Letter

Effectiveness of Weyl gravity in probing quantum corrections to AdS black holes

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(Received 2 April 2024; accepted 24 June 2024; published 22 July 2024)

Computing leading higher curvature contributions to thermodynamic quantities of AdS black hole is drastically simplified once the higher curvature terms are expressed in terms of powers of Weyl tensor by applying proper field redefinitions, avoiding the usual complications caused by higher derivative Gibbons-Hawking-York term or surface counterterms. We establish the method by computing the Euclidean action of general rotating anti–de Sitter (AdS) black holes in five-dimensional quadratic curvature theories with or without supersymmetry and verifying the results numerically. Our result is the state of the art for charged rotating AdS black holes in five-dimensional minimal gauged supergravity including corrections from all three supersymmetric curvature squared terms. Our approach facilitates precision tests in the AdS/CFT correspondence and should be applicable in diverse dimensions.

DOI: 10.1103/PhysRevD.110.L021901

Introduction. The Weyl squared action has played a versatile role in our pursuit of a quantum theory of general relativity. In four dimensions, it admits a convergent Euclidean functional integral [1] and enjoys renormalizability and asymptotic freedom [2]. Interestingly, the quantum fluctuations break the local scale invariance inducing the Einstein-Hilbert term with a calculable Newton's constant [3,4]. In the context of (A)dS/CFT correspondence, Weyl gravity modified by a purely topological contribution from a Gauss-Bonnet term turns out to be equivalent to renormalized Einstein gravity at tree level [5], with suitable boundary conditions chosen [6,7]. The equivalence between Weyl gravity and Einstein gravity underlies the construction of the widely studied critical gravity [8] and admits generalizations also in higher dimensions [6,9]. The principal reason that Weyl gravity has not received general acceptance is because the fourth-order derivatives lead to ghostlike excitations in the linearized theory. However, the violation of tree level unitarity may be cured by invoking the Lee-Wick mechanism [10–12] or adopting the PT symmetric inner product [13,14]. In the full theory, there is the zero energy theorem [15] stating that the exact asymptotically flat solutions in Weyl gravity all have zero energy rendering the ghosts confined in the nonlinear theory at large distances.

The aim of this article is to unveil another instrumental role of Weyl gravity in the framework of effective field theory of quantum gravity. Since the work of Gibbons and Hawking [16], numerous endeavors have been devoted to compute the Euclidean action of spacetime, which plays an important role in the study of black hole thermodynamics, holography and quantum cosmology. For general higher derivative gravities, the task becomes much more difficult since one encounters equations of motion of higher order in partial derivatives. Built upon previous work [17], we find that computation of the leading higher curvature contributions to the Euclidean action of anti–de Sitter (AdS) black hole is drastically simplified once the higher curvature terms are expressed in terms of powers of Weyl tensor by applying proper field redefinitions [18]. The computation of on-shell Euclidean action boils down to simply evaluating the uncorrected solution in the higher derivative action, but without having to concern about the complicated higher derivative Gibbons-Hawking-York or surface counterterms, which would be required in the ordinary approach. In particular, the new approach is convenient for studying Kerr-AdS black holes since to obtain the higher derivative corrections to the Euclidean action, one only needs to evaluate a bulk integral which has no preference on the choice of coordinates that may affect the induced

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metric on the conformal boundary [20]. Our new approach has been verified for static charged AdS black hole in a related work [21] announced recently. (See Ref. [22] for the background subtraction approach.) We have also confirmed the finiteness of Euclidean action for general asymptotically locally AdS spacetime in Einstein gravity perturbed by a Weyl squared term. In this Letter, employing our method, we obtain, for the first time, corrections to thermodynamics of Kerr-AdS black holes from generic quadratic curvature terms. This result is also verified numerically. Equipped with the powerful new method, we also revisit the leading higher derivative corrections to charged rotating black holes in five-dimensional minimal gauged supergravity, using the complete basis of gauged curvature-squared supergravity [23–27]. We show that all three supersymmetric curvature-squared terms contribute to the Euclidean action of the charged rotating AdS black hole regardless of its supersymmetry.

