Toward UV models of kinetic mixing and portal matter. VII. A light dark photon in the $3_c3_L1_A1_B$ model

Thomas G. Rizzo[*]
SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, California 94025, USA

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The kinetic mixing (KM) portal, by which the Standard Model (SM) photon mixes with a light dark photon arising from a new $U(1)_D$ gauge group, allows for the possibility of viable scenarios of sub-GeV thermal dark matter with appropriately suppressed couplings to the SM. This KM can only occur if particles having both SM and dark quantum numbers, here termed portal matter, also exist. The presence of such types of states and the strong suggestion of a need to embed $U(1)_D$ into a non-Abelian gauge structure not too far above the TeV scale based on the renormalization-group equation running of the $U(1)_D$ gauge coupling is potentially indicative of an enlarged group linking together the visible and dark sectors. The gauge group $G = SU(3)_c \times SU(3)_L \times U(1)_A \times U(1)_B = 3_c3_L1_A1_B$ is perhaps the simplest setup wherein the SM and dark interactions are partially unified in a non-Abelian fashion that is not a simple product group of the form $G = G_{\rm SM} \times G_D$ encountered frequently in earlier work. The present paper describes the implications and phenomenology of this type of setup.

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I. INTRODUCTION AND OVERVIEW

As anomalies come and go, the Standard Model (SM) continues to be in exceptional agreement with almost all experimental data. However, many mysteries still remain unexplained with perhaps one of the most obvious being the nature of dark matter (DM). As of today, the existence of DM is only known of through its gravitational interactions derived from astrophysical and cosmological observations and whether or not it has any further interactions with the SM remains unclear. If DM indeed consists of (a set of) new particles (and not, e.g., primordial black holes [1]), then the measured value of its relic density [2] does suggest that some additional, yet unobserved, interactions with at least some subset of the familiar SM particles should be present. Models of particle DM have a long history beginning with either/both the QCD axion [3–5] and thermal weakly interacting massive particles (WIMP) in the few GeV to ~100 TeV mass range [6–8], and broad searches for states such as these continue to push ever further into new parameter space regimes [9–16]. However, the so far null results from these searches have inspired a host of new potential candidate DM models now known to populate an extremely wide spectrum in both mass and coupling that is found to be dauntingly large [17–23]. Attempts to cover even a fraction of this space in the coming years through a variety of different novel search techniques will be quite challenging. In a reasonably large fraction of this space, however, the new interaction(s) between DM and the SM can be described by a set of "portals," i.e., effective field theories (EFTs), all of which predict the existence of a new class of mediator particles [24] beyond just the DM itself and which may or may not be renormalizable depending upon the specific scenario.

One of the most attractive of these general frameworks is an extension of the thermal WIMP [25,26] idea into the few MeV to ~1 GeV mass range which is made possible by introducing the renormalizable kinetic mixing/vector portal setup [27–29]. In these models, the DM field carries a "dark charge," $Q_D \neq 0$, that is linked to a new gauge interaction—most simply a new (dark) $U(1)_D$—which has an associated gauge boson, called the dark photon (DP) V [30–32]. It is assumed that under this new $U(1)_D$ group the SM fields will all have $Q_D = 0$ and so the only way for the SM and DM fields to interact at low energy scales is via the kinetic mixing (KM) of the DP with the SM photon A. Below we will assume that $U(1)_D$ is broken so both the DM and the DP obtain their masses (at least partially in the case of DM) via the vacuum expectation value(s) [VEV(s)] of a, or several, dark Higgs field(s) [33]. Clearly, in order to construct the vacuum polarizationlike diagram(s) that are needed to generate this KM, new heavy particles that carry both SM as well as $U(1)_D$ quantum numbers must exist that are either complex scalars and/or vectorlike fermions (VLFs) [34–41] so as to avoid the well-known unitarity,

[*]Contact author:rizzo@slac.stanford.edu

precision electroweak, and Higgs boson coupling constraints. We will refer to such exotic particles as portal matter (PM) and they have been the subject of much recent attention [42–60].

Given a set of such PM particles, the loop-induced strength of the KM can be expressed via the value of the familiar small dimensionless parameter ϵ as

$$\epsilon = \frac{g_D e}{12\pi^2} \sum_i (\eta_i N_{c_i} Q_{em_i} Q_{D_i}) \ln \frac{m_i^2}{\mu^2}. \quad (1)$$

Here, g_D is identified as the $U(1)_D$ gauge coupling (so that we can likewise define $\alpha_D = g_D^2/4\pi$) and $m_i(Q_{em_i}, Q_{D_i}, N_{c_i})$ are the mass (electric charge, dark charge, number of colors) of the ith PM field. Also, one has that $\eta_i = 1(1/4)$ if the PM is a VLF (complex scalar). In all generality, ϵ is not a finite quantity, but we might imagine that in an at least partially UV-complete theory that group theoretical constraints render the sum

$$\sum_i (\eta_i N_{c_i} Q_{em_i} Q_{D_i}) = 0, \quad (2)$$

so that ϵ will become both finite as well as, in principle, calculable in such a setup. Numerically, for a set of PM fields that have similar masses, such as those we will encounter below, we might expect that, roughly speaking, ϵ lies in the range of $\sim 10^{-(3\text{–}4)}$ so that experimental search constraints can be satisfied while also yielding the observed DM abundance for sub-GeV thermal DM with similar DP masses. It is important to note that once one postulates the necessity of PM particles to generated KM, their existence can also induce other loop-level interactions between (some of) the SM fields and DM which have a strength similar to that induced by KM.

As is well known, there are numerous constraints on these types of setups ranging from direct detection experiments and accelerator searches to cosmology and astrophysics. For example, in addition to the thermal DM velocity-weighted annihilation cross section at freeze-out requirement [25,26], $\sigma v_{\rm rel} \sim 3 \times 10^{-26}$ cm^3 sec^{-1}, the corresponding cross section must be significantly suppressed at later times, i.e., during the cosmic microwave background (CMB) [2,61–64] and today [65,66], implying that such reactions must display a significant temperature (T) dependence. This implies that the annihilation through the DP cannot be via a typical s-wave process and so the DM cannot be an ordinary Dirac fermion. Instead such processes must be, e.g., (i) p wave so that there is a $v^2 \sim T$ suppression at later times, as may be realized in the case of complex scalar or Majorana fermionic DM[1] or may take place (ii) through the coannihilation mechanism, as can be the case with pseudo-Dirac DM with a sizable mass splitting (induced by, e.g., a $Q_D = 2$ dark Higgs VEV), so that annihilation rate is exponentially Boltzmann suppressed at later times due to the much lower temperatures [68–71]. Interestingly, especially in this later case, it has been observed that, over a significant range of low energy couplings, the renormalization-group equation (RGE) running of α_D into the UV indicates that the scale of additional new physics is possibly not very far away due to the eventual loss of perturbativity. This may take place at a scale as low as $\sim$10 TeV mass or so [48,51,72,73], depending, of course, upon the specific details of the low energy field content. The solution to the nonperturbative coupling issue would then be the embedding of the Abelian $U(1)_D$ into a larger non-Abelian, asymptotically free group G_D, which would then reverse the "bad" RGE running *before* this high mass scale is reached thus avoiding this problem.

This observation, combined with the existence of PM—which we can easily imagine might obtain masses as part of the process of $G_D \to U(1)_D$ symmetry breaking—leads to considerations of how this new physics might fit together with the SM in a more unified picture, something that has been the subject of much of our recent work [42,43,45–48,50,51,54,57–59], with the eventual goal being the construction of an at least partially unified UV-complete setup. Following a bottom-up approach, we have previously examined scenarios of the general product form $G_{\rm SM} \times G_D$ where, in the simplest setup considered, it has been assumed that $G_D = SU(2)_I \times U(1)_{Y_I}$ [74], into which $U(1)_D$ can be straightforwardly embedded in a quite familiar fashion. In this class of setups, although the SM $SU(2)_L$ and the dark $SU(2)_I$ act on orthogonal spaces, at least some of the PM fields lie in $SU(2)_I$ doublet representations along with SM fields with which they share common strong and electroweak properties.[2] In such a setup, additional tree-level interactions will exist between the PM and SM fields due to the exchange of the new heavy non-Hermitian gauge boson (NHGB) fields in G_D—interactions beyond those arising just from KM. While such scenarios provide us with important insights into more UV-complete setups, they lack any sort of direct (or even partial) linkage between the SM and the dark sector interactions that we would need to understand better, even if only within the context of a semirealistic toy model.

In this paper, we will explore a quite different type of semiunified structure wherein (at least part of) the electroweak interactions of the SM and the $G_D = SU(2)_I \times U(1)_{Y_I}$-type interactions linking the SM and PM are described by a *single* non-Abelian group whose breaking (eventually) will also lead to a light DP. Perhaps the simplest example of a model of this kind, wherein the SM and these new NHGB-induced interactions arise from a single group, which we will consider

[1]See, however, [67].

[2]This was motivated by our study in earlier work of E_6-type gauge models [75].

below, is based on the gauge structure $SU(3)_c \times SU(3)_L \times U(1)_A \times U(1)_B = 3_c 3_L 1_A 1_B$ [76–85]. This group has previously been examined within the framework of, e.g., $\lesssim$TeV scale DM and a corresponding massive DP, as well as in other interesting contexts, and which might, in the set of models to be discussed below, be regarded as a (very) descoped version of the quartification class of setups examined previously in Ref. [57]. Models based on this gauge group are themselves extensions of the somewhat more familiar and quite well-studied $3_c 3_L 1_X$ scenarios [86–109], but which is not large enough to contain both a light DP as well as the direct link between PM and the usual SM fields that we seek here. Note that in these $3_c 3_L 1_A 1_B$ setups the familiar $U(1)_Y$ and $U(1)_D$ are now both partially contained within the $1_A 1_B$ group factors and we imagine that at some scale, $\gtrsim 10$ TeV, one (or more) Higgs VEV induce the breaking $3_L 1_A 1_B \to 2_L 1_Y 1_D$, generating both the PM and new heavy gauge boson masses in the process.

The outline of this paper is as follows: Following this Introduction, in Sec. II, we present the basic overall group/gauge structure of our $3_c 3_L 1_A 1_B$ setup in generality and review its basic components from the perspective of a sub-GeV DP and DM but with multi-TeV scale PM. This includes a discussion of anomaly cancellations and parameter values as well as choices of the various fermion representations along with the corresponding possible PM electroweak and dark charge assignments. In Sec. III, we discuss the three step symmetry breaking chain $3_L 1_A 1_B \to 2_L 1_Y 1_D \to 1_D 1_{em} \to 1em$ for both the Hermitian and non-Hermitian gauge boson sectors and determine the masses and coupling of the new non-SM states in terms of the high scale model parameters and the set of required Higgs VEVs. Some simple implications of this symmetry breaking relating the heavy gauge boson masses to each other and the impact of their mixings with those of the SM are discussed in Sec. IV along with a restriction on the size of the $U(1)_D$ gauge coupling g_D. In Sec. V, the masses of the various PM fields are analyzed along with their mixing with those of the SM. These mixings are usually critical in allowing for the PM to decay into SM states along with a DP. In Sec. VI, the production and decays of the PM fields and the new heavy gauge bosons at the LHC and the FCC-hh are then analyzed. Finally, a discussion and our conclusions can be found in Sec. VII.

II. REVIEW, PRELIMINARIES, AND GENERIC MODEL FRAMEWORK

As noted above, the well-studied $G = SU(3)_c \times SU(3)_L \times U(1)_A \times U(1)_B = 3_c 3_L 1_A 1_B$ [76–85] general framework of models are obviously extensions of the even more well-studied $3_c 3_L 1_X$ model class [86–109], from which we can extract much important model-building information. The matter content of such models consists of a set of $SU(3)_L$ triplets **3** and/or antitriplets **3*** of fermion fields, together with a set of singlets 1, chosen such that they contain all of the familiar SM fields in their appropriate representations of the $SU(2)_L$ isospin subgroup, i.e., the ordinary $SU(2)_L$ SM isodoublets, are directly embedded into these **3**'s and/or **3***'s. As we will see below, the additional fermion in each of these (anti) triplets will be identified with PM and will prove to be vectorlike with respect to the SM interactions while *not* being generally vectorlike with respect to the larger gauge group. Clearly, since these fundamental triplets and antitriplets, unlike in the case of the $SU(2)_L$ doublets, are inequivalent representations, the most basic constraint on any model building one might consider is that 3_L^3-type gauge anomaly cancellation will require that the number (including color degrees of freedom) of **3** and **3*** must be equal. Still, this cancellation can take place in numerous ways but these can be divided into two main categories depending upon whether or not they cancel within each generation or only when we sum over the contributions of all three generations. In the former case, the leptonic sector of each generation would need to be significantly augmented as, e.g., each color triplet quark **3** would need to have its contribution to this anomaly canceled by three sets of color singlet lepton **3***'s yielding a total of 18 (chiral) triplets plus antitriplets for three generations. In general, in these cases, some of the right-handed (RH) color singlet leptons will also end up in **3**'s or **3***'s. Furthermore, the gauge interactions in such setups could then be trivially arranged so that there are no obvious tree-level flavor changing neutral currents (FCNCs). As has been recently stressed [105], in the later case, when one also demands that asymptotic freedom above the PM scale be maintained then only three generations are allowed—a prediction of such setups. Also in such models, all of the corresponding RH partners of these fermions will lie in 3_L singlets. However, though these types of setups would have fewer degrees of freedom (only 12 chiral triplets/antitriplets instead of 18), the price for this greater simplicity would be that FCNCs at tree level would naturally be induced in the quark sector through the exchange of a new heavy neutral gauge boson that arises from G_D breaking. As has been much discussed in the literature, this happens because, while we can choose all three lepton generations to lie in, e.g., identical **3**'s, then it must be that two of the quark generations would need to be in **3***'s, while the remaining one would instead be in a **3**. FCNCs would then occur for a heavy gauge boson whose couplings would be sensitive to the this difference among the quark generations and this would clearly lead to a lower bound on the mass of such new particles. Representative examples of both of these classes of models will be encountered in our discussion below.

