Mass spectra of strange double charm pentaquarks with strangeness $S = -1$

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The observation of the $T_{c\bar{s}}(2900)$ indicates the potential existence of strange double charm pentaquarks based on the heavy antidiquark symmetry. We systematically study the mass spectra of strange double charm pentaquarks with strangeness $S = -1$ in both molecular and compact structures for quantum numbers $J^P = 1/2^-$, $3/2^-$, $5/2^-$. By constructing the interpolating currents, the mass spectra can be extracted from the two-point correlation functions in the framework of QCD sum rule method. In the molecular picture, we find that the $\Xi_c^{\prime +} D^{*+}$, $\Xi_c^{*+} D^{*+}$, $\Xi_{cc}^{*++} \bar{K}^{*0}$, and $\Omega_{cc}^{*+} \rho^+$ may form molecular strange double charm pentaquarks. In both pictures, the masses of the $J^P = 1/2^-, 3/2^-$ pentaquarks locate within the 4.2–4.6 GeV and 4.2–4.5 GeV regions, respectively. As all of them are above the thresholds of their strong decay channels, they behave as a broad state, making them challenging to be detected experimentally. On the other hand, the strange double charm pentaquark with $J^P = 5/2^-$ lies below its strong decay channel, which may be a very narrow state and easy to identify experimentally. The best observed channel is its semileptonic decay to double charm baryon. As the result, we strongly suggest experiments to search for $J^P = 5/2^-$ strange double charm pentaquarks first.

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I. INTRODUCTION

The study of multiquarks goes back to half a century after the quark model was proposed in 1964 [1,2] and became a hot topic since the observation of the $X(3872)$ in 2003. Due to sufficient experimental statistics, tens of charmoniumlike and bottomoniumlike states were observed by various experimental collaborations [3]. These states could be beyond the conventional quark model and be viewed as exotic candidates. They also provide a novel platform to shed light on the hadronization mechanism. Although numerous theoretical efforts have been put forward to understand their nature [4–16], the hadronization mechanism is still unclear.

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Because of the observation of these exotic candidates, in a more general concept, all the bosons are defined as mesons and all the fermions are defined as baryons. The latter one is more complicated than the former one due to one additional (anti)quark. The first well-established exotic baryon signal was reported by the LHCb Collaboration in the $J/\psi p$ invariant mass distribution of the $\Lambda_b^0 \to J/\psi K^- p$ process [17]. The two structures are called $P_c(4380)$ and $P_c(4450)$. Four years later, with an order-of-magnitude larger statistic data, the LHCb Collaboration further reported their hyperfine structures [18]. The $P_c(4380)$ is split into two structures $P_c(4440)$ and $P_c(4457)$ and a new narrow peak $P_c(4312)$ emerges. In 2020, the LHCb Collaboration observed the strange partner, i.e., the $P_{cs}(4459)$ state, of the P_c states in the $J/\psi\Lambda$ invariant mass distribution of the $\Xi_b^- \to J/\psi\Lambda K^-$ process [19]. Recently, another very narrow resonance $P_{cs}(4338)$ was reported in the $J/\psi\Lambda$ invariant mass of the $B^- \to J/\psi\Lambda\bar{p}$ process, with the preferred $J^P = 1/2^-$ at 90% confidence level [20]. Due to their observed channel, the quark contents of P_c and P_{cs} are $uudc\bar{c}$ and $udsc\bar{c}$, respectively, indicating they are hidden charm pentaquarks.

The recent discovery of the double charm tetraquark $T_{cc}^{+}(3875)$ [21,22] raises the question as to whether double charm pentaquarks exist or not. Some theoretical attempts have been made to consider the mass spectrum from the hadronic molecular picture [23–29] and the compact pentaquark picture [30–34]. Some theoretical attempts have also been made to consider the electromagnetic properties [35,36]. For the double charm pentaquarks, the observed Ξ_{cc} [37,38] provides an important input from the experimental side. In Ref. [24], the authors work on a Bethe-Salpeter equation with the interaction respecting heavy quark spin symmetry and predict a $D^{(*)}\Xi_c^{(\prime *)}$ bound state. Reference [32] works on color-magnetic interaction and predicts several compact $QQqq\bar{q}$ states, which could be searched for in the $\Omega_{cc}\pi$, $\Xi_{cc}K$, and $\Xi_c D$ channels. Reference [31] performs a study within the double heavy triquark-diquark framework respecting SU(3) flavor symmetry; the study finds several stable double charm pentaquarks, for instance a $J^P = 1/2^-$ $cc\bar{s}ud$ double charm pentaquark, against their strong decay channels. Reference [29] considers the potential double charm pentaquarks P_{cc} with quark content $ccud\bar{d}$ in the QCD sum rule method. In this work, we further consider the double charm pentaquarks P_{cc} with quark content $ccus\bar{d}$, based on the heavy antiquark-diquark symmetry (HADS) proposed in Ref. [39]. The HADS states that a color triplet double heavy quarks behaves like a heavy antiquark in color space. In this case, the observed $T_{c\bar{s}0}^{a}(2900)^0$ [40] with quark content $cd\bar{u}\,\bar{s}$ indicates the potential existence of strange double charm pentaquark $ccus\bar{d}$. From the HADS point of view, the mass of double heavy pentaquark satisfies the relation,

$$m(QQqq\bar{q}) - m(QQ\bar{q}\,\bar{q}) = m(qq\bar{q}\,\bar{Q}) - m(\bar{Q}\,\bar{q}\,\bar{q}), \quad (1)$$

by replacing $\bar{Q} \to QQ$, as two heavy quarks in the color antitriplet behave like a steady color source from a heavy antiquark. The essence of Eq. (1) can be traced back to Refs. [41,42]. From this point of view, the recently observed $T_{cc}^{+}(3875)$, as a isospin singlet state, is related to the $\bar{\Lambda}_c$ by the HADS. The $T_{c\bar{s}0}^{0}(2900)$ indicates the existence of strange double charm pentaquarks.