Before we continue, it is worth mentioning some closely related work [28–31]. Both [28] and [29] computed 4-derivative corrections to thermodynamics of charged rotating black holes in five dimensional minimal gauged supergravity using only the supersymmetric Weyl-squared and Ricci scalar-squared action. Surprisingly, it is the bare AdS radius rather than the effective AdS radius that enters the Bogomol'nyi-Prasad-Sommerfield (BPS) relation among conserved charges although the AdS radius and conserved charges are all affected by the curvature squared combinations adopted in [28,29,31]. Thus a third independent check on the thermodynamic quantities is urgently needed. In the very recent work [30], the near horizon geometry of BPS charged rotating black hole was solved perturbatively when the two angular momentum J_1, J_2 are nearly equal, resulting in the BPS black hole entropy expressed order by order in powers of $J_1 - J_2$. Upon taking the BPS limit, our results not only yield the correct black hole entropy in the full parameter region, but also gives rise to a corrected linear relation amongst mass, electric charge, and two angular momentum. As the higher derivative corrections to the BPS relation are fully encoded in the effective AdS radius, our result seems more natural from the point of view of AdS superalgebra compared to the previous result [29] which includes only the bare AdS radius.

Quadratic curvature corrections to Kerr-AdS black holes. We consider the effective theory of the Einstein gravity with a negative cosmological constant in five dimensions extended by the general quadratic curvature terms. In Euclidean signature, the action takes the form

$$I_{QG} = -\frac{\sigma_0}{16\pi}\int d^5x\sqrt{g}(R + 12\ell_0^{-2} + \mathcal{L}_4),$$
$$\mathcal{L}_4 = c_1 R^2 + c_2 r^{\mu\nu} r_{\mu\nu} + c_3 C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma}, \tag{1}$$

where $r_{\mu\nu} = R_{\mu\nu} - \frac{1}{5} g_{\mu\nu} R$, $C_{\mu\nu\rho\sigma}$ is the Weyl tensor, coefficients c_i are of the dimension length squared and we introduced $\sigma_0 = 1/G$ for later convenience. Without higher derivative corrections, the general Kerr-AdS solution has been obtained in [32]. Denote

$$\Xi_{a,0} = 1 - a^2\ell_0^{-2}, \qquad \Xi_{b,0} = 1 - b^2\ell_0^{-2}, \tag{2}$$

the mass, entropy, and two angular momenta are given by [20,32]

$$M_0 = \frac{\sigma_0 \pi m(2\Xi_{a,0} + 2\Xi_{b,0} - \Xi_{a,0}\Xi_{b,0})}{4\Xi_{a,0}^2\Xi_{b,0}^2},$$
$$S_0 = \frac{\sigma_0\pi^2(r_0^2 + a^2)(r_0^2 + b^2)}{2r_0\Xi_a\Xi_b},$$
$$J_{a,0} = \frac{\sigma_0\pi m a}{2\Xi_{a,0}^2\Xi_{b,0}}, \qquad J_{b,0} = \frac{\sigma_0\pi m b}{2\Xi_{b,0}^2\Xi_{a,0}}. \tag{3}$$

where m, a, b are integration constants and r_0 is the radius of the outer horizon determined by

$$(r_0^2 + a^2)(r_0^2 + b^2)(1 + r_0^2\ell_0^{-2}) - 2mr_0^2 = 0. \tag{4}$$

Thermodynamic potentials including temperature and two angular velocities are [20,32]

$$T_0 = \frac{1}{2\pi}\left[\frac{r_0(1 + r_0^2\ell_0^{-2})}{r_0^2 + a^2} + \frac{r_0(1 + r_0^2\ell_0^{-2})}{r_0^2 + b^2} - \frac{1}{r_0}\right],$$
$$\Omega_{a,0} = \frac{a(1 + r_0^2\ell_0^{-2})}{r_0^2 + a^2}, \qquad \Omega_{b,0} = \frac{b(1 + r_0^2\ell_0^{-2})}{r_0^2 + b^2}, \tag{5}$$