More generally, the potential experimental bounds on the masses of the new heavy gauge and scalar fields that can transmit these FCNCs in such classes of models would depend upon exactly how these FCNCs arise in a particular setup, e.g., whether they occur in the u, d quark and/or lepton sector(s), which if any of the generations are treated differently, the nature and size of the mixing between the states that causes them to mix, and whether or not additional discrete symmetries/mixing textures are present that may reduce their influence. Further, since at least some of the PM fields must, in general, mix with their SM analogs (via the $\sim$1 GeV Q_D-violating VEVs to be discussed below) to allow for all of the PM to decay, these PM fields together with the SM Z can *also* transmit FCNCs, making the situation even more complex. In addition to those referenced earlier, this problem has been studied by numerous authors [110] who obtain a wide range of possible constraints, from only a few TeV to well over 100 TeV, under a multitude various assumptions. However, it seems clear that the present experimental constraints will allow for new gauge bosons with masses not far above the $\sim$10 TeV scale without too much effort. A detailed discussion of this complex subject is, however, beyond the scope of the present paper.

As we will also consider further below, the breaking of $3_L 1_A 1_B \to 1_{em}$ will be accomplished in three steps, differing by roughly a factor of $\sim$100 in mass scale: the first occurs through one large VEV, $w \gtrsim 10$ TeV, which breaks $3_L 1_A 1_B \to 2_L 1_Y 1_D$, then through a set of SM breaking VEVs, $v_{1,2} \sim 100$ GeV, as usual, and finally via the VEVs $u_{1,2} \lesssim 1$ GeV which break $U(1)_D$, generating the DP mass as well as PM decay paths through mixing with the SM fields. While all these VEVs arise simultaneously when the scalar potential is minimized, these mass scales are sufficiently widely separated that we can discuss them in sequential steps and, essentially, independently since they are, to a very good approximation, decoupled. Some care is, however, required in the situations where this approximation is not strictly valid. As will be further discussed, these various symmetry breaking steps (as well as the generation of the Dirac fermion masses) can all be accomplished through the introduction of, for simplicity, only three Higgs scalar **3**'s (or **3***'s), each with at least one nonzero VEV. Alternatively, one could assign the 1_D-violating, $u_{1,2}$ VEVs to different **3** (or **3***) representations.

The gauge part of the covariant derivative for the $3_L 1_A 1_B$ subgroup of G, with which we will be most concerned below (since 3_c remains unbroken), can be written as (suppressing Lorentz indices for simplicity)

$$g_L T_{iL} W_{iL} + g_A X_A V_A + g_B X_B V_B, \tag{3}$$

where $g_{L,A,B}$ are the $SU(3)_L, U(1)_A$, and $U(1)_B$ gauge couplings, respectively. Furthermore, for later convenience, we can also define the two coupling constant ratios (in analogy with $t_w = \tan\theta_w = g'/g_L$ in the SM)

$$t_X = g_A/g_L, \qquad t_G = g_B/g_L. \tag{4}$$

Note that, below the 3_L breaking scale, g_L can be directly identified with the usual SM $SU(2)_L$ coupling. Here, the W_i are the eight gauge fields of 3_L with the T_{iL} being the corresponding 3_L generators normalized so that for the fundamental, **3**, representation is just given by $\lambda_i/2$, the λ_i being the familiar 3×3 Gell-Mann matrices; for the antifundamental **3*** representation, one has instead that $T_i = -\lambda_i^*/2$. Recall that both the $T_{3,8L}$ generators are diagonal with T_{3L} being identified with the third component of the weak isospin in the SM. Similarly, $X_{A,B}$ and $V_{A,B}$ are the corresponding gauge charges and gauge fields for the $1_{A,B}$ Abelian groups.

The electric charge $Q = T_{3L} + Y/2$ in the SM can clearly be expressed, in general, as some linear combination of the $3_L 1_A 1_B$ diagonal generators. However, we are free to perform a rotation in $1_A 1_B$ space before symmetry breaking so that the 1_B generator does not contribute to this sum. Also, since T_{3L} is identified with the usual SM generator and we can freely choose the normalization of the X_A charges, we may define Q as just the sum

$$Q = T_{3L} + \frac{\sigma}{\sqrt{3}} T_{8L} + X_A = T_{3L} + \frac{Y}{2}, \tag{5}$$

with the second equality just being the SM relationship and with σ being, *a priori*, an unknown (integer) parameter given this choice of normalization, but, as is well known [86–108], can only take on a rather restricted set of values as we will see below. From the expression above it follows that (at the G breaking scale in the case of running couplings)

$$\frac{1}{g_Y^2} = \frac{\sigma^2}{3g_L^2} + \frac{1}{g_A^2}. \tag{6}$$

Similarly, the dark charge Q_D, which we will normalize here to take on integer values, can also be expressed as a linear sum of the diagonal generators. However, since both fields in the usual SM doublets, i.e., $(\nu, e)_L^T$ (with e standing in for a generic charged lepton) and $(u, d)_L^T$, have $Q_D = 0$, then Q_D cannot depend upon the generator T_{3L}. Thus, since the normalization of the charges X_B is arbitrary, we can express Q_D, in general, as

$$Q_D = \frac{-2\tau}{\sqrt{3}} T_{8L} + 2\lambda X_A + X_B, \tag{7}$$

and, with a further rescaling, we can freely take $\tau = 1$ while λ will, for the moment, still remain arbitrary. We might expect in a more complete UV setup that the value of the parameter λ (not to be confused with any of the Gell-Mann matrices) might be derivable from the other quantities. We can easily invert these relationships and express the $X_{A,B}$ in terms of the other generators; we then see that

$$X_A = \frac{Y}{2} - \frac{\sigma}{\sqrt{3}} T_{8L}$$
$$X_B = Q_D - 2\lambda \frac{Y}{2} + \frac{2}{\sqrt{3}}(1+\lambda\sigma)T_{8L}, \qquad (8)$$

which we can employ to simplify some of the expressions for the covariant derivative and/or couplings of the various gauge bosons that we will encounter below.

Since, in all generality, we will be embedding the familiar SM isodoublets $(\nu, e)_L^T$ and $(u, d)_L^T$ into **3** and/or **3*** representations, it is useful to contemplate the following possible generic basic constructs that will likely appear in any given specific model that we will consider below:

$$q_1 = \begin{pmatrix} u \\ d \\ X_1 \end{pmatrix}_L, \quad q_2^* = \begin{pmatrix} d \\ u \\ X_2 \end{pmatrix}_L, \quad l_1 = \begin{pmatrix} \nu \\ e \\ X_3 \end{pmatrix}_L, \quad l_2^* = \begin{pmatrix} e \\ \nu \\ X_4 \end{pmatrix}_L, \qquad (9)$$

with q_1, l_1 being **3**'s, while q_2^*, l_2^* are instead **3***'s, and with the detailed nature of the fermions X_i being yet unspecified except that we will want to identify them as PM fields having $Q_D \neq 0$. Note that $X_{1,2}$ are necessarily color triplet quarks in the SM sense, while $X_{3,4}$ correspond to color singlet leptons from this same perspective. It is then instructive to act with both the Q and Q_D generators on these four representations and require that we recover the usual electric charges for the SM fields and that these same fields all have $Q_D = 0$. Doing this will then tell us, among other things, the possible values of Q and Q_D for the remaining unfamiliar X_i fields. The first result of this procedure is that we find that uniquely, independent of the values of either σ or λ, that the dark charges of these states are completely fixed,

$$Q_D(X_1) = Q_D(X_2) = -Q_D(X_3) = -Q_D(X_4) = 1. \qquad (10)$$

The electric charges of these states, however, and as is well known in the $3_c 3_L 1_X$ model literature, will remain dependent upon the value of σ, though they are, of course, λ independent and are summarized in Table I. Clearly, as is well known, arbitrary values of σ are excluded and, in fact, only a small set of possibilities are permissible to exclude bizarrely charged states. We observe, e.g., that if we choose $\sigma = \pm 1$ then the X_i will carry values of Q that are the same as the familiar quarks and leptons of the SM. If, instead, one chooses $\sigma = \pm 3$, then we see that the $X_{1,2}$ will have $Q = 5/3$ or $-4/3$ (the choice of which one is identified with X_1 or X_2 being dependent upon the sign of σ), while the $X_{3,4}$ will have similarly $Q = 1$ or -2. Now since we are not identifying any of these X_i with DM but instead as PM, we need all of them to be unstable while they all still carry SM charges as well as having $Q_D = \pm 1$. While some of the heavy 3_L gauge bosons will link these X_i with the ordinary SM fermions occupying the same multiplet, they will not *all* be able to decay this way. At the very least, e.g., the lightest of the PM states will be stable unless we allow it to have $U(1)_D$-violating interactions. In previous discussions, we observed that it was the mixings of the PM fields with the corresponding SM ones having the same color and electroweak quantum numbers, which occurs via the same Q_D-violating Higgs VEVs responsible for generating the DP mass, that was responsible for this. This mixing allowed decays of the general form PM $\to$ SM $+ V, h_D$, where h_D is the dark Higgs, to occur and these were found to be the dominant decay modes for PM in the simplest approaches. In the present case, this requirement would seem to exclude the possibility of the exotic charged states such as $Q = 5/3, -4/3$ as one of the lightest of these might then be stable and so this would restrict us further (or give greater weight) to the choices $\sigma = \pm 1$. Although we will not always impose this requirement in the analysis that follows below, since some clever model building might avoid this apparent outcome, we should remain mindful of it as it will come in at a later stage in our discussion, as all of the heavy charged PM fields must be allowed to decay down to SM states by some means.

TABLE I. Electric charges of the X_i. The electric charges of the new states X_i independent of the value of λ, that lie in the **3** and **3*** representations q_1, q_2^*, l_1, l_2^* as discussed in the text.

State	Q
X_1	$\frac{1}{6}(1-3\sigma)$
X_2	$\frac{1}{6}(1+3\sigma)$
X_3	$-\frac{1}{2}(1+\sigma)$
X_4	$-\frac{1}{2}(1-\sigma)$

How will the DM itself fit into this setup since we recall that it must be both light $\sim$1 GeV as well as a SM singlet with $|Q_D| = 1$? In the fermionic case, there are essentially two possibilities: if it lies in a **3** or **3*** with $Q = T_{3L} = 0$, it must be the lowest multiplet member analogous to, e.g., one of the $X_{2,4}$'s introduced above when $\sigma = \mp 1$. In such setups the DM mass is generally set by the large VEV w discussed above thus having a TeV scale mass [82], a situation that we are not interested in here. A second possibility is that the DM is instead a vectorlike fermion (to avoid any anomalies) which is a singlet under $3_L 1_A$ having $Q_D = X_B$ which becomes a pseudo-Dirac state via a dark Higgs VEV as discussed in the Introduction above and as recently analyzed in, e.g., Ref. [59]. If the DM is a complex scalar without a VEV, it can again be chosen to be a $3_L 1_A$ singlet state, but still carrying a dark charge $Q_D \neq 0$, and can easily satisfy the cross section bounds coming from the CMB mentioned above while still obtaining the required relic density observed by Planck.

We now turn to a discussion of the symmetry breaking and mass generation issues in this setup.

III. THREE STAGES OF GAUGE SYMMETRY BREAKING

A. Hermitian gauge bosons

The goal of this section is to demonstrate that a light DP with a mass $\lesssim 1$ GeV is possible in this type of setup, having all of the required properties, along with the SM photon and Z and, e.g., a new neutral heavy gauge boson Z'_M, which might be accessible at the HL-LHC or at other future colliders. Schematically, the plan is to do this in three distinct stages: the first is via a VEV, $w \sim 10$ TeV, breaking $3_L 1_A 1_B \to 2_L 1_Y 1_D$. Since there is a mass gap until the SM breaking scale is reached, we could imagine that in the energy regime below this large breaking scale we could imagine writing something like an EFT by taking the field content of the original model, now rewriting it in terms of $2_L 1_Y 1_D$ representations, i.e., decomposing the 3_L triplets into 2_L singlets and doublets and removing/integrating out the heavy fields. Then when the electroweak scale is reached a pair of VEVs, $v_{1,2}$, which would now appear in 2_L doublets, will break the SM at ~ 100 GeV as usual, i.e., $2_L 1_Y \to 1_{em}$. Since there is now another large mass gap, below this scale we could again imagine constructing an EFT, consisting only of singlets, by integrating out the SM W and Z, until finally 1_D is broken by a last pair of VEVs, $u_{1,2}$, which we will require to be at the scale $\lesssim 1$ GeV, leaving us with only unbroken QED. Though all of these VEVs are the result of a single minimization of the scalar potential, their large hierarchy allow us to treat their actions sequentially in most cases.

