Test of universality of free fall for neutrinos

Arunava Bhadra,[1,*] Kabita Sarkar,[2,3,†] Ramil N. Izmailov,[4,‡] and Kamal K. Nandi[4,§]

[1]*High Energy and Cosmic Ray Research Centre, University of North Bengal, Siliguri, West Bengal 734013, India*

[2]*Department of Mathematics, Swami Vivekananda Institute of Science and Technology, Dakshin Gobindapur, Kolkata 700145, India*

[3]*IPMDN, M. Akmullah Bashkir State Pedagogical University, 3A, October Revolution Street, Ufa 450008, RB, Russia*

[4]*Zel'dovich International Center for Astrophysics, M. Akmullah Bashkir State Pedagogical University, 3A, October Revolution Street, Ufa 450008, RB, Russia*

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The universality of free fall, fundamental to metric theories of gravity, asserts the equality between inertial and gravitational mass, implying that all particles experience equal gravitational fall regardless of their mass. While it is widely accepted for all particles and photons, its applicability to neutrinos, due to their minimal interactions, remains uncertain. Neutrinos, though predicted to be massless, exhibit flavor-changing behavior and have nonzero mass, making it crucial to determine if they adhere to the principle. This paper introduces evidence from supernova 1987a observations suggesting that the time difference between correlated photons and neutrinos supports the universality of free fall for neutrinos with an accuracy of 2.7 part of a thousand.

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I. INTRODUCTION

The fundamental principle underpinning any metric theory of gravity, such as general relativity (GR), is the equality between inertial mass (m_I) and gravitational mass (m_G) of a particle—known as the universality of free fall [1]. This principle asserts that all particles experience equal gravitational fall, irrespective of their mass, a concept originally inferred from Galileo's experimental observations [2] and later demonstrated by Newton in his pendulum experiment [3], showcasing equality to within one part in a thousand. Numerous subsequent experiments, including the Eötvös experiment, have validated this equality to higher precision, reaching approximately one part in 20,000,000 [4]. Modern experiments have achieved a precision of around 10^{-15} confirming the uniqueness of free fall [5].

According to the weak equivalence principle (WEP), any metric theory, including GR, posits that a photon or a neutrino experiences the same gravitational field as a particle with nonzero mass at a given space-time point due to a gravitating object. The universality of free fall in a sense is integral to the WEP. The WEP encompasses broader principles, such as the requirement for a particle adhering to it to possess equal gravitational and inertial mass, as well as experiencing the same gravitational curvature (induced by a gravitating object) regardless of its energy. While gravitational red-shift experiments indirectly support the uniqueness of free fall for photons, no observational analysis has definitively established whether neutrinos, characterized as "ghost particles" due to their minimal interactions with matter, also adhere to the same gravitational field perception. Neutrinos, with their peculiar flavor-changing behavior and minimal interactions, defy the standard model's zero-mass prediction by exhibiting neutrino oscillations and having a mass upper bound of less than an electron volt (eV) [6]. Consequently, determining whether the uniqueness of free fall holds for neutrinos becomes a crucial question.

This paper aims to present, for the first time, evidence indicating that the time difference in the arrival of correlated photons and neutrinos (of varying energies) from supernova 1987A supports the uniqueness of free fall for neutrinos. It is noteworthy that the reported test of the weak equivalence principle in terms of the curvature effect (through the parametrized post-Newtonian parameter γ) [7,8], based on the mentioned supernova 1987A observations, assumed the uniqueness of free fall for photons and neutrinos without providing any specific reasons. Later on, several researchers have employed the observed arrival time difference between different types of neutral particles/photons/gravitational wave, or photons with

*Contact author: aru_bhadra@yahoo.com
†Contact author: kabita_id@rediffmail.com
‡Contact author: izmailov.ramil@gmail.com
§Contact author: kamalnandi1952@rediffmail.com

varying energies which are emitting simultaneously from astrophysical sources to constrain Einstein's equivalence principle in terms of the parameter γ [9–18]. The forthcoming demonstration illustrates that the gravitational curvature effect is considerably smaller than the impact of the uniqueness of free fall.

