Black holes and naked singularities in four dimensional dS and AdS Chamseddine gravity

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We analyze solutions of Chamseddine's topological gravity in four-dimensional spacetime and discover various black hole solutions, with and without torsion, as well as solutions that describe naked singularities. Since all the solutions belong to the sector with vanishing scalar fields, they share the peculiar trait that all conserved charges are zero.

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I. INTRODUCTION

Black holes (BH) are among the most fascinating objects in Universe. They possess a number of remarkable properties that serve, at the same time, ad guiding principles toward quantum theory of gravitation and the biggest unsolved problems in gravity.

A new chapter in our understanding of black holes, and gravity as a whole, started with the discovery of black hole thermodynamics. Besides energy, angular momentum and electric charge black holes have entropy and temperature, which are both very difficult to understand from the classical point of view. Hawking radiation [1] demonstrated that black holes are not really black and explained the origin of their temperature, but unveiled a new problem, the black hole information paradox. In general relativity (GR) entropy is proportional to the surface area of the event horizon, this is a well-established result that is obtained both by using Wald conserved charge approach [2] and Euclidean action [3]. Up to this day the microscopical understanding of BH entropy remains elusive.

A point that is not often emphasized is that black hole (BH) properties depend crucially on the underlying theory of gravity. As a result, many scenarios that are impossible within general relativity (GR) become possible in other theories. In theories with higher curvature terms and torsion, BH entropy can take a different form, not simply being proportional to the area of the event horizon [4,5], or involving a different proportionality factor than in GR [6–8]. The inclusion of matter leads to interesting results; for example, in [9], a static black hole solution with torsion is obtained for matter described by a conformally coupled scalar field.

Topological theories of gravity have been widely studied in odd dimensions [10–13], where many black hole solutions have been found and their thermodynamic properties studied in great detail. In contrast, Chamseddine's topological gravity [14] in even dimensions has not been explored nearly as extensively. To the authors' knowledge, the only preliminary investigation of black hole solutions was conducted in Ref. [15], while the Hamiltonian structure was studied in Ref. [16]. Recently, however, topological gravity in even dimensions has been explored in the context of holography, yielding a number of interesting results [17]. To gain a better understanding of four-dimensional topological gravity, we begin a preliminary analysis of its solutions, with a particular focus on black holes.

The paper is organized as follows: In Sec. II, we review the basic properties of Chamseddine's topological gravity in four dimensions. In Sec. III, we analyze static solutions (with and without torsion) of dS Chamseddine's gravity, considering spherical, hyperbolic, and planar symmetries. Section IV is dedicated to the AdS sector of the theory, while in Sec. V, we analyze the conserved charges. Section VI concludes the paper with final remarks.

The torsionless solutions, both in the dS and AdS sectors of the theory, do not coincide with the solutions found by Mignemi [15], who analyzed the second-order formulation of the theory in the Riemannian sector. The Appendix contains a brief discussion of the solution inside the horizon.

II. CHAMSEDDINE TOPOLOGICAL GRAVITY IN FOUR DIMENSIONS

Topological gravity in even dimensions was introduced by Chamseddine [14]. *A priori*, it does not include the vielbein as a dynamical variable but only spin connection

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ω^{AB} and scalar field Φ^A. The action in four dimensions is given by

$$S = \epsilon_{ABCDE} F^{AB} F^{CD} \Phi^E, \tag{2.1}$$

where F^{AB} is curvature

$$F^{AB} = d\omega^{AB} + \omega^A{}_C \omega^{CB}. \tag{2.2}$$

Capital Latin indices $A, B, C \ldots$ may correspond to Lorentz or (anti–)de Sitter groups. When the groups in question are (anti–)de Sitter, we can introduce the vielbein as part of the connection, and we will do so explicitly in the following text. It is easy to find the field equations by varying the action with respect to basic dynamical variables.

Field equation for scalar field Φ^A is

$$\epsilon_{ABCDE} F^{AB} F^{CD} = 0. \tag{2.3}$$

Equation of motion for spin connection is given by

$$\epsilon_{ABCDE} F^{CD} D\Phi^E = 0, \tag{2.4}$$

where D is exterior covariant derivative for the connection ω^{AB}.

