Letter

Superrotations at spacelike infinity

Adrien Fiorucci, [1,*] Javier Matulich, [2,†] and Romain Ruzziconi [3,‡]

[1] Institute for Theoretical Physics, TU Wien, Wiedner Hauptstrasse 8–10/136, A-1040 Vienna, Austria
[2] Instituto de Física Teórica UAM/CSIC, Calle Nicolás Cabrera 13-15, Universidad Autónoma de Madrid, Cantoblanco, Madrid 28049, Spain
[3] Mathematical Institute, University of Oxford, Andrew Wiles Building, Radcliffe Observatory Quarter, Woodstock Road, Oxford, OX2 6GG, United Kingdom

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We propose a consistent set of boundary conditions for gravity in asymptotically flat spacetime at spacelike infinity, which yields an enhancement of the Bondi-Metzner-Sachs group with smooth superrotations and new subleading symmetries. These boundary conditions are obtained by allowing fluctuations of the boundary structure which are responsible for divergences in the symplectic form, and a renormalization procedure is required to obtain finite canonical generators. The latter are then made integrable by incorporating boundary terms into the symplectic structure, which naturally derive from a linearized spin-two boundary field on a curved background with positive cosmological constant. Finally, we show that the canonical generators form a nonlinear algebra under the Poisson bracket and verify the consistency of this structure with the Jacobi identity.

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Introduction. Gravity in four-dimensional asymptotically flat spacetime constitutes a model for a large range of phenomena, from the scattering of elementary particles to the description of astrophysical systems below the cosmological scale. It came as a surprise when Bondi, van der Burg, Metzner, and Sachs highlighted that the asymptotic symmetry group of such spacetimes is not only Poincaré, but an infinite-dimensional enhancement of the latter, called the (global) BMS group [1,2], which is a semidirect product between the Lorentz group and the supertranslations.

It was later shown that this asymptotic symmetry group could itself be consistently enhanced by including all the conformal transformations on the two-dimensional celestial sphere, called the superrotations [3–6] (see also [7–10]). Further extensions were obtained by considering asymptotically locally flat spacetimes with fluctuating boundary metric at null infinity, allowing for all the diffeomorphisms on the celestial sphere [11–15], or Weyl symmetries [16,17], see e.g. [18,19] for recent reviews.

The BMS symmetries play a major role in the understanding of the infrared structure of quantum gravity [20]. Remarkably, assuming antipodal matching conditions between past and future null infinity, it was shown that the supertranslation Ward identity is equivalent to Weinberg's leading soft graviton theorem [21,22]. Similarly, superrotation invariance of the gravitational scattering was shown to be related to the subleading soft graviton theorem [23–25] at all orders of perturbation [26–34]. The BMS transformations were also explicitly connected to memory effects, which constitute potential observable phenomena in gravitational astronomy (see e.g. [35]). More precisely, the supertranslations describe a displacement memory effect [36], while the superrotations correspond to spin [37], center-of-mass [38] and superboost/velocity kick memories [13].

The global BMS symmetries were originally discovered by studying the boundary structure of asymptotically flat spacetimes at null infinity. Strikingly, while the Poincaré symmetries had been found for a long time at spacelike infinity [39], it is only recently that the BMS group has been uncovered there [40–43]. The advantage of working at spacelike infinity is that it requires less regularity assumptions on the class of spacetimes that are considered, since it does not assume the existence of a smooth null infinity [44–49]. Furthermore, as discussed in [40–42,50–55], it allows to derive the antipodal matching conditions necessary to establish the equivalence between BMS Ward identities and soft theorems [21,22,24] in a well-posed formulation of the scattering problem.

