Properties of the tensor state $bc\bar{b}\bar{c}$

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(Received 9 January 2025; accepted 31 March 2025; published 18 April 2025)

Spectroscopic parameters and decays of the exotic tensor meson T with content $bc\bar{b}\bar{c}$ are explored in the context of the diquark-antidiquark model. We treat it as a state built of axial-vector diquark $b^T C\gamma_\mu c$ and antidiquark $\bar{b}\gamma_\nu C\bar{c}^T$, where C is the charge conjugation matrix. The mass m and current coupling Λ of this tetraquark are extracted from two-point sum rules. Our result for $m = (12.70 \pm 0.09)$ GeV proves that T is unstable against strong dissociations to two-meson final states. Its dominant decay channels are processes $T \to J/\psi\Upsilon$, $\eta_b\eta_c$, and $B_c^{(*)+}B_c^{(*)-}$. Kinematically allowed transformations of T include also decays $T \to D^{(*)+}D^{(*)-}$ and $D^{(*)0}\bar{D}^{(*)0}$, which are generated by $b\bar{b}$ annihilation inside of T. The full width of T is estimated by considering all of these channels. Their partial widths are calculated by invoking methods of three-point sum rule approach, which are required to evaluate strong couplings at corresponding tetraquark-meson-meson vertices. Our predictions for the mass and width $\Gamma_T = (117.4 \pm 15.9)$ MeV of the tensor state T provide useful information for experimental studies of fully heavy four-quark exotic structures.

DOI: 10.1103/PhysRevD.111.074025

I. INTRODUCTION

During the last years, investigation of fully heavy four-quark mesons has become one of interesting and rapidly growing branches of high energy physics. The main reason for such interest, besides pure theoretical arguments, is observation of four X structures with masses in a range 6.2–7.3 GeV by LHCb-ATLAS-CMS collaborations [1–3]. According to overwhelming opinion they are scalar resonances composed of $cc\bar{c}\bar{c}$ quarks, thought there exist kinematical explanations of their origin as well.

These discoveries generated numerous and interesting publications devoted to study of newly observed structures [4–17]. The X resonances were explored also in the QCD sum rule framework in our articles [18–21], in which we modeled them as diquark-antidiquark and hadronic molecule states. This analysis allowed us to propose our assignments for these resonances. Thus, some of them were interpreted as a pure ground-level diquark-antidiquark [18] and hadronic molecule [19] states, or as admixtures of these two structures [20,21].

Exotic mesons containing only heavy quarks were objects of theoretical investigations starting from first days of the quark model and quantum chromodynamics, which do not forbid existence of hadrons containing four and five quarks, pure gluon or quark-gluon systems. Experimental achievements renewed and intensified interest to these exotic particles. A class of hidden charm-bottom tetraquarks $bc\bar{b}\bar{c}$ are evidently among such hadrons. The structures $bc\bar{b}\bar{c}$ were not discovered yet, but have real chances to be seen in ongoing and future experiments [22,23].

Features of tetraquarks $bc\bar{b}\bar{c}$ with different spin-parities were considered in the literature [10,24–31]. The masses of tetraquarks $bc\bar{b}\,\bar{c}$ are the main parameters calculated in these articles using numerous methods. Information about partial widths of their decay modes is either scarce or absent. In other words, our knowledge about properties of exotic mesons $bc\bar{b}\bar{c}$ is rather limited. These circumstances, as well as discrepancies in predictions for the masses made in the different publications, necessitate detailed studies of the tetraquarks $bc\bar{b}\bar{c}$.

In Refs. [32,33], we investigated the scalar and axial-vector particles $bc\bar{b}\bar{c}$ and determined their masses and widths. In the present paper, we extend our analysis by considering the tensor tetraquark $bc\bar{b}\bar{c}$ with spin-parity $J^{\mathrm{PC}} = 2^+$. For simplicity, we label it T and calculate the mass and full width of this exotic meson. To find the mass m and current coupling Λ, we use the two-point sum rule

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(SR) method [34,35]. Partial widths of numerous decay channels of T are computed by invoking the three-point sum rule approach. This is necessary to estimate strong couplings at relevant tetraquark-meson-meson vertices, which determine widths of the processes under analysis.

There are a few types of decay modes of the tetraquark T. Decays to pairs of quarkonia $J/\psi\Upsilon$ and $\eta_b\eta_c$, as well as processes $T \to B_c^{*+}B_c^{*-}$ and $B_c^+B_c^-$ are dissociations of the initial particle to final-state mesons. These decays are dominant channels of T in which four constituent quarks form the final-state conventional mesons. The second kind of processes is triggered by annihilation in T of $b\bar{b}$ quarks to a pair of light quarks and subsequent generation of DD mesons with suitable electric charges and spin-parities. In the case of the tensor tetraquark, we limit ourselves by investigation of four decays $T \to D^{(*)+}D^{(*)-}$ and $D^{(*)0}\bar{D}^{(*)0}$.

This work is composed of the following parts: In Sec. II, we calculate the mass and current coupling of the tensor state T. Partial widths of decays $T \to J/\psi\Upsilon$ and $\eta_b\eta_c$ are computed in Sec. III. The processes with $B_c^{(*)+}B_c^{(*)-}$ mesons in final states are considered in the next Sec. IV. Partial widths of the decays $T \to D^{(*)+}D^{(*)-}$ and $D^{(*)0}\bar{D}^{(*)0}$ are evaluated in Sec. V. In this section, we also find the full width of the tensor tetraquark T. We make our conclusions in the last part of the paper Sec. VI.

II. MASS m AND CURRENT COUPLING Λ OF THE TETRAQUARK T

Spectroscopic parameters of the tetraquark T are quantities that characterize this particle and determine its possible decay modes. The mass m and current coupling Λ of a particle can be evaluated using different approaches. One of the effective nonperturbative tools to find these parameters is the two-point sum rule method [34,35]. Originally invented to study parameters of the ordinary baryons and mesons, it can be successfully applied for analysis of exotic hadrons as well [36,37].

In the framework of this method one has to extract SRs for m and Λ, which can be done by considering the correlation function

$$\Pi_{\mu\nu\alpha\beta}(p) = i\int d^4x e^{ipx}\langle 0|\mathcal{T}\{J_{\mu\nu}(x)J_{\alpha\beta}^{\dagger}(0)\}|0\rangle, \quad (1)$$

where $J_{\mu\nu}(x)$ is the interpolating current for the tensor tetraquark and $\mathcal{T}$ is the time-ordered product of two currents.

Analytical expression of $J_{\mu\nu}(x)$ depends on a diquark-antidiquark model chosen for the particle. In the present article, we consider T as a diquark-antidiquark state composed of an axial-vector diquark $b^TC\gamma_\mu c$ and antidiquark $\bar{b}\gamma_\nu C\bar{c}^T$. Accordingly, the interpolating current $J_{\mu\nu}(x)$ has the following form:

$$J_{\mu\nu}(x) = b_a^T(x)C\gamma_\mu c_b(x)[\bar{b}_a(x)\gamma_\nu C\bar{c}_b^T(x) - \bar{b}_b(x)\gamma_\nu C\bar{c}_a^T(x)]. \quad (2)$$

Here, C is the charge conjugation matrix, whereas a and b are the color indices. The current $J_{\mu\nu}$ describes the tetraquark with spin-parities $J^P = 2^+$.

To find the sum rules for the mass m and current coupling Λ, we first have to compute the correlation function $\Pi_{\mu\nu\alpha\beta}(p)$ using physical parameters of the tetraquark. For these purposes, we insert into Eq. (1) a full set of states with the quark content and spin-parities of the tetraquark T, and integrate it over the variable x. Then the correlator becomes equal to

$$\Pi_{\mu\nu\alpha\beta}^{\rm Phys}(p) = \frac{\langle 0|J_{\mu\nu}|T(p,\epsilon)\rangle\langle T(p,\epsilon)|J_{\alpha\beta}^{\dagger}|0\rangle}{m^2-p^2} + \cdots, \quad (3)$$

where the term in Eq. (3) is the contribution of the ground-state particle T, whereas the dots show contributions of higher resonances and continuum states. Here, $\epsilon = \epsilon_{\mu\nu}^{(\lambda)}(p)$ is the polarization tensor of the tetraquark T. For further calculations, it is convenient to introduce the matrix element

$$\langle 0|J_{\mu\nu}|T(p,\epsilon(p))\rangle = \Lambda\epsilon_{\mu\nu}^{(\lambda)}(p). \quad (4)$$

To find $\Pi_{\mu\nu\alpha\beta}^{\rm Phys}(p)$ we substitute Eq. (4) into the correlator Eq. (3) and perform summation over polarization tensor using

$$\sum_{\lambda}\epsilon_{\mu\nu}^{(\lambda)}(p)\epsilon_{\alpha\beta}^{*(\lambda)}(p) = \frac{1}{2}(\tilde{g}_{\mu\alpha}\tilde{g}_{\nu\beta} + \tilde{g}_{\mu\beta}\tilde{g}_{\nu\alpha}) - \frac{1}{3}\tilde{g}_{\mu\nu}\tilde{g}_{\alpha\beta}, \quad (5)$$

where

$$\tilde{g}_{\mu\nu} = -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}. \quad (6)$$

Our computations yield

$$\Pi_{\mu\nu\alpha\beta}^{\rm Phys}(p) = \frac{\Lambda^2}{m^2-p^2}\left\{\frac{1}{2}(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}) + \text{other structures}\right\} + \ldots, \quad (7)$$

with ellipses standing for contributions of other structures as well as higher resonances and continuum states. Note that, after application of Eqs. (5) and (6) there appear numerous Lorentz structures in the curly brackets. The term proportional to $(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha})$ contains contribution of only spin-2 particle, whereas remaining components in Eq. (7) are formed due to contributions of spin-0 and -1 states as well. Therefore, in our studies we restrict ourselves by exploring this term and corresponding invariant amplitude $\Pi^{\rm Phys}(p^2)$.

At next phase of investigations, we compute the correlator $\Pi_{\mu\nu\alpha\beta}(p)$ with some accuracy in the operator product expansion (OPE). To this end, we have to insert the explicit expression of the current $J_{\mu\nu}(x)$ into Eq. (1) and contract relevant quark fields to obtain $\Pi_{\mu\nu\alpha\beta}^{\rm OPE}(p)$. As a result, we find

$$\Pi_{\mu\nu\alpha\beta}^{\rm OPE}(p) = i\int d^4x e^{ipx}{\rm Tr}[\gamma_\alpha \tilde{S}_b^{aa'}(x)\gamma_\mu S_c^{bb'}(x)] \times \{{\rm Tr}[\gamma_\nu \tilde{S}_c^{b'b}(-x)\gamma_\beta S_b^{a'a}(-x)] - {\rm Tr}[\gamma_\nu \tilde{S}_c^{a'b}(-x), \times \gamma_\beta S_b^{b'a}(-x)] + {\rm Tr}[\gamma_\nu \tilde{S}_c^{a'a}(-x)\gamma_\beta S_b^{b'b}(-x)] - {\rm Tr}[\gamma_\nu \tilde{S}_c^{b'a}(-x)\gamma_\beta S_b^{a'b}(-x)]\}, \quad (8)$$

where

$$\tilde{S}_{b(c)}(x) = CS_{b(c)}^T(x)C, \quad (9)$$

and $S_{b(c)}(x)$ are b and c quarks' propagators.

