Radiative decay and axial-vector decay behaviors of octet pentaquark states

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In this work, we systematically calculate transition magnetic moments, radiative decay widths, and axial-vector coupling constants of octet hidden-charm molecular pentaquark states with different flavor representations in constituent quark model. We discuss the relations between transition magnetic moments and decay widths for pentaquark states. For octet pentaquark states with the 8_{1f} and 8_{2f} flavor representations, decay widths of the processes $P_\psi|\frac{3}{2}^-\rangle_{(\frac{1}{2}^+\otimes 1^-)} \to P_\psi|\frac{1}{2}^-\rangle_{(\frac{1}{2}^+\otimes 0^-)}\gamma$ and $P_\psi|\frac{1}{2}^-\rangle_{(\frac{1}{2}^+\otimes 1^-)} \to P_\psi|\frac{1}{2}^-\rangle_{(\frac{1}{2}^+\otimes 0^-)}\gamma$ are quite close, and decay widths of the $P_\psi|\frac{3}{2}^-\rangle_{(\frac{1}{2}^+\otimes 1^-)} \to P_\psi|\frac{1}{2}^-\rangle_{(\frac{1}{2}^+\otimes 1^-)}\gamma$ process are close to zero, and we notice that the axial-vector coupling constants of the pentaquark states are generally smaller than that of the nucleon.

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I. INTRODUCTION

In 2003, X (3872) was observed by Belle Collaboration, which led to an explosive exploration of exotic hadronic states [1]. In the past decades, a large number of exotic hadronic states were experimentally observed, which stimulated many discussions [2–11]. In 2015, the pentaquark states $P_\psi(4380)^+$ and $P_\psi(4450)^+$ were observed in the $\Lambda_b^0 \to J/\psi p K^-$ decays by the LHCb Collaboration, which are close to the $\Sigma_c^*\bar{D}$ and $\Sigma_c\bar{D}^*$ thresholds [12]. It is the first time that the pentaquark state was observed experimentally. In 2019, the LHCb Collaboration observed that the $P_\psi(4450)^+$ consists of two narrow overlapping peaks, $P_\psi(4440)^+$ and $P_\psi(4457)^+$, and reported a new state $P_\psi(4312)^+$ near $\Sigma_c\bar{D}$ threshold [13]. Subsequently LHCb Collaboration announced the existence of a series of pentaquark states such as $P_\psi(4459)^+$ [14], $P_\psi(4337)^+$ [15] and $P_\psi(4338)^+$ [16], and these P_ψ states are considered to be hidden-charm pentaquark state candidates. The observation of pentaquark states aroused a great deal of interest among researchers, and theorists engaged in extensive discussions about the nature of pentaquark states [17–36].

Exploring exotic hadronic states can deepen our understanding of the nonperturbative behavior of QCD in the low-energy regime, and help us to understand more about the inner structure of hadrons. The axial-vector coupling constant is another important physical quantity which can help us analyze the electroweak and strong interactions in the Standard Model, and it is also an indicator of nonperturbative QCD chiral symmetry breaking. The inner structure of pentaquark states has been investigated by various methods, such as quark model [37–39], QCD sum rules [40–42], chromomagnetic interaction (CMI) model [43–45], and one-boson-exchange (OBE) model [46–48]. The masses and decay behavior of pentaquark states are studied in some pictures including molecular picture [49–52], diquark–diquark–antiquark picture [53–55], diquark–triquark picture [56–58] and compact pentaquark picture [59–61]. In Ref. [62], the authors systematically investigated magnetic moments, transition magnetic moments and radiative decay behaviors of the S-wave isoscalar $\Xi_c^{(\prime)}\bar{D}^{(\prime)}$ molecular pentaquark states in constituent quark model. In Ref. [63], the authors calculated the effective potentials of the $\Sigma_c\bar{D}^*$ systems with the heavy hadron chiral perturbation theory, and employed quark model with heavy quark spin symmetry to get some relations between different systems. In Ref. [64], the authors studied the mass spectrum of doubly charmed pentaquark states in the $\Lambda_c^{(*)}D^{(*)}$ and $\Sigma_c^{(*)}D^{(*)}$ channels with $J^P = \frac{1}{2}^\pm, \frac{3}{2}^\pm$ and $\frac{5}{2}^\pm$ within the framework of QCD sum rules. In Ref. [65], the authors predicted the mass spectrum of $\Omega_c^{(*)}D_s^{(*)}$-type doubly charmed molecular pentaquark candidates with OBE model considering both the S-D wave mixing effect and the coupled channel effect.

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Previously, we systematically studied magnetic moments and axial-vector coupling constants of the octet hidden-charm pentaquark states in Ref. [66] with constituent quark model. We noticed that the axial-vector coupling constants of octet hidden-charm pentaquark states are generally smaller than that of the nucleon. In this work, we continue to explore further the radiative decay and axial-vector decay behaviors of the octet hidden-charm pentaquark. The properties of these states can provide more information about the inner structure of the hidden-charm pentaquark states, which are also helpful for the studies of chiral effective theory.

This paper is organized as follows. In Sec. II, we construct the wave functions of octet hidden-charm pentaquark states. In Sec. III, we calculate the transition magnetic moments and radiative decay widths of the octet hidden-charm pentaquark states, and we conclude their relations. In Sec. IV, we construct the Lagrangians of the axial-vector decays and calculate the axial-vector coupling constants of the pentaquark states. Finally, we provide a brief summary in Sec. IV.

II. WAVE FUNCTIONS

The total wave function of hadronic state can be expressed as

$$\Psi = \phi_{\mathrm{flavor}}\chi_{\mathrm{spin}}\xi_{\mathrm{color}}\eta_{\mathrm{space}}, \tag{1}$$

where ϕ_{flavor}, χ_{spin}, ξ_{color}, η_{space} are flavor wave function, spin wave function, color wave function, and spatial wave function, respectively. The total wave function of the pentaquark states requires the fermion exchange symmetry to be satisfied, thus the total wave function is required to be antisymmetric. For the S-wave pentaquark states we discussed in this work, the color wave function is antisymmetric and the spatial wave function is symmetric, so the spin-flavor wave function $\phi_{\mathrm{flavor}}\chi_{\mathrm{spin}}$ should be symmetric.

In the molecular model $(q_1q_2Q)(\bar{Q}q_3)$, we construct the wave functions of the pentaquark with $SU(3)$ symmetry. Here, q and Q denote the light quark and the heavy quark, respectively. In the flavor space, the three light quarks $q_1q_2q_3$ form the flavor representations

$$(3\otimes 3)\otimes 3 = (6\oplus \bar{3})\otimes 3 = 10_f \oplus 8_{1f} \oplus 8_{2f} \oplus 1_f, \tag{2}$$

the flavor wave functions of the pentaquark states can be obtained by adding the heavy quark c and the antiquark $\bar{c}$ to the flavor wave functions of the three light quarks. In the molecular model, the pentaquark states are composed of charmed baryons and anticharmed mesons. Based on the flavor symmetry of light diquarks, the charmed baryons can be divided into $\bar{3}_f$ and 6_f flavor representations, $\bar{3}_f$ represents the flavor antisymmetry of light diquark, and 6_f represents the flavor symmetric of light diquark, we list the quark constituents of the charmed baryons and the anticharmed mesons with different flavor representations in Table I. For example, $\Xi_c^{\prime 0}$ represents the S-wave charmed baryon in the 6_f flavor representation, and Ξ_c^0 represents the S-wave charmed baryon in the $\bar{3}_f$ flavor representation. The charmed baryons and anticharmed mesons can form the 8_{1f} and 8_{2f} flavor representations, and we list the flavor wave functions of the octet hidden-charm molecular pentaquark states in Table II.

TABLE I. The quark constituents of charmed baryons and anticharmed mesons.

Hardons	Quark constituents	Hardons	Quark constituents
Σ_c^{++}	uuc	Ξ_c^+	$\sqrt{\frac{1}{2}}(usc-suc)$
Σ_c^+	$\sqrt{\frac{1}{2}}(udc+duc)$	Ξ_c^0	$\sqrt{\frac{1}{2}}(dsc-sdc)$
Σ_c^0	ddc	Λ_c^+	$\sqrt{\frac{1}{2}}(udc-duc)$
$\Xi_c^{\prime +}$	$\sqrt{\frac{1}{2}}(usc+suc)$	$\bar{D}^{(*)0}$	$\bar{c}u$
$\Xi_c^{\prime 0}$	$\sqrt{\frac{1}{2}}(dsc+sdc)$	$D^{(*)-}$	$\bar{c}d$
Ω_c^0	ssc	$D_s^{(*)-}$	$\bar{c}s$

TABLE II. The flavor wave functions of the octet hidden-charm molecular pentaquark states.

States	Flavors	Flavor wave functions
$P_\psi^{N^+}$	8_{1f}	$\frac{1}{\sqrt{3}}\Sigma_c^+\bar{D}^{(*)0}-\sqrt{\frac{2}{3}}\Sigma_c^{++}D^{(*)-}$
	8_{2f}	$\Lambda_c^+\bar{D}^{(*)0}$
$P_\psi^{N^0}$	8_{1f}	$\frac{1}{\sqrt{3}}\Sigma_c^+D^{(*)-}-\sqrt{\frac{2}{3}}\Sigma_c^0\bar{D}^{(*)0}$
	8_{2f}	$\Lambda_c^+D^{(*)-}$
$P_{\psi_s}^{\Sigma^+}$	8_{1f}	$\frac{1}{\sqrt{3}}\Xi_c^{\prime +}\bar{D}^{(*)0}-\sqrt{\frac{2}{3}}\Sigma_c^{++}D_s^{(*)-}$
	8_{2f}	$\Xi_c^+\bar{D}^{(*)0}$
$P_{\psi_s}^{\Sigma^0}$	8_{1f}	$\frac{1}{\sqrt{6}}\Xi_c^{\prime +}D^{(*)-}+\frac{1}{\sqrt{6}}\Xi_c^{\prime 0}\bar{D}^{(*)0}-\sqrt{\frac{2}{3}}\Sigma_c^+D_s^{(*)-}$
	8_{2f}	$\frac{1}{\sqrt{2}}\Xi_c^+D^{(*)-}+\frac{1}{\sqrt{2}}\Xi_c^0\bar{D}^{(*)0}$
$P_{\psi_s}^{\Lambda^0}$	8_{1f}	$\frac{1}{\sqrt{2}}\Xi_c^{\prime +}D^{(*)-}-\frac{1}{\sqrt{2}}\Xi_c^{\prime 0}\bar{D}^{(*)0}$
	8_{2f}	$\frac{1}{\sqrt{6}}\Xi_c^+D^{(*)-}+\frac{1}{\sqrt{6}}\Xi_c^0\bar{D}^{(*)0}-\sqrt{\frac{2}{3}}\Lambda_c^+D_s^{(*)-}$
$P_{\psi_s}^{\Sigma^-}$	8_{1f}	$\frac{1}{\sqrt{3}}\Xi_c^{\prime 0}D^{(*)-}-\sqrt{\frac{2}{3}}\Sigma_c^0D_s^{(*)-}$
	8_{2f}	$\Xi_c^0D^{(*)-}$
$P_{\psi_{ss}}^{N^0}$	8_{1f}	$\frac{1}{\sqrt{3}}\Xi_c^{\prime +}D_s^{(*)-}-\sqrt{\frac{2}{3}}\Omega_c^0\bar{D}^{(*)0}$
	8_{2f}	$\Xi_c^+D_s^{(*)-}$
$P_{\psi_{ss}}^{N^-}$	8_{1f}	$\frac{1}{\sqrt{3}}\Xi_c^{\prime 0}D_s^{(*)-}-\sqrt{\frac{2}{3}}\Omega_c^0D^{(*)-}$
	8_{2f}	$\Xi_c^0D_s^{(*)-}$

TABLE III. The spin wave functions of the octet hidden-charm molecular pentaquark states.

