Letter

CP violation induced by neutral meson mixing interference

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We investigate a long-overlooked *CP* violation effect—the double-mixing *CP* asymmetry—in a type of cascade decay that involves at least two mixing neutral mesons in the decay chain. It is induced by the interference between different oscillation paths of the neutral mesons in the decay process. The double-mixing *CP* asymmetry is of critical importance for phenomenology, providing opportunities for clean determination of Cabibbo-Kobayashi-Maskawa phase angles free of uncertainties induced by the strong dynamics. To illustrate this point, we perform a phenomenological analysis on two examples: $B_s^0 \to \rho^0 K \to \rho^0(\pi^- \ell^+ \nu_\ell)$ and $B^0 \to D^0 K \to D^0(\pi^+ \ell^- \bar{\nu}_\ell)$. Our results demonstrate that the double-mixing *CP* asymmetry can be numerically significant in the absence of strong phases, as shown by the former example. Additionally, the latter example showcases a direct extraction of weak and strong phases from data, without the need for theoretical inputs.

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Introduction. The *CP* violation has always been playing a key role in particle physics. Measurements of *CP* violation effects in flavor processes are crucial to determine the Cabibbo-Kobayashi-Maskawa (CKM) [1,2] matrix, whose unitarity is a critical test of the standard model (SM). Moreover, the observed matter-antimatter asymmetry in the Universe requires the *CP* violation as one of the criteria [3], but the *CP* violation in the SM is too small to be the only source [4,5]. Therefore, precision tests of *CP* asymmetries and searches for their nonstandard source may open a window to physics beyond the SM.

In hadron decays, although the *CP* violation is induced by the weak interaction, its visualization typically requires interplay between weak and strong interactions and thus receives pollution from strong dynamics. Therefore, exploring *CP* asymmetries in diverse physical observables with diverse dependence on strong dynamics would be beneficial to unravel the mystery of the *CP* violation [6]. Particularly, observables free of strong phases provide a clean environment [7,8]. To this end, we propose a long-overlooked *CP* violation observable—double-mixing *CP* asymmetry, which is induced by interferences between different mixing paths in a single cascade decay. It allows for the determination of weak phases avoiding strong-dynamics pollution in appropriate channels.

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The double-mixing *CP* asymmetry exists in cascade decay chains, in which at least two mixing neutral mesons are involved. Such a process typically starts from a primary neutral meson M_1^0, which decays, before or after oscillating to its antiparticle $\bar{M}_1^0$, into a secondary neutral meson $\bar{M}_2^0(M_2^0)$ and other particles. The secondary neutral meson $\bar{M}_2^0(M_2^0)$ also further decays, before or after oscillating, into directly detectable particles by detectors. The process $M_1 \to M_2 \to f$ (the particles produced associated with M_2 in the decay are omitted for simplicity) happens via multiple quantum paths, which interfere with each other. One example is shown in Fig. 1, which allows two oscillation paths $M_1^0 \to \bar{M}_2^0 \to M_2^0$ and $M_1^0 \to \bar{M}_1^0 \to M_2^0$. The interference between the two paths induces the double-mixing *CP* asymmetry, which is our focus in this study.[1] There also exist other interfering paths such as $M_1^0 \to \bar{M}_1^0 \to M_2^0 \to \bar{M}_2^0$ and $M_1^0 \to \bar{M}_2^0$. In cases where M_2^0 and $\bar{M}_2^0$ decay into the same final state, usually a *CP* eigenstate, there are interferences between the four paths, and induce more fruitful *CP* violation observables.

The double-mixing *CP* asymmetry possesses a distinctive phenomenological significance. Its dependence on two time variables, the oscillation time t_1 of M_1^0 and t_2 of M_2^0, allows for a two-dimensional time-dependent analysis on $M_1^0(t_1) \to \bar{M}_2^0(t_2) \to f$, making it a new measurement tool. Practically, the double-mixing *CP* asymmetry can be numerically very significant in certain decay channels, to

[1]If the mixing effect of M_1^0 is very small and neglected, as has been done in certain D meson decays [9,10], only the mixing effect of M_2^0 needs to be taken into account. For more details, refer to [11].