Using (4), one can verify that (3) and (5) obey the first law of thermodynamics. The Gibbs free energy is obtained as

$$G_0 = M_0 - T_0 S_0 - \Omega_{a,0}J_{a,0} - \Omega_{b,0}J_{b,0},$$
$$= \frac{\sigma_0\pi}{4\Xi_{a,0}\Xi_{b,0}}[m - \ell_0^{-2}(r_0^2 + a^2)(r_0^2 + b^2)]. \tag{6}$$

To compute 4-derivative corrections to the thermodynamics of general Kerr-AdS solution, we first perform the field redefinitions

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu} + \lambda_0 g_{\mu\nu} + \lambda_1 R_{\mu\nu} + \lambda_2 g_{\mu\nu} R,$$
$$\lambda_0 = \frac{40}{3}c_1\ell_0^{-2}, \qquad \lambda_1 = -c_2, \qquad \lambda_2 = \frac{2}{3}c_1 + \frac{1}{5}c_2, \tag{7}$$

which transform the general quadratic curvature theory (1) to the Einstein-Weyl theory with equivalent thermodynamic variables [21]

$$I_{\rm EW} = -\frac{\sigma}{16\pi}\int d^5x\sqrt{g}[(R + 12\ell^{-2}) + c_3 C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}]$$
$$- I_{\rm surf}, \tag{8}$$

where the surface term includes the Gibbons-Hawking-York term and counterterms for the 2-derivative bulk action

$$I_{\rm surf} = \frac{\sigma}{16\pi}\int_{z=\epsilon} d^4x\sqrt{h}\left[2K - \left(\frac{6}{\ell}+\frac{\ell}{2}\mathcal{R}\right) + \log\frac{\epsilon^2}{\ell^2}\mathcal{A}_4\right],$$
$$\mathcal{A}_4 = \frac{\ell^3}{8}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{1}{3}\mathcal{R}^2\right) + c_3\frac{\ell}{2}\mathcal{C}^{ijkl}\mathcal{C}_{ijkl}, \tag{9}$$

where K is the extrinsic curvature of the AdS boundary located at $z=\epsilon$ for $\epsilon\to 0$, and $\mathcal{R}$, $\mathcal{R}_{ij}$, and $\mathcal{C}_{ijkl}$ refer to the boundary Ricci scalar curvature, Ricci tensor and Weyl tensor, respectively. The logarithmic terms induced by the Einstein gravity were well known [33]. However, we also find that the bulk Weyl tensor squared also induces a new logarithmic counterterm proportional to c_3. As in the case of 2-derivative pure gravity [34], these surface terms are sufficient to remove all the IR divergences for general asymptotically locally AdS solutions. Notice that the new logarithmic counterterm proportional to c_3 is absent when the bulk spacetime dimension is even (see Ref. [35] for $D=4$ case and [9,36] for $D=6$ case). Moreover, all the logarithmic terms vanish for AdS black holes with $S^1\times M_3$ type boundary topology for M_3 being Einstein which is the case for rotating Kerr-AdS black hole. The coefficients of the logarithmic counterterms also imply the central charges in the dual conformal field theory (CFT) [37]

$$\mathrm{a} = \frac{\pi\ell^3}{8}\sigma, \qquad \mathrm{c} = \frac{\pi\ell^3}{8}\sigma(1+8c_3\ell^{-2}). \tag{10}$$

By treating the 4-derivative terms perturbatively, i.e., their corrections to the solutions vanish smoothly as the 4-derivative couplings are turned off, one can still impose the standard Dirichlet boundary condition on the metric residing on the conformal boundary [34,38,39].

In (8) and (9), various coupling constants in the Einstein-Weyl theory are related to the original ones in (1) by

$$\sigma = \sigma_0 - \frac{40c_1\sigma_0}{\ell_0^2}, \qquad \ell = \ell_0 - \frac{10c_1}{3\ell_0}. \tag{11}$$

For Einstein-Weyl gravity, the quadratic curvature correction to the Euclidean action of Kerr-AdS black hole is obtained by simply plugging the uncorrected solution [32] into Weyl-squared action and performing the integration [21]. With the Dirichlet boundary condition, the resulting Euclidean action is defined in the grand ensemble with fixed temperature and angular velocity. In terms of these variables, the higher derivative interactions only affect the form of the functional dependence of the Euclidean action on T and $\Omega_{a,b}$. Schematically we have

$$I_{\rm EW} = I_0(T,\Omega) + c_3 I_1(T,\Omega) + \frac{3\mathrm{a}}{4\ell}. \tag{12}$$

According to the prescription of [17,21], in terms of the variables r_0, a, b, the form of temperature and angular momenta remain the same, namely

$$T = T_0, \qquad \Omega_a = \Omega_{a,0}, \qquad \Omega_b = \Omega_{b,0}, \tag{13}$$

whereas the conserved charges do receive explicit higher derivative corrections.