States	$\|S,S_3\rangle$	Spin wave functions
P_ψ	$\|\frac{1}{2},\frac{1}{2}\rangle$	$\|\frac{1}{2},\frac{1}{2}\rangle\|0,0\rangle$
		$\sqrt{\frac{1}{3}}\|\frac{1}{2},\frac{1}{2}\rangle\|1,0\rangle-\sqrt{\frac{2}{3}}\|\frac{1}{2},-\frac{1}{2}\rangle\|1,1\rangle$
	$\|\frac{3}{2},\frac{1}{2}\rangle$	$\sqrt{\frac{2}{3}}\|\frac{1}{2},\frac{1}{2}\rangle\|1,0\rangle+\sqrt{\frac{1}{3}}\|\frac{1}{2},-\frac{1}{2}\rangle\|1,1\rangle$

TABLE IV. The spin wave functions of constituent hadrons.

	$\|S,S_3\rangle$	Spin wave functions
6_{baryon}	$\|\frac{1}{2},\frac{1}{2}\rangle$	$\sqrt{\frac{1}{6}}(2\uparrow\uparrow\downarrow-\downarrow\uparrow\uparrow-\uparrow\downarrow\uparrow)$
	$\|\frac{1}{2},-\frac{1}{2}\rangle$	$\sqrt{\frac{1}{6}}(\uparrow\downarrow\downarrow+\downarrow\uparrow\downarrow-2\downarrow\downarrow\uparrow)$
$\bar{3}_{\mathrm{baryon}}$	$\|\frac{1}{2},\frac{1}{2}\rangle$	$\sqrt{\frac{1}{2}}(\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)$
	$\|\frac{1}{2},-\frac{1}{2}\rangle$	$\sqrt{\frac{1}{2}}(\uparrow\downarrow\downarrow-\downarrow\uparrow\downarrow)$
3_{meson}	$\|0,0\rangle$	$\sqrt{\frac{1}{2}}(\uparrow\downarrow-\downarrow\uparrow)$
	$\|1,1\rangle$	$\uparrow\uparrow$
	$\|1,0\rangle$	$\sqrt{\frac{1}{2}}(\uparrow\downarrow+\downarrow\uparrow)$
	$\|1,-1\rangle$	$\downarrow\downarrow$

We construct the spin wave functions of the pentaquark states, the total angular momentum of the pentaquark states can be expressed as $J^P=J_b^P\otimes J_m^P$, where J_b^P and J_m^P are the spins of baryons and mesons, respectively. We consider the spin configurations $\frac{1}{2}^-(\frac{1}{2}^+\otimes 0^-)$, $\frac{1}{2}^-(\frac{1}{2}^+\otimes 1^-)$, and $\frac{3}{2}^-(\frac{1}{2}^+\otimes 1^-)$, the spin wave functions of the pentaquark states are listed in Table III. Since $\phi_{\mathrm{flavor}}\chi_{\mathrm{spin}}=$ symmetric, the spin wave functions of the pentaquark consists of the spin wave functions of the charmed baryons and the anti-charmed mesons, we need to construct the symmetric and antisymmetric spin wave functions of the constituent hadrons, the corresponding spin wave functions with different flavor representations are collected in Table IV. Here, 6_{baryon}, $\bar{3}_{\mathrm{baryon}}$, and 3_{meson} represent the spin wave functions of the hadrons with different flavors.

III. TRANSITION MAGNETIC MOMENTS AND RADIATIVE DECAY WIDTHS

In this section, we present the calculations of the transition magnetic moments of pentaquark states, and we calculate the radiative decay widths of pentaquark states through the results of transition magnetic moments. In the constituent quark model, the transition magnetic moments can be expressed as

$$\mu_{H\to H'}=\langle\psi_{H'}|\hat{\mu}_z e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}|\psi_H\rangle, \tag{3}$$

where $\boldsymbol{k}$ is the emitted photon momentum, and ψ_H and $\psi_{H'}$ denote the spin-flavor wave functions of the initial and final states, respectively. For the S-wave pentaquark state, the magnetic moments operator at the quark level can be expressed as

$$\hat{\mu}=\sum_i\frac{e_i}{2m_i}\hat{\sigma}_i, \tag{4}$$

where e_i and σ_i represent the charge and Pauli's spin operator of the ith constituent of the hadrons, respectively. When the emitted photon momentum is extremely small, the factor $\langle R_{i'}|e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}|R_i\rangle$ is close to zero, and the spatial wave functions has almost no effect on the transition magnetic moments. The Eq. (3) can be simplified as

$$\mu_{H\to H'}=\langle\psi_{H'}|\hat{\mu}_z|\psi_H\rangle. \tag{5}$$

In order to calculate the transition magnetic moments of the S-wave pentaquark states, it is necessary to calculate the magnetic moments of the constituent hadrons and the transition magnetic moments between them. The calculations of the magnetic moments are similar to that of transition magnetic moments. Inserting the z component of the magnetic moment operator into the corresponding spin-flavor wave functions, we obtain the magnetic moments

$$\mu_H=\langle\psi_H|\hat{\mu}_z|\psi_H\rangle, \tag{6}$$

we take the charmed baryon Σ_c^+ as an example to illustrate the calculations of the magnetic moments, the spin-flavor wave function of Σ_c^+ can be expressed as

$$\chi_{\Sigma_c^+}^{|\frac{1}{2},\frac{1}{2}\rangle}=\sqrt{\frac{1}{2}}(udc+duc)\otimes\sqrt{\frac{1}{6}}(2\uparrow\uparrow\downarrow-\downarrow\uparrow\uparrow-\uparrow\downarrow\uparrow). \tag{7}$$

According to Eq. (6), we can obtain the expression for the magnetic moment of Σ_c^+ state as $\frac{2}{3}\mu_u+\frac{2}{3}\mu_d-\frac{1}{3}\mu_c$. With the same method, we list the magnetic moments of the constituent hadrons in Table V. In the numerical calculations, we take the quark masses $m_u=m_d=0.336$ GeV, $m_s=0.540$ GeV, $m_c=1.660$ GeV [67].

In the discussion that follows, we use P_j ($j=1$, 2, 3) to represent octet hidden-charm pentaquark states with different spin configurations, and P_1, P_2, and P_3 represent pentaquark states with spin configurations $J^P=|\frac{1}{2}^-\rangle\times(\frac{1}{2}^+\otimes 0^-)$, $J^P=|\frac{1}{2}^-\rangle(\frac{1}{2}^+\otimes 1^-)$, and $J^P=|\frac{3}{2}^-\rangle(\frac{1}{2}^+\otimes 1^-)$, respectively. We take the process $P_{2\psi}^{N^+}\to P_{1\psi}^{N^+}\gamma$ with 8_{1f} flavor representation as an example to illustrate the calculations of the transition magnetic moments of the S-wave hidden-charm pentaquark states. According to Tables II and III, we can write the spin-flavor wave functions of initial and final states

TABLE V. The magnetic moments of baryons and mesons, the unit is nuclear magnetic moment μ_N.

States	Quantities	Expressions	Results
6_f	Σ_c^{++}	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_c$	2.36
	Σ_c^{+}	$\frac{2}{3}\mu_u + \frac{2}{3}\mu_d - \frac{1}{3}\mu_c$	0.49
	Σ_c^{0}	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_c$	−1.37
	$\Xi_c'^{+}$	$\frac{2}{3}\mu_u + \frac{2}{3}\mu_s - \frac{1}{3}\mu_c$	0.73
	$\Xi_c'^{0}$	$\frac{2}{3}\mu_d + \frac{2}{3}\mu_s - \frac{1}{3}\mu_c$	−1.13
	Ω_c^{0}	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_c$	−0.90
$\bar{3}_f$	Ξ_c^{+}	μ_c	0.38
	Ξ_c^{0}	μ_c	0.38
	Λ_c^{+}	μ_c	0.38
3_f	$\bar{D}^{(*)0}$	$\mu_u + \mu_{\bar{c}}$	1.48
	$D^{(*)-}$	$\mu_d + \mu_{\bar{c}}$	−1.31
	$D_s^{(*)-}$	$\mu_s + \mu_{\bar{c}}$	−0.96

$$\chi_{P_2^{N^+}} = \left[\frac{1}{\sqrt{3}}\Sigma_c^+\bar{D}^{*0} - \sqrt{\frac{2}{3}}\Sigma_c^{++}D^{*-}\right] \otimes \left[\sqrt{\frac{1}{3}}\left|\frac{1}{2},\frac{1}{2}\right\rangle|1,0\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle|1,1\rangle\right], \quad (8)$$

$$\chi_{P_1^{N^+}} = \left[\frac{1}{\sqrt{3}}\Sigma_c^+\bar{D}^{0} - \sqrt{\frac{2}{3}}\Sigma_c^{++}D^{-}\right] \otimes \left|\frac{1}{2},\frac{1}{2}\right\rangle|0,0\rangle. \quad (9)$$

Taking the magnetic moment operator into the spin-flavor wave functions of initial and final states, we can calculate the transition magnetic moment of the process $P_{2\psi}^{N^+} \to P_{1\psi}^{N^+}\gamma$ is $\frac{\sqrt{3}}{9}(\mu_{\bar{D}^{*0}\to\bar{D}^0} + 2\mu_{D^{*-}\to D^-})$. We notice that the transition magnetic moments of pentaquark states are composed of the transition magnetic moments of its constituent hadrons, therefore we present the transition magnetic moment of the process $\bar{D}^{*0} \to \bar{D}^0\gamma$ as an example to illustrate the calculations, the spin-flavor wave functions of $\bar{D}^{*0}$ and $\bar{D}^0$ can be expressed as

$$\chi_{\bar{D}^{*0}}^{|1,0\rangle} = \frac{1}{\sqrt{2}}|\bar{c}u\rangle \otimes |\uparrow\downarrow + \downarrow\uparrow\rangle, \quad (10)$$

$$\chi_{\bar{D}^{0}}^{|0,0\rangle} = \frac{1}{\sqrt{2}}|\bar{c}u\rangle \otimes |\uparrow\downarrow - \downarrow\uparrow\rangle. \quad (11)$$

Taking the above wave functions into Eq. (5), we calculate the transition magnetic moments of the $\bar{D}^{*0} \to \bar{D}^0\gamma$ process as $\mu_{\bar{c}} - \mu_u$, we can obtain the transition magnetic moments of the other constituent hadrons in Table VI.