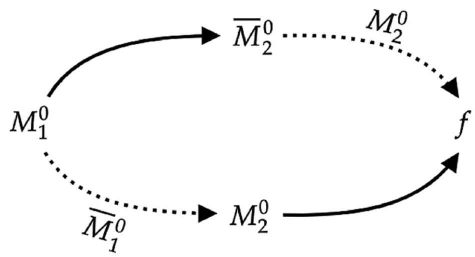

FIG. 1. Interference between two oscillation paths in the cascade decay $M_1 \to M_2 \to f$. The decay products associated with M_2 are not displayed.

be discovered and measured at flavor experiments such as BESIII [12], Belle II [13], LHCb [14], and future lepton colliders [15–18], including the proposed CEPC and FCC-ee, which can also produce fruitful flavor results [19–24]. Furthermore, the double-mixing *CP* asymmetry does not necessitate a nonzero strong phase, thereby circumventing strong-dynamics pollution in specific channels. Even when there is a nonzero strong phase, it turns out that the strong phase can potentially be determined directly from data without theoretical inputs, together with the weak phase. Therefore, by selecting appropriate channels, the double-mixing *CP* asymmetry can enable the extraction of CKM phase angles without hadronic uncertainties, making it sensitive to some dynamics beyond the SM.

In the rest of the paper, we will first present the general formulas for the double-mixing *CP* asymmetry in the process $M_1^0(t_1) \to \bar{M}_2^0(t_2) \to f$. We will then perform the numerical analysis of the $B_s^0 \to \rho^0 \bar{K}^0 \to \rho^0(\pi^- \ell^+ \nu_\ell)$ decay channel, as an example to show that the double-mixing *CP* asymmetry can be very significant in numerics. Additionally, we will analyze the $B^0 \to D^0 K \to D^0(\pi^+ \ell^- \bar{\nu}_\ell)$ decay to exhibit that the involved weak phase can be extracted without any hadronic inputs.

Formulas. In the following derivation, we accept the convention that the mass eigenstates $M_{H,L}$ of the neutral mesons are superpositions of their flavor eigenstates,

$$|M_{H,L}\rangle = p|M^0\rangle \mp q|\bar{M}^0\rangle, \quad (1)$$

where q, p are complex coefficients. The mass and decay width differences are defined as $\Delta m \equiv m_H - m_L$ and $\Delta\Gamma \equiv \Gamma_H - \Gamma_L$ such that the oscillation is formulated by

$$|M^0(t)\rangle = g_+(t)|M^0\rangle - \frac{q}{p} g_-(t)|\bar{M}^0\rangle,$$
$$|\bar{M}^0(t)\rangle = g_+(t)|\bar{M}^0\rangle - \frac{p}{q} g_-(t)|M^0\rangle, \quad (2)$$

$$\text{with} \quad g_\pm(t) = \frac{1}{2}\left[e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t}\right],$$

in the case that the theory is *CPT* invariant.

In this Letter, we will focus on the cascade decay $M_1 \to M_2 \to f$ with the process happening via two oscillating paths $M_1^0 \to \bar{M}_2^0 \to M_2^0$ and $M_1^0 \to \bar{M}_1^0 \to M_2^0$, as shown in Fig. 1. A more comprehensive study, including other cases, can be found in [25]. The two-dimensional time-dependent *CP* asymmetry is defined by

$$A_{CP}(t_1, t_2) \equiv \frac{|\mathcal{M}|^2(t_1, t_2) - |\bar{\mathcal{M}}|^2(t_1, t_2)}{|\mathcal{M}|^2(t_1, t_2) + |\bar{\mathcal{M}}|^2(t_1, t_2)}, \quad (3)$$

where the amplitude $\mathcal{M}(t_1, t_2)$ is the sum of amplitudes of the two paths $M_1^0 \to \bar{M}_2^0 \to M_2^0$ and $M_1^0 \to \bar{M}_1^0 \to M_2^0$, and the amplitude $\bar{\mathcal{M}}(t_1, t_2)$ is the *CP* conjugate of $\mathcal{M}(t_1, t_2)$. The decaying time lengths t_1 and t_2 are defined in the rest frames of M_1 and M_2, respectively, and can be identified in experiments by using vertex detection techniques. To give prominence to the mixing effects, we assume that there are no direct *CP* asymmetries in the decay $M_1 \to M_2$ or $M_2 \to f$, i.e., these decays are tree-amplitude dominant with vanishing or negligible penguin amplitudes.