The massive quark propagator $S_Q(x)$ was calculated using an external field method at the fixed-point gauge (for details, see Ref. [38]). More recent expression for $S_Q(x)$ can be found in Ref. [37], which contains terms $\sim g_s^3 G^3$. It depends only on gluon fields, as a result, the correlator $\Pi_{\mu\nu\alpha\beta}^{\rm OPE}(p)$ contains merely gluon vacuum condensates. In our calculations we take into account nonperturbative contributions $\sim\langle \alpha_s G^2/\pi\rangle$, therefore adopt the following expression for the propagator $S_Q(x)$

$$S_Q^{ab}(x) = i\int \frac{d^4k}{(2\pi)^4} e^{-ikx}\left\{\frac{\delta_{ab}(\not{k}+m_Q)}{k^2-m_Q^2} - \frac{g_s G_{ab}^{\alpha\beta}}{4}\frac{\sigma_{\alpha\beta}(\not{k}+m_Q)+(\not{k}+m_Q)\sigma_{\alpha\beta}}{(k^2-m_Q^2)^2} + \frac{g_s^2 G^2}{12}\delta_{ab} m_Q \frac{k^2+m_Q\not{k}}{(k^2-m_Q^2)^4} + \cdots\right\}. \quad (10)$$

Here, we have introduced the notations

$$G_{ab}^{\alpha\beta} \equiv G_A^{\alpha\beta}\lambda_{ab}^A/2, \qquad G^2 = G_{\alpha\beta}^A G_A^{\alpha\beta}, \qquad A = 1-8, \quad (11)$$

with $G_A^{\alpha\beta}$ being the gluon field-strength tensor, and λ^A–Gell-Mann matrices.

Having extracted the structure $(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha})$ from $\Pi_{\mu\nu\alpha\beta}^{\rm OPE}(p)$ and labeled corresponding invariant amplitude by $\Pi^{\rm OPE}(p^2)$, one can derive SRs for the mass and current coupling of the tetraquark T. In fact, the function $\Pi^{\rm Phys}(p^2)$ can be expressed as the dispersion integral

$$\Pi^{\rm Phys}(p^2) = \int_{4\mathcal{M}^2}^{\infty}\frac{\rho^{\rm Phys}(s)ds}{s-p^2} + \cdots, \quad (12)$$

where $\mathcal{M}^2 = (m_b+m_c)^2$ and the dots indicate subtraction terms required to render finite $\Pi^{\rm Phys}(p^2)$. The spectral density $\rho^{\rm Phys}(s)$ is equal to the imaginary part of $\Pi^{\rm Phys}(p^2)$,

$$\rho^{\rm Phys}(s) = \Lambda^2\delta(s-m^2) + \rho^{\rm h}(s)\theta(s-s_0) \quad (13)$$

Here, $\theta(z)$ is the unit step function, and s_0 is the continuum subtraction parameter. The contribution of the ground-level particle in Eq. (13) is separated from other effects and represented by the pole term. Contributions to $\rho^{\rm Phys}(s)$ coming from higher resonances and continuum states are characterized by an unknown hadronic spectral density $\rho^{\rm h}(s)$. It is clear that $\rho^{\rm Phys}(s)$ leads to the expression

$$\Pi^{\rm Phys}(p^2) = \frac{\Lambda^2}{m^2-p^2} + \int_{s_0}^{\infty}\frac{\rho^{\rm h}(s)ds}{s-p^2}. \quad (14)$$

Theoretically, the amplitude $\Pi^{\rm OPE}(p^2)$ can be calculated in deep Euclidean region $p^2 \ll 0$ using the operator product expansion. The coefficient functions in OPE could be obtained using methods of perturbative QCD, whereas nonperturbative information is contained in the gluon condensate $\langle\alpha_s G^2/\pi\rangle$.

Having continued $\Pi^{\rm OPE}(p^2)$ analytically to the Minkowski domain and found its imaginary part, we get the two-point spectral density $\rho^{\rm OPE}(s)$. In the region $p^2 \ll 0$ we apply the Borel transformation $\mathcal{B}$ to remove subtraction terms in the dispersion integral and suppress contributions of higher resonances and continuum states. For $\mathcal{B}\Pi^{\rm Phys}(p^2)$, we obtain

$$\mathcal{B}\Pi^{\rm Phys}(p^2) = \Lambda^2 e^{-m^2/M^2} + \int_{s_0}^{\infty} ds\rho^{\rm h}(s)e^{-s/M^2}, \quad (15)$$

where M^2 is the Borel parameter. One can write the dispersion representation for the amplitude $\Pi^{\rm OPE}(p^2)$ using $\rho^{\rm OPE}(s)$ as well. Then, by equating the Borel transformations of $\Pi^{\rm Phys}(p^2)$ and $\Pi^{\rm OPE}(p^2)$ and applying the assumption about hadron-parton duality $\rho^{\rm h}(s) \simeq \rho^{\rm OPE}(s)$ in a duality region, we subtract the second term in Eq. (15) from the QCD side of the obtained equality and get

$$\Lambda^2 e^{-m^2/M^2} = \Pi(M^2, s_0). \quad (16)$$

Here,

$$\Pi(M^2, s_0) = \int_{4\mathcal{M}^2}^{s_0} ds\rho^{\rm OPE}(s)e^{-s/M^2} + \Pi(M^2). \quad (17)$$

The nonperturbative function $\Pi(M^2)$ is computed directly from the correlator $\Pi^{\rm OPE}(p)$ and contains contributions that do not enter to the spectral density.

After simple manipulations, we get

$$m^2 = \frac{\Pi'(M^2, s_0)}{\Pi(M^2, s_0)}, \tag{18}$$

and

$$\Lambda^2 = e^{m^2/M^2}\Pi(M^2, s_0), \tag{19}$$

which are the sum rules for m and Λ, respectively. In Eq. (18), we also use the short-hand notation $\Pi'(M^2, s_0) = d\Pi(M^2, s_0)/d(-1/M^2)$. The spectral density $\rho^{\rm OPE}(s)$ contains the perturbative $\rho^{\rm pert}(s)$ and nonperturbative $\rho^{\rm Dim4}(s)$ terms. Explicit expressions for $\rho^{\rm pert}(s)$ and $\Pi(M^2)$ are presented in the Appendix.

We need to specify the input parameters in Eqs. (18) and (19) to perform numerical computations. Some of them are universal quantities and do not depend on a problem under consideration. The masses of b and c quarks and gluon vacuum condensate $\langle \alpha_s G^2/\pi\rangle$ are such parameters. In the present work, we use the following values:

$$m_b = 4.18^{+0.03}_{-0.02}\ \text{GeV}, \qquad m_c = (1.27 \pm 0.02)\ \text{GeV},$$
$$\langle \alpha_s G^2/\pi\rangle = (0.012 \pm 0.004)\ \text{GeV}^4. \tag{20}$$

The m_b and m_c are the running quark masses in the $\overline{\rm MS}$ scheme [39]. The gluon vacuum condensate was extracted from analysis of various hadronic processes in Refs. [34,35].

Contrary, the Borel and continuum subtraction parameters M^2 and s_0 are specific for each problem and should satisfy some standard constraints of SR computations. Dominance of the pole contribution (PC) in extracted quantities and their stability upon variations of M^2 and s_0 as well as convergence of the operator product expansion are important conditions for correct SR analysis. To fulfill these requirements, we impose on the parameters M^2 and s_0 the following restrictions. First, the pole contribution

$$\text{PC} = \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)}, \tag{21}$$

should obey $\text{PC} \geq 0.5$. The convergence of OPE is second important condition in the SR analysis. Because the correlation function contains only the nonperturbative dimension-4 term $\Pi^{\rm Dim4}(M^2, s_0)$, we require fulfilment of the constraint $|\Pi^{\rm Dim4}(M^2, s_0)| = 0.05\Pi(M^2, s_0)$, which ensures the convergence of the operator product expansion. It is worth noting that the maximum of the Borel parameter is determined from Eq. (21), whereas convergence of OPE allows us to fix its minimal value.

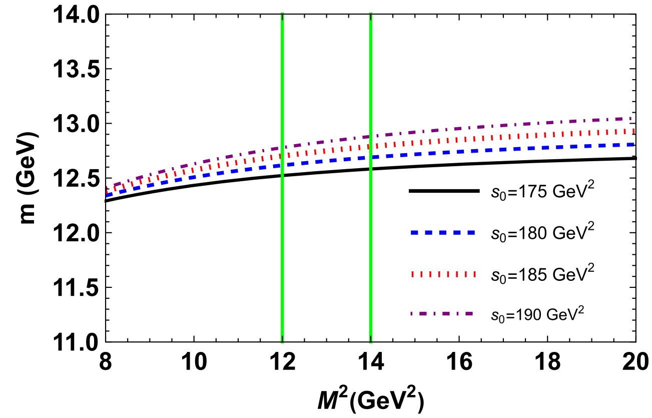

FIG. 1. Mass m of the tetraquark T as a function of M^2 for fixed s_0. The two vertical lines fix values of the Borel parameter, where m is extracted.

Numerical calculations are performed over a wide range of the parameters M^2 and s_0. In Fig. 1, we plot the mass m in the range $M^2 = 8$–$20\ \text{GeV}^2$ at some fixed s_0. Analysis of these results allows us to fix the working windows for M^2 and s_0, where all aforementioned restrictions are obeyed. We find that the regions

$$M^2 \in [12, 14]\ \text{GeV}^2, \qquad s_0 \in [180, 185]\ \text{GeV}^2 \tag{22}$$

comply with these constraints. Indeed, on the average in s_0 at maximal and minimal M^2 the pole contribution is $\text{PC} \approx 0.56$ and $\text{PC} \approx 0.75$, respectively. The nonperturbative term is positive and at $M^2 = 12\ \text{GeV}^2$ forms less than 1% of the whole result. The dependence of PC on the Borel parameter is plotted in Fig. 2, in which all curves exceed the limit line $\text{PC} = 0.5$.

To extract m and Λ, we compute their mean values over the regions Eq. (22) and find

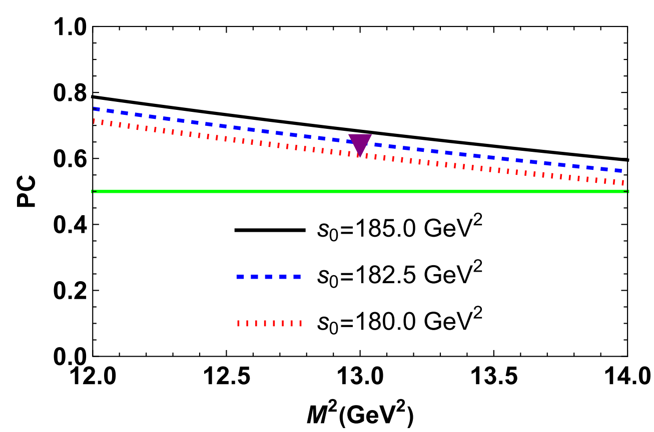

FIG. 2. Dependence of PC on the Borel parameter M^2 at fixed s_0. The horizontal green line corresponds to $\text{PC} = 0.5$. The red triangle shows the point $M^2 = 13\ \text{GeV}^2$ and $s_0 = 182.5\ \text{GeV}^2$.