The linear combination of magnetic moments and transition magnetic moments enables us to calculate the transition magnetic moments of pentaquark states with the 8_{1f} and 8_{2f} flavor representations, we collect their results in Tables VII and VIII.

TABLE VI. The transition magnetic moments of mesons, the unit is nuclear magnetic moment μ_N.

Processes	Expressions	Results
$\bar{D}^{*0} \to \bar{D}^0\gamma$	$\mu_{\bar{c}} - \mu_u$	−2.24
$D^{*-} \to D^-\gamma$	$\mu_{\bar{c}} - \mu_d$	0.55
$D_s^{*-} \to D_s^-\gamma$	$\mu_{\bar{c}} - \mu_s$	0.20

Analyzing the above results for the transition magnetic moments of the hidden-charm pentaquark states, we notice the transition magnetic moments of the hidden-charm pentaquark states are not only related to the transition magnetic moments between their constituent hadrons, but also related to the magnetic moments of the hadrons. With the same flavor representations, the different transition magnetic moments satisfy proportional relations. For example, for state $P_\psi^{N^+}$ with 8_{1f} flavor representation, the transition magnetic moments satisfy the relation

$$\frac{\mu_{\Sigma_c D^*|\frac{3}{2}^-\rangle\to\Sigma_c D|\frac{1}{2}^-\rangle\gamma}}{\mu_{\Sigma_c D^*|\frac{1}{2}^-\rangle\to\Sigma_c D|\frac{1}{2}^-\rangle\gamma}} = \sqrt{2}. \quad (12)$$

And the pentaquark states with different flavor representations have different transition magnetic moments, it implies that the experimental values of transition magnetic moments can identify different flavor representations. For example, for $P_\psi^{N^+}$ state, the transition magnetic moment of the process $\Sigma_c D^*|\frac{3}{2}^-\rangle \to \Sigma_c D^*|\frac{1}{2}^-\rangle\gamma$ with 8_{1f} flavor representation is $1.81\mu_N$, while the transition magnetic moment of the process $\Lambda_c D^*|\frac{3}{2}^-\rangle \to \Lambda_c D^*|\frac{1}{2}^-\rangle\gamma$ with 8_{2f} flavor representation is $-0.33\mu_N$. If the experimental value of transition magnetic moment is around $1.8\mu_N$, we can identify the $P_\psi^{N^+}$ state observed experimentally as 8_{1f} flavor representation. If the experimental value of transition magnetic moment is between the transition magnetic moments of two flavor representations, the physical state should be a combination of pentaquark states with different flavor representations. This is due to the coupling effect between the 8_{1f} and 8_{2f} flavor representations via strong and electromagnetic interactions, and coupling effects between them are dependent on their binding energies. The experimental values for binding energy are currently lacking, when the relevant experimental values are more comprehensive, we will perform more extensive and detailed calculations.

With the transition magnetic moments we obtained in this work, we can calculate the radiative decay widths of the pentaquark states. The radiative decay widths and the transition magnetic moments satisfy relations, which can be expressed as [68]

$$\Gamma_{H\to H'\gamma} = \alpha_{\mathrm{EM}}\frac{E_\gamma^3}{3m_p^2}J_a\frac{|\mu_{H\to H'}|^2}{\mu_N^2} \quad (13)$$

TABLE VII. The transition magnetic moments of pentaquark states with the 8_{1f} flavor representation. Here, the unit of the transition magnetic moment is nuclear magnetic moment μ_N.

States	Processes	Expressions	Results
$P_\psi^{N^+}$	$\Sigma_c D^*\vert\frac{1}{2}^-\rangle \to \Sigma_c D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{3}}{9}(\mu_{\bar{D}^{*0}\to\bar{D}^0} + 2\mu_{D^{*-}\to D^-})$	−0.22
	$\Sigma_c D^*\vert\frac{3}{2}^-\rangle \to \Sigma_c D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{6}}{9}(\mu_{\bar{D}^{*0}\to\bar{D}^0} + 2\mu_{D^{*-}\to D^-})$	−0.31
	$\Sigma_c D^*\vert\frac{3}{2}^-\rangle \to \Sigma_c D^*\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{2}}{9}(2\mu_{\Sigma_c^+} + 4\mu_{\Sigma_c^{++}}) - \frac{\sqrt{2}}{9}(\mu_{\bar{D}^{*0}} + 2\mu_{D^{*-}})$	1.81
$P_\psi^{N^0}$	$\Sigma_c D^*\vert\frac{1}{2}^-\rangle \to \Sigma_c D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{3}}{9}(\mu_{D^{*-}\to D^-} + 2\mu_{\bar{D}^{*0}\to\bar{D}^0})$	−0.76
	$\Sigma_c D^*\vert\frac{3}{2}^-\rangle \to \Sigma_c D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{6}}{9}(\mu_{D^{*-}\to D^-} + 2\mu_{\bar{D}^{*0}\to\bar{D}^0})$	−1.07
	$\Sigma_c D^*\vert\frac{3}{2}^-\rangle \to \Sigma_c D^*\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{2}}{9}(2\mu_{\Sigma_c^+} + 4\mu_{\Sigma_c^0}) - \frac{\sqrt{2}}{9}(\mu_{D^{*-}} + 2\mu_{\bar{D}^{*0}})$	−0.97
$P_{\psi_s}^{\Sigma^+}$	$\Sigma_c D^*\vert\frac{1}{2}^-\rangle \to \Sigma_c D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{3}}{9}(\mu_{\bar{D}^{*0}\to\bar{D}^0} + 2\mu_{D_s^{*-}\to D_s^-})$	−0.35
	$\Sigma_c D^*\vert\frac{3}{2}^-\rangle \to \Sigma_c D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{6}}{9}(\mu_{\bar{D}^{*0}\to\bar{D}^0} + 2\mu_{D_s^{*-}\to D_s^-})$	−0.50
	$\Sigma_c D^*\vert\frac{3}{2}^-\rangle \to \Sigma_c D^*\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{2}}{9}(2\mu_{\Xi_c'^+} + 4\mu_{\Sigma_c^{++}}) - \frac{\sqrt{2}}{9}(\mu_{\bar{D}^{*0}} + 2\mu_{D_s^{*-}})$	1.78
$P_{\psi_s}^{\Sigma^0}$	$\Sigma_c D^*\vert\frac{1}{2}^-\rangle \to \Sigma_c D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{3}}{18}(\mu_{D^{*-}\to D^-} + \mu_{\bar{D}^{*0}\to\bar{D}^0}) + \frac{2\sqrt{3}}{9}\mu_{D_s^{*-}\to D_s^-}$	−0.08
	$\Sigma_c D^*\vert\frac{3}{2}^-\rangle \to \Sigma_c D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{6}}{18}(\mu_{D^{*-}\to D^-} + \mu_{\bar{D}^{*0}\to\bar{D}^0}) + \frac{2\sqrt{6}}{9}\mu_{D_s^{*-}\to D_s^-}$	−0.12
	$\Sigma_c D^*\vert\frac{3}{2}^-\rangle \to \Sigma_c D^*\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{2}}{9}(\mu_{\Xi_c'^+} + \mu_{\Xi_c'^0} + 4\mu_{\Sigma_c^+}) - \frac{\sqrt{2}}{18}(\mu_{D^{*-}} + \mu_{\bar{D}^{*0}} + 4\mu_{D_s^{*-}})$	0.53
$P_{\psi_s}^{\Lambda^0}$	$\Xi_c' D^*\vert\frac{1}{2}^-\rangle \to \Xi_c' D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{3}}{6}(\mu_{D^{*-}\to D^-} + \mu_{\bar{D}^{*0}\to\bar{D}^0})$	−0.49
	$\Xi_c' D^*\vert\frac{3}{2}^-\rangle \to \Xi_c' D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{6}}{6}(\mu_{D^{*-}\to D^-} + \mu_{\bar{D}^{*0}\to\bar{D}^0})$	−0.69
	$\Xi_c' D^*\vert\frac{3}{2}^-\rangle \to \Xi_c' D^*\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{2}}{3}(\mu_{\Xi_c'^+} + \mu_{\Xi_c'^0}) - \frac{\sqrt{2}}{6}(\mu_{D^{*-}} + \mu_{\bar{D}^{*0}})$	−0.61
$P_{\psi_s}^{\Sigma^-}$	$\Sigma_c D^*\vert\frac{1}{2}^-\rangle \to \Sigma_c D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{3}}{9}(\mu_{D^{*-}\to D^-} + 2\mu_{D_s^{*-}\to D_s^-})$	0.18
	$\Sigma_c D^*\vert\frac{3}{2}^-\rangle \to \Sigma_c D^*\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{6}}{9}(\mu_{D^{*-}\to D^-} + 2\mu_{D_s^{*-}\to D_s^-})$	0.26
	$\Sigma_c D^*\vert\frac{3}{2}^-\rangle \to \Sigma_c D^*\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{2}}{9}(2\mu_{\Xi_c'^0} + 4\mu_{\Sigma_c^0}) - \frac{\sqrt{2}}{9}(\mu_{D^{*-}} + 2\mu_{D_s^{*-}})$	−0.71
$P_{\psi_{ss}}^{N^0}$	$\Omega_c D^*\vert\frac{1}{2}^-\rangle \to \Omega_c D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{3}}{9}(\mu_{D_s^{*-}\to D_s^-} + 2\mu_{\bar{D}^{*0}\to\bar{D}^0})$	−0.82
	$\Omega_c D^*\vert\frac{3}{2}^-\rangle \to \Omega_c D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{6}}{9}(\mu_{D_s^{*-}\to D_s^-} + 2\mu_{\bar{D}^{*0}\to\bar{D}^0})$	−1.16
	$\Omega_c D^*\vert\frac{3}{2}^-\rangle \to \Omega_c D^*\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{2}}{9}(2\mu_{\Xi_c'^+} + 4\mu_{\Omega_c^0}) - \frac{\sqrt{2}}{9}(\mu_{D_s^{*-}} + 2\mu_{\bar{D}^{*0}})$	−0.65
$P_{\psi_{ss}}^{N^-}$	$\Omega_c D^*\vert\frac{1}{2}^-\rangle \to \Omega_c D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{3}}{9}(\mu_{D_s^{*-}\to D_s^-} + 2\mu_{D^{*-}\to D^-})$	0.25
	$\Omega_c D^*\vert\frac{3}{2}^-\rangle \to \Omega_c' D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{6}}{9}(\mu_{D_s^{*-}\to D_s^-} + 2\mu_{D^{*-}\to D^-})$	0.36
	$\Omega_c D^*\vert\frac{3}{2}^-\rangle \to \Omega_c' D^*\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{2}}{9}(2\mu_{\Xi_c'^0} + 4\mu_{\Omega_c^0}) - \frac{\sqrt{2}}{9}(\mu_{D_s^{*-}} + 2\mu_{D^{*-}})$	−0.36