However, up to now, only the global version of the BMS group has been found at spacelike infinity. In particular, the

*Contact author: adrien.fiorucci@tuwien.ac.at
†Contact author: javier.matulich@csic.es
‡Contact author: romain.ruzziconi@maths.ox.ac.uk

superrotations are still missing, and due to their crucial role for the physics in asymptotically flat spacetime, it is of paramount importance to find these symmetries at spacelike infinity. In this paper, we address this challenging problem through a purely Hamiltonian approach and derive a phase space at spacelike infinity that accommodates smooth superrotations, coined as *Spi superrotations* (this terminology is introduced by analogy with the Spi supetranslations of [56–58]). To do so, inspired by the analysis at null infinity [11–13], we propose a new set of boundary conditions for asymptotically locally flat spacetimes by relaxing those considered in [40–43] and allowing some fluctuation of the boundary structure. This makes the analysis technically very demanding and requires the use of recently developed methods: (i) a renormalization of the action and the symplectic form is necessary to ensure the finiteness of the canonical generators (see e.g. [13,16,17,59–73]); (ii) a field-dependent redefinition of the symmetry parameters [50,65,74–80], as well as the introduction of boundary degrees of freedom [81–83], are needed to render the canonical generators integrable; (iii) the treatment of nonlinear terms to compute the resulting asymptotic symmetry algebra, which is reminiscent of what happens in higher dimensions where already the global BMS group is realized as a nonlinear algebra at spacelike infinity [84,85].

Note. This paper is accompanied by some Supplemental Material [86] collecting technical and intermediate results that are not essential to follow the reasoning, but shall be quoted for the interested reader. Equations reported in this appended document are numbered as (S.N).

Solution space. The canonical action for Einstein's gravity in four dimensions is given by [87–89]

$$\mathcal{S}=\int \mathrm{d}t\,\mathrm{d}^3x\mathcal{L}_H,\qquad \mathcal{L}_H=\pi^{ij}\dot{g}_{ij}-N\mathcal{H}-N^i\mathcal{H}_i \quad (1)$$

where g_{ij} is the three-dimensional metric on the (spacelike) constant time slices, π^{ij} its conjugate momentum and N, N^i correspond respectively to the lapse and shift functions. We work in units such that $16\pi G=1$. We write R_{ij} the spatial Ricci curvature, R its trace, $_{|i}$ the spatial covariant derivative and $g=\det g_{ij}$. The variation of the action with respect to the lapse and the shift yields the Hamiltonian and momentum constraints

$$\begin{aligned}\mathcal{H}&=-\sqrt{g}R+\frac{1}{\sqrt{g}}\left(\pi^{ij}\pi_{ij}-\frac{1}{2}\pi^2\right)\approx 0,\\ \mathcal{H}_i&=-2\pi_i{}^j{}_{|j}\approx 0.\end{aligned} \quad (2)$$

In the following of the paper, the weak equality symbol $\approx$ denotes the equality on the constraint surface. We introduce spherical coordinates $x^i=(r,x^A)$ on constant time slices and we are interested in the asymptotic behavior of the dynamical fields around spacelike infinity, located on the two sphere of radius $r\to+\infty$. We propose the following boundary conditions:

$$\begin{aligned}g_{rr}&=1+\frac{\bar{h}_{rr}}{r}+\frac{\bar{h}^{(2)}_{rr}}{r^2}+\mathcal{O}(r^{-3}),\\ g_{rA}&=\bar{\lambda}_A+\frac{\bar{h}_{rA}}{r}+\mathcal{O}(r^{-2}),\\ g_{AB}&=r^2\bar{G}_{AB}+r\bar{h}_{AB}+\bar{h}^{(2)}_{AB}+\mathcal{O}(r^{-1}),\\ \pi^{rr}&=r\bar{P}^{rr}+\bar{\pi}^{rr}+\frac{\bar{\pi}^{rr}_{(2)}}{r}+\mathcal{O}(r^{-2}),\\ \pi^{rA}&=\frac{\bar{\pi}^{rA}}{r}+\frac{\bar{\pi}^{rA}_{(2)}}{r^2}+\mathcal{O}(r^{-3}),\\ \pi^{AB}&=\frac{\bar{P}^{AB}}{r}+\frac{\bar{\pi}^{AB}}{r^2}+\frac{\bar{\pi}^{AB}_{(2)}}{r^3}+\mathcal{O}(r^{-4}),\end{aligned} \quad (3)$$