A. de Sitter

Let us first analyze de Sitter group $SO(1,4)$, which represents the group of isometries of the metric $\mathrm{diag}(+,-,-,-,-)$. We can separate indices A into Lorentz a and additional index 4. With this splitting we can define vielbein e^a and separate the scalar field into Lorentz vector scalar field ϕ^a and true scalar ϕ

$$\omega^{a4} = \frac{e^a}{\ell}, \qquad \Phi^a = \phi^a, \qquad \Phi^4 = \phi. \tag{2.5}$$

Here ℓ is the constant with dimension of length, in the rest of the paper we shall set $\ell = 1$. Curvature separates into Lorentz curvature and torsion

$$F^{ab} = R^{ab} + e^a e^b, \tag{2.6a}$$

$$F^{a4} = T^a. \tag{2.6b}$$

We also introduce the notation $\epsilon_{abcd4} = \epsilon_{abcd}$. Field equations for scalar field Φ^A (2.3) after introducing Lorentz indices take the form

$$\epsilon_{abcd}(R^{ab} + e^a e^b)(R^{cd} + e^c e^d) = 0, \tag{2.7a}$$

$$\epsilon_{abcd} T^b (R^{cd} + e^c e^d) = 0. \tag{2.7b}$$

Field equation for spin connection (2.4) in Lorentz indices also splits into two equations

$$\epsilon_{abcd}(R^{bc} + e^b e^c)(\nabla\phi^d - \phi e^d) = 0, \tag{2.8a}$$

$$\epsilon_{abcd}[(R^{cd} + e^c e^d)(d\phi - \phi_i e^i) - 2T^c(\nabla\phi^d - \phi e^d)] = 0. \tag{2.8b}$$

B. Anti–de Sitter

Anti–de Sitter (AdS) group $SO(2,3)$ is isometry of the metric $\mathrm{diag}(+,-,-,-,+)$. As in the dS case we separate indices into Lorentz a and an additional denoted by 4. The curvature splits into Lorentz curvature and torsion

$$F^{ab} = R^{ab} - e^a e^b, \tag{2.9a}$$

$$F^{a4} = T^a. \tag{2.9b}$$

The equation of motion for scalar Φ^A (2.3) again separates into two equations

$$\epsilon_{abcd}(R^{ab} - e^a e^b)(R^{cd} - e^c e^d) = 0, \tag{2.10a}$$

$$\epsilon_{abcd} T^b (R^{cd} - e^c e^d) = 0. \tag{2.10b}$$

The equation of motion for the spin connection (2.4) in Lorentz indices is rewritten as two equations

$$\epsilon_{abcd}(R^{bc} - e^b e^c)(\nabla\phi^d + \phi e^d) = 0, \tag{2.11a}$$

$$\epsilon_{abcd}[(R^{cd} - e^c e^d)(d\phi - \phi_i e^i) - 2T^c(\nabla\phi^d + \phi e^d)] = 0. \tag{2.11b}$$

III. DS CHAMSEDDINE GRAVITY

We shall analyze static solutions of Chamseddine's gravity with high symmetry, namely: spherical, hyperbolic and planar. All the solutions that we obtain are in the sector where all scalars vanish

$$\phi^a = \phi = 0. \tag{3.1}$$

Therefore, we will not explicitly state that the scalars are zero in the subsequent text when describing the solutions.

Due to the complex nature of the equations of motion, the solutions that we have found are either with all scalars equal to zero or with at least one arbitrary function remaining when some scalars are nonzero. Solutions with nonzero scalars and without arbitrary functions are left for future research.

For the simplification of the calculations we use xAct *Mathematica* package [18] that is available at [19].

A. Spherical

Vielbein for static and spherically symmetric space-time is of the form

$$e^0 = B(r)dt, \qquad e^1 = \frac{dr}{B(r)}, \qquad e^2 = rd\theta, \qquad e^3 = r\sin\theta d\varphi. \tag{3.2}$$

The most general static and spherically symmetric ansatz for the connection is of the form

$$\omega^{01} = F_0(r)dt + F_1(r)dr, \qquad \omega^{02} = F_2(r)d\theta, \qquad \omega^{03} = F_2(r)\sin\theta d\varphi$$
$$\omega^{12} = F_3(r)d\theta, \qquad \omega^{13} = F_3(r)\sin\theta d\varphi, \qquad \omega^{23} = \cos\theta d\varphi. \tag{3.3}$$

Since we are not looking for the most general solution, we shall make further restrictions in the ansatz

$$F_1(r) = F_3(r) = 0. \tag{3.4}$$

With this ansatz the solution for metric function is

$$B(r) = r\sqrt{\frac{r^2}{r_h^2} - 1}, \tag{3.5}$$

and nonzero spin connection functions are

$$F_0(r) = -r \qquad F_2(r) = \pm\frac{\sqrt{r^4 - r^2 + k}}{r}, \tag{3.6}$$

where r_h and k are constants, with the condition that $k \neq 0$. If $k = 0$ then we are left with only one independent equation that determines $\phi^1(r)$ in terms of $B(r)$, consequently for $k = 0$ solution is not uniquely determined.