Based on the above arguments, we shall systematically study double charm pentaquarks with quark content $ccus\bar{d}$. To obtain a solid conclusion, we start from both the hadronic molecular currents, i.e,. $\Xi_c^+ D^+$, $\Xi_{cc}^{++}\bar{K}^0$, $\Omega_{cc}^{+}\pi^+$, and the compact pentaquark currents in the QCD sum rule approach [43,44]. The paper is organized as follows. In Sec. II, we construct the local pentaquark interpolating currents for double heavy pentaquark states. Using these currents, we perform the parity-projected QCD sum rules analysis in Sec. III. The numerical analysis follows in Sec. IV. The results and discussions are presented in Sec. V.

II. INTERPOLATING CURRENT FOR DOUBLE HEAVY PENTAQUARK

In this section we systematically construct the local pentaquark interpolating currents with spin-parity $J^P = 1/2^-, 3/2^-, 5/2^-$, since these quantum numbers can be achieved with the S-wave ground heavy (or double heavy) baryon and ground charm (or light) meson with quark content $QQus\bar{d}$. Five flavor configurations $[\bar{d}_d s_d][\epsilon^{abc}Q_a Q_b u_c]$ and $[\bar{d}_d Q_d][\epsilon^{abc}Q_a u_b s_c]$, $[\bar{d}_d u_d] \times [\epsilon^{abc}Q_a Q_b s_c]$, $\epsilon^{aij}\epsilon^{bkl}\epsilon^{abc}[Q_i u_j][Q_k s_l]\bar{d}_c$ and $\epsilon^{aij}\epsilon^{bkl}\epsilon^{abc}\times [Q_i Q_j][u_k s_l]\bar{d}_c$ are considered. Here $a\cdots d$, $i\cdots l$ are color indices, u, d, s represents the up, down, and strange quark, Q represents the heavy quark, i.e., charm or bottom quark. The former three flavor configurations have the same color configuration $\mathbf{1}_c \otimes \mathbf{1}_c$, which can be related by the famous Fierz transformation,

$$\delta^{de}\epsilon^{abc} = \delta^{da}\epsilon^{ebc} + \delta^{db}\epsilon^{aec} + \delta^{dc}\epsilon^{abe}. \quad (2)$$

The latter two flavor configurations have the same color configuration $\bar{\mathbf{3}}_c \otimes \bar{\mathbf{3}}_c \otimes \bar{\mathbf{3}}_c$.

In order to find the correspondence for open charm tetraquark $us\bar{d}\,\bar{c}$ and double charm pentaquark $ccus\bar{d}$ due to HADS, we analyze their spin structure. Insuring the charm diquark has the same color structure, i.e., antisymmetric in color space as anticharm quarks, the spin structure of diquark should be symmetric due to Pauli principle, which requires the spin of charm diquark should be $S_{[cc]} = 1$. As the strange charm tetraquark $T_{c\bar{s}0}^{0}(2900)$ with quark content $[u\bar{c}][s\bar{d}]$ is a spin singlet state [40], the spin structure of diquark and antidiquark should be $[u\bar{c}]_{\mathbf{0}}[s\bar{d}]_{\mathbf{0}}$ or $[u\bar{c}]_{\mathbf{1}}[s\bar{d}]_{\mathbf{1}}$ to form a spin-0 tetraquark state. Thus, for open charm tetraquark with $J^P = 0^+$, the spin structure of corresponding HADS pentaquark partner should be

$$\mathbf{1}_{[cc]} \otimes \frac{\mathbf{1}}{\mathbf{2}}_{[u]} \otimes \mathbf{0}_{[s\bar{d}]} = \frac{\mathbf{1}}{\mathbf{2}}_{[ccus\bar{d}]} \oplus \frac{\mathbf{3}}{\mathbf{2}}_{[ccus\bar{d}]}$$

doublet with spin-0 $[s\bar{d}]$ component or

$$\mathbf{1}_{[cc]} \otimes \frac{\mathbf{1}}{\mathbf{2}}_{[u]} \otimes \mathbf{1}_{[s\bar{d}]} = \frac{\mathbf{1}}{\mathbf{2}}_{[ccus\bar{d}]} \oplus \frac{\mathbf{3}}{\mathbf{2}}_{[ccus\bar{d}]} \oplus \frac{\mathbf{5}}{\mathbf{2}}_{[ccus\bar{d}]}$$

triplet with spin-1 $[s\bar{d}]$ component, which indicates that the HADS partner for the open charm tetraquark with $J^P = 0^+$ should have spin $1/2, 3/2$, or $5/2$. An analogous discussion can also be made for the molecular structure $[s\bar{c}][u\bar{d}]$ and the compact structure $[us][\bar{c}\,\bar{d}]$.

With the above considerations, in this work, we discuss various types of pentaquarks with spin $1/2, 3/2$, and $5/2$. Generally speaking, there is no one to one correspondence between the current and the physical structure. It should be emphasized that some of the interpolating currents in the following subsections can be related by the Fierz transformations in Eq. (2) and hence are not independent.