where the transverse boundary metric $\bar{G}_{AB}$ is a general metric on the two-dimensional sphere. We also impose the following parity conditions:

$$\begin{aligned}&\bar{G}_{AB}\sim\bar{h}_{rr}\sim\text{even},\qquad \bar{P}^{AB}\sim\text{odd},\\ &\bar{\pi}^{rr}-\bar{\pi}+\bar{h}_{rr}\bar{P}-\bar{P}^{AB}\bar{h}_{AB}\sim\text{odd},\end{aligned} \quad (4)$$

under the antipodal map $x^A\mapsto -x^A$ on the sphere. Throughout, we denote the trace of any tensor T_{AB} on the sphere by $T=\bar{G}^{AB}T_{AB}$.

These boundary conditions are weaker than those considered in [40–43] since now, the leading orders $\bar{G}_{AB}$, $\bar{P}_{AB}$ and $\bar{P}^{rr}$ are allowed to fluctuate (by this, we mean that they are not fixed on the solution space). One recovers the boundary conditions of [40–43] by setting $\bar{P}_{AB}=0=\bar{P}^{rr}$ and $\bar{G}_{AB}=\bar{\gamma}_{AB}$, the unit-round sphere metric. In particular, the parity conditions (4) generalize those of these previous references, in which $\bar{h}_{rr}$ and the combination $(\bar{\pi}^{rr}-\bar{\pi})$ were already assumed to have odd parity, and the parity of $\bar{G}_{AB}$ agrees with the one of the unit-round sphere metric. The relaxation of the definition of asymptotic flatness considered here is inspired by the analysis at null infinity, where fluctuations of the transverse boundary metric are required to obtain $\mathrm{Diff}(S^2)$ superrotations [11–13]. Taking into account the falloff conditions

$$N=1+\mathcal{O}(r^{-1}),\quad N^r=\mathcal{O}(r^{-1}),\quad N^A=\mathcal{O}(r^{-2}), \quad (5)$$

on the lapse and the shift one can show that the present boundary conditions are compatible with the typical behavior of asymptotically locally flat spacetime, i.e., the Riemann tensor vanishes when $r\to+\infty$.

The boundary conditions (3) and (4) are invariant under hypersurface deformations generated by

$$\xi^\perp = rb + f + \frac{\epsilon}{r} + \mathcal{O}(r^{-2}),$$
$$\xi^r = W + \frac{\epsilon^r}{r} + \mathcal{O}(r^{-2}),$$
$$\xi^A = Y^A + \frac{\epsilon^A}{r} + \frac{\epsilon^A_{(2)}}{r^2} + \mathcal{O}(r^{-3}), \tag{6}$$

where $\xi^\perp$, ξ^r, and ξ^A denote respectively infinitesimal deformations in the normal, radial and angular directions. In Eq. (6), the parameters f, W, ϵ, ϵ^r, ϵ^A, $\epsilon^A_{(2)}$ are arbitrary functions on the sphere, and b, Y^A are parity-odd functions by consistency with Eq. (4). Acting on the radial expansions in Eq. (3) with the deformations (6) in the standard way, see e.g. [87,88], yields the transformations of each field defined in these expansions.

For the BMS boundary conditions of [40–43], fixing the leading structure yields $\mathcal{L}_Y\bar{\gamma}_{AB} = 0$ and $\bar{D}_A\bar{D}_B b + \bar{\gamma}_{AB} b = 0$ whose solutions define the Lorentz generators (Y^A parametrizes the three rotations and b the three boosts). In our case, Y^A and b are not forced to obey these constraints and contain an infinite tower of modes, which we will identify later as the Spi superrotations. Moreover, it can be shown that one can consistently set $\bar{\lambda}_A = 0 = \bar{h}_{rA}$ by adjusting the subleading parameters ϵ^A and $\epsilon^A_{(2)}$, which we will assume from now on.