Case 1: We consider M_1^0 decaying into $\bar{M}_2^0$ associated with a *CP* eigenstate f_{CP} such as ρ^0. Then, the primary decay in the upper path $M_1^0 \to \bar{M}_2^0 f_{CP}$ and the one in the lower path $\bar{M}_1^0 \to M_2^0 f_{CP}$ are *CP* conjugates of each other. Thus, without direct *CP* violation we have $|\langle \bar{M}_2^0 | M_1^0 \rangle| = |\langle M_2^0 | \bar{M}_1^0 \rangle|$ and the decay amplitudes are related by $\langle \bar{M}_2^0 | M_1^0 \rangle = \langle M_2^0 | \bar{M}_1^0 \rangle e^{2i\omega}$, where ω is a pure weak phase. We further write the mixing parameters of $M_{1,2}$ as $(q/p)_{1,2} = |(q/p)_{1,2}| e^{-i\phi_{1,2}}$. Then, the time-dependent *CP* asymmetry is calculated to be

$$A_{CP}(t_1, t_2) = \frac{|g_{1,+}(t_1)|^2 C_+(t_2) + |g_{1,-}(t_1)|^2 C_-(t_2) + e^{-\Gamma_1 t_1} \sinh\frac{\Delta\Gamma_1 t_1}{2} S_h(t_2) + e^{-\Gamma_1 t_1} \sin(\Delta m_1 t_1) S_n(t_2)}{|g_{1,+}(t_1)|^2 C'_+(t_2) + |g_{1,-}(t_1)|^2 C'_-(t_2) + e^{-\Gamma_1 t_1} \sinh\frac{\Delta\Gamma_1 t_1}{2} S'_h(t_2) + e^{-\Gamma_1 t_1} \sin(\Delta m_1 t_1) S'_n(t_2)}, \quad (4)$$

where

$$C_+(t_2) = |g_{2,-}(t_2)|^2(|p_2/q_2|^2 - |q_2/p_2|^2), \qquad C_-(t_2) = |g_{2,+}(t_2)|^2(|q_1/p_1|^2 - |p_1/q_1|^2), \quad (5)$$

consisting of M_2- and M_1-mixing induced *CP* violation, respectively. The double-mixing *CP* asymmetry is reflected by the terms proportional to

$$S_h(t_2) = \frac{e^{-\Gamma_2 t_2}}{2}\left[-2\sin(\Delta m_2 t_2)\sin(\phi_1+\phi_2+2\omega) + \sinh\frac{\Delta\Gamma_2 t_2}{2}\left(\left|\frac{q_1}{p_1}\right|\left|\frac{p_2}{q_2}\right| - \left|\frac{p_1}{q_1}\right|\left|\frac{q_2}{p_2}\right|\right) \times \cos(\phi_1+\phi_2+2\omega)\right],$$

$$S_n(t_2) = \frac{e^{-\Gamma_2 t_2}}{2}\left[2\sinh\frac{\Delta\Gamma_2 t_2}{2}\sin(\phi_1+\phi_2+2\omega) + \sin(\Delta m_2 t_2)\left(\left|\frac{q_1}{p_1}\right|\left|\frac{p_2}{q_2}\right| - \left|\frac{p_1}{q_1}\right|\left|\frac{q_2}{p_2}\right|\right) \times \cos(\phi_1+\phi_2+2\omega)\right], \tag{6}$$