II. THE HOST GALAXY OF SN1987A AND ITS DISTANCE

The SN1987A resides within the Large Magellanic Cloud (LMC), the principal satellite of our galaxy. Various studies have attempted to gauge the distance to the LMC [19] (and references therein). However, these estimates rely on the calibration of other distance indicators, rendering them indirect and merely statistical in nature. The distinct circumstellar ring encircling SN 1987A has enabled a direct measurement of the distance using a geometric method [20,21]. By observing the radial velocity of SN1987A relative to the LMC's center, the distance to the barycenter of the LMC can be determined. Panagia utilized the angular size of SN 1987A's circumstellar rings in Hubble Space Telescope images and the absolute sizes derived from light curves of narrow UV emission lines to estimate a distance of 51.4 ± 1.2 kpc to SN 1987A and 51.7 ± 1.3 kpc to the LMC barycenter [21]. Recently, Pietrzyński *et al.* [22] employed interferometry-based surface brightness color relation to obtain a distance of 49.59 ± 0.63 kpc to the LMC with an accuracy of 1%. In this study, we adopt this measured distance along with the knowledge that SN1987A is 0.3 kpc closer to the Earth than the LMC barycenter.

Estimating the mass of a galaxy is a challenging task. The situation is particularly intricate for the LMC. Its mass has been inferred from the dynamics of star clusters [23] and rotation curve measurements [24]. However, recent wide-field studies have revealed significant debris along the LMC's periphery, suggesting a potentially larger mass. Various indicators suggest that the LMC's mass exceeds $10^{11} M_\odot$ ($M_\odot$ denotes solar mass). For instance, a mass greater than $10^{11} M_\odot$ is necessary for the gravitational bonding of the LMC and the Small Magellanic Cloud (SMC). Recent investigations have demonstrated that stellar streams offer a novel, independent means of assessing the mass distribution. The gravitational field of the LMC is likely to influence some of the Milky Way's stellar streams. Based on the dynamics of stars in the Orphan stream, recent measurements estimate the mass of the LMC (M_{LMC}) as $1.38 \times 10^{11} M_\odot$, and that of the Milky Way within 50 kpc as $M_{MW} = 3.8 \times 10^{11} M_\odot$ [25].

III. THE TRAVEL TIME OF NEUTRINOS FROM SN1987A

As discussed above, the mass of the LMC cannot be ignored in comparison to that of the Milky Way within 50 kpc. Furthermore, SN1987A is positioned near the barycenter of the LMC, indicating that the gravitational influence of the LMC on particles/photons during their journey from SN1987A to Earth cannot be disregarded. Thus, we treat this as a two-body system and employ the parametric post-Newtonian (PPN) formalism [26].

The PPN metric in Scharzschild coordinates is given by

$$ds^2 = -B(r)c^2 dt^2 + A(r)dr^2 + r^2 d\Omega^2, \tag{1}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ and c the usual speed of light. In the present case

$$B(r) = 1 - \frac{2\alpha G M_{MW}}{c^2 r} + \frac{2\alpha G M_{LMC}}{c^2 (r_{LMC} - r)} + \cdots \tag{2}$$

and

$$A(r) = 1 + \frac{2\gamma G M_{MW}}{c^2 r} - \frac{2\gamma G M_{LMC}}{c^2 (r_{LMC} - r)} + \cdots, \tag{3}$$

where α and γ are PPN parameters and r_{LMC} is the distance of the LMC barycenter. A celestial object's mass is generally measured by studying the acceleration parameter of a test particle. In the process, α is absorbed in GM by setting it as 1 [27]. The uniqueness of free fall, as observed in different experiments to a high accuracy, justifies the choice $\alpha = 1$ to a high accuracy for all particles excluding neutrinos. The gravitational redshift measurements give restrictions on α_γ (the subscript γ refers to photons). Analysis of the Vessot-Levine rocket experiment (Gravity Probe-A) [28] restricts α_γ to $|\alpha_\gamma - 1| \leq 1.4 \times 10^{-4}$. Recently, using the atomic clock data from two satellites of Galileo satellites, $|\alpha_\gamma - 1|$ is constrained to the 2.0×10^{-5} level at 1σ [29,30]. The corresponding PPN parameter for neutrons, α_ν, is not known experimentally so far. In all previous efforts of testing the weak equivalence principle for neutrinos using the observed time difference in the arrival of correlated photons and neutrinos from astrophysical sources, α_ν was taken as 1, and a restriction of $\gamma_\nu - \gamma_\gamma$ was imposed [7,8,13]. The geodesic equations for the motion of a test particle in equatorial plane, for the space time metric given by Eq. (1), leads to the following relation