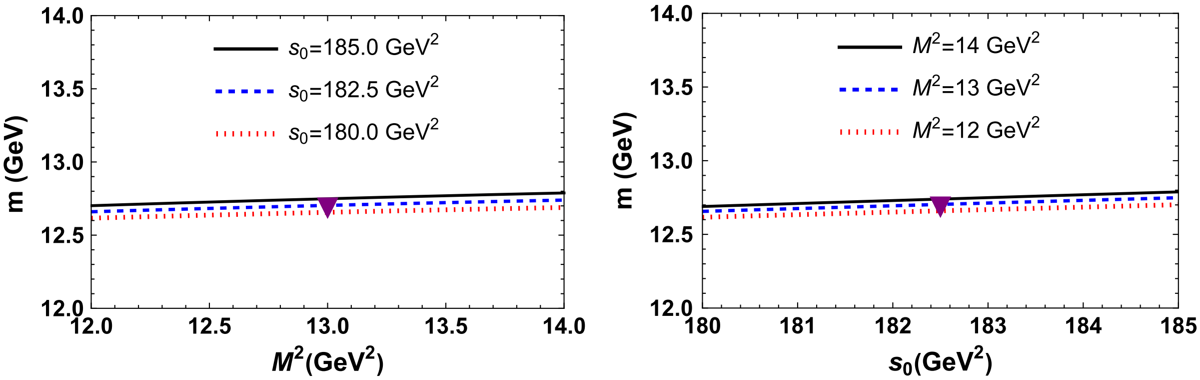

FIG. 3. Mass m as a function of the Borel M^2 (left panel), and continuum threshold s_0 parameters (right panel).

$$m = (12.70 \pm 0.09)\ \mathrm{GeV},$$
$$\Lambda = (2.16 \pm 0.24)\ \mathrm{GeV}^5. \tag{23}$$

Effectively, results in Eq. (23) are equal to SR predictions at the point $M^2 = 13\ \mathrm{GeV}^2$ and $s_0 = 182.5\ \mathrm{GeV}^2$, where $\mathrm{PC} \approx 0.65$, which guarantees the dominance of PC in the extracted parameters. Uncertainties in Eq. (23) are generated mainly by the choices of M^2 and s_0. These theoretical errors form only $\pm 0.7\%$ of the mass m, which demonstrates the high stability of the obtained prediction. Such accuracy of the result is connected with the SR for m, Eq. (18), which determines it as a ratio of the correlation functions. Therefore, changes in the correlators due to M^2, and s_0 compensate each other in m and stabilize in this way the numerical output. In the case of Λ errors amount to $\pm 11\%$ of the central value, but still remain within limits acceptable for the sum rule analysis. In Fig. 3, we show m as a function of M^2 and s_0.

The mass of the tensor tetraquark T was evaluated in the framework of different models and methods [10,24–30]. In the relativistic quark model the authors obtained 12.849 GeV [10]. A considerably larger result, i.e., 13.59–13.599 GeV was found in the color-magnetic interaction model [24]. The mass spectra of all-heavy tetraquarks with different contents were investigated in Ref. [25], in which for the tensor state $bc\bar{b}\,\bar{c}$ the authors got 12.993–13.021 GeV. The nonrelativistic chiral quark model led to 12.809 GeV [26]. In the relativized diquark Hamiltonian model the mass of the tensor tetraquark 2^{++} depending on diquarks' spins and total spin and orbital angular momentum of the tetraquark changes from 12.576 GeV to 13.65 GeV [27]. Prediction 12.582 GeV for m was made in Ref. [28]. In the extended chromomagnetic model [29] this tensor has the mass in the range 12.537 and 12.754 GeV. This problem was addressed also in the SR framework [30]. Values for the mass of the tensor tetraquarks $bc\bar{b}\,\bar{c}$ modeled by a color antitriplet-triplet and sextet-antisextet interpolating currents are equal to 12.30 and 12.35 GeV, respectively.

III. DECAYS $T \to J/\psi\Upsilon$ AND $T \to \eta_b\eta_c$

Information on the mass of the tensor state T permits us to make conclusions about its decay channels. Decays to quarkonium pairs $J/\psi\Upsilon$ and $\eta_b\eta_c$ are among kinematically possible decay modes of T. Indeed, thresholds for creation of these final states are 12.558 GeV and 12.383 GeV, respectively. In this section we study these decay channels of T.

A. Process $T \to J/\psi\Upsilon$

Here, we consider the decay $T \to J/\psi\Upsilon$, the partial width of which, apart from usual input parameters, is determined by the strong coupling g_1 at the vertex $TJ/\psi\Upsilon$. The coupling g_1 can be evaluated using the form factor $g_1(q^2)$ at the mass shell $q^2 = m_{J/\psi}^2$.

We are going to derive the three-point sum rule for the form factor $g_1(q^2)$ from analysis of the correlation function

$$\Pi_{\mu\nu\alpha\beta}(p,p') = i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \langle 0|\mathcal{T}\{J_\mu^{\Upsilon}(y) \times J_\nu^{J/\psi}(0) J_{\alpha\beta}^{\dagger}(x)\}|0\rangle, \tag{24}$$

where $J_\mu^{\Upsilon}(x)$ and $J_\nu^{J/\psi}(x)$ are interpolating currents of the vector quarkonia Υ and J/ψ, respectively. They are defined as

$$J_\mu^{\Upsilon}(x) = \bar{b}_i(x)\gamma_\mu b_i(x), \qquad J_\nu^{J/\psi}(x) = \bar{c}_j(x)\gamma_\nu c_j(x), \tag{25}$$

with i and j being the color indices.

To find the physical side of the sum rule $\Pi_{\mu\nu\alpha\beta}^{\mathrm{Phys}}(p,p')$, we need to rewrite Eq. (24) using the involved particles' physical parameters. By taking into account only contributions of the ground-level states, we recast the correlator $\Pi_{\mu\nu\alpha\beta}(p,p')$ into the form

$$\Pi_{\mu\nu\alpha\beta}^{\rm Phys}(p,p') = \frac{\langle 0|J_\mu^\Upsilon|\Upsilon(p',\varepsilon_1)\rangle}{p'^2-m_\Upsilon^2}\frac{\langle 0|J_\nu^{J/\psi}|J/\psi(q,\varepsilon_2)\rangle}{q^2-m_{J/\psi}^2} \times \langle \Upsilon(p',\varepsilon_1)J/\psi(q,\varepsilon_2)|T(p,\epsilon)\rangle \times \frac{\langle T(p,\epsilon)|J_{\alpha\beta}^\dagger|0\rangle}{p^2-m^2} + \cdots, \tag{26}$$

where $m_\Upsilon = (9460.40 \pm 0.09 \pm 0.04)$ MeV and $m_{J/\psi} = (3096.900 \pm 0.006)$ MeV are masses of the Υ and J/ψ mesons [39]. In the expression above, we denote by ε_1 and ε_2 the polarization vectors of these quarkonia, respectively.

To further simplify Eq. (26), it is convenient to employ the matrix elements of the mesons Υ and J/ψ

$$\langle 0|J_\mu^\Upsilon|\Upsilon(p',\varepsilon_1)\rangle = f_\Upsilon m_\Upsilon \varepsilon_{1\mu}(p'),$$
$$\langle 0|J_\nu^{J/\psi}|J/\psi(q,\varepsilon_2)\rangle = f_{J/\psi} m_{J/\psi}\varepsilon_{2\nu}(q). \tag{27}$$

Here, $f_\Upsilon = (708 \pm 8)$ MeV and $f_{J/\psi} = (411 \pm 7)$ MeV are decay constants of the mesons: Their experimental values are borrowed from Ref. [40].

Besides, one should specify the matrix element $\langle \Upsilon(p',\varepsilon_1)J/\psi(q,\varepsilon_2)|T(p,\epsilon)\rangle$ which can be done by decomposing it in contributions of all possible Lorentz-invariant terms made of the momenta and polarization tensor and vectors of the particles T, Υ, and J/ψ and corresponding form factors. Then, by requiring the gauge-invariance of the matrix element it is possible to express $\langle \Upsilon(p',\varepsilon_1)J/\psi(q,\varepsilon_2)|T(p,\epsilon)\rangle$ using the independent form factors (see, for instance, Ref. [41]). It turns out that a tensor-vector-vector vertex, in general, contains three independent form factors, which correspond to a pair of vector mesons with helicities $\lambda = 0$, ± 1, and ± 2 [41–44]. In two photon decays of a tensor meson the main contribution to the width of this process comes from the amplitude which correspond to a state $\lambda = 2$. Therefore, assuming that the same is true also for the decay $T \to J/\psi\Upsilon$, we consider here a pure $\lambda = 2$ final state for which the relevant vertex acquires the following form [42]:

$$\langle \Upsilon(p',\varepsilon_1)J/\psi(q,\varepsilon_2)|T(p,\epsilon)\rangle = g_1(q^2)\epsilon_{\tau\rho}^{(\lambda)}[(\varepsilon_1^*\cdot q)\varepsilon_2^{\tau *}p'^\rho + (\varepsilon_2^*\cdot p')\varepsilon_1^{*\tau}q^\rho - (p'\cdot q)\varepsilon_1^{\tau *}\varepsilon_2^{\rho *} - (\varepsilon_1^*\cdot\varepsilon_2^*)p'^\tau q^\rho]. \tag{28}$$

As a result, for $\Pi_{\mu\nu\alpha\beta}^{\rm Phys}(p,p')$ we get the expression

$$\Pi_{\mu\nu\alpha\beta}^{\rm Phys}(p,p') = g_1(q^2)\frac{\Lambda f_\Upsilon m_\Upsilon f_{J/\psi}m_{J/\psi}}{(p^2-m^2)(p'^2-m_\Upsilon^2)(q^2-m_{J/\psi}^2)} \times \left[p'_\beta p'_\alpha g_{\mu\nu} + \frac{1}{2}p_\mu p'_\alpha g_{\beta\nu} + \frac{1}{2m^2}p_\beta p_\nu p'_\mu p'_\alpha + \text{other structures}\right] + \cdots. \tag{29}$$

For the QCD side of the sum rule, we obtain

$$\Pi_{\mu\nu\alpha\beta}^{\rm OPE}(p,p') = \int d^4x d^4y e^{ip'y}e^{-ipx}\{\mathrm{Tr}[\gamma_\mu S_b^{ia}(y-x) \times \gamma_\alpha \tilde{S}_c^{jb}(-x)\gamma_\nu \tilde{S}_c^{bj}(x)\gamma_\beta S_b^{ai}(x-y)] - \mathrm{Tr}[\gamma_\mu S_b^{ia}(y-x)\gamma_\alpha \tilde{S}_c^{jb} \times (-x)\gamma_\nu \tilde{S}_c^{aj}(x)\gamma_\beta S_b^{bi}(x-y)]\}. \tag{30}$$

We utilize the structures proportional to $p_\mu p'_\alpha g_{\beta\nu}$ in the correlators and use corresponding amplitudes $\Pi_1^{\rm Phys}(p^2,p'^2,q^2)$ and $\Pi_1^{\rm OPE}(p^2,p'^2,q^2)$ to find SR for the form factor $g_1(q^2)$. After standard operations the sum rule for $g_1(q^2)$ reads

$$g_1(q^2) = \frac{2(q^2-m_{J/\psi}^2)}{\Lambda f_\Upsilon m_\Upsilon f_{J/\psi}m_{J/\psi}} e^{m^2/M_1^2}e^{m_\Upsilon^2/M_2^2}\Pi_1(\mathbf{M}^2,\mathbf{s}_0,q^2). \tag{31}$$

In Eq. (31), $\Pi_1(\mathbf{M}^2,\mathbf{s}_0,q^2)$ is the Borel transformed and subtracted function $\Pi_1^{\rm OPE}(p^2,p'^2,q^2)$. It depends on the parameters $\mathbf{M}^2 = (M_1^2, M_2^2)$ and $\mathbf{s}_0 = (s_0, s'_0)$ where the pairs (M_1^2, s_0) and (M_2^2, s'_0) correspond to the tetraquark and Υ channels, and is given by the following formula:

$$\Pi_1(\mathbf{M}^2,\mathbf{s}_0,q^2) = \int_{4\mathcal{M}^2}^{s_0} ds \int_{4m_b^2}^{s_0} ds'\rho_1(s,s',q^2) \times e^{-s/M_1^2 - s'/M_2^2} + \mathcal{B}\Pi_1^{\rm Dim4}(p^2,p'^2,q^2). \tag{32}$$

The explicit expressions of $\rho_1(s,s',q^2)$ and $\Pi_1^{\rm Dim4}(p^2, p'^2,q^2)$ can be found in the Appendix.