where some doubly suppressed small quantities are neglected. The phase angle dependence on $\phi_1+\phi_2$ clearly indicates that they are induced by the inference between the M_1-oscillating path and the M_2-oscillating path. The S_h and S_n terms have very different time dependence: S_n has a sine dependence on t_1 and a hyperbolic sine dependence on t_2, while S_h has a hyperbolic sine dependence on t_1 and a sine dependence on t_2. The two different types of time dependence can be used to separate S_h and S_n in $A_{CP}(t_1,t_2)$ and analyze the physical implications of each. It is also observed that in the absence of strong phases in decays and mixings, i.e., $|\langle\bar{M}_2^0|M_1^0\rangle| = |\langle M_2^0|\bar{M}_1^0\rangle|$, and $|q/p|=1$, the double-mixing *CP* asymmetries S_h and S_n still exist. Thus, it provides an additional clean environment to determine CKM phases, complementing known clean *CP* violation observables such as the *CP* violation induced by interference between a decay without mixing and a decay with mixing [26].

The terms contributing to the denominator of (4) are given by

$$\begin{aligned} C'_+(t_2) &= 2|g_{2,-}(t_2)|^2, \\ C'_-(t_2) &= 2|g_{2,+}(t_2)|^2, \\ S'_h(t_2) &= e^{-\Gamma_2 t_2}\sinh\frac{\Delta\Gamma_2 t_2}{2}\cos(\phi_1+\phi_2+2\omega), \\ S'_n(t_2) &= e^{-\Gamma_2 t_2}\sin(\Delta m_2 t_2)\cos(\phi_1+\phi_2+2\omega), \end{aligned} \tag{7}$$

with suppressed terms neglected.

Case 2: Another case is that the particles p produced in the primary decay $M_1 \to M_2 p$ are not *CP* eigenstates, e.g., D^0. Then the involved decay $M_1^0 \to \bar{M}_2^0 p$, in the upper path shown by Fig. 1, and the corresponding decay in the lower path $\bar{M}_1^0 \to M_2^0 p$ are not the *CP* conjugations of each other. Therefore, more parameters are necessary to formulate the relations between the primary decay amplitudes in the two paths and their charge conjugations,

$$\begin{aligned} A(\bar{M}_1^0 \to pM_2^0)/A(M_1^0 \to \bar{p}\bar{M}_2^0) &= e^{-2i\omega_1}, \\ A(M_1^0 \to p\bar{M}_2^0)/A(M_1^0 \to \bar{p}\bar{M}_2^0) &= re^{i(\delta+\omega_2)}, \end{aligned} \tag{8}$$

where $\omega_{1,2}$ are the weak phases, r is the magnitude ratio, and δ is the strong phase.

The two-dimensional time-dependent *CP* asymmetry is calculated to be

$$\begin{aligned} A_{CP}(t_1,t_2) &= \frac{e^{-\Gamma_1 t_1}\sinh\frac{\Delta\Gamma_1 t_1}{2}S_h(t_2) + e^{-\Gamma_1 t_1}\sin(\Delta m_1 t_1)S_n(t_2)}{|g_{1,+}(t_1)|^2 r^2 C'_+(t_2) + |g_{1,-}(t_1)|^2 C'_-(t_2) + e^{-\Gamma_1 t_1}\sinh\frac{\Delta\Gamma_1 t_1}{2}S'_h(t_2) + e^{-\Gamma_1 t_1}\sin(\Delta m_1 t_1)S'_n(t_2)}, \\ S_h(t_2) &= -e^{-\Gamma_2 t_2} r\sin\omega'\left[\sin\delta\sinh\frac{\Delta\Gamma_K}{2}t_2 + \cos\delta\sin\Delta m_K t_2\right], \\ S_n(t_2) &= e^{-\Gamma_2 t_2} r\sin\omega'\left[\cos\delta\sinh\frac{\Delta\Gamma_K}{2}t_2 - \sin\delta\sin\Delta m_K t_2\right], \\ S'_h(t_2) &= e^{-\Gamma_2 t_2} r\cos\omega'\left[\cos\delta\sinh\frac{\Delta\Gamma_K}{2}t_2 - \sin\delta\sin\Delta m_K t_2\right], \\ S'_n(t_2) &= e^{-\Gamma_2 t_2} r\cos\omega'\left[\sin\delta\sinh\frac{\Delta\Gamma_K}{2}t_2 + \cos\delta\sin\Delta m_K t_2\right], \end{aligned} \tag{9}$$

where the weak phase $\omega' = 2\omega_1+\omega_2+\phi_1+\phi_2$, and the functions $C'_\pm$ take the same form as in (7). In the calculation, we have neglected the *CP* violation in mixing, i.e., setting $|q_1/p_1| = |q_2/p_2| = 1$.