We first turn to the symmetry breaking for the neutral gauge boson sector; here we will be very generic and assume for simplicity that the only Higgs fields that are present are those that give masses to the quarks and leptons to be discussed below. As noted above and in the previous section, the breaking of $3_L 1_A 1_B \to 1_{em}$ takes place in three distinct, widely separated stages via a set of 3_L Higgs triplets/antitriplets with the first occurring at or above the ~ 10 TeV mass scale where $3_L 1_A 1_B \to 2_L 1_Y 1_D$, i.e., the unbroken SM plus the usual (and at this point massless) DP. The VEV for this breaking, w, must be experienced by one of the components of a complex scalar, which here we will denote as $\chi = (\chi_1, \chi_2, \chi_3)^T = H_X$, having $Q = T_{3L} = Q_D = 0$, as these symmetries must remain unbroken until lower mass scales are reached. These restrictions then tell us that it is the lowest member of this triplet/antitriplet that obtains the VEV, $\langle \chi_3 \rangle = w/\sqrt{2}$, to avoid breaking 2_L and, further, these considerations also completely fix the corresponding values of $X_{A,B}$ for χ in terms of the parameters λ, σ. As we will see later below, w will also end up generating the masses of all of the PM fermion fields as required as well as breaking $3_L 1_A 1_B \to 2_L 1_Y 1_D$. Interestingly, when $\sigma = \pm 1$, one also finds that another one of the components of χ (i.e., χ_1 or χ_2), but now carrying $Q_D = \pm 1$, depending upon whether χ is a **3** or **3***, is also electrically neutral, $Q = 0$, and which also has $T_{3L} = \pm 1/2$. In such a case this component may *also* obtain a VEV, e.g., $\langle \chi_{1(2)} \rangle = u_1/\sqrt{2}$, but we will require it to be $\lesssim 1$ GeV as it leads to a breaking of $U(1)_D$ and generates a DP mass at this scale.

An occurrence of more than one VEV in a single scalar field cannot/is forbidden to happen, e.g., in the SM since there no more than a single component of a Higgs scalar representation can be electrically neutral due to the relationship $Q_{em} = T_{3L} + Y/2$ and the fact that we wish to avoid the possibility of charge breaking minima. However, in more general models with enlarged gauge groups, where Q_{em} is the sum of several generators, it is possible that two or more components of a given scalar representation can be electrically neutral and so obtain VEVs simultaneously. Perhaps the most well-known example where this happen is the Higgs bidoublet in the left-right symmetric model [111], wherein both neutral components obtain VEVs whose ratio is ~ 50 in order to explain the ratio of the top and bottom quark masses at the weak scale. Another such example is provided by the bitriplet Higgs representation appearing in trinification models [112], which obtain three distinct VEVs, which in some cases are very widely separated in scale. We again stress that these multiple distinct VEVs are "generated" simultaneous as part of the minimization of the potential but, due to their hierarchal nature as occurs in the present setup, can be treated as if they occurred sequentially as to their effects. For now, since $w \gg u_1$, we can safely ignore the effects of $u_1 \neq 0$ for the moment, but we will return to it in the later discussion below.

Now consider the piece of the covariant derivative corresponding to the set of four neutral gauge fields $W_{3L,8L}, V_{A,B}$ acting upon the VEV of χ_3 which, since $T_{3L}(\chi_3) = 0$, can be simply written as (note that W_{3L} will not enter here as this VEV has $T_{3L} = 0$)

$$\mp \frac{g_L w}{3\sqrt{2}} \left(\sqrt{3} W_{8L} - \sigma t_X V_A + 2 t_G (1 + \lambda\sigma) V_B\right), \qquad (11)$$

with the overall sign depending upon whether χ is a **3** or **3***. Now let us define the quantities

$$C = [3 + \sigma^2 t_X^2 + 4 t_G^2 (1 + \lambda\sigma)^2]^{1/2}, \quad C' = (3 + \sigma^2 t_X^2)^{1/2}, \qquad (12)$$

so that we can rewrite the expression above as

$$\mp \frac{g_L w}{3\sqrt{2}} C \left[\frac{C' \left(\frac{\sqrt{3} W_{8L} - \sigma t_X V_A}{C'}\right) + 2 t_G (1 + \lambda\sigma) V_B}{C}\right]. \qquad (13)$$

Now we can further introduce the mixing angle factors

$$c_\lambda = \frac{\sqrt{3}}{C'}, \qquad s_\lambda = \frac{-\sigma t_X}{C'},$$
$$c_\phi = \frac{2t_G(1+\lambda\sigma)}{C}, \qquad s_\phi = \frac{C'}{C}, \tag{14}$$

with $t_\lambda = s_\lambda/c_\lambda$, etc., so that we can define the single (normalized) eigenstate that obtains a mass from the VEV w as

$$Z'_M = s_\phi(c_\lambda W_{8L} + s_\lambda V_A) + c_\phi V_B, \tag{15}$$

with a mass-squared value given by

$$M^2_{Z'_M} = \frac{g_L^2 w^2}{9} C^2. \tag{16}$$

More generally then, we can write the necessary orthogonal transformation and its inverse as

$$\begin{pmatrix} K \\ L \\ Z'_M \end{pmatrix} = O \begin{pmatrix} W_{8L} \\ V_A \\ V_B \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} W_{8L} \\ V_A \\ V_B \end{pmatrix} = O^T \begin{pmatrix} K \\ L \\ Z'_M \end{pmatrix}, \tag{17}$$

with the two massless states, K and L, being orthogonal to Z'_M so that, explicitly, one has

$$W_{8L} = -s_\lambda K + c_\lambda(c_\phi L + s_\phi Z'_M),$$
$$V_A = c_\lambda K + s_\lambda(c_\phi L + s_\phi Z'_M),$$
$$V_B = -s_\phi L + c_\phi Z'_M. \tag{18}$$

In terms of these newly defined fields (and reintroducing W_{3L} as well), the couplings of the four Hermitian gauge bosons can be written compactly as

$$g_L\big(T_{3L}W_{3L} + X_K K + X_L L + X_M Z'_M\big), \tag{19}$$

with

$$X_K = c_\lambda t_X \frac{Y}{2} = \tilde{X}_K \frac{Y}{2},$$
$$X_L = \frac{C'}{C} Q_D + X_{LY}\frac{Y}{2}, \tag{20}$$

where, for later use, we have defined the quantity

$$X_{LY} = \frac{2t_G(3\lambda - \sigma t_X^2)}{CC'}, \tag{21}$$

and where we also obtain the result that

$$X_M = C\frac{T_{8L}}{\sqrt{3}} + \frac{2t_G^2(1+\lambda\sigma)}{C} Q_D - \left[\frac{\sigma t_X^2 + 4\lambda t_G^2(1+\lambda\sigma)}{C}\right]\frac{Y}{2}, \tag{22}$$

where we have employed the SM relationship $Y/2 = Q - T_{3L}$.

The next stage of symmetry breaking (for our Hermitian gauge fields) is to, essentially, generate the SM Z mass which will require VEVs with $T_{3L} = \pm 1/2 = -Y/2$ (since these states are electrically neutral), but still having $Q_D = 0$ so that 1_D is not simultaneously broken at the electroweak scale. As will be further emphasized below, in doing this we can ignore the Z'_M to an excellent approximation as $Z - Z'_M$ mixing is suppressed by a factor of $\sim 10^4$, but we will return to this issue in a later discussion. In practice, we will need to employ two distinct Higgs representations to generate, e.g., the u- and d-type SM fermion masses which, again, will be **3**'s or **3***'s of 3_L, since in the models we will examine below the right-handed quarks will all lie in 3_L singlet representations. To be specific, we then need to introduce the two Higgs fields: $\eta = H_u$, one of whose two upper components obtains a VEV $v_1/\sqrt{2}$, which will generally also give mass to SM u-type quarks, and $\rho = H_d$, one of whose upper two components obtains a VEV $v_2/\sqrt{2}$, which will also generally give masses to the SM d-type quarks in complete analogy with the type-II two Higgs doublet model (THDM). So, e.g., if it is η_1, which is the top component that gets the VEV $\sim v_1$, then it is ρ_2, the middle component, that gets the VEV $\sim v_2$.

To proceed further, since $w^2 \gg v_{1,2}^2$, as noted we can to a very good approximation simply decouple Z'_M and limit ourselves to just the basis W_{3L}, K, L. Of course, some care is required since X_M has a term proportional to $Y/2$, so that both $v_{1,2} \neq 0$ will also induce $Z - Z'_M$ mass mixing besides generating the SM Z mass, a subject that we will return to later as noted earlier. Acting on these states, the covariant derivative can be written symbolically using the definitions above as

$$D[\langle\eta\rangle, \langle\rho\rangle] = g_L(T_{3L}W_{3L} + X_K K + X_L L)[H_u, H_d]$$
$$= \frac{g_L}{2\sqrt{2}}(W_{3L} - \tilde{X}_K K - X_{LY} L)[v_1, -v_2], \tag{23}$$

where the relative sign reflects that the two VEVs have opposite values of $T_{3L} = \pm 1/2$. Now let v stand in for either one of $v_{1,2}$; then the expression above up to a sign is just

$$D = \frac{g_L v}{2\sqrt{2}} N_Z Z, \quad N_Z^2 = 1 + N_I^2, \quad N_I^2 = \tilde{X}_K^2 + X_{LY}^2, \tag{24}$$

from which we see that, including now the contributions of both $v_{1,2}$,

$$M_Z^2 = \frac{g_L^2}{4}(v_1^2 + v_1^2) N_Z^2, \tag{25}$$

as we might already have guessed and which looks suspiciously like the SM result that we will reproduce, provided that we can identify $N_Z^2 = 1/c^2$ with $c = c_w$ corresponding to the usual weak mixing angle. To see that is indeed the case, we first define

$$\alpha = \frac{\tilde{X}_K}{N_I} \quad \beta = \frac{X_{LY}}{N_I}, \qquad \alpha^2 + \beta^2 = 1, \tag{26}$$

so that

$$\tilde{X} = \alpha K + \beta L, \qquad \tilde{Y} = -\beta K + \alpha L, \tag{27}$$

and thus we can further define the two orthogonal combinations,

$$Z = cW_{3L} - s\tilde{X}, \qquad \tilde{A} = sW_{3L} + c\tilde{X}, \tag{28}$$

with

$$c = \frac{1}{N_Z}, \qquad s = \frac{N_I}{N_Z}, \qquad c^2 + s^2 = 1, \tag{29}$$

where we still need to show that $c = c_w$ (so that $s = s_w$). To that end, we insert these expressions back into the relevant piece of the covariant derivative at this stage yielding, after some algebra, first finding that

$$t_X^2 = \frac{3s^2}{3 - (3 + \sigma^2)s^2}, \tag{30}$$

and so we arrive at

$$\begin{aligned} &T_{3L}W_{3L} + X_K K + X_L L \\ &= \left[\frac{1}{c}(T_{3L} - s^2 Q) - (\beta s)s_\phi t_G Q_D\right] Z \\ &\quad + [sQ + (\beta c)s_\phi t_G Q_D]\tilde{A} + \alpha s_\phi t_G Q_D \tilde{Y}, \end{aligned} \tag{31}$$

so that Z indeed couples to the SM fields as expected with the identification $s = s_w$, etc., but also apparently couples to SM singlet dark sector fields at this stage as well and which we will return to later below. Now recall that we have yet to break $U(1)_D$ so that neither $\tilde{A}$ nor $\tilde{Y}$ are mass eigenstates. These two states will be seen to mix via this remaining symmetry breaking step leaving us with the massless photon and the massive DP.

We recall from above that two components out of the set of Higgs fields, χ, ρ, η, in the case $\sigma = \pm 1$, will have an additional $Q = 0$ element but with $Q_D = \pm 1$ depending upon whether this Higgs is a **3** or **3*** that may obtain a VEV, $\sim u_{1,2}$, that will break 1_D; further, one of these VEVs, u_1, will also have $T_{3L} \neq 0$. This will *not* happen in the case of $\sigma = \pm 3$ and we would then need to introduce additional Higgs fields to break 1_D. To simplify the analysis that follows we will assume for now that indeed $\sigma = \pm 1$ except as noted (which we will frequently do). Then, similar to what was done for isospin breaking above, since these VEVs $u_{1,2} \lesssim 1$ GeV, we can decouple the heavy SM Z, which is an excellent first approximation since $M_Z \gg 0.1$–1 GeV, so that we can define the mass eigenstates

$$V = \frac{\alpha\tilde{Y} + \beta c\tilde{A}}{N_3}, \quad A = \frac{-\beta c\tilde{Y} + \alpha A}{N_3}, \quad N_3^2 = \alpha^2 + \beta^2 c^2, \tag{32}$$

where we identify $(V), A$ with the usual (dark) photon and with the masses of these states that are just given by

$$M_A^2 = 0, \qquad M_V^2 = (g_L s_\phi t_G)^2 (u_1^2 + u_2^2). \tag{33}$$

These mass eigenstates are then found to couple as

$$g_L \frac{\alpha}{N_3} sQA + g_L \left[N_3 s_\phi t_G Q_D + \frac{sc\beta}{N_3} Q\right] V, \tag{34}$$

so that we must identify

$$e = g_L \frac{\alpha}{N_3} s_w, \tag{35}$$

with the usual electromagnetic coupling to reproduce the familiar QED/SM expression. This all looks rather good, except that we are still left with V having a coupling to Q and the Z having a coupling to Q_D with the common feature that these are both proportional to the parameter β, but neither of which is phenomenologically acceptable if these are both of order unity. We notice that if $\beta = 0$ then $\alpha = N_3 = 1$ and so $e = g_L s_w$ as usual in the SM and we can then define the $U(1)_D$ gauge coupling to be just

$$g_D = g_L s_\phi t_G. \tag{36}$$

Interestingly, these conditions can all be easily achieved simultaneously if we assume the rather simple relationship

$$\lambda\sigma = t_\lambda^2 = \frac{s_\lambda^2}{c_\lambda^2} = \frac{\sigma^2 t_X^2}{3}, \tag{37}$$

with s_λ, c_λ as defined above, and something that we might expect to appear as a signal for and arise from a much more UV-complete picture such as partial unification in quartification [57] or even complete unification in $SU(N)$ [51]. Imposing this condition, up to mass and kinetic mixing corrections which are now all of order $\sim M_Z^2/M_{Z'_M}^2 \sim M_V^2/M_Z^2 \sim \epsilon \sim 10^{-4}$, we would obtain a very SM-like situation—subject to several constraints—but aligned with our hoped for expectations and with $M_Z^2 = g_L^2(v_1^2 + v_2^2)/4c_w^2$ as usual in, e.g., the THDM.