A. Currents for the heavy baryon-heavy meson molecular pentaquark

The currents with color configuration $\epsilon_{abc}[u_a s_b Q_c][\bar{d}_d Q_d]$ could well couple to $\Xi_c^+ D^{+(*)}$ molecular states. In previous QCD sum rules analysis [45], one finds that the currents

$$\frac{1}{\sqrt{2}}\epsilon_{abc}[(u_a^T C\gamma_5 s_b - s_a^T C\gamma_5 u_b)c_c], \tag{3}$$

$$\frac{1}{\sqrt{2}}\epsilon_{abc}[(u_a^T C\gamma_\mu\gamma_5 s_b - s_a^T C\gamma_\mu\gamma_5 u_b)\gamma_\mu c_c], \tag{4}$$

$$\sqrt{\frac{2}{3}}\epsilon_{abc}[(s_a^T C\gamma_\mu u_b)\gamma_5 c_c + (u_a^T C\gamma_\mu c_b)\gamma_5 s_c + (c_a^T C\gamma_\mu s_b)\gamma_5 u_c], \tag{5}$$

can couple well to the Ξ_c^+, $\Xi_c'^+$ and Ξ_c^{*+} states, respectively. Here Ξ_c^+, $\Xi_c'^+$, and Ξ_c^{*+} belong to the SU(4) flavor spin-$\frac{1}{2}$ 20-plet, spin-$\frac{1}{2}$ 20-plet, spin-$\frac{3}{2}$ 20-plet, respectively. Thus, we can construct the following currents to perform QCD sum rule analysis,

$$\begin{aligned}
\eta_1 &= \frac{1}{\sqrt{2}}\epsilon_{abc}[(u_a^T C\gamma_5 s_b - s_a^T C\gamma_5 u_b)Q_c][\bar{d}_d\gamma_5 Q_d], \\
\eta_2 &= \frac{1}{\sqrt{2}}\epsilon_{abc}[(u_a^T C\gamma_\mu\gamma_5 s_b - s_a^T C\gamma_\mu\gamma_5 u_b)\gamma_\mu Q_c][\bar{d}_d\gamma_5 Q_d], \\
\eta_3 &= \frac{1}{\sqrt{2}}\epsilon_{abc}[(u_a^T C\gamma_5 s_b - s_a^T C\gamma_5 u_b)\gamma_\mu Q_c][\bar{d}_d\gamma_\mu Q_d], \\
\eta_{4\mu} &= \frac{1}{\sqrt{2}}\epsilon_{abc}[(u_a^T C\gamma_\nu\gamma_5 s_b - s_a^T C\gamma_\nu\gamma_5 u_b)\gamma_\nu Q_c][\bar{d}_d\gamma_\mu Q_d], \\
\eta_{5\mu} &= \sqrt{\frac{2}{3}}\epsilon_{abc}[(s_a^T C\gamma_\mu u_b)\gamma_5 Q_c + (u_a^T C\gamma_\mu Q_b)\gamma_5 s_c \\
&\quad + (Q_a^T C\gamma_\mu s_b)\gamma_5 u_c][\bar{d}_d\gamma_5 Q_d], \\
\eta_6 &= \sqrt{\frac{2}{3}}\epsilon_{abc}[(s_a^T C\gamma_\mu u_b)\gamma_5 Q_c + (u_a^T C\gamma_\mu Q_b)\gamma_5 s_c \\
&\quad + (Q_a^T C\gamma_\mu s_b)\gamma_5 u_c][\bar{d}_d\gamma_\mu Q_d], \\
\eta_{7,\mu\nu} &= \sqrt{\frac{2}{3}}\epsilon_{abc}[(s_a^T C\gamma_\mu u_b)\gamma_5 Q_c + (u_a^T C\gamma_\mu Q_b)\gamma_5 s_c \\
&\quad + (Q_a^T C\gamma_\mu s_b)\gamma_5 u_c][\bar{d}_d\gamma_\nu Q_d] + (\mu \leftrightarrow \nu),
\end{aligned} \tag{6}$$

where u, d, s denote up, down, and strange quarks, respectively. Here Q denotes heavy quarks c or b, T denotes the transpose of quark field, and C denotes the charge conjugation operator, and the indices a, b, c, d are the color indices of quark fields. One notices that not all of the above currents are related to the strange charm tetraquark by HADS. Further discussions can be found in Sec. V.

It should be noted that the interpolating currents for baryon states could couple to both positive and negative parity states and thus, currents in Eq. (6) could couple to $J^P = 1/2^\pm, 3/2^\pm$, or $5/2^\pm$ pentaquarks. For instance, the current η_1 would couple to both $\Xi_c^+ D^+$ states with $J^P = 1/2^-$ and $\Xi_c^+ D^+$ states with $J^P = 1/2^+$ in P-waves. We will further discuss such an issue in the next section.

B. Currents for the double heavy baryon-light meson molecular pentaquark

We introduce currents with color configuration $\epsilon_{abc}[Q_a Q_b u_c][\bar{d}_d s_d]$ and $\epsilon_{abc}[Q_a Q_b s_c][\bar{d}_d u_d]$ coupling to $\Xi_{cc}^{++}\bar{K}^{0(*)}$ and $\Omega_{cc}^+\pi^+(\rho^+)$ molecular states, respectively. In previous QCD sum rules analysis [45] one suggests that the currents

$$\epsilon_{abc}(c_a^T C\gamma_\mu c_b)\gamma_\mu\gamma_5 u_c, \tag{7}$$

$$\frac{1}{\sqrt{3}}\epsilon_{abc}[2(u_a^T C\gamma_\mu c_b)\gamma_5 c_c + (c_a^T C\gamma_\mu c_b)\gamma_5 u_c], \tag{8}$$

could well couple to Ξ_{cc}^{++} and Ξ_{cc}^{*++} states with $J^P = \frac{1}{2}^+, \frac{3}{2}^+$, respectively. Thus, we can construct the following pentaquark currents to perform QCD sum rules analysis:

$$\begin{aligned}
\xi_1 &= [\epsilon_{abc}(Q_a^T C\gamma_\mu Q_b)\gamma_\mu\gamma_5 u_c][\bar{d}_d\gamma_5 s_d], \\
\xi_{2\mu} &= [\epsilon_{abc}(Q_a^T C\gamma_\nu Q_b)\gamma_\nu\gamma_5 u_c][\bar{d}_d\gamma_\mu s_d], \\
\xi_{3\mu} &= \frac{1}{\sqrt{3}}\epsilon_{abc}[2(u_a^T C\gamma_\mu Q_b)\gamma_5 Q_c \\
&\quad + (Q_a^T C\gamma_\mu Q_b)\gamma_5 u_c][\bar{d}_d\gamma_5 s_d], \\
\xi_4 &= \frac{1}{\sqrt{3}}\epsilon_{abc}[2(u_a^T C\gamma_\mu Q_b)\gamma_5 Q_c \\
&\quad + (Q_a^T C\gamma_\mu Q_b)\gamma_5 u_c][\bar{d}_d\gamma_\mu s_d], \\
\xi_{5,\mu\nu} &= \frac{1}{\sqrt{3}}\epsilon_{abc}[2(u_a^T C\gamma_\mu Q_b)\gamma_5 Q_c + (Q_a^T C\gamma_\mu Q_b)\gamma_5 u_c] \\
&\quad \times [\bar{d}_d\gamma_\nu s_d] + (\mu \leftrightarrow \nu),
\end{aligned} \tag{9}$$

where ξ_i should be the HADS partner of open heavy tetraquark $[u\bar{c}]_{\mathbf{0(1)}}[s\bar{d}]_{\mathbf{0(1)}}$ due to its spin-1 $[cc]$ diquark component and spin-0(1) $[s\bar{d}]$ component.

The interpolating currents for configuration $\epsilon_{abc}[Q_a Q_b s_c][\bar{d}_d u_d]$ are the same as $\epsilon_{abc}[Q_a Q_b u_c][\bar{d}_d s_d]$ with substitution $u \leftrightarrow s$ and we denote them as ψ_i,

$$\psi_i = \xi_i(u \leftrightarrow s), \tag{10}$$

where ψ_i should be the HADS partner of open heavy tetraquark $[s\bar{c}]_{\mathbf{0(1)}}[u\bar{d}]_{\mathbf{0(1)}}$ due to its spin-1 $[cc]$ diquark component and spin-0(1) $[u\bar{d}]$ component.

C. Currents for compact pentaquark

The double heavy pentaquarks with compact structures can be treated by an intuitive picture, in which the heavier component forms a nucleus and the lighter one is in an orbit around this nucleus. Such a picture for compact pentaquark has color configuration $[[cc]_{\bar{3}}\bar{d}_{\bar{3}}]_{3}[us]_{\bar{3}}$, where the heavy diquark forms a color triplet spin-1 state, as suggested previously. Since the $[cc]$ diquark with spin-0 violates the Fermi-Dirac statistic, we only construct the pentaquark with the $[cc]$ diquark with spin-1. To compare with this compact picture, we also consider a more complicated picture, i.e., one heavy quark and one light quark form one color antitriplet diquark and the two heavy-light diquarks combine with the other light antiquark to form a color singlet pentaquark. Such a picture has color configuration $[[cu]_{\bar{3}}[cs]_{\bar{3}}]_{3}\bar{d}_{\bar{3}}$. In this work, we suggest currents with color configuration $\epsilon^{aij}\epsilon^{bkl}\epsilon^{abc}[Q_iu_j][Q_ks_l]\bar{d}_c$ and $\epsilon^{aij}\epsilon^{bkl}\epsilon^{abc}[Q_iQ_j][u_ks_l]\bar{d}_c$ coupling to the two compact pentaquark states above, and we use the following interpolating currents to perform our QCD sum rule analysis:

$$
\begin{aligned}
J_{1,2} &= \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}(Q_i^TC\gamma_\mu Q_j)(u_k^TC\gamma_\mu s_l)\gamma_5 C\bar{d}_c^T, \\
J_{1,3} &= \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}(Q_i^TC\gamma_\mu Q_j)(u_k^TC\gamma_5 s_l)\gamma_\mu C\bar{d}_c^T, \\
J_{1,5\mu} &= \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}(Q_i^TC\gamma_\mu Q_j)(u_k^TC\gamma_5 s_l)\gamma_5 C\bar{d}_c^T, \\
J_{1,8\mu} &= \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}(Q_i^TC\gamma_\nu Q_j)(u_k^TC\gamma_\nu s_l)\gamma_\mu C\bar{d}_c^T, \\
J_{1,9\mu\nu} &= \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}(Q_i^TC\gamma_\mu Q_j)(u_k^TC\gamma_\nu s_l)\gamma_5 C\bar{d}_c^T \\
&\quad + (\mu \leftrightarrow \nu),
\end{aligned} \tag{11}
$$