Requirements which should be satisfied by the auxiliary parameters $\mathbf{M}^2$ and $\mathbf{s}_0$ are universal for all SR computations and have been explained in the previous section. Numerical analysis shows that the regions in Eq. (22) for the parameters (M_1^2, s_0) and

$$M_2^2 \in [10, 12]\ \mathrm{GeV}^2, \qquad s'_0 \in [98, 100]\ \mathrm{GeV}^2 \tag{33}$$

for (M_2^2, s'_0) satisfy all these requirements. Because the form factor $g_1(q^2)$ depends on the mass and current coupling of the tetraquark T, this choice for (M_1^2, s_0) excludes also additional uncertainties in m and Λ, as well as in $g_1(q^2)$, which may appear beyond the regions

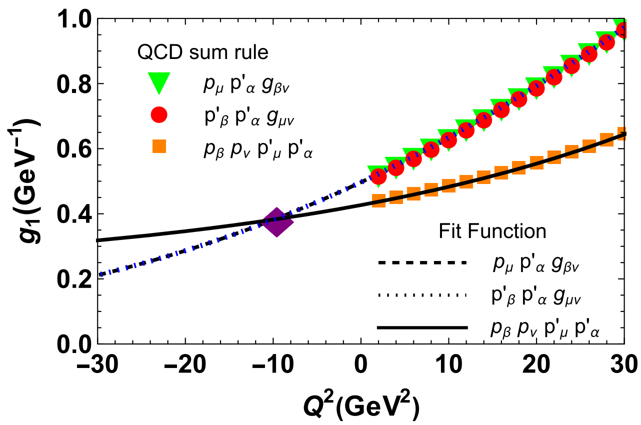

FIG. 4. Sum rule's data and fit functions for $g_1(Q^2)$. The diamond fixes the point $Q^2 = -m_{J/\psi}^2$ where g_1 has been estimated.

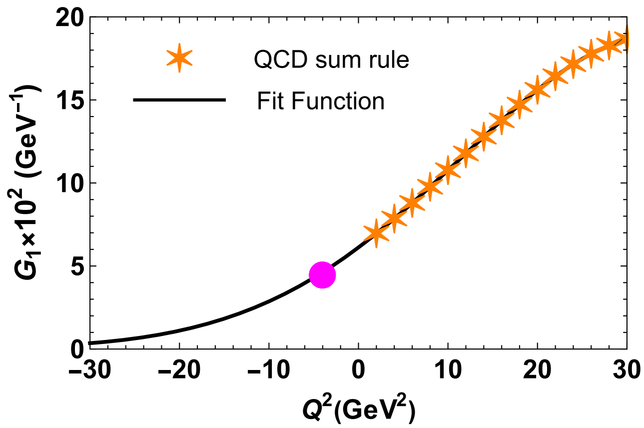

FIG. 5. Sum rule data and extrapolating function for the form factor $G_1(Q^2)$. The circle shows the point $Q^2 = -m_{D^{*0}}^2$.

Eq. (22). It is worth noting that s_0' is limited by the mass $m_{\Upsilon(2S)} = (10023.4 \pm 0.5)$ MeV of the radially excited state $\Upsilon(2S)$, i.e., $s_0' < m_{\Upsilon(2S)}^2$.

The SR method leads to reliable predictions for the form factor $g_1(q^2)$ in the Euclidean region $q^2 < 0$. But $g_1(q^2)$ determines the strong coupling g_1 at the mass shell $q^2 = m_{J/\psi}^2$. Therefore, it is convenient to introduce the function $g_1(Q^2)$ with $Q^2 = -q^2$ and use it in our analysis. The results obtained for $g_1(Q^2)$ are plotted in Fig. 4, where Q^2 varies inside the limits $Q^2 = 2$–$30\ \mathrm{GeV}^2$.

As it has been emphasized above, the strong coupling g_1 should be extracted at $q^2 = m_{J/\psi}^2$, i.e., at $Q^2 = -m_{J/\psi}^2$ where the SR method does not work. Therefore, we introduce the fit function $\mathcal{G}_1(Q^2)$ that at momenta $Q^2 > 0$ gives the same SR data, but can be extrapolated to the domain of negative Q^2. For these purposes, we utilize the function

$$\mathcal{G}_i(Q^2) = \mathcal{G}_i^0 \exp\left[c_i^1 \frac{Q^2}{m^2} + c_i^2 \left(\frac{Q^2}{m^2}\right)^2\right], \qquad (34)$$

where $\mathcal{G}_i^0$, c_i^1, and c_i^2 are fitted constants. Then, having compared QCD output and Eq. (34), it is easy to find

$$\mathcal{G}_1^0 = 0.50\ \mathrm{GeV}^{-1}, \quad c_1^1 = 4.10, \quad \text{and} \quad c_1^2 = -2.66. \qquad (35)$$

This function is also shown in Fig. 5, where a nice agreement of $\mathcal{G}_1(Q^2)$ and QCD data is clear. For the strong coupling g_1, we find

$$g_1 \equiv \mathcal{G}_1(-m_{J/\psi}^2) = (3.9 \pm 0.9) \times 10^{-1}\ \mathrm{GeV}^{-1}. \qquad (36)$$

The form factor $g_1(Q^2)$ and coupling g_1 can also be extracted from alternative SRs. To this end, we have used the amplitudes that in the correlators $\Pi_{\mu\nu\alpha\beta}^{\mathrm{Phys}}(p, p')$ and $\Pi_{\mu\nu\alpha\beta}^{\mathrm{OPE}}(p, p')$ correspond to structures $p_\beta' p_\alpha' g_{\mu\nu}$ and $p_\beta p_\nu p_\mu' p_\alpha'$, respectively. Numerical predictions for $g_1'(Q^2)$ and $g_1''(Q^2)$ found using these new SRs are depicted in Fig. 4 as well. It is seen that, in the case of the structure $p_\beta' p_\alpha' g_{\mu\nu}$, the sum rule data almost coincide with ones extracted above for $g_1(Q^2)$. Consequently the parameters of the extrapolating function $\mathcal{G}_1'(Q^2)$, and $g_1' = \mathcal{G}_1'(-m_{J/\psi}^2)$ with a high accuracy are identical to our results from Eqs. (35) and (36). The sum rule that corresponds to the structure $p_\beta p_\nu p_\mu' p_\alpha'$ leads for $g_1''(q^2)$ to different predictions. These SR points can be extrapolated by employing $\mathcal{G}_1''(Q^2)$ with parameters $\mathcal{G}_1^{0\prime\prime} = 0.43\ \mathrm{GeV}^{-1}$, $c_1^{1\prime\prime} = 1.90$, and $c_1^{2\prime\prime} = 1.72$. Though QCD data differ from each other the fitting function $\mathcal{G}_1''(Q^2)$ gives at the mass shell $Q^2 = -m_{J/\psi}^2$

$$g_1'' \equiv \mathcal{G}_1''(-m_{J/\psi}^2) = (3.8 \pm 0.9) \times 10^{-1}\ \mathrm{GeV}^{-1}, \qquad (37)$$

which is very close to Eq. (36). In other words, three different structures in the correlation functions $\Pi_{\mu\nu\alpha\beta}^{\mathrm{Phys}}(p, p')$ and $\Pi_{\mu\nu\alpha\beta}^{\mathrm{OPE}}(p, p')$, and corresponding SRs lead almost to the same result for the strong coupling g_1 at the vertex $TJ/\psi\Upsilon$. Because uncertainty in g_1 generated by a choice of the different structures is considerably smaller than theoretical errors of the SR method itself, it can be safely neglected.

The partial width of the decay $T \to J/\psi\Upsilon$ is determined by the expression

$$\Gamma[T \to J/\psi\Upsilon] = g_1^2 \frac{\lambda_1}{40\pi m^2} |M_1|^2, \qquad (38)$$

where

$$|M_1|^2=\frac{1}{6m^4}[m_{J/\psi}^8+m_{J/\psi}^6(m^2-4m_\Upsilon^2)+(m^2-m_\Upsilon^2)^2 \times(6m^4+3m^2m_\Upsilon^2+m_\Upsilon^4)+m_{J/\psi}^4(m^4-m^2m_\Upsilon^2+6m_\Upsilon^4) -m_{J/\psi}^2(9m^6-34m^4m_\Upsilon^2+m^2m_\Upsilon^4+4m_\Upsilon^6)], \quad (39)$$

and $\lambda_1=\lambda(m,m_\Upsilon,m_{J/\psi})$

$$\lambda(x,y,z)=\frac{\sqrt{x^4+y^4+z^4-2(x^2y^2+x^2z^2+y^2z^2)}}{2x}. \quad (40)$$

Then, we obtain

$$\Gamma[T\to J/\psi\Upsilon]=(27.7\pm 9.1)\ \mathrm{MeV}. \quad (41)$$

B. Decay $T\to\eta_b\eta_c$

The partial width of the process $T\to\eta_b\eta_c$ is governed by the strong coupling g_2 at the vertex $T\eta_b\eta_c$. In the framework of the SR method the relevant form factor $g_2(q^2)$ can be obtained from analysis of the correlator

$$\Pi_{\mu\nu}(p,p')=i^2\int d^4xd^4ye^{ip'y}e^{-ipx}\langle 0|\mathcal{T}\{J^{\eta_b}(y) \times J^{\eta_c}(0)J_{\mu\nu}^{\dagger}(x)\}|0\rangle. \quad (42)$$

The interpolating currents of the quarkonia η_c and η_b in Eq. (42) are

$$J^{\eta_c}(x)=\bar{c}_i(x)i\gamma_5c_i(x), \qquad J^{\eta_b}(x)=\bar{b}_j(x)i\gamma_5b_j(x). \quad (43)$$