Symplectic structure. Taking one variation of the Lagrangian density $\boldsymbol{\mathcal{L}}_H = \mathcal{L}_H \mathrm{d}t\mathrm{d}^3x$ gives

$$\delta\boldsymbol{\mathcal{L}}_H = (A^{ij}\delta g_{ij} + B_{ij}\delta\pi^{ij})\mathrm{d}t\mathrm{d}^3x - \mathrm{d}\boldsymbol{\Theta} \tag{7}$$

where $A^{ij} = 0$, $B_{ij} = 0$ are the Hamilton (Einstein) equations and the three-form $\boldsymbol{\Theta}$ is the presymplectic potential obtained by keeping the boundary terms in the integrations by parts and from which the presymplectic current $\boldsymbol{\omega} \equiv \delta\boldsymbol{\Theta}$ derives. Given any Cauchy slice Σ, the presymplectic structure $\Omega = \int_\Sigma \boldsymbol{\omega}$ is then given by [87,88]

$$\Omega[\delta g_{ij}, \delta\pi^{ij}] = \int_\Sigma \mathrm{d}^3x \delta\pi^{ij} \wedge \delta g_{ij}. \tag{8}$$

For our boundary conditions (3), the latter is linearly divergent in r due to the fluctuation of the leading fields $\bar{G}_{AB}, \bar{P}^{AB}$. To renormalize these divergences, we supplement the action with some appropriate boundary term $\boldsymbol{\mathcal{B}}$, i.e. $\bar{\boldsymbol{\mathcal{L}}}_H \equiv \boldsymbol{\mathcal{L}}_H - \mathrm{d}\boldsymbol{\mathcal{B}}$ [60–64]. We have

$$\delta\boldsymbol{\mathcal{B}} = \frac{\delta\boldsymbol{\mathcal{B}}}{\delta\phi}\delta\phi - \mathrm{d}\boldsymbol{\theta}_b, \qquad \boldsymbol{\omega}_b = \delta\boldsymbol{\theta}_b \tag{9}$$

where ϕ collectively denotes relevant boundary degrees of freedom. The incorporation of the boundary term then modifies the presymplectic potential as [60–64]

$$\bar{\boldsymbol{\Theta}} = \boldsymbol{\Theta} - \delta\boldsymbol{\mathcal{B}} - \mathrm{d}\boldsymbol{\theta}_b, \qquad \bar{\boldsymbol{\omega}} = \boldsymbol{\omega} + \mathrm{d}\boldsymbol{\omega}_b \tag{10}$$

such that the new symplectic structure

$$\bar{\Omega}[\delta g_{ij}, \delta\pi^{ij}] = \int_\Sigma \mathrm{d}^3x\, \delta\pi^{ij} \wedge \delta g_{ij} + \oint_{\partial\Sigma} \boldsymbol{\omega}_b \tag{11}$$

is modified by a surface term on the sphere at infinity. Divergent parts in $\boldsymbol{\mathcal{B}}$ can be fixed in such a way that (i) the modified symplectic structure (11) is finite in the $r \to +\infty$ limit, hence ensuring finiteness of the canonical generators, and (ii) the variation of the action is finite on-shell. We refer to Eq. (S.20) for the explicit expression.