TABLE VIII. The transition magnetic moments of pentaquark states with the 8_{2f} flavor representation.

States	Processes	Expressions	Results
$P_\psi^{N^+}$	$\Lambda_c D^*\vert\frac{1}{2}^-\rangle \to \Lambda_c D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{3}}{3}\mu_{\bar{D}^{*0}\to\bar{D}^0}$	−1.29
	$\Lambda_c D^*\vert\frac{3}{2}^-\rangle \to \Lambda_c D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{6}}{3}\mu_{\bar{D}^{*0}\to\bar{D}^0}$	−1.83
	$\Lambda_c D^*\vert\frac{3}{2}^-\rangle \to \Lambda_c D^*\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{2}}{3}(2\mu_{\Lambda_c^+} - \mu_{\bar{D}^{*0}})$	−0.33
$P_\psi^{N^0}$	$\Lambda_c D^*\vert\frac{1}{2}^-\rangle \to \Lambda_c D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{3}}{3}\mu_{D^{*-}\to D^-}$	0.32
	$\Lambda_c D^*\vert\frac{3}{2}^-\rangle \to \Lambda_c D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{6}}{3}\mu_{D^{*-}\to D^-}$	0.45
	$\Lambda_c D^*\vert\frac{3}{2}^-\rangle \to \Lambda_c D^*\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{2}}{3}(2\mu_{\Lambda_c^+} - \mu_{D^{*-}})$	0.97
$P_{\psi_s}^{\Sigma^+}$	$\Xi_c D^*\vert\frac{1}{2}^-\rangle \to \Xi_c D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{3}}{3}\mu_{\bar{D}^{*0}\to\bar{D}^0}$	−1.29
	$\Xi_c D^*\vert\frac{3}{2}^-\rangle \to \Xi_c D\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{6}}{3}\mu_{\bar{D}^{*0}\to\bar{D}^0}$	−1.83
	$\Xi_c D^*\vert\frac{3}{2}^-\rangle \to \Xi_c D^*\vert\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{2}}{3}(2\mu_{\Xi_c^+} - \mu_{\bar{D}^{*0}})$	−0.34

(Table continued)

TABLE VIII. *(Continued)*

States	Processes	Expressions	Results
$P_{\psi_s}^{\Sigma^0}$	$\Xi_cD^*\|\frac{1}{2}^-\rangle \to \Xi_cD\|\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{3}}{6}(\mu_{D^{*-}\to D^-}+\mu_{\bar{D}^{*0}\to\bar{D}^0})$	−0.49
	$\Xi_cD^*\|\frac{3}{2}^-\rangle \to \Xi_cD\|\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{6}}{6}(\mu_{D^{*-}\to D^-}+\mu_{\bar{D}^{*0}\to\bar{D}^0})$	−0.69
	$\Xi_cD^*\|\frac{3}{2}^-\rangle \to \Xi_cD^*\|\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{2}}{3}(\mu_{\Xi_c^+}+\mu_{\Xi_c^0})-\frac{\sqrt{2}}{6}(\mu_{D^{*-}}+\mu_{\bar{D}^{*0}})$	0.31
$P_{\psi_s}^{\Lambda^0}$	$\Lambda_cD^*\|\frac{1}{2}^-\rangle \to \Lambda_cD\|\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{3}}{18}(\mu_{D^{*-}\to D^-}+\mu_{\bar{D}^{*0}\to\bar{D}^0})+\frac{2\sqrt{3}}{9}\mu_{D_s^{*-}\to D_s^-}$	−0.08
	$\Lambda_cD^*\|\frac{3}{2}^-\rangle \to \Lambda_cD\|\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{6}}{18}(\mu_{D^{*-}\to D^-}+\mu_{\bar{D}^{*0}\to\bar{D}^0})+\frac{2\sqrt{6}}{9}\mu_{D_s^{*-}\to D_s^-}$	−0.12
	$\Lambda_cD^*\|\frac{3}{2}^-\rangle \to \Lambda_cD^*\|\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{2}}{9}(\mu_{\Xi_c^+}+\mu_{\Xi_c^0}+4\mu_{\Lambda_c^+})-\frac{\sqrt{2}}{18}(\mu_{D^{*-}}+\mu_{\bar{D}^{*0}}+4\mu_{D_s^{*-}})$	0.64
$P_{\psi_s}^{\Sigma^-}$	$\Xi_cD^*\|\frac{1}{2}^-\rangle \to \Xi_cD\|\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{3}}{3}\mu_{D^{*-}\to D^-}$	0.32
	$\Xi_cD^*\|\frac{3}{2}^-\rangle \to \Xi_cD\|\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{6}}{3}\mu_{D^{*-}\to D^-}$	0.45
	$\Xi_cD^*\|\frac{3}{2}^-\rangle \to \Xi_cD^*\|\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{2}}{3}(2\mu_{\Xi_c^0}-\mu_{D^{*-}})$	0.97
$P_{\psi_{ss}}^{N^0}$	$\Xi_cD^*\|\frac{1}{2}^-\rangle \to \Xi_cD\|\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{3}}{3}\mu_{D_s^{*-}\to D_s^-}$	0.12
	$\Xi_cD^*\|\frac{3}{2}^-\rangle \to \Xi_cD\|\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{6}}{3}\mu_{D_s^{*-}\to D_s^-}$	0.17
	$\Xi_cD^*\|\frac{3}{2}^-\rangle \to \Xi_cD^*\|\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{2}}{3}(2\mu_{\Xi_c^+}-\mu_{D_s^{*-}})$	0.81
$P_{\psi_{ss}}^{N^-}$	$\Xi_cD^*\|\frac{1}{2}^-\rangle \to \Xi_cD\|\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{3}}{3}\mu_{D_s^{*-}\to D_s^-}$	0.12
	$\Xi_cD^*\|\frac{3}{2}^-\rangle \to \Xi_cD\|\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{6}}{3}\mu_{D_s^{*-}\to D_s^-}$	0.17
	$\Xi_cD^*\|\frac{3}{2}^-\rangle \to \Xi_cD^*\|\frac{1}{2}^-\rangle\gamma$	$\frac{\sqrt{2}}{3}(2\mu_{\Xi_c^0}-\mu_{D_s^{*-}})$	0.81

TABLE IX. The radiative decay widths of pentaquark states with the 8_{1f} and 8_{2f} flavor representations. Here, the radiative decay width is in units of keV.