B. Non-Hermitian gauge bosons

We now turn to the parallel analysis for the mass generation for the three non-Hermitian gauge bosons that appear in this framework. Fortunately, this is far simpler than in the Hermitian gauge boson case. The symmetry breaking implications for the "off-diagonal" non-Hermitian gauge bosons are relatively straightforward to analyze as this breaking is contained entirely within the 3_L sector of the model. The interactions of such fields can be expressed simply through the off-diagonal matrix acting upon, e.g., **3** fields, here, in particular, the Higgs scalars, as (we stress that the NHGB field A appearing here is not to be confused with the SM photon above)

$$g_L T_{iL} W_{iL}|_{\text{off-diagonal}} = \frac{g_L}{\sqrt{2}} \begin{pmatrix} 0 & W & A \\ W^\dagger & 0 & B \\ A^\dagger & B^\dagger & 0 \end{pmatrix}, \quad (38)$$

where we need to consider the individual contributions of each of the three Higgs fields, χ, ρ, and η above to the NHGB mass-squared matrix and then simply combine them. Note that while the SM W carries $Q = 1$, when $\sigma = \pm 1$ we see that one of A or B is electrically neutral while the other also carries a $Q = 1$ charge. If we make the alternate choice of $\sigma = \pm 3$, then one of the new NHGBs would have $Q = 2$ instead of being neutral. Further, we see that both A, B will carry a nonzero value of Q_D so that the W might then mix with the $Q = 1$ NHGB state via one of the, e.g., 1_D breaking VEVs, $u_{1,2}$. Clearly, in any of these cases we expect that to leading order in the VEVs and in the absence of any mixing, $M^2_{A,B} \simeq g_L^2 w^2/4$ while $M_W^2 \simeq g_L^2(v_1^2 + v_2^2)/4$. As an example of these mixing effects, let us be specific and consider the case with $\sigma = -1$ and with the Higgs in triplets; then using the expression above, the mass-squared matrix for the NHGB in the (W, A, B) basis would then become

$$\mathcal{M}^2_{\text{NHBG}} = \frac{g_L^2}{4} \begin{pmatrix} v_1^2 + v_2^2 + u_1^2 & 0 & v_1 u_2 + w u_1 \\ 0 & w^2 + v_1^2 + u_1^2 + u_2^2 & 0 \\ v_1 u_2 + w u_1 & 0 & w^2 + v_2^2 + u_2^2 \end{pmatrix}. \quad (39)$$

Here we see that our naive expectations are met and that W-B mixing is dominated (assuming that roughly $u_1 \simeq u_2$) by the product of VEVs wu_1, since $w \gg v_{1,2}$, and that the relevant mixing angle would then be $\theta_{WB} \simeq u_1/w \lesssim 10^{-4}$, which is phenomenologically negligible for most considerations. The result we have obtained is typical of what one would find in the other sample cases. For example, when $\sigma = \pm 3$, the W will still mix with the other $Q = 1$ gauge boson via the 1_D breaking VEVs but in this case leaving the $Q = 2$ state unmixed. This mixing is also seen to induce a small downward shift in the expected W mass by roughly the same fractional amount, i.e., $\delta M_W^2/M_W^2 \simeq -u_1/w$.

IV. SOME IMPLICATIONS

If we demand that the relationship $\lambda\sigma = t_\lambda^2$ holds (especially in the cases where $\sigma^2 = 1$, which we will generally assume henceforth except where noted), then there are many simplifications in the expressions above and the results of the previous analysis become much more transparent with a number of interesting implications. A simple example is provided by Eq. (33) above; defining the abbreviations

$$\kappa_L = \frac{g_D}{g_L}, \qquad r = \frac{4(1 - x_w)}{[3 - (3 + \sigma^2)x_w]}, \quad (40)$$

with $x_w = s_w^2 \simeq 0.2315$ at the weak scale as usual, we find that we can express t_G simply as

$$t_G^2 = \frac{\kappa_L^2}{1 - \kappa_L^2 r} \geq 0, \quad (41)$$

implying that κ_L is "bounded" from above, i.e.,

$$\kappa_L \leq \frac{1}{\sqrt{r}} \simeq 0.821(0.269), \quad (42)$$

for $|\sigma| = 1(3)$ or $g_D \lesssim 0.535(0.176)$, again employing suggestive weak scale input values. If g_D runs to smaller values as the $\sim$1 GeV mass scale is approached from above, as we might perhaps expect, this can have implications for low energy dark sector searches. Another simple example is provided by the mass ratio of the new heavy Hermitian gauge boson Z'_M to those of the new NHGB encountered in the previous section (in the limit where the subleading $v_{1,2}^2$ contributions can be neglected) is just

$$\frac{M^2_{Z'_M}}{M^2_{\text{NHGB}}} = \frac{r}{1 - \kappa_L^2 r}, \quad (43)$$

which is always greater than unity and has important phenomenological implications as it determines whether or not NHGB pairs can be produced resonantly via the Z'_M, i.e., when the relation $M_{Z'_M} > 2M_{\text{NHGB}}$ holds. This requirement is shown in the upper panel of Fig. 1 in the case when $|\sigma| = 1$ and we observe that $\kappa_L \gtrsim 0.652$ must be satisfied for such processes to occur. Interestingly, when $|\sigma| = 3$,

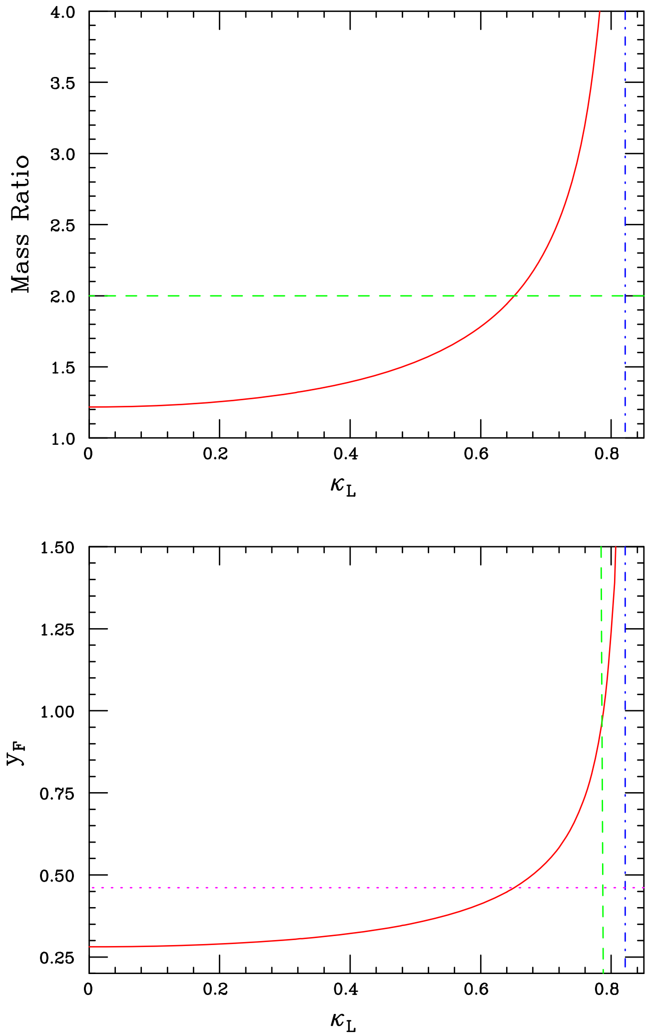

FIG. 1. Top: ratio of the mass of the neutral Z'_M gauge boson to that of either of the NHGBs (solid red curve) as a function of κ_L when $|\sigma| = 1$ in the $v_{1,2}^2/w^2 \to 0$ limit. The vertical dash-dotted blue line on the right-hand side of the figure show the upper bound on $\kappa_L \simeq 0.821$ in this case as discussed in the text. The horizontal green dashed line shows the mass threshold beyond which Z'_M is kinematically allowed to decay to NHGB pairs, which occurs when κ_L exceeds $\simeq$0.652. Bottom: maximum value of the PM (F) generic Yukawa coupling y_F as a function of κ_L above which the decay $Z'_M \to \bar{F}F$ is kinematically forbidden. The vertical green dashed line on the right-hand side of the panel corresponds to the value of $\kappa_L \simeq 0.788$ when $y_F = 1$. The magenta dotted line indicates the corresponding maximum value of $y_F \simeq 0.462$ beyond which NHGB decays to $\bar{F}f$ are kinematically forbidden. As in the upper panel, the blue vertical dash-dotted line shows the upper bound on κ_L.

this mass ratio requirement is *always* satisfied for all physical κ_L values, as r in this case is so much larger than when $|\sigma| = 1$.

Finally, we consider the case of $Z - Z'_M$ mixing as alluded to previously; this quantity partly arises from the terms in X_M, as introduced above, which are proportional to both T_{8L} and also to $Y/2 = -T_{3L}$, with the last equality holding when acting on the $Q_D = Q = 0$ components of the Higgs fields that obtain the electroweak scale VEVs, $v_{1,2}$. Note that these VEVs have $T_{8L}/\sqrt{3} = 1/6$, $T_{3L} = \pm 1/2$, respectively. Employing the same notation as above, for simplicity we first introduce the abbreviations

$$a = \frac{3}{2}\left(\frac{r}{1 - r\kappa_L^2}\right)^{1/2}, \quad b = -\frac{1}{4a\sigma}\left(r - \frac{4}{3}\right)\left[\frac{1 + 8r\kappa_L^2}{1 - r\kappa_L^2}\right]. \quad (44)$$

Note that, for $\sigma = \pm 1$, $r \simeq 1.482$ so that b is generally small in comparison to a. Then, to leading order in the ratio of squared VEVs, $v_{1,2}^2/w^2$, the $Z - Z'_M$ mass-squared submatrix is given by

$$\mathcal{M}^2_{Z-Z'_M} \simeq \begin{pmatrix} M^2_{Z'_M} & M^2_{\rm int} \\ M^2_{\rm int} & M_Z^2 \end{pmatrix}, \quad (45)$$

where both M_Z^2 and $M^2_{Z'_M}$ are given above and $M^2_{\rm int}$ is given in terms of the parameters a, b by

$$M^2_{\rm int} = \frac{g_L^2}{2c_w}\left[\frac{b}{2}(v_1^2 + v_2^2) + \frac{a}{6}(v_2^2 - v_1^2)\right], \quad (46)$$

which leads to an explicit expression for the $Z - Z'_M$ mixing angle to leading order in the VEV ratios given by

$$\theta_{\rm mix} \simeq \frac{M^2_{\rm int}}{M^2_{Z'_M}} = \frac{3}{4ac_w}\left[\frac{3b}{a}\left(\frac{v_1^2 + v_1^2}{w^2}\right) + \frac{v_2^2 - v_1^2}{w^2}\right] \sim 10^{-4}, \quad (47)$$

with the magnitude being as expected. This induces, at the same level of approximation, a small downward fractional shift in the SM Z mass explicitly given by

$$\frac{\delta M_Z^2}{M_Z^2} \simeq -\frac{1}{36w^2}\left[\left(1 - \frac{3b}{a}\right)^2 v_1^2 + \left(1 + \frac{3b}{a}\right)^2 v_2^2\right] \sim 10^{-4}, \quad (48)$$

which we see is again of the same small magnitude as found in the case of W-NHGB mixing.

V. FERMION PM MASSES

Given the setup above we can now more explicitly discuss the Dirac masses of the PM fields, all of which lie at the scale of the VEV $\sim w$. In the case where the anomalies cancel between the fermions among the three generations, the PM will consist of some set of the X_i introduced above as left-handed **3**'s or **3***'s and right-handed singlets. When the anomalies instead cancel within each of the generations, then the quark (i.e., color triplet) sector will still appear as a set of the $X_{1,2}$ as just described, but then the lepton sector is necessarily more complex as was discussed above.