and

$$
\begin{aligned}
J_{2,1} &= \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}(Q_i^TC\gamma_5 u_j)(Q_k^TC\gamma_5 s_l)\gamma_5 C\bar{d}_c^T, \\
J_{2,2} &= \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}(Q_i^TC\gamma_\mu u_j)(Q_k^TC\gamma_\mu s_l)\gamma_5 C\bar{d}_c^T, \\
J_{2,3} &= \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}(Q_i^TC\gamma_\mu u_j)(Q_k^TC\gamma_5 s_l)\gamma_\mu C\bar{d}_c^T, \\
J_{2,4} &= \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}(Q_i^TC\gamma_5 u_j)(Q_k^TC\gamma_\mu s_l)\gamma_\mu C\bar{d}_c^T, \\
J_{2,5\mu} &= \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}(Q_i^TC\gamma_\mu u_j)(Q_k^TC\gamma_5 s_l)\gamma_5 C\bar{d}_c^T, \\
J_{2,6\mu} &= \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}(Q_i^TC\gamma_5 u_j)(Q_k^TC\gamma_\mu s_l)\gamma_5 C\bar{d}_c^T, \\
J_{2,7\mu} &= \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}(Q_i^TC\gamma_5 u_j)(Q_k^TC\gamma_5 s_l)\gamma_\mu C\bar{d}_c^T, \\
J_{2,8\mu\nu} &= \epsilon_{aij}\epsilon_{bkl}\epsilon_{abc}(Q_i^TC\gamma_\mu u_j)(Q_k^TC\gamma_\nu s_l)\gamma_5 C\bar{d}_c^T \\
&\quad + (\mu \leftrightarrow \nu),
\end{aligned} \tag{12}
$$

where currents $J_{1,i}$ are heavy diquark coupled and $J_{2,i}$ are heavy diquark decoupled. For convenience, we call these two types of currrents Type-I and Type-II. The currents $J_{1,2}, J_{1,3}, J_{1,5\mu}, J_{1,8\mu}, J_{1,9\mu\nu}$ should couple to the HADS partners of open heavy tetraquark $[us\bar{d}\,\bar{c}]$ due to their spin-1 $[cc]$ diquark components.

III. QCD SUM RULES

In this section, we shall investigate the currents using the method of QCD sum rules. Symbols J, J_μ, and $J_{\mu\nu}$ are assigned to denote the currents with spin $J = 1/2, 3/2, 5/2$, respectively. The two-point correlation functions obtained by the currents can be written as [46]

$$
\begin{aligned}
\Pi(q^2) &= i\int d^4x e^{iq\cdot x}\langle 0|T[J(x)\bar{J}(0)]|0\rangle \\
&= (\not{q} + M_X)\Pi^{1/2}(q^2), \\
\Pi_{\mu\nu}(q^2) &= i\int d^4x e^{iq\cdot x}\langle 0|T[J_\mu(x)\bar{J}_\nu(0)]|0\rangle \\
&= \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)(\not{q} + M_X)\Pi^{3/2}(q^2) + \cdots, \\
\Pi_{\mu\nu\alpha\beta}(q^2) &= i\int d^4x e^{iq\cdot x}\langle 0|T[J_{\mu\nu}(x)\bar{J}_{\alpha\beta}(0)]|0\rangle \\
&= (g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha})(\not{q} + M_X)\Pi^{5/2}(q^2) + \cdots,
\end{aligned} \tag{13}
$$

where $\cdots$ contains the other coupling states, M_X denotes the mass of physical state X. In this work, we will only use the structures 1, $g_{\mu\nu}$, and $g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}$ for the correlation functions $\Pi(p^2)$, $\Pi_{\mu\nu}(p^2)$, and $\Pi_{\mu\nu\alpha\beta}(p^2)$, respectively, to study the $J^P = 1/2^-, 3/2^-$, and $5/2^-$ double charm pentaquark states. We assume that the current couples to the physical state X through

$$
\begin{aligned}
\langle 0|J|X_{1/2}\rangle &= f_X u(p), \\
\langle 0|J_\mu|X_{3/2}\rangle &= f_X u_\mu(p), \\
\langle 0|J_{\mu\nu}|X_{5/2}\rangle &= f_X u_{\mu\nu}(p),
\end{aligned} \tag{14}
$$

where f_X denotes the coupling constant, $u(p)$ denotes the Dirac spinor and $u_\mu(p), u_{\mu\nu}(p)$ denotes the Rarita-Schwinger vector and tensor, respectively.

For the convenience of discussing the parity of hadron currents, we assumed that the hadron state X has the same parity as its current J, and used the non-γ_5 coupling relation in (14). Meanwhile, the γ_5 coupling relation also exists,

$$
\begin{aligned}
\langle 0|J|X'_{1/2}\rangle &= f_X\gamma_5 u(p), \\
\langle 0|J_\mu|X'_{3/2}\rangle &= f_X\gamma_5 u_\mu(p), \\
\langle 0|J_{\mu\nu}|X'_{5/2}\rangle &= f_X\gamma_5 u_{\mu\nu}(p),
\end{aligned} \tag{15}
$$

where X' has the opposite parity of X. Equations (14) and (15) indicate the fact that two states with opposite parity could couple to the same current. These relations also suggest that the current $j \equiv \gamma_5 J$ with opposite parity can couple to the state X. In the following discussion we will denote the currents with positive parity as J and the currents with negative parity as j. The parity issue will be further discussed at the end of this section.

At the hadron level, two-point correlation function can be written as

$$\Pi(q^2) = \frac{1}{\pi}\int_{s_<}^{\infty} \frac{\mathrm{Im}\Pi(s)}{s - q^2 - i\epsilon} ds, \tag{16}$$

where we have used the form of the dispersion relation, and $s_<$ denotes the physical threshold. The imaginary part of the correlation function is defined as the spectral function, which is usually evaluated at the hadron level by inserting intermediate hadron states $\sum_n |n\rangle\langle n|$,

$$\begin{aligned}\rho(s) &\equiv \frac{1}{\pi}\mathrm{Im}\Pi(s) = \sum_n \delta(s - M_n^2)\langle 0|J|n\rangle\langle n|\bar{J}|0\rangle \\ &= f_X^{-2}(\not{p} + m_X^-)\delta(s - m_X^{-2}) + f_X^{+2}(\not{p} - m_X^+)\delta(s - m_X^{+2}) \\ &\quad + \text{continuum}. \end{aligned}\tag{17}$$