	8_{1f}		8_{2f}	
States	Processes	Radiative decay widths	Processes	Radiative decay widths
$P_{\psi}^{N^+}$	$\Sigma_cD^*\|\frac{1}{2}^-\rangle \to \Sigma_cD\|\frac{1}{2}^-\rangle\gamma$	1.07	$\Lambda_cD^*\|\frac{1}{2}^-\rangle \to \Lambda_cD\|\frac{1}{2}^-\rangle\gamma$	37.61
	$\Sigma_cD^*\|\frac{3}{2}^-\rangle \to \Sigma_cD\|\frac{1}{2}^-\rangle\gamma$	1.07	$\Lambda_cD^*\|\frac{3}{2}^-\rangle \to \Lambda_cD\|\frac{1}{2}^-\rangle\gamma$	37.84
$P_{\psi}^{N^0}$	$\Sigma_cD^*\|\frac{1}{2}^-\rangle \to \Sigma_cD\|\frac{1}{2}^-\rangle\gamma$	13.05	$\Lambda_cD^*\|\frac{1}{2}^-\rangle \to \Lambda_cD\|\frac{1}{2}^-\rangle\gamma$	2.25
	$\Sigma_cD^*\|\frac{3}{2}^-\rangle \to \Sigma_cD\|\frac{1}{2}^-\rangle\gamma$	12.94	$\Lambda_cD^*\|\frac{3}{2}^-\rangle \to \Lambda_cD\|\frac{1}{2}^-\rangle\gamma$	2.22
$P_{\psi_s}^{\Sigma^+}$	$\Sigma_cD^*\|\frac{1}{2}^-\rangle \to \Sigma_cD\|\frac{1}{2}^-\rangle\gamma$	3.28	$\Xi_cD^*\|\frac{1}{2}^-\rangle \to \Xi_cD\|\frac{1}{2}^-\rangle\gamma$	37.69
	$\Sigma_cD^*\|\frac{3}{2}^-\rangle \to \Sigma_cD\|\frac{1}{2}^-\rangle\gamma$	3.35	$\Xi_cD^*\|\frac{3}{2}^-\rangle \to \Xi_cD\|\frac{1}{2}^-\rangle\gamma$	37.92
$P_{\psi_s}^{\Sigma^0}$	$\Sigma_cD^*\|\frac{1}{2}^-\rangle \to \Sigma_cD\|\frac{1}{2}^-\rangle\gamma$	0.15	$\Xi_cD^*\|\frac{1}{2}^-\rangle \to \Xi_cD\|\frac{1}{2}^-\rangle\gamma$	5.36
	$\Sigma_cD^*\|\frac{3}{2}^-\rangle \to \Sigma_cD\|\frac{1}{2}^-\rangle\gamma$	0.17	$\Xi_cD^*\|\frac{3}{2}^-\rangle \to \Xi_cD\|\frac{1}{2}^-\rangle\gamma$	5.31
$P_{\psi_s}^{\Lambda^0}$	$\Xi'_cD^*\|\frac{1}{2}^-\rangle \to \Xi'_cD\|\frac{1}{2}^-\rangle\gamma$	5.36	$\Lambda_cD^*\|\frac{1}{2}^-\rangle \to \Lambda_cD\|\frac{1}{2}^-\rangle\gamma$	0.15
	$\Xi'_cD^*\|\frac{3}{2}^-\rangle \to \Xi'_cD\|\frac{1}{2}^-\rangle\gamma$	5.32	$\Lambda_cD^*\|\frac{3}{2}^-\rangle \to \Lambda_cD\|\frac{1}{2}^-\rangle\gamma$	0.17
$P_{\psi_s}^{\Sigma^-}$	$\Sigma_cD^*\|\frac{1}{2}^-\rangle \to \Sigma_cD\|\frac{1}{2}^-\rangle\gamma$	0.75	$\Xi_cD^*\|\frac{1}{2}^-\rangle \to \Xi_cD\|\frac{1}{2}^-\rangle\gamma$	2.25
	$\Sigma_cD^*\|\frac{3}{2}^-\rangle \to \Sigma_cD^*\|\frac{1}{2}^-\rangle\gamma$	0.78	$\Xi_cD^*\|\frac{3}{2}^-\rangle \to \Xi_cD\|\frac{1}{2}^-\rangle\gamma$	2.23
$P_{\psi_{ss}}^{N^0}$	$\Omega_cD^*\|\frac{1}{2}^-\rangle \to \Omega_cD\|\frac{1}{2}^-\rangle\gamma$	15.46	$\Xi_cD^*\|\frac{1}{2}^-\rangle \to \Xi_cD\|\frac{1}{2}^-\rangle\gamma$	0.34
	$\Omega_cD^*\|\frac{3}{2}^-\rangle \to \Omega_cD\|\frac{1}{2}^-\rangle\gamma$	15.47	$\Xi_cD^*\|\frac{3}{2}^-\rangle \to \Xi_cD\|\frac{1}{2}^-\rangle\gamma$	0.34
$P_{\psi_{ss}}^{N^-}$	$\Omega_cD^*\|\frac{1}{2}^-\rangle \to \Omega_cD\|\frac{1}{2}^-\rangle\gamma$	1.41	$\Xi_cD^*\|\frac{1}{2}^-\rangle \to \Xi_cD\|\frac{1}{2}^-\rangle\gamma$	0.34
	$\Omega_cD^*\|\frac{3}{2}^-\rangle \to \Omega'_cD\|\frac{1}{2}^-\rangle\gamma$	1.46	$\Xi_cD^*\|\frac{3}{2}^-\rangle \to \Xi_cD\|\frac{1}{2}^-\rangle\gamma$	0.34

where the electromagnetic fine structure constant $\alpha_{\rm EM}$ is $\frac{1}{137}$, m_p is the mass of proton with $m_p = 0.938$ GeV, the angular momentum coefficient J_a ($a = 1, 2, 3$) can be written

$$\begin{cases} J_1 = \frac{J_H+1}{J_H} & \text{when } J_H = J_{H'} \\ J_2 = J_H & \text{when } J_H = J_{H'}+1. \\ J_3 = \frac{(J_H+1)(2J_H+3)}{2J_H+1} & \text{when } J_H = J_{H'}-1 \end{cases} \quad (14)$$

For the $H \to H'\gamma$ process, E_γ is the momentum of the emitted photon, which can be written as

$$E_\gamma = \frac{m_H^2 - m_{H'}^2}{2m_H}, \quad (15)$$

where m_H and $m_{H'}$ represent the masses of the corresponding hadrons. Taking the results of the transition magnetic moments into the Eq. (13), we can obtain the numerical results of the radiative decay widths in Tables IX.

According to the results of the radiative decay widths in Table IX, several noteworthy points can be identified:

(i) For decay process $\Sigma_c D^*|\frac{3}{2}^-\rangle \to \Sigma_c D^*|\frac{1}{2}^-\rangle\gamma$, we can obtain its radiative decay width less than 0.06 keV, which is due to the fact that the masses of $\Sigma_c D^*|\frac{3}{2}^-\rangle$ and $\Sigma_c D^*|\frac{1}{2}^-\rangle$ are very close to each other. In other flavor representations, the radiative decay widths of this group are also almost 0 due to the same reasons.

(ii) For decay processes $\Sigma_c D^*|\frac{1}{2}^-\rangle \to \Sigma_c D|\frac{1}{2}^-\rangle\gamma$ and $\Sigma_c D^*|\frac{3}{2}^-\rangle \to \Sigma_c D|\frac{1}{2}^-\rangle\gamma$, their transition magnetic moments are not close, but their radiative decay widths are quite close, which is due to the joint effects of the angular momentum coefficient J_a and the transition magnetic moments in Eq. (13). For example, for $P_\psi^{N^+}$ state with 8_{1f} flavor representation, their transition magnetic moments satisfy the relation

$$\mu_{\Sigma_c D^*|\frac{3}{2}^-\rangle \to \Sigma_c D|\frac{1}{2}^-\rangle\gamma} = \sqrt{2}\mu_{\Sigma_c D^*|\frac{1}{2}^-\rangle \to \Sigma_c D|\frac{1}{2}^-\rangle\gamma}, \quad (16)$$

while the coefficients of angular momentum J_a satisfy the relation $\frac{J_2}{J_1} = \frac{1}{2}$. The relations lead to quite close radiative decay widths in Eq. (13).

(iii) We notice that the radiative decay widths of the $P_\psi^{N^0}$ and $P_{\psi_{ss}}^{N^0}$ states in 8_{1f} flavor representation are larger than 12.0 keV, the decay widths of the $P_\psi^{N^+}$ and $P_{\psi_s}^{\Sigma^+}$ states in 8_{2f} flavor representation are larger than 37.0 keV, and the decay widths of the remaining states are around 5.0 keV. This is helpful for distinguish the pentaquark states experimentally.

IV. THE AXIAL-VECTOR COUPLING CONSTANTS OF PENTAQUARK STATES

In this section, we present the calculations of the axial-vector coupling constants of pentaquark states. The Lagrangian in the chiral quark model is

$$\begin{aligned}\mathcal{L}_{\rm quark} &= \frac{1}{2}g_q\bar{\psi}_q\gamma^\mu\gamma_5\partial_\mu\phi\psi_q \sim \frac{1}{2}g_q\bar{\psi}_q\sigma_z\partial_z\phi\psi_q \\ &= \frac{1}{2}\frac{g_q}{f_\pi}(\bar{u}\sigma_z\partial_z\pi_0 u - \bar{d}\sigma_z\partial_z\pi_0 d) + \ldots,\end{aligned} \quad (17)$$

where g_q is the coupling coefficient, $f_\pi = 92$ MeV is the decay constant of the meson, ψ_q and $\bar{\psi}_q$ are the quark and antiquark fields, respectively. ϕ denotes the pseudoscalar meson field in the SU(2) flavor symmetry.

$$\phi = \frac{1}{f_\pi}\begin{pmatrix} \pi_0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi_0 \end{pmatrix}. \quad (18)$$

The nucleon N at quark level

$$\left\langle N, j_3 = +\frac{1}{2}; \pi_0 \left| \frac{1}{i}\frac{1}{2}\frac{g_q}{f_\pi}(\bar{u}\sigma_z\partial_z\pi_0 u - \bar{d}\sigma_z\partial_z\pi_0 d) \right| N, j_3 = +\frac{1}{2} \right\rangle = \frac{5}{6}\frac{q_z}{f_\pi}g_q, \quad (19)$$

where q_z is the external momentum of π_0, at the hadron level

$$\left\langle N, j_3 = +\frac{1}{2}; \pi_0 \left| \frac{1}{i}\frac{g_A}{f_\pi}\bar{N}\frac{\sum_{N_z}}{2}\partial_\mu\phi N \right| N, j_3 = +\frac{1}{2} \right\rangle = \frac{1}{2}\frac{q_z}{f_\pi}g_A, \quad (20)$$

where g_A is the axial-vector charge of the nucleon. According to Eqs. (19) and (20), we can obtain $g_q = \frac{3}{5}g_A$.