Expressing parameters of Einstein-Weyl theory back to those of the original theory, we obtain the Euclidean action of Kerr-AdS black hole with general quadratic curvature corrections. Using the standard relation between Euclidean action and Gibbs free energy [16] and omitting the central charge, we obtain Gibbs free energy of Kerr-AdS black hole in the grand canonical ensemble

$$G_{QG} = G_0 + \delta G_1 + \delta G_2, \tag{14}$$

in which δG_1 and δG_2 are corrections caused by the Ricci scalar squared and Weyl-squared

$$\delta G_1 = -c_1\frac{5\pi(\hat a^2+1)(\hat b^2+1)\hat r_0^2\sigma_0}{6\hat\Xi_{a,0}^2\hat\Xi_{b,0}^2}\left[7\hat r_0^6\hat a^2\hat b^2 - 8\hat r_0^4(\hat a^2\hat b^2+\hat a^2+\hat b^2) + \hat r_0^2(7\hat a^2+7\hat b^2+9) - 6\right],$$
$$\delta G_2 = -c_3\frac{\pi(\hat r_0^2+1)^2\sigma_0[\hat a^2\hat b^2(\hat a^2\hat b^2-20) + 2(\hat a^2+\hat b^2)(1-3\hat a^2\hat b^2) + \hat a^4+\hat b^4+9]}{4(\hat a^2+1)(\hat b^2+1)\hat\Xi_{a,0}\hat\Xi_{b,0}}, \tag{15}$$

where we introduced dimensionless variables $\hat r_0 = r_0\ell_0^{-1}$, $\hat a = ar_0^{-1}$, $\hat b = br_0^{-1}$ and accordingly $\hat\Xi_{a,0} = 1-\hat a^2\hat r_0^2$, $\hat\Xi_{b,0} = 1-\hat b^2\hat r_0^2$. It is interesting to see that $r_{\mu\nu}r^{\mu\nu}$ does not contribute. The other thermodynamic variables are obtained from the Gibbs free energy via standard relations

$$M = G_{QG} + TS + \Omega_a J_a + \Omega_b J_b,$$
$$S = -\frac{\partial G_{QG}}{\partial T}\bigg|_{\Omega_{a,b}}, \qquad J_{a(b)} = -\frac{\partial G_{QG}}{\partial\Omega_{a(b)}}\bigg|_{T,\Omega_{b(a)}}. \tag{16}$$

To test results above, we consider Kerr-AdS black hole with equal rotation which is cohomogenity-1 allowing us to solve the 4-derivative field equations numerically. From the numerical solution, we extract the mass and angular momenta using the generalized Ashtekar-Magnon-Das formula for quadratic curvature theories [40–42] and compare them to the analytical results obtained above. For a nonextremal Kerr-AdS solution, the comparison is still not easy as all the thermodynamic quantities depend on two variables. To simplify the comparison further, we

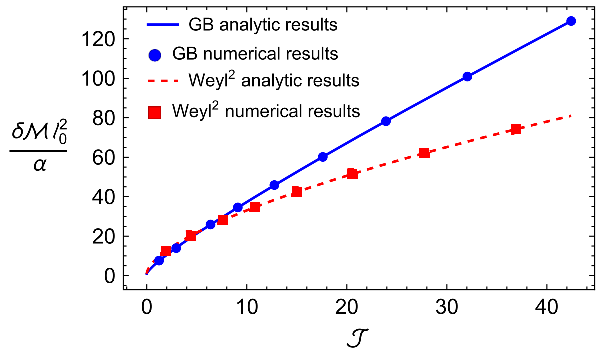

FIG. 1. For extremal Kerr-AdS black hole with equal rotation, we exhibit matching between analytical and numerical results for $5D$ Einstein-Gauss-Bonnet gravity and Einstein-Weyl gravity.