For the decay chain constructed by the oscillation paths $M_1^0 \to M_2^0 \to \bar{M}_2^0$ and $M_1^0 \to \bar{M}_1^0 \to \bar{M}_2^0$, the results can be obtained by taking (4) and replacing q_2/p_2 with p_2/q_2, i.e., $|q_2/p_2| \to |p_2/q_2|$ and $\phi_2 \to -\phi_2$.

Phenomenology. We perform the phenomenological analysis of two decay channels, as the examples correspond to the two cases above. For case 1, $B_s^0 \to \rho^0 K \to \rho^0(\pi^-\ell^+\nu_\ell)$

is analyzed, showing that the value of the double-mixing CP asymmetry in this channel is very significant, in the absence of strong phases. For case 2, $B^0 \to D^0 K \to D^0(\pi^+\ell^-\bar{\nu}_\ell)$ is analyzed, showing that the strong phase and the weak phase can be simultaneously determined by measurements of the double-mixing CP asymmetry.

Example channel 1: The $B_s^0 \to \rho^0 K \to \rho^0(\pi^-\ell^+\nu_\ell)$ decay channel is taken as the first example. The process has $B_s^0 \to \rho^0 \bar{K}^0 \to \rho^0 K^0 \to \rho^0(\pi^-\ell^+\nu_\ell)$ and $B_s^0 \to \bar{B}_s^0 \to \rho^0 K^0 \to \rho^0(\pi^-\ell^+\nu_\ell)$ as the two paths shown in Fig. 1. Because both B_s^0 and K^0 have large mixing effects, their interference could also be large.

Despite the small Wilson coefficients of the penguin operator contributions to $B_s^0 \to \rho^0 \bar{K}^0$ [27,28], such contributions may not be negligible due to large corrections possibly from power corrections. However, in order to clearly illustrate the double-mixing CP violation, we have opted to disregard the penguin contributions, i.e., the direct CP asymmetry, and explore their potential impact in the Supplemental Material [29]. Then, the decay amplitude $\langle \rho^0 \bar{K}^0 | B_s^0 \rangle$ and its charge conjugation $\langle \rho^0 K^0 | \bar{B}_s^0 \rangle$ share the same magnitude, and the phase difference is given by $e^{i2\omega} = -(V_{ub}^* V_{ud})/(V_{ub} V_{ud}^*)$, where the minus sign is caused by $CP|\rho^0 K^0\rangle = -|\rho^0 \bar{K}^0\rangle$ in a pseudoscalar meson decay. We also neglect the indirect CP asymmetries induced by B_s and K mixings, i.e., taking $|q/p| = 1$, and then the phases of the mixing coefficients are approximately given by $e^{-i\phi_1} = e^{-i\phi_{B_s}} = (V_{tb}^* V_{ts})/(V_{tb} V_{ts}^*)$ and $e^{-i\phi_2} = e^{-i\phi_K} = (V_{cd}^* V_{cs})/(V_{cb} V_{cs}^*)$.[2] With these approximations, the only nonvanishing CP asymmetry is induced by the double-mixing interference, contained in S_h and S_n given by (6).