$$\frac{A(r)}{B(r)^2}\left(\frac{dr}{dt}\right)^2 + \frac{J^2}{r^2} - \frac{c^2}{B(r)} = -\eta c^2, \tag{4}$$

where J and η correspond to the constants associated with the motion. J is related to the specific angular momentum of the particle, defined as $J \equiv r^2 \frac{d\phi}{dt}/B(r)$, ϕ is the azimuthal angle. On the other hand, η ($\equiv \frac{d\tau}{dt}/B(r)$) relates to the specific energy of the particle. For particle with nonzero rest mass m, $\eta > 0$; whereas for particle with zero rest mass, $\eta = 0$. At the distance of closest approach r_o, $\frac{dr}{dt}$ must vanish, i.e., $\frac{dr}{dt}|_{r=r_o} = 0$. This gives

$$J^2 = c^2\left[-\eta + \frac{1}{B(r_o)}\right]r_o^2. \tag{5}$$

The time required by a particle to traverse a distance from r to r_o in the gravitational field described by Eq. (1) is obtained from Eq. (4) using Eq. (5), which is given by

$$\Delta t(r, r_o) = \frac{1}{c}\int_{r_o}^{r}\sqrt{P(r,\eta)}dr, \tag{6}$$

where

$$P(r,\eta) = \frac{A(r)/B(r)}{\left[1 - \eta B(r) + \frac{r_o{}^2}{r^2}\left(\eta B(r) - \frac{B(r)}{B(r_o)}\right)\right]}. \tag{7}$$

In the trajectory of photons/neutrinos from SN 1987A to us, the closest approach (r_o) can be taken as the distance of the Earth (r_E) from the Galactic Center [7]. Accordingly, $\eta = \frac{m^2c^4}{\epsilon^2 B(r_E)}$, m and ϵ are the mass and energy of the particle. One thus gets from Eq. (7) that the coordinate time required by the particle traveling from the SN 1987A to the Earth is given by

$$\Delta t(r_{SN}, r_E) = \frac{1}{c\sqrt{1-\eta}}\left[\sqrt{r_{SN}^2 - r_E^2} + \Delta t_o\right], \tag{8}$$

where

$$\begin{aligned}\Delta t_o = {} & \frac{(\alpha + \gamma - (2\alpha+\gamma)\eta)\mu_{MW}}{1-\eta} ln\frac{r_{SN} + \sqrt{r_{SN}^2 - r_E^2}}{r_E} + \frac{\alpha\mu_{MW}}{1-\eta}\left(\frac{r_{SN}-r_E}{r_{SN}+r_E}\right)^{1/2} \\ & + \frac{(\alpha + \gamma - (2\alpha+\gamma)\eta)\mu_{LMC}}{1-\eta} ln\frac{r_{SN} + \sqrt{r_{SN}^2 - r_E^2}}{r_E} - \frac{\alpha\mu_{LMC}r_E^2}{(1-\eta)(r_{LMC}^2 - r_E^2)}\left(\frac{r_{SN}-r_E}{r_{SN}+r_E}\right)^{1/2} \\ & - \frac{(\alpha + \gamma - (2\alpha+\gamma)\eta)\mu_{LMC}}{1-\eta}\frac{r_{LMC}}{\sqrt{r_{LMC}^2 - r_E^2}} ln\left[\frac{r_{LMC}r_{SN} - r_E^2}{r_E(r_{LMC} - r_{SN})} + \frac{2\sqrt{r_{LMC}^2 - r_E^2}\sqrt{r_{SN}^2 - r_E^2}}{(r_{LMC} - r_{SN})r_E}\right] \\ & + \frac{\alpha\mu_{LMC}}{1-\eta}\frac{r_{LMC}r_E^2}{(r_{LMC}^2 - r_E^2)^{3/2}} ln\left[\frac{\left(\sqrt{r_{LMC}^2 - r_E^2}\sqrt{r_{SN}^2 - r_E^2} + r_{LMC}r_{SN} - r_E^2\right)(r_{LMC} - r_E)}{(r_{LMC} - r_{SN})(r_{LMC}r_{SN} - r_E^2)}\right],\end{aligned} \tag{9}$$