The $SU(3)$ invariant Lagrangian of pentaquark state for $P_2(\frac{1}{2}^-)P_1(\frac{1}{2}^-)\pi_0$ in 8_{1f} flavor representation reads

$$\mathcal{L}_\rho^{\frac{1}{2}^- \to \frac{1}{2}^-} = {\rm Tr}(g_\rho\bar{P}_1\gamma_\mu\gamma^5\{\partial^\mu\Phi, P_2\} + f_\rho\bar{P}_1\gamma_\mu\gamma^5[\partial^\mu\Phi, P_2]), \quad (21)$$

where $\{\partial^\mu\Phi, P_j\} = \partial^\mu\Phi P_j + P_j\partial^\mu\Phi$, $[\partial^\mu\Phi, P_j] = \partial^\mu\Phi P_j - P_j\partial^\mu\Phi$, and g_ρ, f_ρ are independent axial-vector coupling constants of pentaquark states for $P_2(\frac{1}{2}^-)P_1(\frac{1}{2}^-)\pi_0$ in 8_{1f} flavor representation. Here, P_j represent octet hidden-charm molecular pentaquark states with different spin configurations

$$P_j(j=1,2,3)=\begin{pmatrix} \frac{1}{\sqrt{2}}P_{\psi_s}^{\Sigma^0}+\frac{1}{\sqrt{6}}P_{\psi_s}^{\Lambda^0} & P_{\psi_s}^{\Sigma^+} & P_{\psi}^{N^+} \\ P_{\psi_s}^{\Sigma^-} & -\frac{1}{\sqrt{2}}P_{\psi_s}^{\Sigma^0}+\frac{1}{\sqrt{6}}P_{\psi_s}^{\Lambda^0} & P_{\psi}^{N^0} \\ P_{\psi_{ss}}^{N^-} & P_{\psi_{ss}}^{N^0} & \frac{2}{\sqrt{3}}P_{\psi_s}^{\Lambda^0} \end{pmatrix}. \tag{22}$$

Φ represents the pseudoscalar meson field in $SU(3)$ flavor symmetry

$$\Phi=\sum_{a=1}^{8}\lambda_a\Phi_a\equiv\begin{pmatrix} \pi_0+\frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi_0+\frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}. \tag{23}$$

Take Φ, P_j into Eq. (21) and expand it. Here, we merely consider π_0 term, we obtain

$$\begin{aligned}\mathcal{L}_{\rho}^{\frac{1}{2}^-\to\frac{1}{2}^-}=&\frac{2}{\sqrt{3}}\frac{1}{f_\pi}g_\rho(\bar{P}_1^{\Sigma^0}\Sigma_z\partial_z\pi^0P_2^{\Lambda^0}+\bar{P}_1^{\Lambda^0}\Sigma_z\partial_z\pi^0P_2^{\Sigma^0})-\frac{1}{f_\pi}(f_\rho+g_\rho)\bar{P}_1^{N^0}\Sigma_z\partial_z\pi^0P_2^{N^0}+\frac{1}{f_\pi}(g_\rho+f_\rho)\bar{P}_1^{N^+}\Sigma_z\partial_z\pi^0P_2^{N^+}\\ &+\frac{1}{f_\pi}(g_\rho-f_\rho)\bar{P}_1^{N^-}\Sigma_z\partial_z\pi^0P_2^{N^-}+\frac{1}{f_\pi}(f_\rho-g_\rho)\bar{P}_1^{N^0}\Sigma_z\partial_z\pi^0P_2^{N^0}+\frac{1}{f_\pi}2f_\rho\bar{P}_1^{\Sigma^+}\Sigma_z\partial_z\pi^0P_2^{\Sigma^+}-\frac{1}{f_\pi}2f_\rho\bar{P}_1^{\Sigma^-}\Sigma_z\partial_z\pi^0P_2^{\Sigma^-}.\end{aligned} \tag{24}$$

At the hadron level, for $P_{\psi}^{N^+}$ state with 8_{1f} flavor representation, the Lagrangian for the decay process $P_2\to P_1\pi_0$ reads

$$\begin{aligned}\mathcal{L}_{P_{\psi}^{N^+}}^{\frac{1}{2}^-\to\frac{1}{2}^-}&=\frac{1}{2}(g_\rho+f_\rho)\bar{P}_1^{N^+}\gamma^\mu\gamma_5\partial_\mu\Phi P_2^{N^+}\\ &\sim\frac{g_\rho+f_\rho}{f_\pi}\bar{P}_1^{N^+}\frac{\Sigma_z}{2}\partial_z\pi_0P_2^{N^+}.\end{aligned} \tag{25}$$

For $P_{\psi_{ss}}^{N^0}$ state with 8_{1f} flavor representation, the Lagrangian for the process $P_2\to P_1\pi_0$ reads

$$\begin{aligned}\mathcal{L}_{P_{\psi_{ss}}^{N^0}}^{\frac{1}{2}^-\to\frac{1}{2}^-}&=\frac{1}{2}(f_\rho-g_\rho)\bar{P}_1^{N^0}\gamma^\mu\gamma_5\partial_\mu\Phi P_2^{N^0}\\ &\sim\frac{f_\rho-g_\rho}{f_\pi}\bar{P}_1^{N^0}\frac{\Sigma_z}{2}\partial_z\pi_0P_2^{N^0},\end{aligned} \tag{26}$$

where $\frac{\Sigma_z}{2}$ is the spin operator of the hidden-charm pentaquark states. At the hadron level, the axial-vector coupling constants read

$$\langle P_{\psi}^{N^+};\pi_0|\frac{g_\rho+f_\rho}{f_\pi}\bar{P}_{\psi}^{N^+}\frac{\Sigma_z}{2}\partial_z\pi_0P_{\psi}^{N^+}|P_{\psi}^{N^+}\rangle=\frac{g_\rho+f_\rho}{2}\frac{q_z}{f_\pi}, \tag{27}$$

$$\langle P_{\psi_{ss}}^{N^0};\pi_0|\frac{f_\rho-g_\rho}{f_\pi}\bar{P}_{\psi_{ss}}^{N^0}\frac{\Sigma_z}{2}\partial_z\pi_0P_{\psi ss}^{N^0}|P_{\psi_{ss}}^{N^0}\rangle=\frac{f_\rho-g_\rho}{2}\frac{q_z}{f_\pi}. \tag{28}$$

At the quark level, we notice that the π meson interactions only exist between light quarks. We take $P_{\psi}^{N^+}$ state with 8_{1f} flavor representation as example to illustrate the calculations, its wave function is

$$|P_{\psi}^{N^+}\rangle=\left|\sqrt{\frac{1}{6}}(udc)(\bar{c}u)+\sqrt{\frac{1}{6}}(duc)(\bar{c}u)-\sqrt{\frac{2}{3}}(uuc)(\bar{c}d)\right\rangle \tag{29}$$

$$\otimes|[(q_1q_2)_{s_1}\otimes c]_{s_2}\otimes(\bar{c}q_3)_{s_3}\rangle_J^{J_z}, \tag{30}$$

where s_1 is the spin of the light diquark q_1q_2, which couples to the spin of charm quark to form the spin s_2. s_3 is the spin of the meson $\bar{c}q_3$, which couples to the baryon spin s_2 to form the total angular momentum J, and J_z is its third component. For the process $P_2\to P_1\pi_0$, the wave functions of initial and final states can be written as

$$\begin{aligned}|P_2^{N^+}\rangle=&\left|\sqrt{\frac{1}{6}}(udc)(\bar{c}u)+\sqrt{\frac{1}{6}}(duc)(\bar{c}u)-\sqrt{\frac{2}{3}}(uuc)(\bar{c}d)\right\rangle\\ &\otimes\left[\frac{1}{6}(2|\uparrow\uparrow\downarrow\rangle-|\downarrow\uparrow\uparrow\rangle-|\uparrow\downarrow\uparrow\rangle)(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle)-\frac{1}{3}(|\uparrow\downarrow\downarrow\rangle+|\downarrow\uparrow\downarrow\rangle-2|\downarrow\downarrow\uparrow\rangle)|\uparrow\uparrow\rangle\right],\end{aligned} \tag{31}$$

$$|P_1^{N^+}\rangle = \left|\sqrt{\frac{1}{6}}(udc)(\bar{c}u) + \sqrt{\frac{1}{6}}(duc)(\bar{c}u) - \sqrt{\frac{2}{3}}(uuc)(\bar{c}d)\right\rangle \otimes \left[\frac{\sqrt{3}}{6}(2|\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\rangle - |\uparrow\downarrow\uparrow\rangle)(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)\right]. \quad (32)$$

For the above $P_\psi^{N^+}$ molecular state, its axial-vector coupling constant at the quark level can be expressed as

$$\left\langle P_1^{N^+}, +\frac{1}{2}; \pi_0 \left| \frac{1}{i}\frac{1}{2}\frac{g_q}{f_\pi}(\bar{u}\sigma_z\partial_z\pi_0 u - \bar{d}\sigma_z\partial_z\pi_0 d)\right| P_2^{N^+}, +\frac{1}{2}\right\rangle = \frac{1}{2}\left(-\frac{\sqrt{3}}{18} - \frac{\sqrt{3}}{18} + \frac{2\sqrt{3}}{9}\right)\frac{q_z}{f_\pi}g_q = \frac{\sqrt{3}}{18}\frac{q_z}{f_\pi}g_q. \quad (33)$$

Additionally, the axial-vector coupling constant can also be expressed in another way, and Eq. (33) can be written as

$$\begin{aligned}\left\langle P_1^{N^+}, +\frac{1}{2}; \pi_0 \left| \frac{1}{i}\frac{1}{2}\frac{g_q}{f_\pi}(\bar{u}\sigma_z\partial_z\pi_0 u - \bar{d}\sigma_z\partial_z\pi_0 d)\right| P_2^{N^+}, +\frac{1}{2}\right\rangle &= \frac{\sqrt{3}}{9}\langle \bar{D}^0; \pi_0 | \frac{1}{i}\frac{1}{2}\frac{g_q}{f_\pi}(\bar{u}\sigma_z\partial_z\pi_0 u - \bar{d}\sigma_z\partial_z\pi_0 d)|\bar{D}^{*0}\rangle \\ &\quad + \frac{2\sqrt{3}}{9}\langle \bar{D}^-; \pi_0 | \frac{1}{i}\frac{1}{2}\frac{g_q}{f_\pi}(\bar{u}\sigma_z\partial_z\pi_0 u - \bar{d}\sigma_z\partial_z\pi_0 d)|\bar{D}^{*-}\rangle \\ &= \frac{\sqrt{3}}{18}\frac{q_z}{f_\pi}g_q. \end{aligned} \quad (34)$$

With the same method, the axial-vector coupling constant for the process $P_2 \to P_1\pi_0$ of $P_{\psi_{ss}}^{N^0}$ state read

$$\left\langle P_1^{N^0}, +\frac{1}{2}; \pi_0 \left| \mathcal{L}_{\rm quark} \right| P_2^{N^0}, +\frac{1}{2}\right\rangle = -\frac{\sqrt{3}}{9}\frac{q_z}{f_\pi}g_q. \quad (35)$$

Compare with the axial-vector charge of the nucleon

$$\frac{\frac{1}{2}g_A}{\frac{5}{6}g_q} = \frac{\frac{g_\rho+f_\rho}{2}}{\frac{\sqrt{3}}{18}g_q} = \frac{\frac{f_\rho-g_\rho}{2}}{-\frac{\sqrt{3}}{9}g_q}. \quad (36)$$

We obtain $g_\rho = \frac{\sqrt{3}}{10}g_A$ and $f_\rho = -\frac{\sqrt{3}}{30}g_A$. We can calculate the axial-vector coupling constants for the octet pentaquark states via the combination of the coupling constants g_ρ and f_ρ. With the same methods, we obtain the axial-vector coupling constants of the pentaquark states with other flavor representations.