The matrix elements

$$\langle 0|J^{\eta_b}|\eta_b(p')\rangle=\frac{f_{\eta_b}m_{\eta_b}^2}{2m_b}, \qquad \langle 0|J^{\eta_c}|\eta_c(q)\rangle=\frac{f_{\eta_c}m_{\eta_c}^2}{2m_c} \quad (44)$$

are necessary to calculate $\Pi_{\mu\nu}^{\mathrm{Phys}}(p,p')$ with f_{η_b}, m_{η_b}, and f_{η_c}, m_{η_c} being the decay constants and masses of the mesons η_b and η_c, respectively. The vertex $T\eta_b\eta_c$ is given by the expression [44]

$$\langle\eta_b(p')\eta_c(q)|T(p,\epsilon)\rangle=g_2(q^2)\epsilon_{\alpha\beta}^{(\lambda)}(p)p'^{\alpha}p'^{\beta}. \quad (45)$$

For the correlator $\Pi_{\mu\nu}^{\mathrm{Phys}}(p,p')$, we find

$$\Pi_{\mu\nu}^{\mathrm{Phys}}(p,p')=g_2(q^2)\frac{\Lambda f_{\eta_b}m_{\eta_b}^2f_{\eta_c}m_{\eta_c}^2}{4m_bm_c(p^2-m^2)(p'^2-m_{\eta_b}^2)}\frac{1}{(q^2-m_{\eta_c}^2)}\left[\frac{m^4-2m^2(m_{\eta_b}^2+q^2)+(m_{\eta_b}^2-q^2)^2}{12m^2}g_{\mu\nu} + p'_\mu p'_\nu+\text{other terms}\right]. \quad (46)$$

The QCD side of the sum rule $\Pi_{\mu\nu}^{\mathrm{OPE}}(p,p')$ has the form

$$\Pi_{\mu\nu}^{\mathrm{OPE}}(p,p')=i\int d^4xd^4ye^{ip'y}e^{-ipx}\{\mathrm{Tr}[\gamma_5S_b^{ia}(y-x) \times\gamma_\mu\tilde{S}_c^{jb}(-x)\gamma_5\tilde{S}_c^{bj}(x)\gamma_\nu S_b^{ai}(x-y)] -\mathrm{Tr}[\gamma_5S_b^{ia}(y-x)\gamma_\mu\tilde{S}_c^{jb} \times(-x)\gamma_5\tilde{S}_c^{aj}(x)\gamma_\nu S_b^{bi}(x-y)]\}. \quad (47)$$

The functions $\Pi_{\mu\nu}^{\mathrm{Phys}}(p,p')$ and $\Pi_{\mu\nu}^{\mathrm{OPE}}(p,p')$ have the same Lorentz structures. We consider terms $\sim p'_\mu p'_\nu$ and use corresponding invariant amplitudes $\Pi_2^{\mathrm{Phys}}(p^2,p'^2,q^2)$ and $\Pi_2^{\mathrm{OPE}}(p^2,p'^2,q^2)$ to derive the sum rule for the form factor $g_2(q^2)$

$$g_2(q^2)=\frac{4m_cm_b(q^2-m_{\eta_c}^2)}{\Lambda f_{\eta_c}m_{\eta_c}^2f_{\eta_b}m_{\eta_b}^2}e^{m^2/M_1^2}e^{m_{\eta_b}^2/M_2^2} \times\Pi_2(\mathbf{M}^2,\mathbf{s}_0,q^2), \quad (48)$$

where $\Pi_2(\mathbf{M}^2,\mathbf{s}_0,q^2)$ is the amplitude $\Pi_2^{\mathrm{OPE}}(p^2,p'^2,q^2)$ after Borel transformations and continuum subtractions.

The remaining manipulations are usual prescriptions of the SR method, which have been explained above. In numerical computations, for the masses of the quarkonia η_c and η_b, we use $m_{\eta_c}=(2984.1\pm 0.4)$ MeV, $m_{\eta_b}=(9398.7\pm 2.0)$ MeV from PDG [39]. The decay constant $f_{\eta_c}=(421\pm 35)$ MeV was extracted from SR analysis [45], whereas for f_{η_b} we employ 724 MeV. We have utilized also the following windows for M_2^2, and s_0' in the η_b channel

$$M_2^2\in[10,12]\ \mathrm{GeV}^2, \qquad s_0'\in[95,99]\ \mathrm{GeV}^2. \quad (49)$$

The extrapolating function $\mathcal{G}_2(Q^2)$ and parameters $\mathcal{G}_2^0=33.63\ \mathrm{GeV}^{-1}$, $c_2^1=8.31$, and $c_2^2=-13.55$ lead to reasonable agreement with SR data. Then the strong coupling g_2 amounts to

$$g_2 \equiv \mathcal{G}_2(-m_{\eta_c}^2) = (20.4 \pm 4.9)\ \mathrm{GeV}^{-1}. \quad (50)$$

The partial width of this process is equal to

$$\Gamma[T \to \eta_b \eta_c] = g_2^2 \frac{\lambda_2}{40\pi m^2} |M_2|^2, \quad (51)$$

where

$$|M_2|^2 = \frac{[m^4 + (m_{\eta_b}^2 - m_{\eta_c}^2)^2 - 2m^2(m_{\eta_b}^2 + m_{\eta_c}^2)]^2}{24m^4}, \quad (52)$$

and $\lambda_2 = \lambda(m, m_{\eta_b}, m_{\eta_c})$.

The width of the decay $T \to \eta_b \eta_c$ is

$$\Gamma[T \to \eta_b \eta_c] = (21.1 \pm 7.2)\ \mathrm{MeV}. \quad (53)$$

IV. MODES $T \to B_c^{*+} B_c^{*-}$ AND $B_c^+ B_c^-$

Here, we consider decays of the tensor tetraquark T to $B_c^{*+} B_c^{*-}$ and $B_c^+ B_c^-$ final states. It is known that experimental information about B_c mesons is limited by the mass of $B_c^\pm$ and its first radial excitation $B_c^\pm(2S)$ [39]. Therefore, for parameters of the $c\bar{b}$ ($b\bar{c}$) mesons with other spin-parities one should use theoretical predictions. In the case of the vector meson $B_c^{*\pm}$, for its mass and decay constant we employ

$$m_{B_c^*} = 6338\ \mathrm{MeV}, \qquad f_{B_c^*} = 471\ \mathrm{MeV} \quad (54)$$

from Refs. [46,47], respectively. We also utilize the experimental value $m_{B_c} = (6274.47 \pm 0.27 \pm 0.17)$ MeV for the mass of $B_c^\pm$ and decay constant $f_{B_c} = (371 \pm 37)$ MeV from Ref. [48]. It is not difficult to see that the processes $T \to B_c^{*+} B_c^{*-}$ and $B_c^+ B_c^-$ are permitted decay modes of the tensor diquark-antidiquark state T, because thresholds for production of the $B_c^{*+} B_c^{*-}$ and $B_c^+ B_c^-$ final-states 12.68 GeV and 12.55 GeV are below its mass m.

A. $T \to B_c^{*+} B_c^{*-}$

Analysis of this decay goes in line with a scheme presented and explained above. Therefore, we write down principal formulas and final results.

The correlation function to derive SR for the form factor $\tilde{g}_1(q^2)$ responsible for strong interaction at the vertex $TB_c^{*+} B_c^{*-}$ is

$$\tilde{\Pi}_{\mu\nu\alpha\beta}(p, p') = i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \langle 0 | \mathcal{T}\{ J_\mu^{B_c^{*+}}(y) \times J_\nu^{B_c^{*-}}(0) J_{\alpha\beta}^\dagger(x)\} | 0 \rangle. \quad (55)$$

Here, $J_\mu^{B_c^{*+}}$ and $J_\nu^{B_c^{*-}}$ are the interpolating currents of B_c^{*+} and B_c^{*-} mesons, which are determined by the expressions

$$J_\mu^{B_c^{*+}}(x) = \bar{b}_i(x)\gamma_\mu c_i(x), \qquad J_\nu^{B_c^{*-}}(x) = \bar{c}_j(x)\gamma_\nu b_j(x). \quad (56)$$

In terms of the physical parameters of the particles, $\tilde{\Pi}_{\mu\nu\alpha\beta}(p, p')$ acquires the following form:

$$\tilde{\Pi}_{\mu\nu\alpha\beta}(p, p') = \frac{\langle 0 | J_\mu^{B_c^{*+}} | B_c^{*+}(p', \varepsilon_1)\rangle}{p'^2 - m_{B_c^*}^2} \frac{\langle 0 | J_\nu^{B_c^{*-}} | B_c^{*-}(q, \varepsilon_2)\rangle}{q^2 - m_{B_c^*}^2} \times \langle B_c^{*+}(p', \varepsilon_1) B_c^{*-}(q, \varepsilon_2) | T(p, \epsilon)\rangle \times \frac{\langle T(p, \epsilon) | J_{\alpha\beta}^\dagger | 0\rangle}{p^2 - m^2} + \cdots. \quad (57)$$

Subsequent calculations are carried out using the matrix elements

$$\langle 0 | J_\mu^{B_c^{*+}} | B_c^{*+}(p', \varepsilon_1)\rangle = f_{B_c^*} m_{B_c^*} \varepsilon_{1\mu}(p'),$$
$$\langle 0 | J_\nu^{B_c^{*-}} | B_c^{*-}(q, \varepsilon_2)\rangle = f_{B_c^*} m_{B_c^*} \varepsilon_{2\nu}(q), \quad (58)$$

where $\varepsilon_{1\mu}(p')$ and $\varepsilon_{2\nu}(q)$ are the polarization vectors of B_c^{*+} and B_c^{*-}, respectively. The vertex $TB_c^{*+} B_c^{*-}$ is considered in the form Eq. (28) with replacement $g_1(q^2) \to \tilde{g}_1(q^2)$.

Then $\tilde{\Pi}_{\mu\nu\alpha\beta}^{\mathrm{Phys}}(p, p')$ in terms of the physical parameters of the tetraquark T and mesons $B_c^{*\pm}$ reads

$$\tilde{\Pi}_{\mu\nu\alpha\beta}^{\mathrm{Phys}}(p, p') = \frac{\tilde{g}_1(q^2) \Lambda f_{B_c^*}^2 m_{B_c^*}^2}{(p^2 - m^2)(p'^2 - m_{B_c^*}^2)} \frac{1}{(q^2 - m_{B_c^*}^2)} \left[\frac{1}{2} p_\mu p'_\alpha g_{\nu\beta} + \frac{m^2 + m_{B_c^*}^2 - q^2}{4m^2} p'_\mu p_\alpha g_{\beta\nu} + p'_\beta p'_\alpha g_{\mu\nu} + \text{other structures} \right] + \cdots. \quad (59)$$

The function $\tilde{\Pi}_{\mu\nu\alpha\beta}(p, p')$ computed in terms of the quark propagators is equal to

$$\tilde{\Pi}_{\mu\nu\alpha\beta}^{\mathrm{OPE}}(p, p') = i \int d^4x d^4y e^{ip'y} e^{-ipx} \{ \mathrm{Tr}[\gamma_\mu S_b^{ia}(y - x) \times \gamma_\alpha \tilde{S}_c^{jb}(-x) \gamma_\nu \tilde{S}_b^{aj}(x) \gamma_\beta S_c^{bi}(x - y)] - \mathrm{Tr}[\gamma_\mu S_b^{ia}(y - x) \gamma_\alpha \tilde{S}_c^{jb} \times (-x) \gamma_\nu \tilde{S}_b^{bj}(x) \gamma_\beta S_c^{ai}(x - y)] \}. \quad (60)$$

The sum rule for $\tilde{g}_1(q^2)$

$$\tilde{g}_1(q^2) = \frac{2(q^2 - m_{\eta_c}^2)}{\Lambda f_{B_c^*}^2 m_{B_c^*}^2} e^{m^2/M_1^2} e^{m_{B_c^*}^2/M_2^2} \times \tilde{\Pi}_1(\mathbf{M}^2, \mathbf{s}_0, q^2) \quad (61)$$

is derived using invariant amplitudes $\tilde{\Pi}_1^{\mathrm{Phys}}(p^2, p'^2, q^2)$ and $\tilde{\Pi}_1^{\mathrm{OPE}}(p^2, p'^2, q^2)$, which correspond to terms $\sim p_\mu p'_\alpha g_{\nu\beta}$ in the correlators $\tilde{\Pi}^{\rm Phys}_{\mu\nu\alpha\beta}(p,p')$ and $\tilde{\Pi}^{\rm OPE}_{\mu\nu\alpha\beta}(p,p')$, respectively. Above $\tilde{\Pi}_1(\mathbf{M}^2,\mathbf{s}_0,q^2)$ is the amplitude $\tilde{\Pi}_1^{\rm OPE}(p^2,p'^2,q^2)$ obtained after relevant transformations.