The spectral density $\rho(s)$ can also be evaluated at the quark-gluon level via the operator product expansion (OPE). After performing the Borel transform at both the hadron and quark-gluon levels, the two-point correlation function can be expressed as

$$\Pi(M_B^2) \equiv \mathcal{B}_{M_B^2}\Pi(p^2) = \int_{s_<}^{\infty} e^{-s/M_B^2}\rho(s)ds. \tag{18}$$

Finally, we assume that the contribution from the continuum states can be approximated well by the OPE spectral density above a threshold value s_0 (duality), and arrive at the sum rule relation which can be used to perform numerical analysis,

$$M_X^2(s_0, M_B) = \frac{\int_{s_<}^{s_0} e^{-s/M_B^2}\rho(s)sds}{\int_{s_<}^{s_0} e^{-s/M_B^2}\rho(s)ds}. \tag{19}$$

To further discuss the parity of the hadron states, we assume that the correlation function of J is given by

$$\Pi_+(p^2) = \not{p}\Pi_1(p^2) + \Pi_2(p^2),$$

each scalar functions $\Pi_{1,2}(p^2)$ in the equation above can construct a sum rule with (19) separately. Meanwhile, the correlation function of j can be written as

$$\Pi_-(p^2) = \not{p}\Pi_1(p^2) - \Pi_2(p^2).$$

The difference between the correlation function of J and j appears only in the sign in front of $\Pi_2(p^2)$ due to the γ_5 coupling. Thus, the same functions Π_1 and Π_2 appear in Π_+ and Π_- bring us no independent sum rule from j. Here we use the method of parity projected sum rule to obtain two independent sum rules with different parity [47,48].

In the zero-width resonance approximation, the imaginary part of the correlation function in the rest frame $\vec{p} = 0$ is considered as

$$\begin{aligned}\frac{\mathrm{Im}\Pi(p_0)}{\pi} &= \sum_n \bigg[(\lambda_n^+)^2\frac{\gamma_0 + 1}{2}\delta(p_0 - m_n^+) \\ &\qquad + (\lambda_n^-)^2\frac{\gamma_0 - 1}{2}\delta(p_0 - m_n^-)\bigg] \\ &\equiv \gamma_0 p_0 \rho_A(p_0) + \rho_B(p_0), \end{aligned}\tag{20}$$

where $\lambda^\pm$ are coupling constants and $m^\pm$ denote the mass of positive or negative parity state. $\rho_A(p_0), \rho_B(p_0)$ are defined by

$$p_0\rho_A(p_0) \equiv \frac{1}{2}\sum_n [(\lambda_n^+)^2\delta(p_0 - m_n^+) + (\lambda_n^-)^2\delta(p_0 - m_n^-)],$$

$$\rho_B(p_0) \equiv \frac{1}{2}\sum_n [(\lambda_n^+)^2\delta(p_0 - m_n^+) - (\lambda_n^-)^2\delta(p_0 - m_n^-)].$$

The combination $p_0\rho_A(p_0) + \rho_B(p_0)$ and $p_0\rho_A(p_0) - \rho_B(p_0)$ contain contributions only from the positive- or negative-parity states obviously; thus, we can establish the corresponding parity projected sum rules,

$$\begin{aligned}\mathcal{L}_k(s_0^+, M_B^2, +) &\equiv \frac{1}{2}\int_{s_<}^{s_0^+} e^{-s/M_B^2}[\sqrt{s}\rho_A^{\mathrm{OPE}}(s) + \rho_B^{\mathrm{OPE}}(s)] \\ &\quad \times s^k ds = \lambda_+^2 m_+^{2k+1}\exp\left[-\frac{m_+^2}{M_B^2}\right], \\ \mathcal{L}_k(s_0^-, M_B^2, -) &\equiv \frac{1}{2}\int_{s_<}^{s_0^-} e^{-s/M_B^2}[\sqrt{s}\rho_A^{\mathrm{OPE}}(s) - \rho_B^{\mathrm{OPE}}(s)] \\ &\quad \times s^k ds = \lambda_-^2 m_-^{2k+1}\exp\left[-\frac{m_-^2}{M_B^2}\right], \end{aligned}\tag{21}$$

where $s_0^\pm$ denote the threshold of positive or negative parity state. We can extract the mass for positive and negative parity states by

$$m_\pm(s_0^\pm, M_B) = \sqrt{\frac{\mathcal{L}_1(s_0^\pm, M_B^2, \pm)}{\mathcal{L}_0(s_0^\pm, M_B^2, \pm)}}. \tag{22}$$

We shall discuss the detail to obtain suitable parameter working regions in QCD sum rule analysis in next section.

Using the OPE method, the two-point function can also be evaluated at the quark-gluonic level as a function of various QCD parameters. To evaluate the Wilson coefficients, we adopt the quark propagator in momentum space and the propagator,

$$iS_Q^{ab}(p) = \frac{i\delta^{ab}}{\not{p} - m_Q} + \frac{i}{4}g_s\frac{\lambda_{ab}^n}{2}G_{\mu\nu}^n\frac{\sigma^{\mu\nu}(\not{p} + m_Q) + (\not{p} + m_Q)\sigma^{\mu\nu}}{(p^2 - m_Q^2)^2} + \frac{i\delta^{ab}}{12}\langle g_s^2 GG\rangle m_Q\frac{p^2 + m_Q\not{p}}{(p^2 - m_Q^2)^4}, \tag{23}$$