Charge algebra. Contracting the renormalized symplectic form (11) with a gauge transformation δ_ξ yields the following canonical generator

$$\delta \int_\Sigma \mathrm{d}^3x(\xi^\perp\mathcal{H} + \xi^i\mathcal{H}_i) = \iota_{\delta_\xi}\bar{\Omega} + \lim_{r\to+\infty} \bar{\mathcal{K}}_\xi. \tag{12}$$

The surface charge $\bar{\mathcal{K}}_\xi$ is finite but nonintegrable, i.e. it is not a δ-exact term on the phase space. Analogously to [40–43] (see also [50,65,74–76,78–80]), we perform a field-dependent redefinition of the parameters $f \mapsto T$, $\epsilon \mapsto \tilde{\epsilon}$, given explicitly by Eq. (S.15), and we require that the new parameters $T, \tilde{\epsilon}$ are field-independent, $\delta T = 0 = \delta\tilde{\epsilon}$, so that the remaining obstruction to integrability only involves variations of the fluctuating boundary structure $\bar{G}_{AB}$, $\bar{P}^{AB}$.

To resolve this remaining issue, we introduce a couple of boundary fields, i.e. a symmetric tensor C_{AB} and a symmetric tensor density F^{AB}, and modify the boundary symplectic structure as

$$\boldsymbol{\omega}_b \mapsto \boldsymbol{\omega}_b + \mathrm{d}^2x(\delta\bar{P}^{AB} \wedge \delta C_{AB} + \delta F^{AB} \wedge \delta\bar{G}_{AB}). \tag{13}$$

Then, we are free to prescribe the transformation laws of the new fields (C_{AB}, F^{AB}) in such a way that the modification brought by (13) to the charge absorbs the remaining nonintegrability, and we refer to Eqs. (S.24) and (S.25) for the explicit expressions. Furthermore, the new term in Eq. (13) brings a supplementary integrable piece in the charge that reads as

$$\bar{\mathcal{K}}^{(C,F)}_{(b,Y)} = -\delta \oint_{\partial\Sigma} \mathrm{d}^2x[b\mathcal{H}^{(C,F)} + Y^A\mathcal{H}^{(C,F)}_A]. \tag{14}$$

Here, $\mathcal{H}^{(C,F)}$ and $\mathcal{H}^{(C,F)}_A$ are given by

$$\mathcal{H}^{(C,F)} = \sqrt{\bar{G}}(\bar{D}^A\bar{D}^B C_{AB} - (\triangle + 1)C) - \frac{2}{\sqrt{\bar{G}}}\left[F^A_B\bar{P}^B_A - \bar{P}F - \frac{1}{4}(\bar{P}^A_B\bar{P}^B_A - \bar{P}^2)C\right],$$
$$\mathcal{H}^{(C,F)}_A = -2\bar{D}_B F^B{}_A + \bar{P}^B_C(\bar{D}_A C^C_B - 2\bar{D}_B C^C_A), \tag{15}$$

which remarkably correspond to the Hamiltonian and momentum constraints for three-dimensional linearized gravity on a curved background with positive cosmological constant [90], the latter being described by the spatial metric $\bar{G}_{AB}$ and the canonical momentum $\bar{P}^{AB}$.

The fact that the background has a positive cosmological constant should be expected from the asymptotic analyses of Beig [56], Beig and Schmidt [57], Ashtekar and Romano [58] and Friedrich [91] where, using projective geometry, spacelike infinity is a de Sitter hyperboloid. Moreover, from (14), b and Y^A may respectively be thought as the parameters of normal and tangential deformations of the boundary theory.

Incorporating all the modifications, the final expression of the charge is integrable and found to be

$$\begin{aligned}\bar{\mathcal{K}}_\xi[g_{ij},\pi^{ij}]=\delta\oint_{\partial\Sigma}\mathrm{d}^2x\Big[&2T\sqrt{\bar{G}}\bar{h}_{rr}+2Y^A(\bar{G}_{AB}\bar{\pi}^{rB}_{(2)}+\bar{h}_{AB}\bar{\pi}^{rB})\\&+2W(\bar{\pi}^{rr}-\bar{\pi}+\bar{h}_{rr}\bar{P}-\bar{P}^{AB}\bar{h}_{AB})\\&+b\sqrt{\bar{G}}(2\bar{k}^{(2)}+\bar{k}^2+\bar{k}^{AB}\bar{k}_{AB}-3\bar{h}_{rr}\bar{k})\\&+\frac{2b}{\sqrt{\bar{G}}}\bar{\pi}^{rA}\bar{\pi}^r_A+\tilde{\epsilon}\sqrt{\bar{G}}+\epsilon^r\bar{P}\Big]+\bar{\mathcal{K}}^{(C,F)}_{(b,Y)},\end{aligned}\tag{16}$$