With the numerical inputs listed in Table I, where the neutral meson average decay width is defined by $\Gamma_M \equiv (\Gamma_{M,H} + \Gamma_{M,L})/2$, we calculate the numerical results for the two-dimensional time-dependent CP violation observable $A_{CP}(t_1, t_2)$ (4), and the contributions $A_h(t_1, t_2)$ and $A_n(t_1, t_2)$ proportional to the S_h and S_n terms (6), respectively. The result for the $A_{CP}(t_1, t_2)$ dependence on t_1 and t_2 is displayed in the left panel of Fig. 2. It can be observed that the magnitude of the peak values can exceed 50%. If the data sample is not large enough for a two-dimensional time-dependence analysis, we can also integrate out one time dimension and get the evolution along the remaining one. Integrating t_2 from 0 to the kaon average lifetime $\tau_K \equiv 1/\Gamma_K$, we obtain the t_1 dependence of A_{CP} displayed in the middle panel of Fig. 2; integrating t_1 from τ_{B_s} to $5\tau_{B_s}$ with $\tau_{B_s} \equiv 1/\Gamma_{B_s}$, we obtain the t_2 dependence of A_{CP} displayed in the right panel of Fig. 2. Approximately, τ_K is twice the K_S lifetime. The contributions A_h and A_n are displayed by dashed and dotted curves, respectively. The time evolution along t_2 is dominated by A_h, because A_n highly oscillates along the integrated time t_1. The time evolution along t_1 is dominated by A_n, but the contribution of A_h is also considerable. The discovery of this channel and its double-mixing CP violation is more promising at LHCb, given the substantial production of B_s mesons at this facility.

TABLE I. The input parameters and their values, with $x_M \equiv \Delta m_M/\Gamma_M$ and $y_M \equiv \Delta\Gamma_M/(2\Gamma_M)$, respectively.

Parameter	Value	Parameter	Value
ϕ_{B_s}	$(-2.106 \pm 0.135)^\circ$ [30]	ϕ_B	$(44.4 \pm 1.4)^\circ$ [31]
x_{B_s}	27.01 ± 0.10 [30]	x_B	0.769 ± 0.004 [30]
y_{B_s}	-0.064 ± 0.003 [30]	y_B	-0.0005 ± 0.0050 [30]
ϕ_K	$(0.176 \pm 0.001)^\circ$ [30]	2ω	$(-48.907 \pm 3.094)^\circ$ [30]
x_K	0.946 ± 0.002 [30]	$2\omega_1$	0° [30]
y_K	-0.996506 ± 0.000016 [30]	ω_2	$(65.54 \pm 1.55)^\circ$ [30]

Example channel 2: The $B^0 \to D^0 K \to D^0(\pi^+\ell^-\bar{\nu}_\ell)$ decay channel is taken as the second example, showing how CKM phases can be determined by measurements of the double-mixing CP asymmetry. Although a strong phase is involved in this channel, it can be extracted together with the weak phase from data without any theoretical input needed.

The $B^0 \to D^0 K \to D^0(\pi^+\ell^-\bar{\nu}_\ell)$ channel has two oscillating paths, $B^0 \to D^0 K^0 \to D^0 \bar{K}^0$ and $B^0 \to \bar{B}^0 \to D^0 \bar{K}^0$, which have a nonzero relative strong phase between each other, as parametrized in (8). The mixing of D^0 is disregarded in the calculation owing to the minuscule mixing parameters $x_D, y_D \sim 5 \times 10^{-3}$ [30], and again the mixing CP asymmetries of the involved neutral mesons are neglected.[3] Comparing to the setup in case 2, we have $p = D^0$ and $\bar{M}_2^0 = K^0$. To apply (9), we need to additionally flip the sign of ϕ_2, i.e., the total weak phase is $\omega' = 2\omega_1 + \omega_2 + \phi_1 - \phi_2$, with

$$2\omega_1 = \arg\frac{V_{cb}^* V_{us}}{V_{cb} V_{us}^*}, \qquad \omega_2 = \arg\frac{V_{ub}^* V_{cs}}{V_{cb}^* V_{us}},$$
$$\phi_1 = \phi_B = -\arg\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}, \qquad \phi_2 = \phi_K. \qquad (10)$$

It can be checked that ω' is approximately $2\beta + \gamma$ in the conventional parametrization of the CKM phase angles [30]. With the values of the input parameters listed in Table I, and choosing the reference values for the ratio

[2]The discussion of the numerical magnitudes of these approximations will be presented in Supplemental Material [29]. Actually, the neglected direct CP asymmetry has the potential to induce substantial modifications, and we will defer its investigation to a future study.