The nonzero components of torsion are given by

$$T^0 = -\frac{2B(r)}{r}e^0e^1, \qquad T^1 = 0,$$
$$T^2 = B(r)e^1e^2 + F_2(r)e^0e^2,$$
$$T^3 = B(r)e^1e^3 + F_2(r)e^0e^3, \tag{3.7}$$

while Cartan curvature reads

$$R^{01} = e^0e^1, \qquad R^{02} = \frac{B(r)F_2(r)'}{r}e^1e^2,$$
$$R^{12} = \frac{F_2(r)}{B(r)}e^0e^2, \qquad R^{03} = \frac{B(r)F_2(r)'}{r}e^1e^3,$$
$$R^{12} = \frac{F_2(r)}{B(r)}e^0e^3, \qquad R^{23} = -\frac{F_2(r)^2+1}{r^2}e^2e^3, \tag{3.8}$$

where $F_2(r)' \coloneqq dF_2(r)/dr$. Then Cartan curvature scalar is

$$R = 2\left(2 - r^2 - \frac{k+1}{r^2}\right), \tag{3.9}$$

and it is divergent for $r = 0$. The quadratic curvature invariant reads

$$R^{ijkl}R_{ijkl} = 2r^2\left(1 + \frac{(1+F_2(r)^2)^2}{r^4} - \frac{2F_2(r)^2}{B(r)^2} - \frac{2B(r)^2(F_2(r)')^2}{r^2}\right), \tag{3.10}$$

and it is also divergent for $r = 0$. The important point is that quadratic curvature invariant is *a priori* divergent at the horizon $r = r_h$, for arbitrary r_h and k. By demanding that it is finite at the horizon we determine value of the constant k as function of location of the horizon r_h

$$k = r_h^2 - r_h^4. \tag{3.11}$$

Finally, the Ricci scalar of the Riemannian curvature is given by

$$\tilde{R} = -12 - \frac{2}{r} + \frac{30r^2}{r_h^2}, \tag{3.12}$$

and it is divergent at $r = 0$, meaning that this solution describes spherically symmetric black hole. It is worth noting, that in order to obtain Ricci scalar, we continued metric inside the horizon, as it is normally done when studding BH solutions.

The spin connection is singular inside of the horizon, because the square root becomes complex for $r < r_h$. We note that the same applies for metric function $B(r)$, indicating that the obtained solution is valid only outside the horizon, i.e., for $r \geq r_h$. Therefore, we only need to require that the spin connection is real outside the horizon. The solution inside the horizon is discussed in the Appendix.

B. Hyperbolic

The vielbein is given by

$$e^0 = B(r)dt, \qquad e^1 = \frac{dr}{B(r)},$$
$$e^2 = rd\theta, \qquad e^3 = r\sinh\theta d\varphi. \tag{3.13}$$

The hyperbolic connection takes form

$$\omega^{01} = F_0(r)dt + F_1(r)dr, \qquad \omega^{02} = F_2(r)d\theta, \qquad \omega^{03} = F_2(r)\sinh\theta d\varphi,$$
$$\omega^{12} = F_3(r)d\theta, \qquad \omega^{13} = F_3(r)\sinh\theta d\varphi, \qquad \omega^{23} = \cosh\theta d\varphi. \tag{3.14}$$

The metric function is the same as in spherically symmetric case

$$B(r) = r\sqrt{\frac{r^2}{r_h^2} - 1}. \tag{3.15}$$

Nonzero spin connection functions are

$$F_0(r) = -r, \qquad F_2(r) = \pm\frac{\sqrt{r^4 + r^2 + k}}{r}, \tag{3.16}$$

where r_h and k are constants and $k \neq 0$ for the same reason as in spherical case.