where r_{SN}, r_E, and r_{LMC} are, respectively, the radial coordinate distances of the SN 1987A, the Earth, and the barycenter of LMC from the center of the Milky Way, and $\mu_i = M_i G/c^2$, the subscript i refers to Milky Way (MW), LMC. The above expressions show that an extra time [the second term of the right-hand side of Eq. (8)] is required by a particle for traversing along a trajectory in a massive object's gravitational field over the required time for the travel between the same two points in Minkowski spacetime. This extra time is the well-known Shapiro time delay. However, the gravitational field of the LMC acts on the opposite direction to that of the Milky Way and consequently the net time delay is reduced. The above expressions have been used in the literature (with $\alpha = 1$ and neglecting the gravitational field of the LMC) to test the weak equivalence principle from the difference in arrival times of photons and neutrinos from SN 1987A [7,8,31] that imposed restrictions on $\gamma_\gamma - \gamma_\nu$. Note that [7,8] additionally imposed $\eta = 0$ for netrinos, i.e., they treated neutrino as massless.

The difference in proper time between transmission and reception of the signal to be measured by the observer at r_E is

$$\Delta\tau = \sqrt{B(r_E)}\Delta t(r_{SN}, r_E). \tag{10}$$

Since the observer is at higher gravitational field compared to that in the major part of the trajectory, there will be a net negative time delay (time advancement) [32–34] in this case. The difference in time of travel between photons [is obtained by putting $\eta = 0$ in Eqs. (8), (9)] and neutrinos for the journey from the SN 1987 to the Earth is

$$\begin{aligned}\Delta\tau^\nu - \Delta\tau^\gamma \approx {} & -(\alpha_\nu - \alpha_\gamma)\left(\frac{\mu_{SN}}{r_E} - \frac{\mu_{LMC}}{r_{LMC} - r_E}\right)\frac{D}{c} \\ & - c_o(\Delta t_{o\nu} - \Delta t_{o\gamma}),\end{aligned} \tag{11}$$

where $D = \sqrt{r_{SN}^2 - r_E^2}$ is the distance of the SN 1987A from the Earth. $c_o \simeq 1 - (\frac{\mu_{SN}}{r_E} - \frac{\mu_{LMC}}{r_{LMC} - r_E})$, $\Delta t_{o\nu}$, and $\Delta t_{o\gamma}$ are, respectively, the expressions given by Eq. (9) for neutrinos with mass m and energy ϵ and for photons (where $\eta = 0$). An intriguing point is that the above time difference in the leading order does not depend on the mass of the neutrinos (particles). For SN1987A emission, the above equation leads to

$$\Delta\tau^{\nu} - \Delta\tau^{\gamma} \approx [-7.93(\alpha_{\nu} - \alpha_{\gamma}) + 1.2(\gamma_{\nu} - \gamma_{\gamma})] \times 10^6 \text{ s.} \tag{12}$$

The time difference of emission between neutrinos and correlated photons is not known properly. The neutrinos are expected to emit in the first few seconds of the collapse. The electromagnetic bursts may occur after the shock waves in the core collapse reach the stellar surface, which may take a couple of hours of the collapse. The difference in optical brightening of the source from the first detection of neutrino burst from SN1987A at the Kamioka and IMB (Irvine-Michigan-Brookhaven) detectors was less than three hours [35–37]. So a conservative assumption is that the difference in propagation time of photons and neutrinos is within six hours as taken in previous studies in different contexts [7,38]. Assuming that the γ_{ν} is close to 1, the above equations along with the restrictions on α_{γ} by redshift measurements imply $|\alpha_{\nu} - 1.000| \le 2.7 \times 10^{-3}$. If no *a priori* restriction is applied on γ_{ν}, then it is not possible to impose an individual constraint on α_{ν}. In that case, Eq. (12) gives $|(\alpha_{\nu} - 1.000) - 0.15(\gamma_{\nu} - 1.000)| \le 2.7 \times 10^{-3}$. It was further found that the dispersion in arrival times for neutrinos and antineutrinos from the SN1987A with energies between roughly 7.5 and 40 MeV was less than 10 s, which implies $\alpha_{\nu} - \alpha_{\bar{\nu}} \le 1.48 \times 10^{-6}$.

IV. CONCLUSION

We conclude that the observed difference of arrival times of neutrinos (and antineutrinos) and gamma rays from SN1987A constitutes the first direct test of uniqueness of free fall with neutrinos. The principle is validated with an accuracy of 2.7 part of a thousand. In this current analysis, we confine our focus solely to a local astrophysical source (SN1987A), due to certain concerns regarding the application of standard time delay calculations on cosmological scales [39]. Additionally, considering the gravitational influence of the host galaxy of sources appears to be crucial. We intend to undertake a comprehensive examination of these aspects in the near future.

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