[3]The discussion of the numerical magnitudes of these approximations will be presented in Supplemental Material [29].

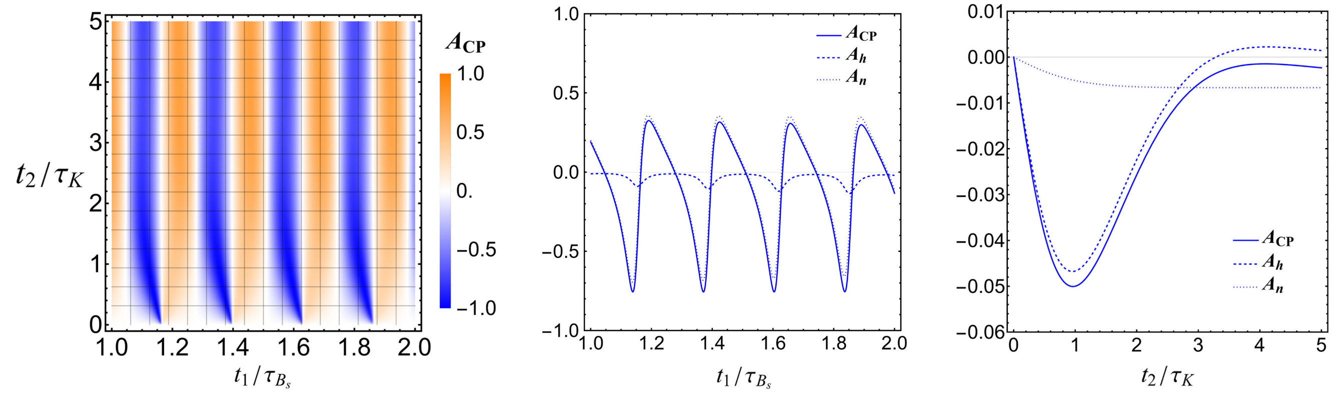

FIG. 2. Time dependence of the double-mixing *CP* asymmetry A_{CP} in $B_s^0(t_1) \to \rho^0 K(t_2) \to \rho^0(\pi^- \ell^+ \nu_\ell)$. The left panel displays the two-dimensional time dependence. The middle panel and the right panel display the dependence on t_1 (with t_2 integrated from 0 to τ_K) and t_2 (with t_1 integrated from τ_{B_s} to $5\tau_{B_s}$), respectively.

$r = 0.366$ and the strong phase $\delta = 164°$ (close to the measured values of the corresponding parameters for $B^0 \to DK^*$ [32,33]), we obtain the numerical result for the two-dimensional time-dependent *CP* asymmetry $A_{CP}(t_1, t_2)$ and display it in the left panel of Fig. 3. Integrating out t_1 from 0 to $3\tau_B$, the t_2 dependence of A_{CP} is retained, as shown in the middle panel of Fig. 3. It is observed that in both cases the *CP* violation effects are considerable.

To show how the weak phase is extracted from measurements of the double-mixing *CP* asymmetry of this channel, we simulate the events for both $B^0 \to D^0 K \to D^0(\pi^+ \ell^- \bar{\nu}_\ell)$ and its charge-conjugate process. According to the branching ratio of neutral K mesons [30], the ratio between the semileptonic and $\pi\pi$ decay modes of kaons is approximately 10^{-3}. By combining the corresponding branching ratio of B^0 mesons [30] with the denominator in Eq. (9), and assuming that semileptonic decays of kaons from B mesons, which occur within $2\tau_{K_S}$ in the rest frame, can be perfectly detected by Belle II with a spatial resolution better than 1 mm, we estimate that the event numbers for such decay channels are around $\mathcal{O}(10^3)$. Based on these assumptions, we simulate 3000 $B^0 \to D^0 K \to D^0(\pi^+ \ell^- \bar{\nu}_\ell)$ events along with the corresponding charge conjugate events. Assuming 100% reconstruction efficiency for the decay process, zero background, and zero systematic uncertainty, the simulated t_2-dependent *CP* asymmetry is shown in the red histogram in the middle panel of Fig. 3. Afterward, the ratio r and the strong and weak phases δ and ω' are treated as unknown parameters to be determined. By fitting the formulas for the double-mixing *CP* asymmetry (8) and the *CP*-averaged branching ratio to the simulated events, the three parameters are extracted to be

$$r = 0.367 \pm 0.014, \qquad \delta = (164.1 \pm 4.1)°,$$
$$\omega' = (108.9 \pm 4.8)°, \qquad (11)$$