where the fields $\bar{k}_{AB}$ and $\bar{k}^{(2)}_{AB}$ appear in the radial expansion of the extrinsic curvature K_{AB} of the sphere,

$$K^A_B=-\frac{1}{r}\delta^A_B+\frac{1}{r^2}\bar{k}^A_B+\frac{1}{r^3}\bar{k}^{(2)A}{}_B+\mathcal{O}(r^{-4}).\tag{17}$$

The terms collected in square brackets in Eq. (16) formally reproduce the result of [41–43], except for the last two terms in the first line, which are sensitive to the boundary momentum $\bar{P}^{AB}$, and the last two terms in the second line, which involve subleading symmetry parameters. As a consequence of the parity conditions (4), only the parity-even part of T and the parity-odd part of W generate improper gauge transformations, the BMS supertranslations. However, by contrast with the analysis of [41–43], b and Y^A are now arbitrary parity-odd functions that no longer obey the conformal Killing equations on the sphere. The additional modes in b and Y^A compared to the Lorentz generators are identified with the Spi superrotations. Finally, we also find new large gauge symmetries associated with the parity-even part of the subleading parameter ϵ and the parity-odd part of ϵ^r in the charge expression (16).

Writing $\bar{\mathcal{K}}_\xi=\delta\bar{K}_\xi$ and defining the generators

$$\mathcal{G}_\xi[g_{ij},\pi^{ij}]\equiv\int_\Sigma\mathrm{d}^3x(\xi^\perp\mathcal{H}+\xi^i\mathcal{H}_i)+\bar{K}_\xi[g_{ij},\pi^{ij}]\tag{18}$$

of the asymptotic symmetries, an intricate and lengthy computation shows that

$$\{\mathcal{G}_{\xi_1},\mathcal{G}_{\xi_2}\}[g_{ij},\pi^{ij}]\approx\mathcal{G}_{\hat{\xi}}[g_{ij},\pi^{ij}]+\Lambda_{\xi_1,\xi_2}[g_{ij},\pi^{ij}]\tag{19}$$

on the constraint surface (2), where $\{\mathcal{G}_{\xi_1},\mathcal{G}_{\xi_2}\}\equiv\delta_{\xi_2}\mathcal{G}_{\xi_1}$ is the Poisson bracket and $\hat{\xi}$ is parametrized by

$$\begin{aligned}\hat{Y}^A&=Y^B_1\partial_BY^A_2+\bar{G}^{AB}b_1\partial_Bb_2-(1\leftrightarrow2),\\\hat{b}&=Y^A_1\partial_Ab_2-(1\leftrightarrow2),\quad\hat{W}=Y^A_1\partial_AW_2-b_1T_2-(1\leftrightarrow2),\\\hat{T}&=Y^A_1\partial_AT_2-3b_1W_2-\bar{G}^{AB}\partial_Ab_1\partial_BW_2-b_1\bar{G}^{AB}\bar{D}_A\bar{D}_BW_2\\&\quad-(1\leftrightarrow2),\\\hat{\epsilon}&=Y^B_1\partial_B\epsilon_2+b_1\bar{G}^{AB}\bar{D}_A\bar{D}_B\epsilon^r_2+4b_1\epsilon^r_2-b_1\epsilon^r_2\bar{R}-(1\leftrightarrow2),\\\hat{\epsilon}^r&=Y^B_1\partial_B\epsilon^r_2+b_1\epsilon_2-(1\leftrightarrow2).\end{aligned}\tag{20}$$