restrict to the extremal Kerr-AdS solution in which all the thermodynamic quantities depend on a single variable. In particular, we consider mass as a function of the angular momentum

$$M_{\rm ex}(J_{\rm ex}) = M_0(J_{\rm ex}) + \delta M(J_{\rm ex}), \tag{17}$$

of which $M_0(J_{\rm ex})$ can be derived from the uncorrected quantities given in (3), the explicit expression of $M_{\rm ex}(J_{\rm ex})$ is a bit lengthy and will be postponed to the Supplemental Material [43]. It is the latter which will be subject to numerical tests.

In Fig. 1 below, we show that the analytical and numerical results indeed match in a wide range of variables for the well studied Gauss-Bonnet combination corresponding to $c_1 = \frac{3}{10}\alpha, c_2 = -\frac{8}{3}\alpha, c_3 = \alpha$ and Einstein-Weyl (EW) gravity corresponding to $c_1 = c_2 = 0, c_3 = \alpha$. In the plot, we have defined the dimensionless mass $\mathcal{M} = M_{\rm ex}/(\pi\sigma_0\ell_0^2)$ and angular $\mathcal{J} = J_{\rm ex}/(\pi\sigma_0\ell_0^3)$. The plot is independent of choice of α as $\delta\mathcal{M}$ depends on α linearly.

Quadratic curvature corrections to charged rotating black holes in 5D minimal gauged supergravity. The complete basis of curvature squared supergravity in 5D minimal gauged supergravity was presented in [25,26]. After applying field redefinitions preserving black hole thermodynamics [21], we obtain the action below:

$$\begin{aligned} I_{5D,N=1} = {}& -\frac{\sigma}{16\pi}\int d^5x\sqrt{g}\bigg(R + \frac{12}{\ell^2} - \frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} \\ &+ \frac{\mathrm{i}}{12\sqrt{3}g^3}\epsilon^{\mu\nu\rho\sigma\delta}F_{\mu\nu}F_{\rho\sigma}A_\delta + c_1\mathcal{L}_{\rm Weyl^2}\bigg) - I_{\rm surf}, \end{aligned} \tag{18}$$

where the surface term includes additional logarithmic counterterm proportional to $F_{ij}F^{ij}$ [48]. We recall that in the original Lagrangian, there are supersymmetric Ricci tensor squared and Ricci scalar squared actions. Denoting their coefficients by c_2 and c_3, respectively, the coupling constants in the action (18) are related to the original ones via (see the Supplemental Material [43] for further details)

$$\begin{aligned} \sigma &= \sigma_0 - 24\sigma_0(c_2 + c_3)\ell_0^{-2}, \\ g &= g_0\Big(1 + 4(c_2 + c_3)\ell_0^{-2}\Big), \\ \ell &= \ell_0\Big(1 - 4(c_2 + c_3)\ell_0^{-2}\Big), \end{aligned} \tag{19}$$

where g_0 is the U(1) coupling before the field redefinition. It is a bookkeeping parameter introduced via $A_\mu \to A_\mu/g_0$ in the standard supergravity action and does not affect physical quantities. The on-shell Weyl-squared supergravity action $\mathcal{L}_{\rm Weyl^2}$ is given by

$$\begin{aligned} \mathcal{L}_{\rm Weyl^2} = {}& -\frac{2}{g^2\ell^2}F_{\mu\nu}F^{\mu\nu} - \frac{\mathrm{i}}{\sqrt{3}g^3\ell^2}\epsilon^{\mu\nu\rho\sigma\delta}F_{\mu\nu}F_{\rho\sigma}A_\delta \\ &+ C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - \frac{1}{2g^2}C_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} + \frac{13}{96g^4}(F_{\mu\nu}F^{\mu\nu})^2 \\ &- \frac{13}{24g^4}F^4 + \frac{\sqrt{3}i}{6g}\epsilon^{\mu\nu\rho\sigma\alpha}A_\mu C_{\nu\rho}{}^{\beta\gamma}C_{\sigma\alpha\beta\gamma}, \end{aligned} \tag{20}$$