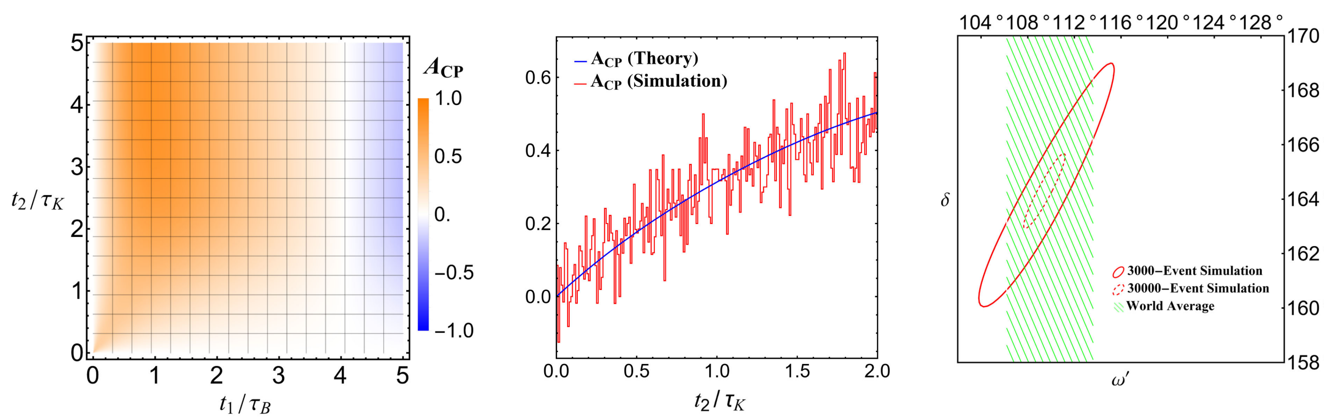

FIG. 3. Numerical results for the double-mixing *CP* asymmetry A_{CP} in $B^0(t_1) \to D^0 K(t_2) \to D^0(\pi^+ \ell^- \bar{\nu}_\ell)$ and the extraction of the weak and strong phases from its numerical simulation. The left panel displays the two-dimensional time dependence of A_{CP}. The middle panel displays the t_2 dependence of A_{CP} (with t_1 integrated from 0 to $3\tau_B$) and the corresponding simulated result. The right panel presents the strong phase δ and weak phase ω' at 68% confidence level determined by the simulated data samples, with 3000 and 30000 simulated events, respectively, compared to the world average for the weak phase ω' at 68% confidence level.

where only the statistical uncertainties are considered. The 68% confidence interval for $\delta - \omega'$ is shown in the right panel of Fig. 3 by the red solid contour. The precision of the weak phase $\omega' \approx 2\beta + \gamma$ is comparable to the world average value of all the current experiments, $(109.9 \pm 3.7)^\circ$. If 10 times events are collected, the precision can be further improved by about 3 times, as shown by the red dashed contour in the right panel of Fig. 3.

Conclusion. In conclusion, we have investigated a historically overlooked CP violation effect, the double-mixing CP asymmetry, which exists in cascade decays involving two neutral mesons oscillating. This effect is dependent on two time variables, allowing for a two-dimensional time dependence analysis. Unlike direct CP asymmetries, the existence of the double-mixing CP asymmetry does not require a nonzero strong phase. Even in the presence of a strong phase, it can be directly extracted from data with the corresponding weak phase in appropriate channels. As a result, the double-mixing CP asymmetry provides a means to directly extract weak phases without any pollution from strong dynamics, which is crucial for CKM matrix determination and new physics search. Furthermore, the double-mixing CP asymmetries can be numerically significant in certain channels, making them very promising for experimental measurement.

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