The Lagrangian of pentaquark states for the process $P_3 \to P_1\pi_0$ in 8_{1f} flavor representation reads

$$\mathcal{L}_\alpha^{\frac{3}{2}^- \to \frac{1}{2}^-} = {\rm Tr}(g_\alpha \bar{P}_1\gamma_\mu\gamma^5\{\partial^\mu\Phi, P_{3\nu}\} + f_\alpha\bar{P}_1\gamma_\mu\gamma^5[\partial^\mu\Phi, P_{3\nu}]). \quad (37)$$

The Lagrangian of pentaquark states for the process $P_3 \to P_2\pi_0$ in 8_{1f} flavor representation reads

$$\mathcal{L}_\beta^{\frac{3}{2}^- \to \frac{1}{2}^-} = {\rm Tr}(g_\beta \bar{P}_2\gamma_\mu\gamma^5\{\partial^\mu\Phi, P_{3\nu}\} + f_\beta\bar{P}_2\gamma_\mu\gamma^5[\partial^\mu\Phi, P_{3\nu}]). \quad (38)$$

The Lagrangian of pentaquark states for the process $P_2 \to P_1\pi_0$ in 8_{2f} flavor representation reads

$$\mathcal{L}_\kappa^{\frac{1}{2}^- \to \frac{1}{2}^-} = {\rm Tr}(g_\kappa \bar{P}_1\gamma_\mu\gamma^5\{\partial^\mu\Phi, P_2\} + f_\kappa\bar{P}_1\gamma_\mu\gamma^5[\partial^\mu\Phi, P_2]). \quad (39)$$

The Lagrangian of pentaquark states for the process $P_3 \to P_1\pi_0$ in 8_{2f} flavor representation reads

$$\mathcal{L}_\tau^{\frac{3}{2}^- \to \frac{1}{2}^-} = {\rm Tr}(g_\tau \bar{P}_1\gamma_\mu\gamma^5\{\partial^\mu\Phi, P_{3\nu}\} + f_\tau\bar{P}_1\gamma_\mu\gamma^5[\partial^\mu\Phi, P_{3\nu}]). \quad (40)$$

The Lagrangian of pentaquark states for the process $P_3 \to P_2\pi_0$ in 8_{2f} flavor representation reads

$$\mathcal{L}_\omega^{\frac{3}{2}^- \to \frac{1}{2}^-} = {\rm Tr}(g_\omega \bar{P}_2\gamma_\mu\gamma^5\{\partial^\mu\Phi, P_{3\nu}\} + f_\omega\bar{P}_2\gamma_\mu\gamma^5[\partial^\mu\Phi, P_{3\nu}]). \quad (41)$$

With the same method as above, we can calculate the values of the coupling constants g_i and f_i ($i = \rho, \alpha, \beta, \kappa, \tau, \omega$) of the pentaquark states with different flavor representations, and we list them in Table X. With the coupling constants g_i

TABLE X. The transition coupling constants of the pentaquark states with different flavor representations.

Constants	Results	Constants	Results
g_ρ	$\frac{\sqrt{3}}{10}g_A$	f_ρ	$-\frac{\sqrt{3}}{30}g_A$
g_α	$\frac{3}{10}g_A$	f_α	$-\frac{1}{10}g_A$
g_β	$\frac{7\sqrt{3}}{30}g_A$	f_β	$\frac{17\sqrt{3}}{90}g_A$
g_κ	$-\frac{\sqrt{3}}{10}g_A$	f_κ	$-\frac{\sqrt{3}}{10}g_A$
g_τ	$-\frac{3}{10}g_A$	f_τ	$-\frac{3}{10}g_A$
g_ω	$-\frac{\sqrt{3}}{10}g_A$	f_ω	$-\frac{\sqrt{3}}{10}g_A$

TABLE XI. The axial-vector coupling constants of the octet hidden-charm pentaquark states with the 8_{1f} flavor representation.

Processes			Coefficients	Results
$\frac{1}{2}^- \to \frac{1}{2}^-$	$P_\psi^{N^+}$	$\langle \Sigma_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Sigma_c D^*, ^2S_{\frac{1}{2}} \rangle$	$g_\rho + f_\rho$	$\frac{\sqrt{3}}{15} g_A$
	$P_\psi^{N^0}$	$\langle \Sigma_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Sigma_c D^*, ^2S_{\frac{1}{2}} \rangle$	$-g_\rho - f_\rho$	$-\frac{\sqrt{3}}{15} g_A$
	$P_{\psi_s}^{\Sigma^+}$	$\langle \Sigma_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Sigma_c D^*, ^2S_{\frac{1}{2}} \rangle$	$2f_\rho$	$-\frac{\sqrt{3}}{15} g_A$
	$P_{\psi_s}^{\Sigma^0}$	$\langle \Sigma_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Sigma_c D^*, ^2S_{\frac{1}{2}} \rangle$	$\frac{2}{\sqrt{3}} g_\rho$	$\frac{1}{5} g_A$
	$P_{\psi_s}^{\Lambda^0}$	$\langle \Xi_c' D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Xi_c' D^*, ^2S_{\frac{1}{2}} \rangle$	$\frac{2}{\sqrt{3}} g_\rho$	$\frac{1}{5} g_A$
	$P_{\psi_s}^{\Sigma^-}$	$\langle \Xi_c' D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Xi_c' D^*, ^2S_{\frac{1}{2}} \rangle$	$-2f_\rho$	$-\frac{\sqrt{3}}{15} g_A$
	$P_{\psi_{ss}}^{N^0}$	$\langle \Omega_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Omega_c D^*, ^2S_{\frac{1}{2}} \rangle$	$-g_\rho + f_\rho$	$-\frac{2\sqrt{3}}{15} g_A$
	$P_{\psi_{ss}}^{N^-}$	$\langle \Omega_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Omega_c D^*, ^2S_{\frac{1}{2}} \rangle$	$g_\rho - f_\rho$	$\frac{2\sqrt{3}}{15} g_A$
Processes			Coefficients	Results
$\frac{3}{2}^- \to \frac{1}{2}^-$	$P_\psi^{N^+}$	$\langle \Sigma_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Sigma_c D^*, ^4S_{\frac{3}{2}} \rangle$	$g_\alpha + f_\alpha$	$\frac{1}{5} g_A$
		$\langle \Sigma_c D^*, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Sigma_c D^*, ^4S_{\frac{3}{2}} \rangle$	$g_\beta + f_\beta$	$\frac{19\sqrt{3}}{45} g_A$
	$P_\psi^{N^0}$	$\langle \Sigma_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Sigma_c D^*, ^4S_{\frac{3}{2}} \rangle$	$-g_\alpha - f_\alpha$	$-\frac{1}{5} g_A$
		$\langle \Sigma_c D^*, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Sigma_c D^*, ^4S_{\frac{3}{2}} \rangle$	$-g_\beta - f_\beta$	$-\frac{19\sqrt{3}}{45} g_A$
	$P_{\psi_s}^{\Sigma^+}$	$\langle \Sigma_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Sigma_c D^*, ^4S_{\frac{3}{2}} \rangle$	$2f_\alpha$	$-\frac{1}{5} g_A$
		$\langle \Sigma_c D^*, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Sigma_c D^*, ^4S_{\frac{3}{2}} \rangle$	$2f_\beta$	$\frac{17\sqrt{3}}{45} g_A$
	$P_{\psi_s}^{\Sigma^0}$	$\langle \Sigma_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Sigma_c D^*, ^4S_{\frac{3}{2}} \rangle$	$\frac{2}{\sqrt{3}} g_\alpha$	$\frac{\sqrt{3}}{5} g_A$
		$\langle \Sigma_c D^*, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Sigma_c D^*, ^4S_{\frac{3}{2}} \rangle$	$\frac{2}{\sqrt{3}} g_\beta$	$\frac{21}{45} g_A$
	$P_{\psi_s}^{\Lambda^0}$	$\langle \Xi_c' D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Xi_c' D^*, ^4S_{\frac{3}{2}} \rangle$	$\frac{2}{\sqrt{3}} g_\alpha$	$\frac{\sqrt{3}}{5} g_A$
		$\langle \Xi_c' D^*, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Xi_c' D^*, ^4S_{\frac{3}{2}} \rangle$	$\frac{2}{\sqrt{3}} g_\beta$	$\frac{16}{45} g_A$
	$P_{\psi_s}^{\Sigma^-}$	$\langle \Sigma_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Sigma_c D^*, ^4S_{\frac{3}{2}} \rangle$	$-2f_\alpha$	$\frac{1}{5} g_A$
		$\langle \Sigma_c D^*, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Sigma_c D^*, ^4S_{\frac{3}{2}} \rangle$	$-2f_\beta$	$-\frac{17\sqrt{3}}{45} g_A$
	$P_{\psi_{ss}}^{N^0}$	$\langle \Omega_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Omega_c D^*, ^4S_{\frac{3}{2}} \rangle$	$-g_\alpha + f_\alpha$	$-\frac{2}{5} g_A$
		$\langle \Omega_c D^*, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Omega_c D^*, ^4S_{\frac{3}{2}} \rangle$	$-g_\beta + f_\beta$	$-\frac{2\sqrt{3}}{45} g_A$
	$P_{\psi_{ss}}^{N^-}$	$\langle \Omega_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Omega_c D^*, ^4S_{\frac{3}{2}} \rangle$	$g_\alpha - f_\alpha$	$\frac{2}{5} g_A$
		$\langle \Omega_c D^*, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Omega_c D^*, ^4S_{\frac{3}{2}} \rangle$	$g_\beta - f_\beta$	$\frac{2\sqrt{3}}{45} g_A$

TABLE XII. The axial-vector coupling constants of the octet hidden-charm pentaquark states with the 8_{2f} flavor representation.