The torsion and Cartan curvature are given by the same expressions as in the spherical case, Eqs. (3.7) and (3.8), with the vielbein and spin connection replaced by their hyperbolic counterparts. The quadratic curvature invariant has the same form as in the spherical case, Eq. (3.10), and from the condition that it remains regular at the horizon, we can determine the value of the constant k

$$k = -r_h^2 - r_h^4. \tag{3.17}$$

The Ricci scalar is given by

$$\tilde{R} = -12 + \frac{2}{r} + \frac{30r^2}{r_h^2}, \tag{3.18}$$

and it is divergent at the origin, proving that this solution describes black hole with hyperbolic symmetry.

C. Planar

The vielbein with planar symmetry is given by

$$e^0 = B(r)dt, \qquad e^1 = \frac{dr}{B(r)}, \qquad e^2 = rdx, \qquad e^3 = rdy. \tag{3.19}$$

The nonzero components of spin connection are assumed to be of the same form as in previous cases

$$\omega^{01} = F_0(r)dt, \qquad \omega^{02} = F_2(r)dx, \qquad \omega^{03} = F_2(r)dy. \tag{3.20}$$

The metric function is, again, the same

$$B(r) = r\sqrt{\frac{r^2}{r_h^2} - 1}, \tag{3.21}$$

while the spin connection functions are

$$F_0(r) = -r, \qquad F_2(r) = \pm\frac{\sqrt{r^4 + k}}{r}, \tag{3.22}$$

where r_h and k are constants. The crucial point, as before, is that k is nonvanishing. The torsion and Cartan curvature are again of the same form as in the spherical case, Eqs. (3.7) and (3.8), with the vielbein and spin connection adapted for the planar case. The quadratic curvature invariant is the same as in the spherical case, Eq. (3.10), and from the requirement that it remains regular at the horizon, we can determine the value of the constant k

$$k = -r_h^4. \tag{3.23}$$

The Cartan curvature scalar

$$R = -\frac{k}{r^4}, \tag{3.24}$$

and it is divergent for $r = 0$.

The Ricci scalar is regular

$$\tilde{R} = -12 + \frac{30r^2}{r_h^2}. \tag{3.25}$$

Since everything is regular at the surface $r = r_h$, it represents a coordinate singularity. The divergence of the Cartan curvature scalar at $r = 0$ indicates that this solution corresponds to a black hole with a planar horizon.

D. Solutions without torsion

The form of the spherical (3.2), hyperbolic (3.13) and planar (3.19) vielbein is the same as before. The torsion zero condition, $T^i = 0$, determines the value of the spin connection in terms of metric function, and the only equation of motion that is not identically satisfied leads to the solution for the metric function

$$B(r) = \sqrt{1 - r^2 \pm \sqrt{a + br}}. \tag{3.26}$$

The solution (3.26) is not identical to the solution found by Mignemi [15]. The discrepancy arises from the fact that Mignemi's solution [15] does not represent the solution of the equations of motion (2.7). Mignemi focused on the torsionless sector of the theory by considering the second order formulation of the action. At one point, an additional term was introduced into the action, leading to the equations of motion which are nonequivalent to the original ones.

The actions of the gravitational theory in the first and second order formulation may not lead to the same equations of motion and the theories may have different number of degrees of freedom. For example, in the Riemannian sector of the topological three-dimensional Mielke-Baeckler model [20] the equations of motion are equivalent to ones of three-dimensional GR. However, if we simply substitute Levi-Chivita (Riemannian) connection into the action (instead of the Lorentz connection) we obtain the action of the topologically massive gravity, a theory that possesses a propagating degree of freedom and whose equations of motion contain an additional term—the Cotton tensor.

The function $B(r)$ has only one real zero and, since for the large values of r the term $-r^2$ dominates, this zero represents cosmological horizon. The spin connection is given by

$$F_1 = F_2 = 0, \tag{3.27a}$$

$$F_0 = -B(r)B'(r) = r \mp \frac{b}{4\sqrt{a+br}}, \tag{3.27b}$$

$$F_3(r) = B(r) = \sqrt{1 - r^2 \pm \sqrt{a+br}}. \tag{3.27c}$$

The reality condition for the metric at $r = 0$ implies

$$a \geq 0, \tag{3.28}$$

while for the choice of minus sign in $B(r)$ we must also ensure that a is not greater than one. In the spherically symmetric case the Ricci scalar R diverges at $r = 0$

$$R = -12 \pm \frac{8a^2 + 24abr + 15b^2r^2}{4r^2(a+br)^{3/2}}, \tag{3.29}$$

unless both a and b are zero. Consequently, this solution represents a naked singularity, except when $a = b = 0$ in which case it reduces to dS spacetime.