Numerical computations have been carried out by employing the following values for the parameters M_2^2 and s_0' in the B_c^{*+} channel

$$M_2^2\in[6.5,7.5]\ \mathrm{GeV}^2,\qquad s_0'\in[49,51]\ \mathrm{GeV}^2. \tag{62}$$

The constants of the function $\tilde{\mathcal{G}}_1(Q^2)$ are equal to $\tilde{\mathcal{G}}_1^0=0.31\ \mathrm{GeV}^{-1}$, $\tilde{c}_1^1=0.19$, and $\tilde{c}_1^2=3.37$. We find for the strong coupling $\tilde{g}_1$

$$\tilde{g}_1\equiv\tilde{\mathcal{G}}_1(-m_{B_c^*}^2)=(3.6\pm0.9)\times10^{-1}\ \mathrm{GeV}^{-1}. \tag{63}$$

The partial width of the decay $T\to B_c^{*+}B_c^{*-}$ is equal to

$$\Gamma[T\to B_c^{*+}B_c^{*-}]=\frac{\tilde{g}_1^2\tilde{\lambda}_1}{80\pi m^2}(m^4-3m^2m_{B_c^*}^2+6m_{B_c^*}^4), \tag{64}$$

where $\tilde{\lambda}_1=\lambda(m,m_{B_c^*},m_{B_c^*})$. Alternatively, the width of this decay can be obtained from Eqs. (38) and (39) upon replacement $m_{J/\psi}=m_\Upsilon\to m_{B_c^*}$.

Numerical calculations yield

$$\Gamma[T\to B_c^{*+}B_c^{*-}]=(20.1\pm6.6)\ \mathrm{MeV}. \tag{65}$$

B. $T\to B_c^+B_c^-$

The process $T\to B_c^+B_c^-$ is investigated in analogous manner. We consider the correlation function

$$\tilde{\Pi}_{\mu\nu}(p,p')=i^2\int d^4xd^4ye^{ip'y}e^{-ipx}\langle0|\mathcal{T}\{J^{B_c^+}(y)\times J^{B_c^-}(0)J^\dagger_{\mu\nu}(x)\}|0\rangle, \tag{66}$$

with $J^{B_c^+}(x)$ and $J^{B_c^-}(x)$ being the interpolating currents of the B_c^+ and B_c^- mesons

$$J^{B_c^+}(x)=\bar{b}_i(x)i\gamma_5c_i(x),\qquad J^{B_c^-}(x)=\bar{c}_j(x)i\gamma_5b_j(x). \tag{67}$$

The matrix elements of the $B_c^\pm$ mesons are

$$\langle0|J^{B_c^\pm}|B_c^\pm\rangle=\frac{f_{B_c}m_{B_c}^2}{m_b+m_c}. \tag{68}$$

The vertex $\langle B_c^+(p')B_c^-(q)|T(p,\epsilon)\rangle$ has the form

$$\langle B_c^+(p')B_c^-(q)|T(p,\epsilon)\rangle=\tilde{g}_2(q^2)\epsilon^{(\lambda)}_{\alpha\beta}(p)p'^\alpha p'^\beta. \tag{69}$$

The $\tilde{\Pi}^{\rm Phys}_{\mu\nu}(p,p')$ obtained using these matrix elements after some substitutions [$m_{\eta_b}^2$ and $m_{\eta_c}^2\to m_{B_c}^2$, $4m_cm_b\to(m_b+m_c)^2$, etc.] is given by Eq. (46), whereas the QCD side of SR is defined by Eq. (47). The SR for the form factor $\tilde{g}_2(q^2)$ is

$$\tilde{g}_2(q^2)=\frac{(m_b+m_c)^2(q^2-m_{B_c}^2)}{\Lambda f_{B_c}^2m_{B_c}^4}e^{m^2/M_1^2}e^{m_{B_c}^2/M_2^2}\times\tilde{\Pi}_2(\mathbf{M}^2,\mathbf{s}_0,q^2). \tag{70}$$

In numerical analysis, we have used the following parameters:

$$M_2^2\in[6.5,7.5]\ \mathrm{GeV}^2,\qquad s_0'\in[45,47]\ \mathrm{GeV}^2. \tag{71}$$

Computations of the form factor $\tilde{g}_2(q^2)$ and coupling $\tilde{g}_2$ lead to the prediction

$$\tilde{g}_2\equiv\tilde{\mathcal{G}}_2(-m_{B_c}^2)=(26.6\pm6.4)\ \mathrm{GeV}^{-1}, \tag{72}$$

where $\tilde{\mathcal{G}}_2(Q^2)$ is the fitting function with parameters $\tilde{\mathcal{G}}_2^0=29.89\ \mathrm{GeV}^{-1}$, $\tilde{c}_2^1=0.95$, and $\tilde{c}_2^2=1.90$.

The width of the decay $T\to B_c^+B_c^-$ can be computed by means of the expression

$$\Gamma[T\to B_c^+B_c^-]=\tilde{g}_2^2\frac{\tilde{\lambda}_2}{960\pi m^2}(m^2-4m_{B_c}^2)^2, \tag{73}$$

where $\tilde{\lambda}_2=\lambda(m,m_{B_c},m_{B_c})$. This formula in the limit m_{η_b} and $m_{\eta_c}\to m_{B_c}$ can be obtained from Eq. (52). Our computations yield

$$\Gamma[T\to B_c^+B_c^-]=(20.1\pm6.9)\ \mathrm{MeV}. \tag{74}$$

V. DECAYS DUE TO $b\bar{b}$ ANNIHILATIONS

It has been emphasized above that the tetraquark T can transform to conventional mesons also due to annihilation of $b\bar{b}$ to light quark-antiquark pairs [7,8,49] and creation of DD mesons with required electric charges and spin-parities. Here, we consider decays of the tetraquark T to $D^{*0}\bar{D}^{*0}$, $D^0\bar{D}^0$, $D^{*+}D^{*-}$, and D^+D^- mesons.

It is clear that these decays are kinematically possible modes for transformation of the tetraquark T to ordinary mesons. We study these processes in the same context of the three-point sum rule approach. But here we encounter a situation when relevant correlation functions contain $\bar{b}b$ quarks' vacuum matrix element $\langle\bar{b}b\rangle$ [49]. In calculations, we replace this matrix element with known value of the gluon condensate $\langle\alpha_sG^2/\pi\rangle$.

A. Decays $T \to D^{*0}\bar{D}^{*0}$ and $D^0\bar{D}^0$

Let us analyze the process $T \to D^{*0}\bar{D}^{*0}$. To find the coupling G_1 of particles at the vertex $TD^{*0}\bar{D}^{*0}$, we start from the correlation function

$$\hat{\Pi}_{\mu\nu\alpha\beta}(p,p') = i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \langle 0|\mathcal{T}\{J_\mu^{\bar{D}^{*0}}(y) \times J_\nu^{D^{*0}}(0) J_{\alpha\beta}^\dagger(x)\}|0\rangle, \quad (75)$$

where $J_\mu^{\bar{D}^{*0}}(x)$ and $J_\nu^{D^{*0}}(x)$ are interpolating currents for the mesons $\bar{D}^{*0}$ and D^{*0}

$$J_\mu^{\bar{D}^{*0}}(x) = \bar{c}_i(x)\gamma_\mu u_i(x), \qquad J_\nu^{D^{*0}}(x) = \bar{u}_j(x)\gamma_\nu c_j(x). \quad (76)$$

The expression of the correlation function $\hat{\Pi}_{\mu\nu\alpha\beta}(p,p')$ in terms of T, D^{*0}, and $\bar{D}^{*0}$ particles' parameters reads

$$\hat{\Pi}^{\rm Phys}_{\mu\nu\alpha\beta}(p,p') = \frac{\langle 0|J_\mu^{\bar{D}^{*0}}|\bar{D}^{*0}(p',\varepsilon_1)\rangle}{p'^2 - m^2_{D^{*0}}} \frac{\langle 0|J_\nu^{D^{*0}}|D^{*0}(q,\varepsilon_2)\rangle}{q^2 - m^2_{D^{*0}}} \times \langle \bar{D}^{*0}(p',\varepsilon_1) D^{*0}(q,\varepsilon_2)|T(p,\epsilon)\rangle \times \frac{\langle T(p,\epsilon)|J_\mu^\dagger|0\rangle}{p^2 - m^2} + \cdots, \quad (77)$$

where $m_{D^{*0}} = (2006.85 \pm 0.05)$ MeV is the mass of the mesons $\bar{D}^{*0}$ and D^{*0}, whereas $\varepsilon_{1\mu}$ and $\varepsilon_{2\nu}$ are their polarization vectors.

The matrix elements which are required to calculate $\hat{\Pi}^{\rm Phys}_{\mu\nu\alpha\beta}(p,p')$ are

$$\langle 0|J_\mu^{\bar{D}^{*0}}|\bar{D}^{*0}(p',\varepsilon_1)\rangle = f_{D^*} m_{D^{*0}} \varepsilon_{1\mu}(p'),$$
$$\langle 0|J_\nu^{D^{*0}}|D^{*0}(q,\varepsilon_2)\rangle = f_{D^*} m_{D^{*0}} \varepsilon_{2\nu}(q), \quad (78)$$

with $f_{D^*} = (252.2 \pm 22.66)$ MeV being the decay constant of the mesons D^{*0} and $\bar{D}^{*0}_D$. The vertex $\langle \bar{D}^{*0}(p',\varepsilon_1) D^{*0}(q,\varepsilon_2)|T(p,\epsilon)\rangle$ is modeled in the form of Eq. (28).