This being the case, we will begin with the "simpler" quark sector where, ignoring flavor issues, there are two subcases, denoted as $Q = q_1, q_2^*$ above, depending upon whether the u or d quark, respectively, has a PM partner field with which it mixes via one or more of the Q_D-violating VEVs. For example, taking $\sigma = -1$, if we make the choice $Q = q_2^*$, we see that the quark mass terms can be written as

$$\mathcal{L}_{\text{Yukawa}} = y_d \bar{Q} d_R \eta^* + y_u \bar{Q} d_R \rho^* + y_D \bar{Q} D_R \chi^* + \text{H.c.}, \quad (49)$$

which, once all of the VEVs are turned on, yields the 2×2 mass matrix

$$\mathcal{M}_d = \frac{1}{\sqrt{2}} (\, \bar{d}_L, \quad \bar{D}_L \,) \begin{pmatrix} y_d v_1 & y_D u_2 \\ y_d u_1 & y_D w \end{pmatrix} \begin{pmatrix} d_R \\ D_R \end{pmatrix}, \quad (50)$$

while $m_u = y_u v_2/\sqrt{2}$. $\mathcal{M}_d$ is, as usual, diagonalized by a biunitarity transformation, $M_{\text{diag}} = U_L \mathcal{M}_d U_R^\dagger$ which, if we neglect any phases for simplicity, can be parametrized by employing two small mixing angles $\theta^d_{(L,R)} \simeq (u_2, y_d u_1/y_D)/w \lesssim 10^{-4}$, since we might expect that $y_d/y_D < 1$, to leading order in the VEV ratios. This is roughly the same magnitude as was found for both of the $Z - Z_M'$ and W-NHGB mixing factors obtained above. Note that if we had instead chosen $Q = q_1$, then essentially the roles of d and u would be interchanged with the replacement $D \to U$. Similarly, when the left-handed leptonic multiplet is either of l_1, l_2^*, as is the case when the anomalies cancel among the three generations, an essentially identical set of results are found to hold. As alluded to previously, these rather small mixings between the PM and the corresponding SM particles with which it shares a multiplet are of great phenomenological relevance as they generate the PM dominant decays, such as $D \to dV, h_D$, with partial rates proportional to the combinations $\sim(\theta_{L,R} m_D/m_V)^2$, which we see is $\sim O(1)$ as was shown in very early work on PM [42]. Clearly, this important mixing cannot occur when X_i has an exotic electric charge, as will be the case when the corresponding choice $|\sigma| = 3$ is made.

Of course, in the leptonic sector, the situation is a bit more complex when the gauge anomalies cancel generation by generation due to the required augmentation mentioned above, i.e., three leptonic **3**'s are needed to cancel the contribution to the 3_L^3 anomaly of a single quark $\mathbf{3}^*$ for each generation and vice versa. In models such as this, fields with SM charges but with $Q_D = \pm 2$ can occur. The simplest examples of this, when $\sigma = 1$, are the $S_{9,10}$ sets of lepton representations as given in [102] for a single generation that is quite representative of this possibility and so we will examine them briefly here. In the case of S_9, all of the leptonic fields lie in three $\mathbf{3}^*$ representations, R_{1-3}, *including* the conjugates of the left-handed fields, e.g., e_L^c, etc., which can be written in the form

$$S_9: R_1 = \begin{pmatrix} e^- \\ \nu \\ N_1 \end{pmatrix}_L, \quad R_2 = \begin{pmatrix} E \\ N_2 \\ N_3 \end{pmatrix}_L, \quad R_3 = \begin{pmatrix} N_2^c \\ E^c \\ e^c \end{pmatrix}_L, \quad (51)$$

where we see that the leptonic PM fields consist of a $Q = Q_D = -1$ Dirac state, $E = E^-$, as well as the four neutral fermions, N_{1-3}, N_2^c, with various transformation properties, e.g., $Q_D(N_1) = Q_D(N_2) = -1$ while $Q_D(N_3) = -2$. Given our previous discussion, such a state as N_3 would necessarily decay via a NHGB exchange if it does not mix with any of the others. Also note that only a conjugate field exists for N_2 to form a Dirac mass term. In such a case, as there are no right-handed singlets present, the relevant fermion mass terms must be generated via the asymmetric triple product of pairs of the R's, with the set of (conjugated) Higgs fields, $H^*_{A,B,C}$ with H's being generally similar to one of the members of the set (χ, η, ρ) introduced in Sec. II above. This results in a set of general interactions of the form

$$\mathcal{L}_{S_9} = \epsilon_{ijk} \left(\kappa_A R_2^i R_3^j H_A^{k*} + \kappa_B R_1^i R_3^j H_B^{k*} + \kappa_C R_1^i R_2^j H_C^{k*} \right) + \text{H.c.}, \quad (52)$$

where the κ's are a set of Yukawa couplings and with the (i, j, k)'s labeling the various components of the relevant representations. Here we observe, for example, that $H_A \sim \chi$ and $H_B \sim \eta$ above, while we will see that H_C is necessarily somewhat different.

For the charged leptons, after the Higgs fields obtain their VEVs, this set of couplings results in the 2×2 mass matrix

$$\mathcal{M}_e = \frac{1}{\sqrt{2}} (\, e, \quad E \,)_L \begin{pmatrix} -\kappa_B v_1 & \kappa_B u_2 \\ -\kappa_A u_1 & \kappa_A w \end{pmatrix} \begin{pmatrix} e^c \\ E^c \end{pmatrix}_L, \quad (53)$$

which is quite similar to the matrix $\mathcal{M}_d$ discussed previously above. We also obtain the following 4×4 Dirac mass matrix for the neutral fields given by

$$\mathcal{M}_\nu = \frac{1}{\sqrt{2}} (\, \nu, \quad N_1, \quad N_2, \quad N_3 \,)_L \begin{pmatrix} 0 & 0 & -\kappa_B u_2 & 0 \\ 0 & 0 & \kappa_B v_1 & 0 \\ 0 & 0 & -\kappa_A w & 0 \\ 0 & 0 & \kappa_A u_1 & 0 \end{pmatrix} \begin{pmatrix} \nu^c \\ N_1^c \\ N_2^c \\ N_3^c \end{pmatrix}_L, \quad (54)$$

where we see that only one combination, essentially $N_2 N_2^c$ to leading order in the VEVs, gets a Dirac mass at this stage, as the fields $\nu^c, N_{1,3}^c$ do not appear among the S_9 set. Two Majorana mass terms are also found to be potentially generated by the last term in these couplings,

$$\frac{\kappa_C \tilde{u}}{\sqrt{2}}(\nu N_3 - N_1 N_2) + \text{H.c.}, \tag{55}$$

but this requires that the top component of H_C^* with $Q = 0$ obtain a small VEV, $\tilde{u} \lesssim 1$ GeV, while this same field simultaneously must have $Q_D = 2$. This is perhaps not so strange as a $Q_D = 2$ Higgs VEV is also needed to generate pseudo-Dirac dark matter to satisfy the CMB constraints, as mentioned in the Introduction. While such a field will contribute to the DP mass, if it exists it will not significantly influence the discussion of the gauge boson masses presented in the previous sections. Clearly, the neutral lepton sector of this type of setup deserves some further study.

In the case of S_{10}, the leptons of a single generation (and their conjugates) now lie in three different **3** representations, R_{1-3}, as well as three additional 3_L singlets, which we can, in terms of left-handed fields, write as

$$S_{10}\colon R_1 = \begin{pmatrix} \nu \\ e \\ E_1 \end{pmatrix}_L, \quad R_2 = \begin{pmatrix} E_2^c \\ N_1^c \\ N_2^c \end{pmatrix}_L,$$
$$R_3 = \begin{pmatrix} N_1 \\ E_2 \\ E_3 \end{pmatrix}_L, \quad e_L^c, \quad E_{1L}^c, \quad E_{3L}^c, \tag{56}$$

where we now see that the PM states consist of the three Dirac charged states, E_{1-3}^-, as well as two neutral states, $N_{1,2}$, all with various transformation properties. In principle, due to the parameter freedom in this model, unusual Q_D quantum numbers can arise in this case. With the SM fields having $Q_D = 0$ as usual, we see that the general assignments $Q_D(E_1) = 1$, $Q_D(E_2, N_1) = q$, $Q_D(N_2^c) = 1 - q$, and $Q_D(E_3) = 1 + q$ are possible for arbitrary values of q, with the choice $q = 1$ being the simplest one that we will consider below. In such a case, we see that we can make the important identification $N_2^c = \nu^c$ so that the SM neutrinos can obtain an ordinary Dirac mass. Here the SM and PM masses are not only the result of the triple product of fields, as was the case with S_9 above, but are also due to the more familiar couplings of the quarklike **313*** product variety already encountered above. A minimal set of such couplings (without, e.g., introducing any additional $|Q_D| = 2$ Higgs fields) is given by

$$\begin{aligned}\mathcal{L}_{S_{10}} = \epsilon_{ijk}(y_\nu R_1^i R_2^j H(\nu)^k + y_D R_2^i R_3^j H(N_1, E_2)^k) \\ + y_e R_1 e^c H(e) + y_{E_1} R_1 E_1^c H(E_1) \\ + y_{E_3} R_3 E_3^c H(E_3) + \text{H.c.},\end{aligned} \tag{57}$$

where as before i, j, k label the multiplet member and the H's are potentially different Higgs representations denoted by the fields to which they give mass. Some algebra tells us, in the notation above, that in terms of transformation properties and VEVs, $H(E_1) = H(E_3) \sim \chi^*$, $H(e) \sim \eta^*$, $H(N_1, E_2) \sim \chi$, and $H(\nu) \sim \eta$. From this interaction, after the all the Higgs fields obtain their VEVs, the following 4×4 mass matrix is generated for the $|Q| = 1$ fermions:

$$\mathcal{M}_e = \frac{1}{\sqrt{2}}(\, e, \quad E_1, \quad E_2, \quad E_3\,)_L \begin{pmatrix} y_e v_1 & y_{E_1} u_2 & -y_\nu u_1 & 0 \\ y_e u_1 & y_{E_1} w & y_\nu v_1 & 0 \\ 0 & 0 & y_D w & y_{E_3} u_2 \\ 0 & 0 & -y_D u_2 & y_{E_3} w \end{pmatrix} \begin{pmatrix} e^c \\ E_1^c \\ E_2^c \\ E_3^c \end{pmatrix}_L. \tag{58}$$

Here we see that the PM fields obtain their usual large masses from the w VEV and that both $e - E_1$ and $E_2 - E_3$ will mix via the $U(1)_D$-violating VEVs $u_{1,2}$, while $E_1 - E_2$ mixing is generated by the electroweak scale VEV v_1. These mixings are observed to be sufficient to allow the E_i to all eventually decay down to the electron plus DP final state as required. Similarly, the corresponding 2×2 Dirac mass matrix for the neutral fields, after now making the identification $\nu^c = N_2^c$, is then given by

$$\mathcal{M}_\nu = \frac{1}{\sqrt{2}}(\nu, \ N_1)_L \begin{pmatrix} -y_\nu v_1 & -y_\nu u_1 \\ y_D u_2 & -y_D w \end{pmatrix} \begin{pmatrix} \nu^c \\ N_1^c \end{pmatrix}_L, \tag{59}$$

while we see that no Majorana mass terms are generated from just these terms alone without further extending $\mathcal{L}_{S_{10}}$, which is certainly possible. This is similar to that obtained for $e - E$ mixing in the case of the S_9 setup previously discussed, as well as what was obtained for the quark sector, except that y_ν must be highly suppressed to explain the SM neutrino masses, while we might expect $y_D \sim O(1)$. This implies that $\theta_L^\nu \simeq (y_\nu / y_D) u_1 / w \ll \theta_R^\nu \simeq u_2 / w \sim 10^{-4}$, but still allows for the decay $N_1 \to \nu V$ to occur.

One of the obvious results of the discussion above is that we find, as expected, that the masses of all of the PM fermions F are given generically by a relation of the form $m_F \simeq y_F w / \sqrt{2}$ to leading order in the VEVs, where we might expect the various y_F's to be $O(1)$. An important question to ask, especially when we address the phenomenological collider signatures below, is whether or not the new heavy gauge bosons Z'_M and the NHGB above can decay into such states, i.e., $Z'_M \to \bar{F}F$, NHGB $\to \bar{F}f(\text{or}\,\bar{f}F)$ as these would then be potentially resonantly enhanced cross sections, particularly in the Z'_M case. The case for the NHGB is simple as, to leading order in the VEVs, $M_{\text{NHGB}} \simeq g_L w/2$ so that we see that this decay channel is open provided that $y_F < \sqrt{2}(\sqrt{2}G_F M_W^2)^{1/2} = \sqrt{2}x \simeq 0.462$, a bound that is shown in the lower panel of Fig. 1. In the case of $Z'_M \to \bar{F}F$, the corresponding bound on y_F will clearly be κ_L dependent and is given instead by

$$y_F < \frac{x}{\sqrt{2}}\left[\frac{r}{1-r\kappa_L^2}\right]^{1/2}, \tag{60}$$

so that, e.g., if $y_F = 1$ (similar to that of the top quark on the SM) and $\sigma = \pm 1$, then $\kappa_L > 0.788$ is needed for this decay to be kinematically allowed, a value not much smaller that the upper bound of $\simeq$0.821 previously obtained on κ_L above. This constraint on y_F, as a function of κ_L, is also shown in the lower panel of Fig. 1. In the case where $\sigma = \pm 3$, r is significantly larger by almost an order of magnitude such than even potentially perturbatively troublesome values of y_F would be still allowed, as long as we stayed away from the corresponding upper bound on the value of κ_L in this case, i.e., $\simeq$0.269.