Processes			Coefficients	Results
$\frac{1}{2}^- \to \frac{1}{2}^-$	$P_\psi^{N^+}$	$\langle \Lambda_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Lambda_c D^*, ^2S_{\frac{1}{2}} \rangle$	$g_\kappa + f_\kappa$	$-\frac{\sqrt{3}}{5} g_A$
	$P_\psi^{N^0}$	$\langle \Lambda_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Lambda_c D^*, ^2S_{\frac{1}{2}} \rangle$	$-g_\kappa - f_\kappa$	$\frac{\sqrt{3}}{5} g_A$
	$P_{\psi_s}^{\Sigma^+}$	$\langle \Xi_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Xi_c D^*, ^2S_{\frac{1}{2}} \rangle$	$2f_\kappa$	$-\frac{\sqrt{3}}{5} g_A$
	$P_{\psi_s}^{\Sigma^0}$	$\langle \Xi_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Xi_c D^*, ^2S_{\frac{1}{2}} \rangle$	$\frac{2}{\sqrt{3}} g_\kappa$	$-\frac{1}{5} g_A$
	$P_{\psi_s}^{\Lambda^0}$	$\langle \Lambda_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Lambda_c D^*, ^2S_{\frac{1}{2}} \rangle$	$\frac{2}{\sqrt{3}} g_\kappa$	$-\frac{1}{5} g_A$
	$P_{\psi_s}^{\Sigma^-}$	$\langle \Xi_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Xi_c D^*, ^2S_{\frac{1}{2}} \rangle$	$-2f_\kappa$	$\frac{\sqrt{3}}{5} g_A$
	$P_{\psi_{ss}}^{N^0}$	$\langle \Xi_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Xi_c D^*, ^2S_{\frac{1}{2}} \rangle$	$-g_\kappa + f_\kappa$	0
	$P_{\psi_{ss}}^{N^-}$	$\langle \Xi_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Xi_c D^*, ^2S_{\frac{1}{2}} \rangle$	$g_\kappa - f_\kappa$	0
Processes			Coefficients	Results
$\frac{3}{2}^- \to \frac{1}{2}^-$	$P_\psi^{N^+}$	$\langle \Lambda_c D, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Lambda_c D^*, ^4S_{\frac{3}{2}} \rangle$	$g_\tau + f_\tau$	$-\frac{3}{5} g_A$
		$\langle \Lambda_c D^*, ^2S_{\frac{1}{2}} \vert \mathcal{L} \vert \Lambda_c D^*, ^4S_{\frac{3}{2}} \rangle$	$g_\omega + f_\omega$	$-\frac{\sqrt{3}}{5} g_A$

(Table continued)

TABLE XII. *(Continued)*

Processes		Coefficients	Results
$P_{\psi}^{N^0}$	$\langle\Lambda_c D, {}^2S_{\frac{1}{2}}\vert\mathcal{L}\vert\Lambda_c D^*, {}^4S_{\frac{3}{2}}\rangle$	$-g_\tau - f_\tau$	$\frac{3}{5}g_A$
	$\langle\Lambda_c D^*, {}^2S_{\frac{1}{2}}\vert\mathcal{L}\vert\Lambda_c D^*, {}^4S_{\frac{3}{2}}\rangle$	$-g_\omega - f_\omega$	$\frac{\sqrt{3}}{5}g_A$
$P_{\psi_s}^{\Sigma^+}$	$\langle\Xi_c D, {}^2S_{\frac{1}{2}}\vert\mathcal{L}\vert\Xi_c D^*, {}^4S_{\frac{3}{2}}\rangle$	$2f_\tau$	$-\frac{3}{5}g_A$
	$\langle\Xi_c D^*, {}^2S_{\frac{1}{2}}\vert\mathcal{L}\vert\Xi_c D^*, {}^4S_{\frac{3}{2}}\rangle$	$2f_\omega$	$-\frac{\sqrt{3}}{5}g_A$
$P_{\psi_s}^{\Sigma^0}$	$\langle\Xi_c D, {}^2S_{\frac{1}{2}}\vert\mathcal{L}\vert\Xi_c D^*, {}^4S_{\frac{3}{2}}\rangle$	$\frac{2}{\sqrt{3}}g_\tau$	$-\frac{\sqrt{3}}{5}g_A$
	$\langle\Xi_c D^*, {}^2S_{\frac{1}{2}}\vert\mathcal{L}\vert\Xi_c D^*, {}^4S_{\frac{3}{2}}\rangle$	$\frac{2}{\sqrt{3}}g_\omega$	$-\frac{1}{5}g_A$
$P_{\psi_s}^{\Lambda^0}$	$\langle\Lambda_c D, {}^2S_{\frac{1}{2}}\vert\mathcal{L}\vert\Lambda_c D^*, {}^4S_{\frac{3}{2}}\rangle$	$\frac{2}{\sqrt{3}}g_\tau$	$-\frac{\sqrt{3}}{5}g_A$
	$\langle\Lambda_c D^*, {}^2S_{\frac{1}{2}}\vert\mathcal{L}\vert\Lambda_c D^*, {}^4S_{\frac{3}{2}}\rangle$	$\frac{2}{\sqrt{3}}g_\omega$	$-\frac{1}{5}g_A$
$P_{\psi_s}^{\Sigma^-}$	$\langle\Xi_c D, {}^2S_{\frac{1}{2}}\vert\mathcal{L}\vert\Xi_c D^*, {}^4S_{\frac{3}{2}}\rangle$	$-2f_\tau$	$\frac{3}{5}g_A$
	$\langle\Xi_c D^*, {}^2S_{\frac{1}{2}}\vert\mathcal{L}\vert\Xi_c D^*, {}^4S_{\frac{3}{2}}\rangle$	$-2f_\omega$	$\frac{\sqrt{3}}{5}g_A$
$P_{\psi_{ss}}^{N^0}$	$\langle\Xi_c D, {}^2S_{\frac{1}{2}}\vert\mathcal{L}\vert\Xi_c D^*, {}^4S_{\frac{3}{2}}\rangle$	$-g_\tau + f_\tau$	0
	$\langle\Xi_c D^*, {}^2S_{\frac{1}{2}}\vert\mathcal{L}\vert\Xi_c D^*, {}^4S_{\frac{3}{2}}\rangle$	$-g_\omega + f_\omega$	0
$P_{\psi_{ss}}^{N^-}$	$\langle\Xi_c D, {}^2S_{\frac{1}{2}}\vert\mathcal{L}\vert\Xi_c D^*, {}^4S_{\frac{3}{2}}\rangle$	$g_\tau - f_\tau$	0
	$\langle\Xi_c D^*, {}^2S_{\frac{1}{2}}\vert\mathcal{L}\vert\Xi_c D^*, {}^4S_{\frac{3}{2}}\rangle$	$g_\omega - f_\omega$	0

and f_i, we can obtain the axial-vector coupling constants of the octet hidden-charm pentaquark in the 8_{1f} and 8_{2f} flavor representations, and their results are listed in Tables XI and XII respectively.

Analyzing the numerical results in Tables XI and XII, we can summarize the several points:

(i) Comparing with our previous work [66], we notice that the axial-vector coupling constants of the pentaquark states are generally smaller than that of the nucleon g_A, which is due to the fact that the contributions of the three light quarks to the axial-vector coupling constants interfere with each other, resulting in the smaller axial-vector coupling constants.

(ii) The axial-vector coupling constants of the pentaquark states with different flavor representations are not the same in the identical decay modes. With different flavor representations, the pentaquark states with the same angular momentum J have different spin-flavor wave functions, and the axial-vector coupling constants of the hidden-charm pentaquark states are sensitive to the flavor and spin configurations.

(iii) For the $P_{\psi_{ss}}^{N^0}$, $P_{\psi_{ss}}^{N^-}$ states with the 8_{2f} flavor representation, due to the coupling constants $g_\kappa = f_\kappa$, $g_\tau = f_\tau$ and $g_\omega = f_\omega$, the axial-vector coupling constants of the processes $P_\psi|\frac{1}{2}^-\rangle \to P_\psi|\frac{1}{2}^-\rangle\pi_0$, $P_\psi|\frac{3}{2}^-\rangle \to P_\psi|\frac{1}{2}^-\rangle\pi_0$ are equal to 0.

V. SUMMARY

In recent years, the research about hidden-charmed pentaquark states had great progress, most of the studies were focused on the properties of pentaquark states such as mass and decay width. Inspired by these studies, we systematically investigated the radiative decay and axial-vector decay behaviors of octet hidden-charmed pentaquark states.

In this work, we construct the wave functions of the octet hidden-charm pentaquark states with constituent quark model, we calculate the transition magnetic moments and radiative decay widths of the octet pentaquark states. The numerical results show that the transition magnetic moments and radiative decay widths of pentaquark states with the same flavor representations satisfy some relations. For example, for the $P_\psi^{N^+}$ state with 8_{1f} flavor representation, their transition magnetic moments satisfy the relation $\frac{\mu_{\Sigma_c D^*|\frac{3}{2}^-\rangle \to \Sigma_c D|\frac{1}{2}^-\rangle\gamma}}{\mu_{\Sigma_c D^*|\frac{1}{2}^-\rangle \to \Sigma_c D|\frac{1}{2}^-\rangle\gamma}} = \sqrt{2}$, their radiative decay widths $\Gamma_{\Sigma_c D^*|\frac{1}{2}^-\rangle \to \Sigma_c D|\frac{1}{2}^-\rangle\gamma}$ and $\Gamma_{\Sigma_c D^*|\frac{3}{2}^-\rangle \to \Sigma_c D|\frac{1}{2}^-\rangle\gamma}$ are quite close, and the decay width $\Gamma_{\Sigma_c D^*|\frac{3}{2}^-\rangle \to \Sigma_c D^*|\frac{1}{2}^-\rangle\gamma}$ is close to 0. We also calculated the axial-vector coupling constants for the transition processes in the chiral quark model, we notice that the π meson interaction only exists among light quarks, and the axial-vector coupling constants are quite sensitive to the flavor and spin configurations. Among the pentaquark states we discussed, for the $P_{\psi_{ss}}^{N^0}$ and $P_{\psi_{ss}}^{N^-}$ states with 8_{2f} flavor representation, the axial-vector coupling constants of the processes $P_\psi|\frac{1}{2}^-\rangle \to P_\psi|\frac{1}{2}^-\rangle\pi_0$, $P_\psi|\frac{3}{2}^-\rangle \to P_\psi|\frac{1}{2}^-\rangle\pi_0$ are all 0.

The transition magnetic moments and decay widths play important role in the studies of inner structure of pentaquark states, and can also distinguish experimentally between pentaquark states with different configurations. The axial-vector coupling constants can help us understand the strong interactions, and are also helpful for the calculations of chiral effective theory. We hope that the present work will inspire more exploration of the properties related to octet hidden-charmed pentaquark states.

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