In the hyperbolic case the Ricci scalar is divergent even when $a = b = 0$

$$R = -12 \mp \frac{b^2}{4(a+br)^{3/2}} \pm \frac{2(a + 2br + 2\sqrt{a+br})}{r^2\sqrt{a+br}}. \tag{3.30}$$

In the planar case the situation is the same as in the hyperbolic case

$$R = -12 \mp \frac{b^2}{4(a+br)^{3/2}} + \frac{2(\pm a \pm 2br + 2\sqrt{a+br})}{r^2\sqrt{a+br}}, \tag{3.31}$$

the solution has naked singularity at $r = 0$ even when $a = b = 0$.

IV. ADS CHAMSEDDINE GRAVITY

A. Black hole solutions without torsion

We use the same ansatz for vielbein as in dS case, namely (3.2) for spherical, (3.13) for hyperbolic and (3.19) for planar.

The only equation of motion that is not identically satisfied leads to the solution for metric function

$$B(r) = \sqrt{1 + r^2 \pm \sqrt{a+br}}. \tag{4.1}$$

Again, as in the dS sector, the solution is different from the one found by Mignemi.

The torsion zero condition gives the value of the spin connection

$$F_1(r) = F_2(r) = 0, \tag{4.2a}$$

$$F_0(r) = -B(r)B'(r) = -r \mp \frac{b}{4\sqrt{a+br}}, \tag{4.2b}$$

$$F_3(r) = B(r)\sqrt{1 + r^2 \pm \sqrt{a+br}}. \tag{4.2c}$$

For the choice of the minus sign in the metric function B, we can obtain black hole solutions with one or two horizons, as well as naked singularities. The presence of two parameters allows for tuning in such a way that a horizon appears. However, for the plus sign, the solutions represent naked singularities for all values of the parameters a and b.

In spherically symmetric case the Ricci scalar for the Riemannian curvature $\tilde{R}$ diverges at $r = 0$

$$\tilde{R} = 12 \pm \frac{8a^2 + 24abr + 15b^2r^2}{4r^2(a+br)^{3/2}}, \tag{4.3}$$

unless both a and b are zero, which describes AdS spacetime.

In the hyperbolic case the Ricci scalar is divergent even when $a = b = 0$

$$\tilde{R} = 12 \mp \frac{b^2}{4(a+br)^{3/2}} \pm \frac{2(a + 2br + 2\sqrt{a+br})}{r^2\sqrt{a+br}}. \tag{4.4}$$

In the planar case we encounter the same situation as in the hyperbolic case

$$\tilde{R} = 12 \mp \frac{b^2}{4(a+br)^{3/2}} + \frac{2(\pm a \pm 2br + \sqrt{a+br})}{r^2\sqrt{a+br}}, \tag{4.5}$$

i.e., the solution has a singularity at $r = 0$, even when $a = b = 0$.

Solutions with the minus sign in front of the square root can have one or two horizons, depending on the values of the parameters a and b, as we have verified for certain choices.

B. Solutions with torsion

We use the same ansatz for vielbein and spin connection as in the dS case.

The solution for the metric function reads

$$B(r) = r\sqrt{1 \pm \frac{r^2}{r_h^2}}, \tag{4.6}$$

while the nonzero spin connection functions are

$$F_0(r) = r, \qquad F_2(r) = \pm\frac{\sqrt{-r^4 + sr^2 + k}}{r}, \tag{4.7}$$

where r_h and k are constants and s is equal to 0 for planar, $+1$ for hyperbolic and -1 for spherical case. The crucial point, as before, is that we must have $k \neq 0$ to obtain a unique solution. The signs in the metric function B and spin connection function F_2 are independent, i.e., for any choice of signs there is a solution.

In spherical and hyperbolic case the Ricci scalar R is divergent at $r = 0$

$$R_{\text{Spherical}} = 12 - \frac{2}{r^2} \pm 30\frac{r^2}{r_h^2}, \tag{4.8}$$

$$R_{\text{Hyperbolic}} = 12 + \frac{2}{r^2} \pm 30\frac{r^2}{r_h^2}, \tag{4.9}$$

meaning that these solutions describe naked singularities. In the planar case the Ricci scalar is regular everywhere

$$R = 12 \pm 30\frac{r^2}{r_h^2}. \tag{4.10}$$

V. CHARGES

The simplest and most straightforward way to determine the charges in the case where all scalars are zero is by using the covariant phase space formalism [21].

