+\to\pi^+} = (1/2f_\pi^{\,2})\Gamma_{D^0}$ and $\omega_{D^0\to\pi^+} = (1/2f_\pi^{\,2})\Gamma_{D^+}$. Also K^0, $\bar{K}^0$, and K^- production, but not K^+ production, would be suppressed in the soft-kaon limit.