$$iS_q^{ab}(x) = \frac{i\delta^{ab}}{2\pi^2 x^4}\not{x} - \frac{\delta^{ab}}{12}\langle\bar{q}q\rangle + \frac{i}{32\pi^2}\frac{\lambda^n_{ab}}{2}g_s G^n_{\mu\nu}\frac{1}{x^2}(\sigma^{\mu\nu}\not{x} + \not{x}\sigma^{\mu\nu}) + \frac{\delta^{ab}x^2}{192}\langle\bar{q}g_s\sigma\cdot Gq\rangle - \frac{m_q\delta^{ab}}{4\pi^2 x^2} + \frac{i\delta^{ab}m_q\langle\bar{q}q\rangle}{48}\not{x} - \frac{im_q\langle\bar{q}g_s\sigma\cdot Gq\rangle\delta^{ab}x^2\not{x}}{1152}, \tag{24}$$

where Q represents the heavy quark c or b, q represents the light quark u, d, s, the superscripts a, b denote the color indices. In this work, we evaluate Wilson coefficients of the correlation function up to dimension nine condensates at the leading order in α_s. The OPE results for the currents in Eqs. (6), and (9)–(12) are too lengthy, thus we collect these results in the supplementary document "OPEresult.nb" [49].

IV. NUMERICAL ANALYSIS

In this section we perform the QCD sum rule analysis for double heavy molecular pentaquark systems using the interpolating currents in Eqs. (6) and (9)–(12). We use the standard values of various QCD condensates as $\langle\bar{q}q\rangle\times(1\ \mathrm{GeV}) = -(0.24\pm0.03)^3\ \mathrm{GeV}^3$, $\langle\bar{q}g_s\sigma\cdot Gq\rangle(1\ \mathrm{GeV}) = -M_0^2\langle\bar{q}q\rangle$, $M_0^2 = (0.8\pm0.2)\ \mathrm{GeV}^2$, $\langle\bar{s}s\rangle/\langle\bar{q}q\rangle = 0.8\pm0.1$, $\langle g_s^2 GG\rangle(1\ \mathrm{GeV}) = (0.48\pm0.14)\ \mathrm{GeV}^4$ at the energy scale $\mu = 1$ GeV [50–57] and $m_s(2\ \mathrm{GeV}) = 95^{+9}_{-3}$ MeV, $m_c(m_c) = 1.27^{+0.03}_{-0.04}$ GeV, $m_b(m_b) = 4.18^{+0.04}_{-0.03}$ GeV from the Particle Data Group [3]. We also take into account the energy-scale dependence of the above parameters from the renormalization group equation,

$$\begin{aligned}
m_s(\mu) &= m_s(2\ \mathrm{GeV})\left[\frac{\alpha_s(\mu)}{\alpha_s(2\ \mathrm{GeV})}\right]^{\frac{12}{33-2n_f}},\\
m_c(\mu) &= m_c(m_c)\left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right]^{\frac{12}{33-2n_f}},\\
m_b(m_b) &= m_b(m_b)\left[\frac{\alpha_s(\mu)}{\alpha_s(m_b)}\right]^{\frac{12}{33-2n_f}},\\
\langle\bar{q}q\rangle(\mu) &= \langle\bar{q}q\rangle(1\ \mathrm{GeV})\left[\frac{\alpha_s(1\ \mathrm{GeV})}{\alpha_s(\mu)}\right]^{\frac{12}{33-2n_f}},\\
\langle\bar{s}s\rangle(\mu) &= \langle\bar{s}s\rangle(1\ \mathrm{GeV})\left[\frac{\alpha_s(1\ \mathrm{GeV})}{\alpha_s(\mu)}\right]^{\frac{12}{33-2n_f}},\\
\langle\bar{q}g_s\sigma\cdot Gq\rangle(\mu) &= \langle\bar{q}g_s\sigma\cdot Gq\rangle(1\ \mathrm{GeV})\left[\frac{\alpha_s(1\ \mathrm{GeV})}{\alpha_s(\mu)}\right]^{\frac{2}{33-2n_f}},\\
\langle\bar{s}g_s\sigma\cdot Gs\rangle(\mu) &= \langle\bar{s}g_s\sigma\cdot Gs\rangle(1\ \mathrm{GeV})\left[\frac{\alpha_s(1\ \mathrm{GeV})}{\alpha_s(\mu)}\right]^{\frac{2}{33-2n_f}},\\
\alpha_s(\mu) &= \frac{1}{b_0 t}\left[1 - \frac{b_1}{b_0}\frac{\log t}{t} + \frac{b_1^2(\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2}\right],
\end{aligned} \tag{25}$$

where $t = \log\frac{\mu^2}{\Lambda^2}$, $b_0 = \frac{33-2n_f}{12\pi}$, $b_1 = \frac{153-19n_f}{24\pi^2}$, $b_2 = \frac{2857-\frac{5033}{9}n_f+\frac{325}{27}n_f^2}{128\pi^3}$, $\Lambda = 210$ MeV, 292 MeV, and 332 MeV for the flavors $n_f = 5, 4$, and 3, respectively. In this work, we evolve all the input parameters to the energy scale $\mu = 2m_c$ for our sum rule analysis.

To establish a stable mass sum rule, one should find the appropriate parameter working regions at first, i.e., for the continuum threshold s_0 and the Borel mass M_B^2. The threshold s_0 can be determined via the minimized variation of the hadronic mass m_X with respect to the Borel mass M_B^2. The lower bound on the Borel mass M_B^2 can be fixed by requiring a reasonable OPE convergence, while its upper bound is determined through a sufficient pole contribution. The pole contribution (PC) is defined as

$$\mathrm{PC}(s_0^{\pm}, M_B^2, \pm) = \frac{\mathcal{L}_0(s_0^{\pm}, M_B^2, \pm)}{\mathcal{L}_0(\infty, M_B^2, \pm)}, \tag{26}$$

where $\mathcal{L}_0$ has been defined in Eq. (21).