where we define $F^4 \coloneqq F_{\mu\nu}F^{\nu\lambda}F_{\lambda\delta}F^{\delta\mu}$. Different from the ungauged supergravity, in the gauged case, the curvature squared supergravity actions contain also 2-derivative terms. It is worth mentioning that in the frame we have chosen, the parameter ℓ already represents the effective AdS radius. This is quite different from the setup adopted in [28,29] where the curvature squared combinations do renormalize the bare AdS radius [49].

The general charged rotating black hole solution in 2-derivative $5D$ minimal gauged supergravity was obtained in [50]. Regularity of the solution determines the inverse temperature to be

$$\beta = \frac{2\pi r_0(abq + (r_0^2 + a^2)(r_0^2 + b^2))}{r_0^4[1 + (a^2 + b^2 + 2r_0^2)\ell^{-2}] - (ab + q)^2}, \tag{21}$$

where r_0 is the radius of the outer horizon, a, b, q are parameters related to angular velocities and electrostatic potential given by

$$\begin{aligned} \Omega_a &= \frac{a(r_0^2 + b^2)(1 + \ell^{-2}r_0^2) + bq}{abq + (r_0^2 + a^2)(r_0^2 + b^2)}, \\ \Omega_b &= \frac{b(r_0^2 + a^2)(1 + \ell^{-2}r_0^2) + aq}{abq + (r_0^2 + a^2)(r_0^2 + b^2)}, \\ \Phi_e &= \frac{g\sqrt{3}qr_0^2}{abq + (r_0^2 + a^2)(r_0^2 + b^2)}. \end{aligned} \tag{22}$$

Analogous to the discussion in the previous section, these variables characterize the Euclidean action of charged rotating black holes as the current choice of boundary condition specifies the grand canonical ensemble.

With supersymmetric curvature squared corrections switched on, in principle one would have to first solve for the modified field equations before computing the corrected on-shell action. However, as established in [17,21] and here, this step can be circumvented if the higher curvature terms are expressed in terms of the Weyl tensor. The fully corrected on-shell action can be obtained by simply evaluating the modified action on the uncorrected solution. Adopting this method, we obtain the Euclidean action for the general charged rotating AdS black holes in 5D minimal gauged supergravity extended by all three curvature squared invariants. The full result is given in the Supplemental Material [43]. Similar to the nonsupersymmetric case, the thermodynamic variables $\beta, \Omega_{a,b}, \Phi_e$ remains the same form as in (22). Thus the conserved charges can be obtained by differentiating the Euclidean action with respect to $\beta, \Omega_{a,b}, \Phi_e$.

We now apply the result to under the entropy of the supersymmetric charged rotating AdS black hole [51,52] which admits microscopic description in terms of index of the dual $4D, N=1$ superconformal field theory [53–59]. To proceed, we impose supersymmetry condition

$$q = -(a - ir_0)(b - ir_0)(1 - ir_0\ell^{-1}). \tag{23}$$

Note that the BPS limit requires also zero temperature and can be arrived via $r_0 \to \sqrt{\ell(a+b)+ab}$ [50]. Different from previous work [28,29], the supersymmetric condition now is corrected by the 4-derivative terms whose effect is fully encoded in the effective AdS radius ℓ. Subsequently, one can define thermodynamic potentials

$$\begin{aligned}\omega_a &= \beta_s(\Omega_{a,s} - \Omega_{a,*}) = \frac{2\pi(b - ir_0)(a-\ell)}{\Xi},\\ \omega_b &= \beta_s(\Omega_{b,s} - \Omega_{b,*}) = \frac{2\pi(a - ir_0)(b-\ell)}{\Xi},\\ \varphi &= \beta_s(\Phi_{e,s} - \Phi_{e,*}) = \frac{6\pi g\ell(a - ir_0)(b - ir_0)}{\sqrt{3}\Xi},\end{aligned} \tag{24}$$

where $\Xi = 2r_0(\ell + a + b) + i(\ell(a+b) + ab) - 3ir_0^2$ and they satisfy $\omega_a + \omega_b - \frac{\sqrt{3}}{g\ell}\varphi - 2\pi i = 0$. Here "$s$" means that the supersymmetry condition (23) has been applied and "*" denotes values of these variable in the BPS limit

$$\Omega_{a,*} = \ell^{-1}, \qquad \Omega_{b,*} = \ell^{-1}, \qquad \Phi_{e,*} = \sqrt{3}g, \tag{25}$$