Despite the resemblance with the results of [41], the parameters b and Y^A now contain an infinite tower of Spi superrotations modes. We also find two Abelian ideals associated with ϵ and ϵ^r, on which the superrotations are acting nontrivially. Notice that the structure constants (20) now depend explicitly on the particular solution through the presence of $\bar{G}_{AB}$. Technically, this mathematical structure is referred to as a Lie algebroid [92–94], and naturally appears when the boundary structure is allowed to fluctuate on the phase space [63,64,95]. Moreover, the asymptotic symmetry algebra (19) admits the following nonlinear contribution:

$$\Lambda_{\xi_1,\xi_2}=2\oint_{\partial\Sigma}\mathrm{d}^2x(b_1T_2-b_2T_1)\bar{P}\bar{h}_{rr}.\tag{21}$$

This term would be invisible in a linear treatment of infinity. It is reminiscent of the appearance of a field-depend two-cocycle in the charge algebra at null infinity in presence of superrotations [6,13,93]. As a nontrivial consistency check, we verified explicitly that the Jacobi identity

$$\{\mathcal{G}_{\xi_1},\{\mathcal{G}_{\xi_2},\mathcal{G}_{\xi_3}\}\}+\mathrm{cyclic}(1,2,3)\approx0\tag{22}$$

is satisfied. Interestingly, the presence of the nonlinear contribution (21) is absolutely essential for this computation to work, due to the field-dependence of the structure constants in Eq. (20).

Discussion. In this work, we proposed a consistent set of boundary conditions at spacelike infinity allowing for an enhancement of BMS symmetries with smooth Spi superrotations. An interesting observation is that these symmetries have the same parity as the Lorentz symmetries, which was shown in [40–43,51,52] to be consistent with the antipodal matching conditions advocated in [21,22,24]. Therefore, our analysis extends the compatibility with the antipodal matching conditions for superrotation symmetries.

We believe that the Spi superrotations identified here can be matched with the generalized BMS symmetries [11–13] uncovered at null infinity, and which manifest here in a unusual basis adapted to Hamiltonian decomposition in time and space. The precise relation between spacelike and null infinity requires using suitable coordinate systems as Beig-Schmidt [56,57] or Friedrich [91] gauges, along the lines of [40,41,51,96], which has still to be understood with our relaxed boundary conditions. In this setup, it would also be beneficial to identify the geometric structure associated with the boundary conditions discussed in this paper. These important questions will be addressed elsewhere.

Besides the Spi superrotations, we also found two infinite towers of charges associated with the subleading symmetry parameters ϵ and ϵ^r. These subleading symmetries are Abelian and in a semidirect sum with the Spi superrotations. This echoes some recent results obtained at null infinity by relaxing Bondi gauge fixing conditions [5,9,16,68–70,97–100]. It would be interesting to further explore this intriguing resemblance.

Interestingly, in the process of rendering the charges integrable, the self-consistency of the Hamiltonian analysis led us to add new fields at the boundary, see Eq. (13). The latter were reinterpreted as the canonical variables for a linearized spin-two field theory at the boundary. The emergence of these boundary degrees of freedom is reminiscent of the edge mode fields [101–103], which have been argued in [104,105] to be useful to obtain integrable charges. Our analysis constitutes an explicit realization of this proposal.

Finally, let us emphasize that this work combined the powerful machinery of the Hamiltonian formalism previously applied at spacelike infinity, together with covariant phase space techniques developed in parallel mostly at null infinity. In particular, this is the first time that the renormalization of the symplectic structure is applied in the Hamiltonian formalism at spacelike infinity. This somehow concludes a long programme of finding the complete set of BMS symmetries at spacelike infinity, which started with the seminal work of Regge and Teitelboim [39], and has then known a decisive turning point with the beautiful series of papers of Henneaux and Troessaert [41–43].

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