The correlator $\hat{\Pi}^{\rm Phys}_{\mu\nu\alpha\beta}(p,p')$ is a sum of different components. The SR for the form factor $G_1(q^2)$ is obtained by employing the invariant amplitude $\hat{\Pi}_1^{\rm Phys}(p^2,p'^2,q^2)$ that corresponds to the structure $p_\mu p'_\alpha g_{\beta\nu}$. The same correlation function $\hat{\Pi}_{\mu\nu\alpha\beta}(p,p')$ computed using the heavy and light quark propagators is

$$\hat{\Pi}^{\rm OPE}_{\mu\nu\alpha\beta}(p,p') = \frac{2}{3}\langle \bar{b}b\rangle \int d^4x d^4y e^{ip'y} e^{-ipx} \times {\rm Tr}[\gamma_\mu S_u^{ij}(y)\gamma_\nu S_c^{jb}(-x)\gamma_\alpha\gamma_\beta S_c^{bi}(x-y)], \quad (79)$$

where $S_u(x)$ is the u quark's propagator [37]. In what follows, the function $\hat{\Pi}_1^{\rm OPE}(p^2,p'^2,q^2)$ is the invariant amplitude that corresponds in $\hat{\Pi}^{\rm OPE}_{\mu\nu\alpha\beta}(p,p')$ to the term $p_\mu p'_\alpha g_{\beta\nu}$.

For further studies, we make use of the relation between condensates

$$\langle \bar{b}b\rangle = -\frac{1}{12 m_b}\left\langle \frac{\alpha_s G^2}{\pi}\right\rangle \quad (80)$$

derived in Ref. [34] from the sum rule analysis. This expression was obtained at the leading order of the perturbative QCD and is valid as far as higher order corrections in m_b^{-1} are very small.

The SR for the coupling $G_1(q^2)$ reads

$$G_1(q^2) = \frac{2(q^2 - m^2_{D^{*0}})}{\Lambda f^2_{D^*} m^2_{D^{*0}}} e^{m^2/M_1^2} e^{m^2_{D^{*0}}/M_2^2} \times \hat{\Pi}_1(\mathbf{M}^2, \mathbf{s}_0, q^2), \quad (81)$$

where $\hat{\Pi}_1(\mathbf{M}^2,\mathbf{s}_0,q^2)$ is the amplitude $\hat{\Pi}_1^{\rm OPE}(p^2,p'^2,q^2)$ undergone to Borel transformations and continuum subtractions.

To extract $G_1(q^2)$ from this SR, we carry out standard manipulations and skip further details: In the $\bar{D}^{*0}$ meson channel, we have used the parameters

$$M_2^2 \in [2,3]\ {\rm GeV}^2, \qquad s'_0 \in [5.7, 5.8]\ {\rm GeV}^2. \quad (82)$$

The coupling G_1 has been evaluated by employing SR data for $Q^2 = 2$–30 GeV^2 and the extrapolating function with parameters $\hat{\mathcal{G}}_1^0 = 0.06\ {\rm GeV}^{-1}$, $\hat{c}_1^1 = 10.67$, and $\hat{c}_1^2 = -25.14$. The SR data and fit function $\hat{\mathcal{G}}_1(Q^2)$ are plotted in Fig. 5. The coupling G_1 has been computed at the mass shell $q^2 = m^2_{D^{*0}}$ and amounts to

$$G_1 \equiv \hat{\mathcal{G}}_1(-m^2_{D^{*0}}) = (4.62 \pm 1.11) \times 10^{-2}\ {\rm GeV}^{-1}. \quad (83)$$

The width of the decay $T \to D^{*0}\bar{D}^{*0}$ is

$$\Gamma[T \to D^{*0}\bar{D}^{*0}] = (7.7 \pm 2.6)\ {\rm MeV}. \quad (84)$$

The second process $T \to D^0\bar{D}^0$ is considered starting from the correlator

$$\hat{\Pi}_{\mu\nu}(p,p') = i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \langle 0|\mathcal{T}\{J^{\bar{D}^0}(y) \times J^{D^0}(0) J^\dagger_{\mu\nu}(x)\}|0\rangle, \quad (85)$$

where the currents $J^{\bar{D}^0}(x)$ and $J^{D^0}(x)$ are defined by expressions

$$J^{\bar{D}^0}(x) = \bar{c}_i(x) i\gamma_5 u_i(x), \quad J^{D^0}(x) = \bar{u}_j(x) i\gamma_5 c_j(x). \quad (86)$$

To get the sum rule for the form factor $G_2(q^2)$ responsible for strong interaction of particles at the vertex $TD^0\bar{D}^0$, we calculate $\hat{\Pi}_{\mu\nu}^{\rm Phys}(p,p')$ and $\hat{\Pi}_{\mu\nu}^{\rm OPE}(p,p')$.

We determine $\hat{\Pi}_{\mu\nu}^{\rm Phys}(p,p')$ using the following matrix elements:

$$\langle 0|J^{\bar{D}^0}|\bar{D}^0\rangle = \langle 0|J^{D^0}|D^0\rangle = \frac{f_D m_{D^0}^2}{m_c}, \quad (87)$$

and

$$\langle \bar{D}^0(p')D^0(q)|T(p,\epsilon)\rangle = G_2(q^2)\epsilon_{\alpha\beta}^{(\lambda)}(p)p'^{\alpha}p'^{\beta}, \quad (88)$$

with $m_{D^0} = (1864.84 \pm 0.05)$ MeV and $f_D = (211.9 \pm 1.1)$ MeV being the mass and decay constant of mesons D^0 and $\bar{D}^0$ [39,50]. As a result, we obtain

$$\hat{\Pi}_{\mu\nu}^{\rm Phys}(p,p') = \frac{G_2(q^2)\Lambda f_D^2 m_{D^0}^4}{m_c^2(p^2-m^2)(p'^2-m_{D^0}^2)(q^2-m_{D^0}^2)} \times \left[\frac{m^4-2m^2(m_{D^0}^2+q^2)+(m_{D^0}^2-q^2)^2}{12m^2}g_{\mu\nu} + p'_\mu p'_\nu - \frac{m^2+m_{D^0}^2-q^2}{2m^2}p_\mu p'_\nu + \text{other terms}\right]. \quad (89)$$

For $\hat{\Pi}_{\mu\nu}^{\rm OPE}(p,p')$, we find

$$\hat{\Pi}_{\mu\nu}^{\rm OPE}(p,p') = \frac{2}{3}\langle \bar{b}b\rangle \int d^4x d^4y e^{ip'y}e^{-ipx} \times {\rm Tr}[\gamma_5 S_u^{ij}(y)\gamma_5 S_c^{jb}(-x)\gamma_\mu\gamma_\nu S_c^{bi}(x-y)]. \quad (90)$$

We extract SR for $G_2(q^2)$ using the amplitudes $\hat{\Pi}_2^{\rm Phys}(p^2,p'^2,q^2)$ and $\hat{\Pi}_2^{\rm OPE}(p^2,p'^2,q^2)$ corresponding to structures $p'_\mu p'_\nu$ and get

$$G_2(q^2) = \frac{m_c^2(q^2-m_{D^0}^2)}{\Lambda f_D^2 m_{D^0}^4} e^{m^2/M_1^2} e^{m_{D^0}^2/M_2^2} \times \hat{\Pi}_2(\mathbf{M}^2, \mathbf{s}_0, q^2), \quad (91)$$

with $\hat{\Pi}_2(\mathbf{M}^2, \mathbf{s}_0, q^2)$ being the transformed function $\hat{\Pi}_2^{\rm OPE}(p^2,p'^2,q^2)$.

In numerical calculations we employed the parameters

$$M_2^2 \in [1.5, 3]\ \text{GeV}^2, \quad s_0' \in [5, 5.2]\ \text{GeV}^2. \quad (92)$$

We have found the coupling G_2 by means of the of the function $\hat{\mathcal{G}}_2(Q^2)$ with $\hat{\mathcal{G}}_2^0 = 0.20\ \text{GeV}^{-1}$, $\hat{c}_2^1 = 10.72$, and $\hat{c}_2^2 = -26.80$

$$G_2 \equiv \hat{\mathcal{G}}_2(-m_{D^0}^2) = (0.16 \pm 0.04)\ \text{GeV}^{-1}. \quad (93)$$

The partial width of the decay $T \to D^0\bar{D}^0$ is equal to

$$\Gamma[T \to D^0\bar{D}^0] = (6.5 \pm 2.3)\ \text{MeV}. \quad (94)$$

B. Processes $T \to D^{*+}D^{*-}$ and D^+D^-

The modes $T \to D^{*+}D^{*-}$ and D^+D^- are explored in accordance with the scheme explained above. Let us analyze the process $T \to D^{*+}D^{*-}$. The strong form factor $G_3(q^2)$ at the vertex $TD^{*+}D^{*-}$ is extracted from the correlation function

$$\Pi'_{\mu\nu\alpha\beta}(p,p') = i^2 \int d^4x d^4y e^{ip'y}e^{-ipx}\langle 0|\mathcal{T}\{J_\mu^{D^{*+}}(y) \times J_\nu^{D^{*-}}(0)J_{\alpha\beta}^\dagger(x)\}|0\rangle, \quad (95)$$

where currents for the mesons D^{*+} and D^{*-} are given by the formulas

$$J_\mu^{D^{*+}}(x) = \bar{d}_i(x)\gamma_\mu c_i(x), \quad J_\nu^{D^{*-}}(x) = \bar{d}_j(x)\gamma_\nu d_j(x). \quad (96)$$

The matrix elements of these particles and the vertex are similar to ones introduced above. Therefore, we omit these expressions and write down the QCD side of the SR

$$\bar{\Pi}'^{\rm OPE}_{\mu\nu\alpha\beta}(p,p') = -\frac{i}{18m_b}\left\langle\frac{\alpha_s G^2}{\pi}\right\rangle \int d^4x d^4y e^{ip'y}e^{-ipx} \times {\rm Tr}[\gamma_\mu S_d^{ij}(y)\gamma_\nu S_c^{jb}(-x)\gamma_\alpha\gamma_\beta S_c^{bj}(x-y)]. \quad (97)$$

As usual, we utilize invariant amplitudes corresponding to the structures $p_\mu p'_\alpha g_{\beta\nu}$. In numerical calculations the Borel and continuum subtraction parameters in the D^{*+} channel are fixed as in Eq. (82). The mass of $D^{*\pm}$ mesons is $m_{D^*} = (2010.26 \pm 0.05)$ MeV, whereas for their decay constants we use $f_{D^*} = (252.2 \pm 22.66)$ MeV.

The function $\hat{\mathcal{G}}_3(Q^2)$ with the constants $\hat{\mathcal{G}}_3^0 = 0.06\ \text{GeV}^{-1}$, $\hat{c}_3^1 = 10.65$, and $\hat{c}_3^2 = -25.11$ leads to coupling G_3

$$G_3 \equiv \hat{\mathcal{G}}_3(-m_{D^*}^2) = (4.63 \pm 1.11) \times 10^{-2}\ \text{GeV}^{-1}. \quad (98)$$

For the partial width of the mode $T \to D^{*+}D^{*-}$, we get

$$\Gamma[T \to D^{*+}D^{*-}] = (7.7 \pm 2.7)\ \text{MeV}. \quad (99)$$

The process $T \to D^+D^-$ is explored in similar way. The coupling G_4 describing the strong interaction of the particles at the vertex TD^+D^- is

$$G_4 \equiv \tilde{\mathcal{G}}_4(-m_D^2) = (0.16 \pm 0.04)\ \text{GeV}^{-1}. \quad (100)$$

For the width of this decay, we find

$$\Gamma[T \to D^{+}D^{-}] = (6.5 \pm 2.3)\ \mathrm{MeV}. \tag{101}$$

Computations performed in present paper allow us to estimate the full width of the axial-vector tetraquark T with content $bc\bar{b}\,\bar{c}$. As a result, we obtain

$$\Gamma[T] = (117.4 \pm 15.9)\ \mathrm{MeV}. \tag{102}$$

VI. CONCLUSIONS

In present article, we have calculated the mass and full width of the tensor tetraquark $bc\bar{b}\,\bar{c}$. Analyses have been performed in the framework of QCD sum rule method. To evaluate the mass of T, we have applied the two-point SR method, whereas its decays have been studied by invoking the three-point SR approach.