Imposing the supersymmetric condition (23), we find that the Euclidean action drastically simplifies

$$I_{\mathrm{ren},s} = \frac{\pi\sigma\varphi^3(1 - \frac{12c_1}{\ell^2})}{12\sqrt{3}g^3\omega_a\omega_b} + \frac{c_1\pi\sigma\varphi(\omega_a^2 + \omega_b^2 - 4\pi^2)}{\sqrt{3}g\omega_a\omega_b}. \tag{26}$$

In terms of the a and c central charges of the dual theory given in (10) with c_3 replaced by c_1, the supersymmetric on-shell action indeed takes the form as its counterpart in the dual field theory [60]. Although our supersymmetric action takes the same form as previous results in [28,29], the details are different as our $\omega_{a,b}, \varphi$ defined in (24) depend on the effective AdS radius, while those in [28,29] use bare AdS radius instead. Taking the BPS limit, we find the conserved charges obey the linear relation [61]

$$M_* - \ell^{-1}J_{a,*} - \ell^{-1}J_{b,*} - \frac{3}{2}\ell^{-1}Q_R = 0, \tag{27}$$

where $Q_R := \frac{2g\ell}{\sqrt{3}}Q_{e,*}$ is the canonically normalized U(1) R-charge in the dual SCFT. This equality leads to vanishing Gibbs free energy. In BPS limit, the entropy of the charged rotating black hole also reproduces the microscopic result [28,29]. Namely, up to $\mathcal{O}(c_i)$ it is given by

$$S_* = \pi\sqrt{3Q_R^2 - 8\mathrm{a}(J_{a,*} + J_{b,*}) - 16\mathrm{a}(\mathrm{a} - \mathrm{c})\frac{(J_{a,*} - J_{b,*})^2}{Q_R^2 - 2\mathrm{a}(J_{a,*} + J_{b,*})}}. \tag{28}$$

Conclusion and outlook. So far we have showed that Weyl gravity offers an efficient way of computing the leading higher curvature contributions to thermodynamic quantities of general rotating AdS black holes in 5D quadratic gravity theories with or without supersymmetry. In fact, this approach also applies to more general quadratic gravity theories with matter couplings in other dimensions simply due to the fact that Weyl tensor vanishes sufficiently fast near the AdS boundary. We believe our approach can be pushed forward to the next to next to leading order higher curvature corrections as already achieved in the asymptotically flat case [62]. Together with the first law of thermodynamics, our results also imply that in the basis of Weyl tensor the leading higher derivative corrections to the black entropy can be readily computed via $\delta S = -I_{\mathrm{hd}}|_{(T,\Phi_e,\Omega_{a,b})}$, which should be useful in the discussion of the AdS counterpart of weak gravity conjecture. To further establish the effectiveness of Weyl gravity in AdS quantum gravity through holography, it should be very interesting to consider other solutions such AdS black strings [63–65] and compute correlation functions as well as various transport coefficients.

Acknowledgments. We are grateful to D. Cassani, M. Ozkan, A. Ruiprez, G. Tartaglino-Mazzucchelli, and E. Turetta for useful discussions. In particular, we thank D. Cassani, A. Ruiprez, and E. Turetta for communications regarding the dependence of the on-shell action on the combination of $c_2 + c_3$. We also thank N. Bobev for useful correspondence. Y. P. is supported by the National Key Research and Development Program No. 2022YFE0134300 and the NSFC Grant No. 12175164. The work of Y. P. is also partially supported by Peng Huanwu Center for Fundamental Theory, under Grant No. 12247103. H. L. is supported in part by the National Natural Science Foundation of China (NSFC) Grants No. 11935009 and No. 12375052.

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