The variation of the action reads

$$\delta S = \int \text{EOM} + d(\epsilon_{ABCDE}\delta\omega^{AB}F^{CD}\Phi^E), \tag{5.1}$$

from which we determine the presymplectic potential current

$$\theta = \epsilon_{ABCDE}\delta\omega^{AB}F^{CD}\Phi^E. \tag{5.2}$$

In the sector of the theory where all scalars are equal to zero presymplectic potential current vanishes

$$\theta = 0. \tag{5.3}$$

As a direct consequence, presymplectic form current is also zero in this sector

$$\omega = \delta\theta = 0. \tag{5.4}$$

This implies that all the charges are zero when all scalars vanish, and that the number of degrees of freedom is also zero in this sector. For explicit calculations of charges from the symplectic form, see, for example, [22].

VI. CONCLUDING REMARKS

We investigated the solutions of Chamseddine's topological gravity in four dimensions. Since the equations of motion are quite complicated, we were only able to find solutions in which all scalar fields vanish. This corresponds to the trivial sector of the theory, as all charges and degrees of freedom are zero. Despite being trivial from the perspective of charges, this sector still features a rich solution space.

To simplify the equations as much as possible, we focused on highly symmetric solutions and obtained static solutions with spherical, hyperbolic, and planar symmetries. In the de Sitter case, the solutions with torsion represent black holes, while those without torsion describe naked singularities. In contrast, in the anti–de Sitter sector, solutions without torsion correspond to black holes. The Riemannian solutions in both the de Sitter and anti–de Sitter sectors differ from those found in [15] due to Mignemi's consideration of the second-order formulation of the theory.

Black holes in de Sitter Chamseddine gravity feature an arbitrary nonzero constant k in spin connection. By requiring regularity of the solution at $r = r_h$, from quadratic Cartan curvature invariant we determined the value of k in terms of r_h. Only solutions with that special value of k describe black holes; otherwise there is a singularity at $r = r_h$. Additionally, black holes in dS Chamseddine gravity exhibit the peculiar property that one cannot shrink the horizon radius to zero and obtain regular solutions that are not black holes.

More general solutions, with nonzero scalar fields, which correspond to the more interesting sector of the theory, are left for further study.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX: SOLUTION INSIDE OF THE HORIZON

1. dS black holes

We search for a solution inside of the horizon by first extending metric $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$ to have the same functional form inside of the horizon as it has outside. Because the metric components g_{tt} and g_{rr} change signs inside of the horizon, effectively r becomes time coordinate and t space coordinate.

The vielbein inside of the horizon $r < r_h$ is determined from the metric inside of the horizon in the standard way. Vielbeins e^2 and e^3 remain the same as outside of the horizon, while e^0 and e^1 are given by

$$e^0 = \frac{dr}{r\sqrt{1-\frac{r^2}{r_h^2}}}, \qquad e^1 = r\sqrt{1-\frac{r^2}{r_h^2}}dt. \tag{A1}$$

The form of the spin connection remains the same as the ansatz for connection, while the value of functions are obtained by demanding the same form of torsion as outside of the horizon. The solution that fulfills this requirements is

$$F_0(r) = r, \qquad F_3(r) = \mp\frac{\sqrt{-r^4 - sr^2 - k}}{r}, \tag{A2}$$

where s is ± 1 or zero depending on the topology of the horizon, as used previously in the text.

One could have guessed the result by noticing that inside of the horizon we changed places of e^1 and e^0, effectively making a replacement of indices $0 \leftrightarrow 1$. With the addition that functions under square root should change sign to become positive the same way as in vielbein.

2. AdS black holes

The vielbein inside of the horizon $r < r_h$ is

$$e^0 = \frac{dr}{\sqrt{-1 - r^2 + \sqrt{a+br}}},$$
$$e^1 = \sqrt{-1 - r^2 + \sqrt{a+br}}dt, \tag{A3}$$

while e^2 and e^3 remain the same as outside of the horizon. The condition that torsion is zero determines spin connection functions

$$F_0(r) = r - \frac{b}{4\sqrt{a+br}}$$
$$F_2(r) = -\sqrt{-1 - r^2 + \sqrt{a+br}}. \tag{A4}$$

Note that we assume the values of a and b are such that there is only one horizon. As mentioned earlier, for certain choices, there may be no horizon at all, or there could even be two horizons.

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