VI. COLLIDER SIGNATURES FROM NEW HEAVY STATES

Having the results of the analyses above in hand, we can now turn to a discussion of some of the collider implications of this setup resulting from the production of the new heavy gauge bosons as well as the PM fields; here we will many times limit ourselves to the cases where $\sigma = \pm 1$ for definiteness, but occasionally we will note the differences were we to instead choose $|\sigma| = 3$. Of course, in addition to new physics at colliders, the existence of these new states can also impact phenomena at lower energies, e.g., flavor physics. However, the level of this impact depends upon the model details of both the generational dependence of the PM-SM mixings and the associated couplings to the new heavy gauge bosons. Because of this significant model dependence (that we have so far avoided), we will not consider these interesting possibilities here.

The most obvious place to begin is with the Drell-Yan production of the Z'_M in $\bar{q}q$ annihilation at hadron colliders, e.g., the LHC and the 100 TeV FCC-hh, as this state can couple to a purely SM initial state, be resonantly produced on shell, and decay into lepton pairs with a significant branching fraction. In the narrow width approximation (NWA), the signal rate is proportional to the Z'_M resonant production cross section times its leptonic branching fraction B_l. For the SM fields, which we recall all have $Q_D = 0$, we can write the relevant couplings in terms of the parameters (a, b) that were introduced previously [which we recall are functions of both r (and so also σ) and κ_L] as just

$$g_L\left(a\frac{T_{8L}}{\sqrt{3}} + b\frac{Y}{2} + \frac{2}{3}a\kappa_L^2 Q_D\right)Z'_M, \tag{61}$$

with the last term just being zero in the case of the SM fields. We also recall from above, for numerical purposes, that for $\sigma = \pm 1$, b is relatively small in comparison to a so that we expect reduced sensitivity to the sign of σ in production cross sections. Since the values of both T_{8L} and $Y/2$ are fixed for the SM fields, all of the relevant couplings are known quantities apart from the values of κ_L and σ. For later usage, we note that the partial width for the Z'_M decay into (massless) left-handed SM leptons (which we recall have $T_{8L}/\sqrt{3} = 1/6$) is given, for simplicity, in the approximate $|b/a| \to 0$ limit by

$$\Gamma_l = \Gamma(Z'_M \to l^+ l^-) \simeq \frac{g_L^2 M_{Z'_M}}{864\pi}a^2. \tag{62}$$

Further, apart from the overall QCD/color factors, i.e., $N_c = 3$, and small QCD corrections, $\simeq(1 + \alpha_s(M_{Z'_M})/\pi)$, that appear for the quarks, all of the SM fermions will essentially have this same partial width in the $|b/a| \to 0$ limit when the top mass can be safely neglected. Observe that when a^2 gets sufficiently large near the upper limit of the allowed range for κ_L we will loose perturbativity and so we can no longer be able to trust our results and certainly not the NWA to be employed below. Clearly, the largest that B_l can be is when the Z'_M decays *only* into just the SM fermion final states, as any additional modes would increase its total decay width. If this condition is satisfied, then we find that $B_l \simeq 0.04$ but, more generally, B_l is instead

$$B_l = \frac{\Gamma_l}{\Gamma_T + \mathcal{N}\Gamma_l} \simeq \frac{1}{25 + \mathcal{N}}, \tag{63}$$

where we have parametrized any additional contributions to the total Z'_M decay width via the quantity $\mathcal{N} = \Gamma(Z'_M \to \text{new})/\Gamma(Z'_M \to l^+ l^-)$, which is zero in the simplest case, with Γ_T being the total width in this case. Thus, we will be interested in knowing how any additional partial decay widths will scale with respect to that for the SM leptons given above in what follows. Certainly, if $\mathcal{N}$ becomes very large, the NWA approximation will also fail, as the Z'_M total width to mass ratio will be become too large—as may also happen when a becomes too large even without the introduction of any new additional decay modes as already noted. Below, we frequently employ this simplifying assumption, i.e., the absence of any additional new physics, so that if $|b/a| \ll 1$ then we will have $B_l \simeq 4\%$ as previously noted. Any bound on $\mathcal{N}$ will be much more easily saturated when $\sigma = \pm 3$ than if $\sigma = \pm 1$, as much larger values of a^2 are obtained in that case.

Figure 2 shows the rate for Z'_M production as a function of its mass under the assumption that only SM fermion final states can appear for either choice of $\sigma = \pm 1$ and with different selections of the value of κ_L at both the LHC and the 100 TeV FCC-hh. Here we see that, as expected, this rate is relatively insensitive to b and, hence, the sign of σ. We also see that it grows as κ_L approaches its maximum value, $1/\sqrt{r} \simeq 0.821$, due to the factor of $(1-\kappa_L^2 r)^{1/2}$ appearing in the denominator of the definition of a. When $\sigma = \pm 3$, for a given value of $\kappa \lesssim 0.269$, both r and a are now larger, which would then result in an enhanced production cross section and a potentially increased influence of the sign of σ as r would be somewhat larger in such a case. In practice, however, we still find that this sign will have little practical influence on the search reaches.

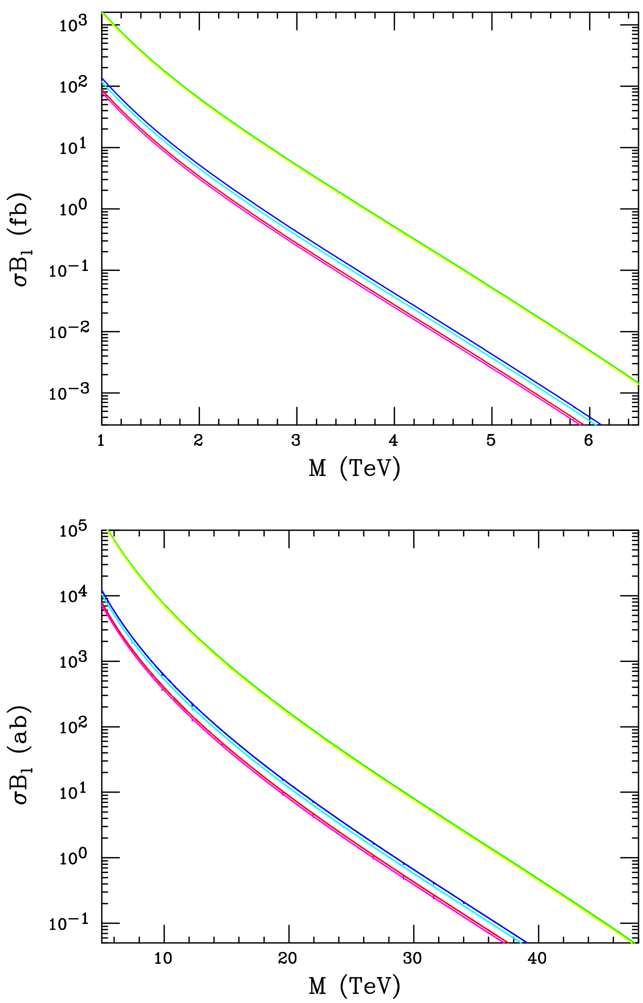

FIG. 2. Production cross section times leptonic branching fraction for the new heavy gauge boson Z'_M, as a function of its mass employing the narrow width approximation as discussed in the text for (top) the 13 TeV LHC and (bottom) the 100 TeV FCC-hh. The red (blue, green) curves correspond to $\kappa_L = 0.1, \sqrt{x_w}$ (i.e., $g_D = e$), and 0.8, respectively, with the choice of $\sigma = 1$. The magenta (cyan, yellow) curves are the corresponding results assuming instead that $\sigma = -1$, and we observe very little difference. In all cases, only Z'_M decays to SM fermions have been assumed to be kinematically accessible.

Both ATLAS [113] and CMS [114] have performed Z' searches in the dilepton channel at the 13 TeV LHC employing an integrated luminosity of 139 fb^{-1}, which we can apply to the case at hand; here we will employ the specific results obtained by ATLAS. Similarly, we can follow the analysis as presented in Ref. [115] to obtain the corresponding expected reach for the 100 TeV FCC-hh assuming 30 ab^{-1} of integrated luminosity. The results of these analyses are shown in Fig. 3 as functions of κ_L assuming that $\sigma = \pm 1$.[3] Here we see that the bounds at the 13 TeV LHC are relatively modest and increase slowly at first as κ increases away from zero with the choice of $\sigma = 1$ yielding the slightly larger values; as a point of comparison, a heavy SM-like Z', $Z'_{\rm SSM}$, is constrained to lie above $\simeq$5.1 TeV from this search [113]. However, again due to the factor of $(1-\kappa_L^2 r)^{1/2}$, when κ_L exceeds roughly $\simeq$0.64, the limit strengthens significantly due to the now much more rapid growth of the parameter a. Assuming no signals are found, the 14 TeV HL-LHC with a luminosity of 3 ab^{-1} is expected to increase these bounds by roughly $\sim$10%–15%. The corresponding results for the 100 TeV FCC-hh in the lower panel of the figure show quite similar behavior; in this case, we note that the $Z'_{\rm SSM}$ bound is determined to be $\simeq$42 TeV from Ref. [115] for comparison purposes.

The corresponding bounds obtained in the case of $\sigma = \pm 3$ are shown in Fig. 4. Here we see that, as expected, substantially greater exclusion and search reaches are obtained, by roughly $\simeq$20%–25%, since the values of a are now much larger. With the ratio of $|b/a|$ being somewhat larger in this case, as was noted above, we also observe a somewhat greater sensitivity to the sign of σ, also as expected.

One simple and immediate application of the Z'_M mass constraints obtained above is that, within the present model context, we can translate these bounds into the corresponding, yet "indirect," constraints on the NHGB using the mass relationships discussed above. These are potentially important as they can, in some parameter space regions, supersede those from direct production to be discussed further below. In Fig. 5, these constraints are shown, assuming $\sigma = \pm 1$, using the results as displayed in Figs. 1 and 3 as input. These constraints are seen to be relatively strong for small values of κ_L but then weaken substantially as κ_L increases toward its upper bound as we might have

[3] As we will see below, when $\sigma = \pm 3$, these cross sections will we seen to increase by an order of magnitude or more, which results in an increased mass reach of $\simeq$20%–25%.

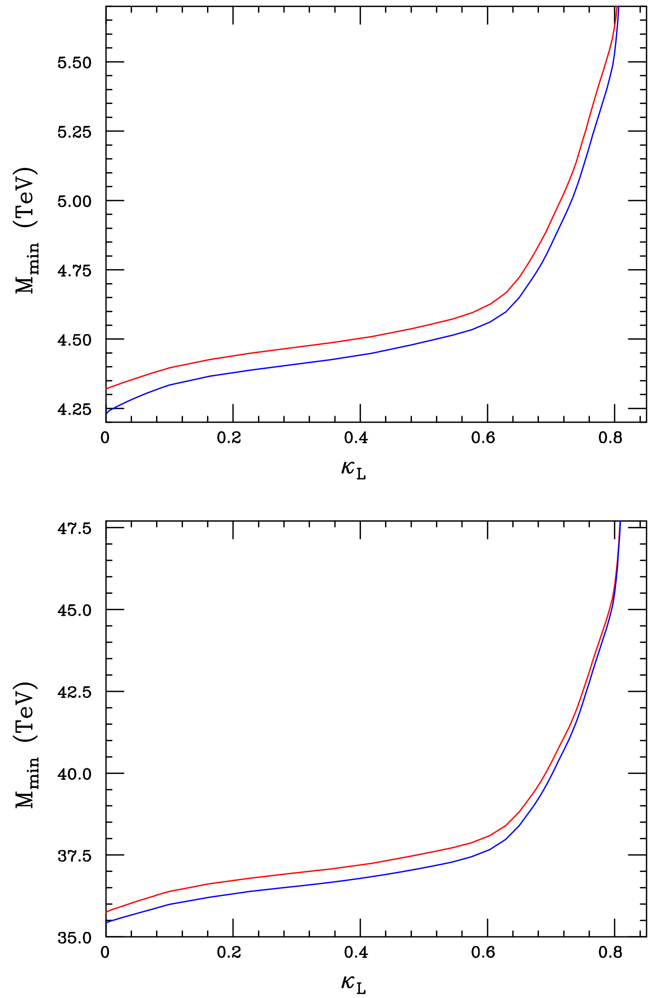

FIG. 3. Z'_M mass bounds as functions of κ_L with $\sigma = 1(-1)$ corresponding to the red (blue) curve, following the analysis described in the text for (top) the 13 TeV LHC employing the results from ATLAS [113] and (bottom) for the 100 TeV FCC-hh assuming an integrated luminosity of 30 ab^{-1} and employing the analysis as presented in Ref. [115], respectively. The NWA approximation is employed in obtaining these results and assumes decays only to the SM fermions.