The mass m of the tensor tetraquark T was evaluated in different articles, sometimes with contradictory results [10,24–30]. Our prediction $m = (12.70 \pm 0.09)$ GeV is smaller than those reported in publications [10,24–26]. In Refs. [27–30] the authors found the mass of this state in most of cases below m. Thus, m evaluated in the present work is somewhere between these two groups of predictions.

The results of current paper demonstrate that the tensor state T can decay to ordinary mesons through the strong fall-apart mechanism. In almost all articles cited above authors made similar conclusions: Only in Ref. [30] T was predicted to be stable against two-meson strong dissociations. But let us emphasize that structures $bc\bar{b}\,\bar{c}$, due to $b\bar{b}$ and $c\bar{c}$ annihilations and generations of ordinary heavy-light mesons, are always strong-interaction unstable particles.

We have calculated partial widths of the four processes $T \to J/\psi\Upsilon$, $\eta_b\eta_c$ and $B_c^{(*)+}B_c^{(*)-}$, which are dominant decay channels of T. We have evaluated also widths of modes triggered by $b\bar{b}$ annihilations inside of T and containing at the final states $D^{(*)+}D^{(*)-}$ and $D^{(*)0}\bar{D}^{(*)0}$ mesons. It is worth noting that the contribution of these processes is not small and forms approximately 24% of the T tetraquark's full width.

Our predictions characterize T as a wide diquark-antidiquark state, which can decay to two-meson final states through both fall-apart and $b\bar{b}$ annihilation mechanisms. The tensor tetraquark T, as well as the scalar and axial-vector tetraquarks $bc\bar{b}\,\bar{c}$, establish a family of fully heavy exotic mesons with different spin-parities. Having compared m with masses of the scalar and axial-vector states $m_{\mathrm{S}} = (12.697 \pm 0.090)$ GeV and $m_{\mathrm{AV}} = (12.715 \pm 0.090)$ GeV [32,33], one sees that they form almost a degenerate system of particles.

Heavy tetraquarks are an inseparable part of the exotic hadron spectroscopy. Structures $bc\bar{b}\,\bar{c}$ were not observed yet, but they can be seen in the future runs of the LHC and Future Circular Collider [22,23]. Publications devoted to fully heavy four-quark states are concentrated on analysis of their masses. Decays of these states, including $bc\bar{b}\,\bar{c}$ ones, did not become objects of detailed investigations. But besides masses, all conclusions about nature of discovered resonances have to be also based on knowledge about their decay channels and widths: This information is required for reliable interpretation of collected data and for planning new measurements.

DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX: THE CORRELATION FUNCTIONS $\Pi(M^2,s_0)$ AND $\Pi_1(\mathbf{M}^2,\mathbf{s}_0,q^2)$

This appendix contains the correlation functions $\Pi(M^2, s_0)$ and $\Pi_1(\mathbf{M}^2, \mathbf{s}_0, q^2)$, which have been applied to compute the mass of the tensor tetraquark T and partial width of the decay $T \to J/\psi\Upsilon$.

The correlation function $\Pi(M^2, s_0)$, which appears in the SRs has been presented in Eq. (17),

$$\Pi(M^2, s_0) = \int_{4\mathcal{M}^2}^{s_0} ds \rho^{\mathrm{OPE}}(s) e^{-s/M^2} + \Pi(M^2).$$

The components of the spectral density $\rho^{\mathrm{OPE}}(s)$ are given by the general expression

$$\rho(s) = \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \int_0^{1-\alpha-\beta} d\gamma \rho(s, \alpha, \beta, \gamma), \tag{A1}$$

where α, β, and γ are the Feynman parameters. The function $\Pi(M^2)$ is also determined by an Eq. (A1)-type formula with the integrand $\Pi(M^2, \alpha, \beta, \gamma)$.

The perturbative function $\rho^{\mathrm{pert}}(s, \alpha, \beta, \gamma)$ is given by the formula

$$\rho^{\mathrm{pert}}(s, \alpha, \beta, \gamma) = \frac{N^2\theta(N)}{512C^4A^4\pi^6}\{12C^2L_1^3s^2\alpha^3\beta^3\gamma^3 + 4A^2Cs\alpha\beta\gamma(3BCm_bm_c - 4L_1^2N\alpha\beta\gamma) + A^4(-4Cm_bm_c(3Cm_bm_c + N\alpha\beta) + N\gamma L_1(4Cm_bm_c + N\alpha\beta))\}. \tag{A2}$$

Here

$$N = -C[s\alpha\beta\gamma L_1 + A(m_c^2 L_3 - m_b^2(\beta+\gamma))]/A^2, \tag{A3}$$

and

$$A = \beta\gamma L_3 + \alpha^2(\beta+\gamma) + \alpha[\beta(\beta+2\gamma-1)+\gamma(\gamma-1)], \qquad B = \alpha^2(\beta-\gamma) - \gamma L_3^2 \\ + \alpha(\beta^2 - 2\gamma(\gamma-1) - \beta(\gamma+1)), \qquad C = \alpha\beta + \alpha\gamma + \beta\gamma. \tag{A4}$$

We also use the notations

$$L_1 = \alpha+\beta+\gamma-1, \qquad L_2 = \alpha+\beta-1, \qquad L_3 = \beta+\gamma-1, \qquad L_4 = \alpha+\gamma-1. \tag{A5}$$

The function $\Pi(M^2,\alpha,\beta,\gamma)$ has the following form:

$$\Pi(M^2,\alpha,\beta,\gamma) = -\frac{\langle\alpha_s G^2/\pi\rangle C}{384A^4 L_1\pi^4\beta\gamma}\exp\left[-\frac{A(m_b^2(\beta+\gamma)-L_3 m_c^2)}{M^2 L_1\alpha\beta\gamma}\right][m_b^2(\beta+\gamma)-L_3 m_c^2]^2 \\ \times (L_3^2 m_c^4 L_1\beta\gamma(L_3^2+3L_3\alpha+3\alpha^2) + L_1 m_b^4\beta\gamma^4(\beta+\gamma) - m_b^2 m_c^2\beta\gamma L_1(\beta(\beta-1)(2\alpha^2+2\alpha(L_2-\alpha) \\ + (\beta-1)^2) + \gamma((\beta-1)^2(3\beta-1) + \alpha^2(5\beta-2) + \alpha(\beta-1)(7\beta-2)) + \gamma^2(3-5\alpha+3\alpha^2 \\ + 8\beta(\alpha-1) + 5\beta^2) + \gamma^3(-4+3\alpha+5\beta) + 2\gamma^4) + m_b^3 m_c\beta\gamma(\alpha^3(\beta+\gamma)^2 + L_3\alpha\gamma(\beta+\gamma)(-3+3\beta+2\gamma) \\ + \alpha^2 L_3(\beta+\gamma)(\beta+3\gamma) + \gamma L_3(\beta(\beta-1)^2+\gamma+\beta\gamma(2\beta-3)+\gamma^2(2\beta-1))) \\ - m_b m_c^3(L_3^2\alpha\beta(\beta(\beta-1)^2+\gamma+\beta\gamma(6\beta-7)+\gamma^2(9\beta-5)+2\gamma^3) + \alpha^2(\beta^2(\alpha^2+2\alpha(L_2-\alpha) \\ + 2(\beta-1)^2)(-2+\alpha+2\beta) + \beta(2\alpha^3+7\alpha(\beta-1)(3\beta-1)+2\alpha^2(5\beta-3)+4(\beta-1)^2(5\beta-1))\gamma \\ + (\alpha+\alpha^3+2\alpha^2(4\beta-1)+4\beta(\beta-1)(9\beta-4)+\alpha\beta(25\beta-18))\gamma^2 + (2\alpha(\alpha-1)+\beta(11\alpha-19) \\ + 27\beta^2)\gamma^3 + \gamma^4(\alpha+7\beta)) + L_3^3\beta\gamma(-\gamma+\beta(-1+\beta+2\gamma)))). \tag{A6}$$

An explicit formula for $\rho^{\mathrm{Dim4}}(s,\alpha,\beta,\gamma)$ is rather cumbersome, therefore we do not write down it here.

The correlator $\Pi_1(\mathbf{M}^2,\mathbf{s}_0,q^2)$ is given by the formula Eq. (32)

$$\Pi_1(\mathbf{M}^2,\mathbf{s}_0,q^2) = \int_{4\mathcal{M}^2}^{s_0} ds\int_{4m_b^2}^{s_0} ds'\rho_1(s,s',q^2)e^{-s/M_1^2 - s'/M_2^2} + \mathcal{B}\Pi_1^{\mathrm{Dim4}}(p^2,p'^2,q^2). \tag{A7}$$

Here, the function $\rho(s,s',q^2)$ is determined by the expression

$$\rho_1(s,s',q^2) = \frac{3m_b m_c}{32\pi^4}\int_0^1 d\alpha\int_0^{1-\alpha} d\beta\int_0^{1-\alpha-\beta} d\gamma\frac{\theta(\Delta)}{(\alpha+\gamma)^2 L_4^2}, \tag{A8}$$

where

$$\Delta = -m_b^2 - \frac{1}{L_4}\left[q^2\frac{\alpha\gamma}{\alpha+\gamma} + m_c^2(\alpha+\gamma) - s'\beta L_1\right]. \tag{A9}$$

The dimension 4 function $\Pi_1^{\mathrm{Dim4}}(p^2,p'^2,q^2)$ is given by the formula

$$\Pi_1^{\mathrm{Dim4}}(p^2,p'^2,q^2) = \frac{\langle\alpha_s G^2/\pi\rangle m_b m_c}{192\pi^2}\int_0^1 d\alpha\int_0^{1-\alpha} d\beta\int_0^{1-\alpha-\beta}\frac{d\gamma}{\Delta^3 L_4^8(\alpha+\gamma)^5}\{-\Delta L_4^2(\alpha+\gamma)^2 \\ \times [2\alpha^4 + 6\beta^4 + \alpha(12\beta^3 + 30\beta^2(\gamma-1) + 27\beta(\gamma-1)^2 + 8(\gamma-1)^3) + 12\beta^3(\gamma-1) + 15\beta^2(\gamma-1)^2 \\ + 9\beta(\gamma-1)^3 + 2(\gamma-1)^4 + 3L_3\alpha^2(5\beta+4\gamma-4) + \alpha^3(9\beta+8\gamma-8)] + 2(\alpha+\gamma)^3 L_1\beta[m_b^2 L_4^2 \\ - p'^2 L_1\beta][2L_3\alpha + \alpha^2 + 2\beta^2 + 2\beta(\gamma-1) + (\gamma-1)^2] + 2L_4^3\alpha\gamma(q^2\alpha\gamma + m_c^2(\alpha+\gamma)^2)\}. \tag{A10}$$

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