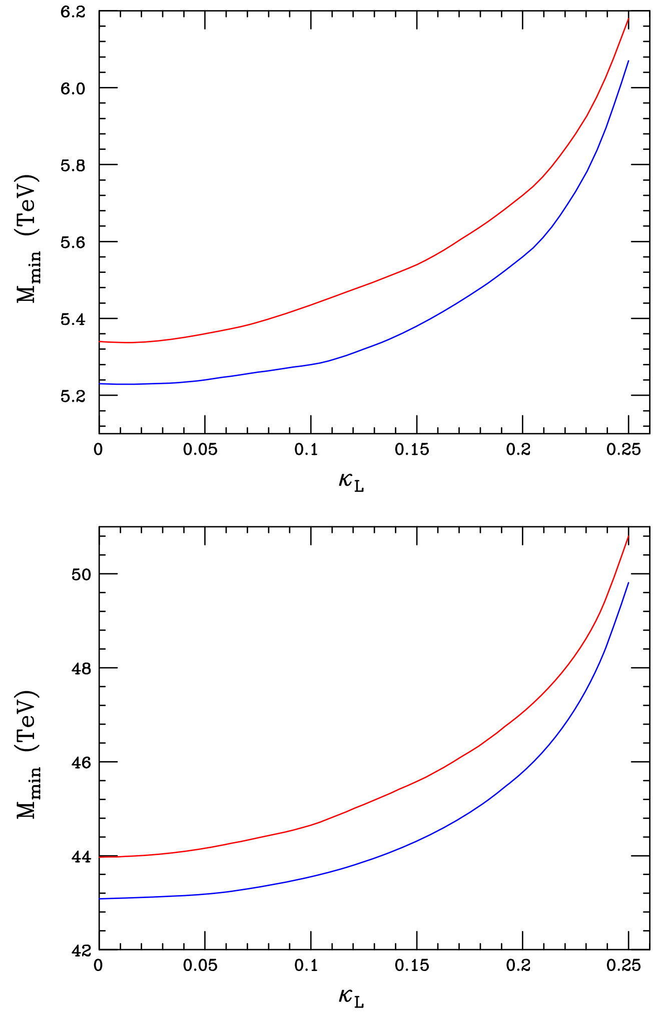

FIG. 4. Same as the previous figure, but now assuming that $\sigma = \pm 3$.

expected. Correspondingly, indirect bounds are similarly obtainable for the case of $\sigma = \pm 3$ and display a comparable overall behavior.

What about other Z'_M decay modes beyond those to the SM fermions which may be significant? Such modes can exist even if we remain restricted to the familiar set of SM final states. Any such modes will lower B_l and so lead to a (generally modest) reduction in the corresponding search reach using the Drell-Yan channel, but they are also important to consider on their own. Interestingly, as is well known [75], the small mixing of a new gauge boson such as Z'_M with the SM Z can induce other decay modes which may have significant partial widths, which can be comparable to those for the SM leptons. A classic example of this is the mode $Z'_M \to W^+W^-$ which is induced via this angle, θ_{mix}, as encountered above in Eqs. (44)–(46); in the present case, one finds that this partial width is given, to leading order in the small mass ratios, by

$$\Gamma(Z'_M \to W^+W^-) \simeq \frac{g_L^2 M_{Z'_M}}{192\pi c_w^2}\left(\theta_{\mathrm{mix}} \frac{M^2_{Z'_M}}{M_Z^2}\right)^2, \qquad (64)$$

where (using $M_W = M_Z c_w$ which remains true to leading order in the small VEV ratios) we see the familiar result that the small mixing angle is offset by the square of the large ratio of the Z'_M and SM Z. Using the expression above that relates this mixing angle to the other model parameters, $\theta_{\mathrm{mix}} \simeq M^2_{\mathrm{int}}/M^2_{Z'_M}$, we obtain from Eqs. (44)–(46) that

$$\mathcal{R}_W = \frac{B(Z'_M \to W^+W^-)}{B_l} \simeq \frac{1}{4} \cdot \left[\frac{v_2^2 - v_1^2}{v_2^2 + v_1^2} + \frac{3b}{a}\right]^2, \qquad (65)$$

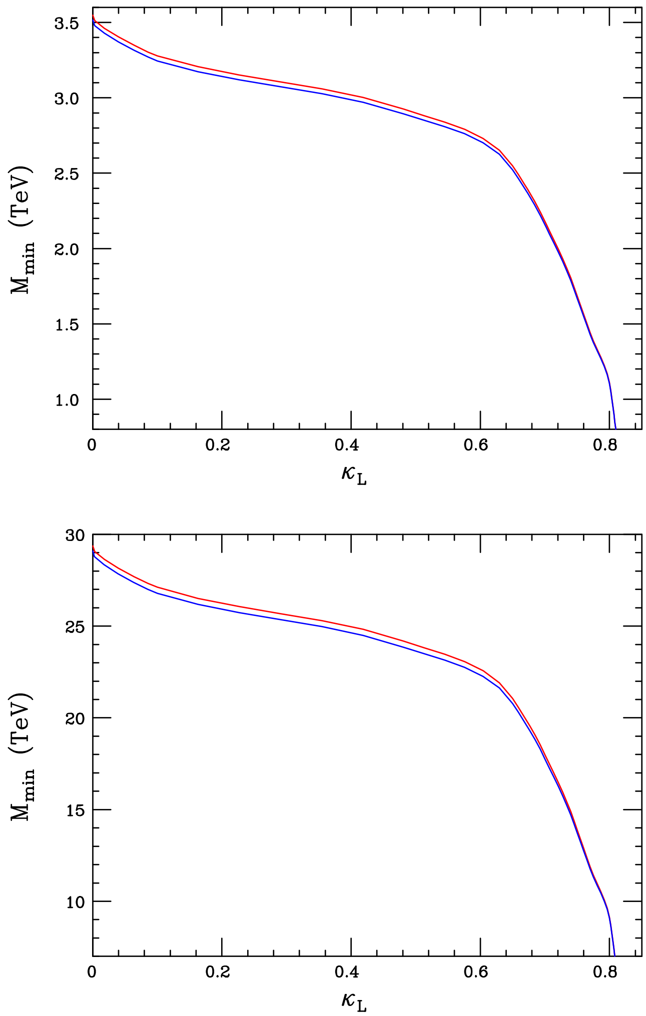

FIG. 5. Indirect mass bounds on the NHGBs obtained by employing the model-dependent mass relationship given in the text and the Z'_M mass constraints from the previous figures, here assuming that $\sigma = \pm 1$.

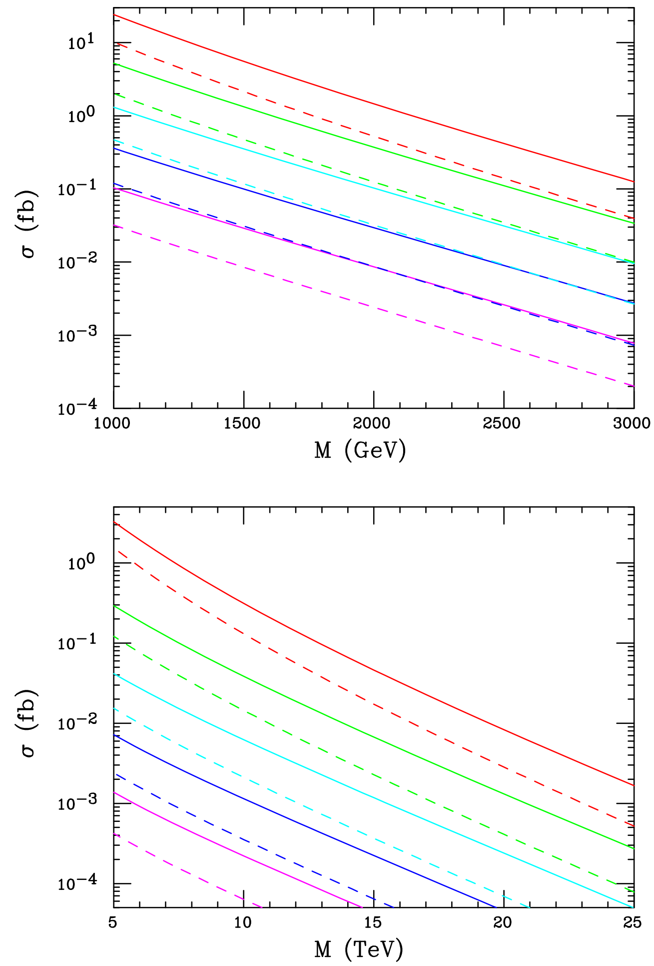

FIG. 6. $gu(gd)$-initiated NHGB plus PM associated production cross section represented by the solid (dashed) curves at the (top) 13 TeV LHC and (bottom) 100 TeV FCC-hh as functions of the appropriate NHGB mass. In the top panel, from top to bottom, the curves are for a PM mass of 1, 1.5, 2, 2.5, and 3 TeV, respectfully, whereas in the bottom panel, the corresponding curves label the results for PM masses of 5, 10, 15, 20, and 25 TeV, respectively.

so that $\mathcal{R}_W$ may be appreciable $\simeq 0.25$ or so as the expression in brackets if $O(1)$. Clearly, a relative branching fraction of this magnitude would have essentially no impact on the searches above employing the dilepton channel. If the dilepton production rate is significant, then the W^+W^- mode can also be sufficiently large to be used to probe this gauge boson mixing and the symmetry breaking structure of the model. The cleanest signature for the observation of this decay, since the W's are expected to be highly boosted, would likely be an almost collinear lepton + missing transverse energy (MET) combination plus two collimated jets in the opposite hemisphere with both systems reconstructing to the W mass.

The NHGBs, which we recall carry $Q_D \neq 0$, can be produced at hadron colliders via two mechanisms: generically, the first of these, associated production, can be described by the $gq(\bar{q}) \to \text{NHGB} + \text{PM}$ process where g is a gluon and $q(\bar{q})$ is a either a SM u- or d-type (anti) quark—not dissimilar to a previously examined production process studied for both the LHC and FCC-hh [57]. Depending upon whether the quarks lie in a q_1 or a q_2^* representation, as well as the value of σ, several distinct channels are possible, e.g., $gu \to B^+D, A^0U$ in the language employed above. Since the overall coupling g_L is fixed, only the PM and NHGB masses are *a priori* unknown. Figure 6 shows the relevant cross sections for both the gu- and gd-initiated processes at the 13 TeV LHC and at the 100 TeV FCC-hh as functions of the masses of these two heavy particles in the final state. Certainly the signatures for this production process will depend somewhat on the model details such as the mass ordering of the PM and NHGB, neither of which are likely to be very boosted since they are expected to be quite massive. For example, in the simple scenario where the NHGB is the heavier of the two, then we can have decays such as $\text{NHGB} \to \text{PM} + \bar{q}$ while

PM $\to qV$, which leads to a $3j+$ MET final state with the MET coming from the two DP in the final state, which are assumed to be either long-lived or which decay into DM. Many other similarly interesting modes can occur, as any given model will have both color triplet as well as color singlet PM field content. For example, the NHGB may instead dominantly decay to a leptonic $\bar{e}E$ pair followed by the $E \to eV$ leading to a $1j+$ a nonresonant, opposite sign dilepton pair $+$ MET final state. By combining several such modes, a substantial discovery reach is very likely obtainable. From Fig. 6, we see that regions of parameter space corresponding to the sum of the NHGB and PM masses up to (very) roughly $\simeq 4.0(3.5)$ and $\simeq 27(25)$ TeV for the $gu(d)$-initiated process may be obtainable at these two colliders, respectively.

The second mechanism for making these NHGBs is pair production via s-channel exchanges, i.e., $q\bar{q} \to \gamma, Z, Z'_M \to \mathrm{NHGB} + \mathrm{NHGB}^\dagger$, where the SM γ exchange may or may not contribute depending upon the particular model details and the specific NHGB under consideration. Recall that both of the NHGBs carry weak isospin having $T_{3L} = \pm 1/2$, and at least one of them has a nonzero electric charge. Since, unlike in the case of associated production, pair production is a purely electroweak process, the cross section in this case is generally small. Importantly, however, the rate for this reaction can also be potentially Z'_M resonance enhanced as was discussed earlier provided that $M_{Z'_M} > 2M_{\mathrm{NHGB}}$. As we saw above, when $|\sigma| = 1$, one finds that κ_L is restricted to a narrow range to allow for this possibility, whereas for, $|\sigma| = 3$, there is no such restriction. It thus behooves us to examine the ratio of branching fractions $\mathcal{R}_N = B(Z'_M \to \mathrm{NHGB} + \mathrm{NHGB}^\dagger)/B_l$ above this threshold, which is similar to that for the W^+W^- final state above except that there is no longer any small mixing angle suppression and that the Z'_M and NHGB masses are now more than likely be comparable. Quite generally, we may write (again for simplicity assuming that we may approximately take $|b/a| \to 0$ as above)

$$\Gamma(Z'_M \to \mathrm{NHGB} + \mathrm{NHGB}^\dagger) = \frac{g_L^2 M_{Z'_M}}{192\pi} P^2 \frac{1}{X^2} (1-4X)^{3/2} (1+20X+12X^2), \quad (66)$$

where, from the above discussion and Eq. (42) one obtains in the present setup that

$$X = \frac{M^2_{\mathrm{NHGB}}}{M^2_{Z'_M}} = \frac{1-\kappa_L^2 r}{r} \le \frac{1}{4}, \quad (67)$$

and we find, employing Eqs. (14) and (43) as well as the subsequent discussion, that the parameter $P^2 = 9/(4a^2) = X$. Thus, we finally arrive at the required expression

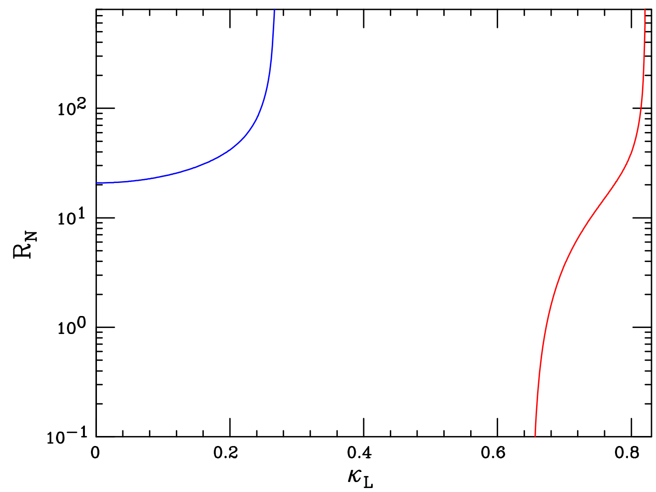

FIG. 7. The ratio $\mathcal{R}_N$ described in the text as a function of the parameter κ_L, for both $|\sigma| = 1$ (red) and $|\sigma| = 3$ (blue). Recall that in both cases the allowed range of κ is restricted from above.

$$\mathcal{R}_N = \frac{1}{X} (1-4X)^{3/2} (1+20X+12X^2), \quad (68)$$

which we see can very easily be larger than unity when X becomes small. For example, taking $|\sigma| = 1$ and $\kappa_L = 0.7$, one finds that $\mathcal{R}_N \simeq 3.7$ for *each* of the NHGB which is quite substantial; when $|\sigma| = 3$, even larger values of this ratio are obtained. Figure 7 shows the values of $\mathcal{R}_N$ as a function of κ_L when $|\sigma| = 1$ or 3; the reader is reminded of the restricted ranges of κ_L in these two cases as discussed above when examining the results as presented in this figure.

On the positive side, this enhancement is quite advantageous as it allows us to study the production of the NHGB up to quite large masses assuming that they can be pair produced on shell. Simultaneously, however, the relatively large partial width for this process begins to significantly reduce the value of B_l which will negatively impact the discovery reach for the Z'_M in the dilepton channel when very large values of $\mathcal{R}_N$ are realized. However, even when rather sizable values of this ratio are indeed obtained, e.g., 25–50, the actual search reach is found to be not very much degraded, roughly by less than $\sim$5%–10%, in practice.

Turning now to the heavy PM fields, in the case of the U- or D-type color triplets, these states can be pair produced though QCD in analogy to the top quark in the SM via gg and $\bar{q}q$. As mentioned previously, these particles will dominantly decay to the analogous SM states with which they share a $\mathbf{3}$ or $\bar{\mathbf{3}}$ multiplet together with a DP so that the simplest final state (for non-b or t quarks) is just $2j+$ MET, similar to that of squark production. As discussed in earlier work [48], these types of searches at the 13 TeV LHC already exclude U/D masses below $\simeq$1.5–1.8 TeV, depending upon the flavor of the SM quarks in the final state, but at the FCC-hh the corresponding search reaches increase substantially to roughly $\simeq$10 TeV. Analogously, the heavy PM fields which are color singlets, N and E, can only be produced by electroweak exchanges, i.e., $\bar{q}q \to \gamma, Z, Z'_M \to \bar{N}N, E^+E^-$ and so their cross sections are relatively small. In the case of E, LHC searches employing the assumed dominant eV (or eh_D) decay mode and producing an opposite sign lepton plus MET signature already exclude PM masses up to roughly $\simeq 1$ TeV [48]. However, if these PM leptons can appear in the decays of Z'_M, as was the case with the NHGB, we will have a resonance enhanced search reach. In the case of $|\sigma| = 1$ where the anomalies cancel among the three generations, these PM leptons, N, E, will lie in either l_1- or l_2^*-like triplet/antitriplet representations in a manner similar to that of U, D just discussed. In such a case, where the right-handed components of these PM leptons are singlets of 3_L, the relevant quantity of interest is then just the ratio $\mathcal{R}_L = B(Z'_M \to \bar{N}N \text{ or } E^+E^-)/B_l$ which, where if we again make the now familiar approximation $|b/a| \to 0$ and recall that $|Q_D(N,E)| = 1$, is given by an identical expression for both N, E types of PM leptons,

$$\mathcal{R}_L = 2\beta\left[\beta^2 + \frac{3-\beta^2}{2}(1+4\kappa_L^2)^2\right], \tag{69}$$

with $\beta^2 = 1\text{–}4m^2/M^2_{Z'_M}$ and where m is now either $m_{N,E}$. As the top panel of Fig. 8 shows, $\mathcal{R}_L$ can be quite large, even near the mass threshold, when κ_L becomes large. A similar expression would hold if Z'_M were allowed to decay into pairs of the color triplet PM fields, apart from an overall factor of $\simeq 3(1+\alpha_s(M_{Z'_M})/\pi)$, which under some circumstances may match or exceed the rate that is expected from the pure QCD production process already mentioned, especially at the 100 TeV FCC-hh. This could push the discovery reach for these states up to roughly the kinematic limit $\simeq 0.5 m_{Z'_M}$, similar to that for the color singlet states.

If the vectorlike (with respect to 3_L) leptons of the type appearing in $S_{9,10}$ (i.e., N_2, E in S_9 or N_1, E_2 in S_{10}, respectively) are also/instead of encountered, then the corresponding cross sections are somewhat reduced in comparison to the previous result but can still be quite significant and we would now obtain

$$\mathcal{R}_L = 2\beta\left[\frac{3-\beta^2}{2}\right](1+\kappa_L^2)^2, \tag{70}$$

as is shown in the bottom panel of this same figure. These models with the anomalies canceling within each generation will potentially also lead to more complex signatures in the leptonic sector due to cascading decay effects mentioned above, especially if a significant fraction of the color singlet states are kinematically accessible in Z'_M decays.

Clearly, the production of the color singlet and triplet PM states as well as the new heavy gauge bosons associated with the enlarged gauge symmetries can lead to some interesting collider phenomenology.

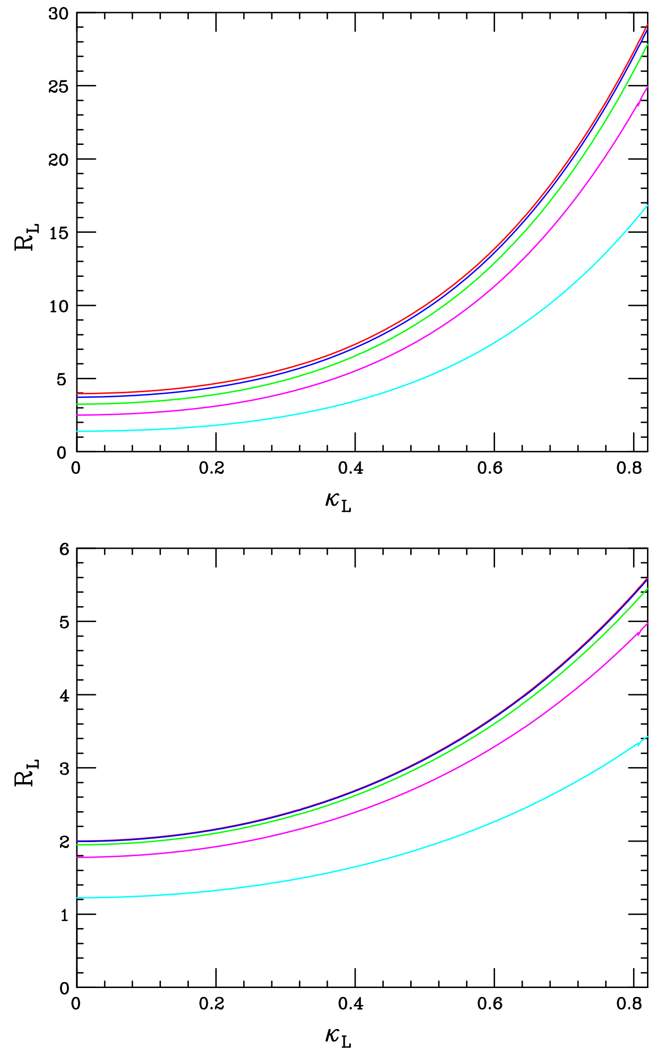

FIG. 8. The ratio $\mathcal{R}_L$ of the leptonic PM Z'_M branching fraction to that for ordinary SM charged leptons, described in the text, as a function of the parameter κ_L assuming, from top to bottom, $m/M_{Z'_M} = 0.05$ (red), 0.15 (blue), 0.25 (green), 0.35 (magenta), and 0.45 (cyan), respectively, where m is the N or E PM mass in the case where such fields are (top) right-handed singlets or are (bottom) vectorlike with respect to 3_L as discussed in the text.

VII. DISCUSSION AND CONCLUSION

The kinetic mixing between the dark photon, arising from a $U(1)_D$ gauge symmetry, and the SM photon to mediate the interactions between a light DM and ordinary particles provides a simple yet elegant portal to explain the observed relic density with potentially numerous phenomenological implications. However, KM necessitates the existence of a set of fields that carry both SM and dark sector quantum numbers, i.e., portal matter, which must consist of scalars and/or fermions which are vectorlike with respect to their SM interactions due to numerous unitarity, Higgs, and precision measurement constraints. The combination of the constraints arising from the CMB and the requirement that the coupling of the $U(1)_D$ gauge group associated with the DP remain perturbative up to some large scale, $\sim$ tens of TeV, possibly hints at the existence of some enlarged non-Abelian group, G_D. When this gets broken down to (at least) $U(1)_D$, it provides the masses for the PM fields as well as to a set of new heavy gauge bosons, some of which will carry dark quantum numbers. It is then natural to ask how these ideas can fit together with the SM in a more unified framework. Is there a way to understand how the PM fields carrying both SM and dark quantum numbers arise in such a setup, what their properties might be, and how they and the new heavy gauge bosons might be directly discovered at colliders?

In our previously considered bottom-up approaches that attempted to address these and related questions, we examined an enlarged gauge group G, essentially consisting of a product of the SM and dark gauge groups, wherein, e.g., the $SU(2)_L$ part of the SM and $U(1)_D$ were totally uncorrelated and independent. One might easily imagine that this would no longer be true in a more fully unified picture so it behooves us to examine alternative possibilities. In the present work, following a parallel bottom-up approach, we have explored a (relatively) simple toy setup wherein the $SU(2)_L$ part of the SM gauge group as well as part of the $U(1)_D$ gauge group associated with the DP are unified within a single $SU(3)_L$ factor. This $SU(3)_L$ is also accompanied by two $U(1)$ factors, $1_A 1_B$, thus providing for partial unification of the dark and SM electroweak gauge interactions. Explicitly, in such a setup the PM fields naturally (mostly) lie in common representations of 3_L along with the corresponding SM fields having similar quantum numbers. In this setup, the Higgs fields responsible for breaking $3_L 1_A 1_B$ down to the $\mathrm{SM} \times U(1)_D$ are directly responsible for generating the PM masses, whereas the Higgs VEVs that break 1_D also lead to a mixing of the PM fermions with analog SM fields, thus providing them with a likely dominant decay path and a unique collider signature. This is perhaps the simplest scenario that is not a product group of the form $G_D \times G_{\mathrm{SM}}$ which allows for a (at least partial) unification of the SM electroweak with the dark sector. If we consider the breaking $3_L \to 2_L 1_L$, with 2_L identified as the usual SM group in this setup, then the SM hypercharge 1_Y is embedded as a linear combination of the two factors $1_L 1_A$, while 1_D is instead embedded as an orthogonal combination of all three factors, $1_L 1_A 1_B$.

Although several variants of this general setup are possible, depending upon the assumed value of the parameter σ and the manner in which the anomalies are canceled, they all have most of their features in common, which we take advantage of in the current study. First, given the extended group structure, a new heavy Hermitian and two new non-Hermitian gauge bosons must exist in the spectrum with masses comparable to the PM fields. In particular, the required hierarchal breaking of the various gauge symmetries as well as the generation of all of the SM and PM (Dirac) masses and the mixings necessary for all the PM fermions to decay (unless exotically charged fields are present that are the lightest ones among the set of PM) can be accomplished via the VEVs of just three Higgs (anti) triplets of 3_L having differing $1_A 1_B$ quantum numbers. Second, these PM fermion fields necessarily consist of both color singlet as well as color triplet states, at least some of which will share 3_L (anti)triplet representations with the SM fermions and so are connected to them via the two new heavy NHGBs. PM decays via these NHGBs can compete with the familiar and usually dominant SM-PM mixing-induced mode provided they are kinematically allowed. Third, at the high scale, the $U(1)_D$ gauge coupling g_D is found to be bounded from above in a manner dependent upon the value of $|\sigma|$ but is always constrained to be $\lesssim 0.83 g_L$. This bound can have important implications for low energy dark sector searches if g_D runs to even smaller values as the $\sim$1 GeV scale is approached from above, as we might anticipate. Finally, if kinematically allowed, PM and/or NHGBs appearing as final states in resonant Z'_M decay can have enhanced production rates and correspondingly extended mass reaches beyond the typical naive estimates.

PM models with extended gauge sectors linking the dark and SM sectors can lead to a wide spectrum of interesting and complex phenomenology at existing and future colliders. Hopefully, signals of the physics of the dark sector will soon be observed.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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