As an example, we use the current $J_{1,2}(x)$ with $J^P = 1/2^-$ in c sector to show the details of the numerical analysis. As we mainly focus on the negative parity states in this work, we will omit the parity superscript without ambiguity in the following discussion. For this current, the dominant nonperturbative contribution to the correlation function comes from the quark condensate, which is proportional to the charm quark mass m_c. In Fig. 1, we show the contributions of the perturbative term and various condensate terms to the correlation function with respect to M_B^2 when s_0 tends to infinity. It is clear that the Borel mass M_B^2 should be large enough to ensure the convergence of

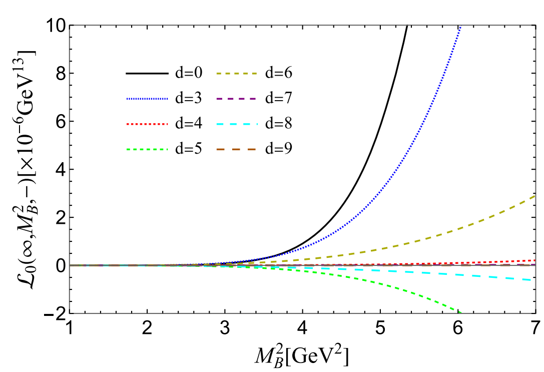

FIG. 1. Contributions of various OPE terms with dimension $d = 0$ to 9 in Eq. (21) for the current $J_{1,2}(x)$ with negative parity in c sector, as a function of M_B^2 when $s_0 \to \infty$.

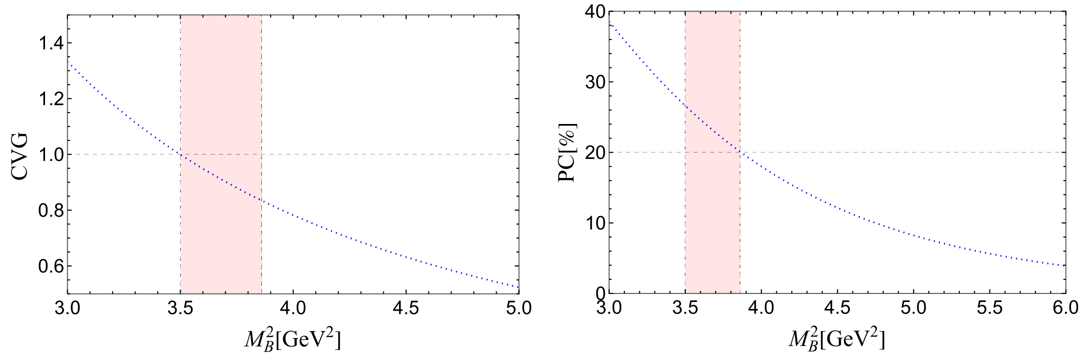

FIG. 2. Convergence (left) and pole contribution (right) for the interpolating current $J_{1,2}(x)$ with $J^P = 1/2^-$ in c sector. The band shows the working Borel window 3.50 GeV$^2 \leq M_B^2 \leq$ 3.86 GeV^2.

the OPE series. In this work, we require that the contribution of the perturbative term is larger than the contribution of the quark condensate term, which is

$$\mathrm{CVG}(M_B^2, \pm) = \frac{\mathcal{L}_0^{\langle\bar{q}q\rangle}(\infty, M_B^2, \pm)}{\mathcal{L}_0^{\mathrm{pert}}(\infty, M_B^2, \pm)} \leq 1, \qquad (27)$$

providing the lower bound of the Borel mass $M_B^2 \geq$ 3.50 GeV^2. After studying the pole contribution defined in Eq. (26), one finds that the PC is small for pentaquark system due to the high dimension of the interpolating current. To find an upper bound of the Borel mass, we require that the pole contribution to be larger than 20%. As the right panel of Fig. 2 shown, the upper bound of the Borel mass is determined as 3.86 GeV^2. As a result, a reasonable Borel window for the current $J_{1,2}(x)$ is obtained as 3.50 GeV$^2 \leq M_B^2 \leq$ 3.86 GeV^2.

As mentioned above, the variation of the extracted hadron mass m with respect to M_B^2 should be minimized to obtain the optimal value of the continuum threshold s_0. We define the following hadron mass $\bar{m}_X$ and quantity $\chi^2(s_0)$ to study the stability of mass sum rules

$$\bar{m}(s_0, \pm) = \sum_{i=1}^{N} \frac{m(s_0, M_{B,i}^2, \pm)}{N}, \qquad (28)$$

$$\chi^2(s_0, \pm) = \sum_{i=1}^{N} \left[\frac{m(s_0, M_{B,i}^2, \pm)}{\bar{m}(s_0, \pm)} - 1\right]^2, \qquad (29)$$

where the $M_{B,i}^2 (i = 1, 2, \ldots, N)$ represent N definite values for the Borel parameter M_B^2 in the Borel window. According to the above definition, the optimal choice for the continuum threshold s_0 in the QCD sum rule analysis can be obtained by minimizing the quantity $\chi^2(s_0)$, which is only the function of s_0. In this example, there is a minimum point around $s_0 \approx 22.3$ GeV^2 in χ^2 function and we show the variation of m with s_0 in the left panel of Fig. 3, from which we can find that the optimized value of the continuum threshold can be chosen as $s_0 \approx 22.3$ GeV^2 indeed. In the right panel of Fig. 3, the mass sum rules are established to be very stable in the above parameter regions

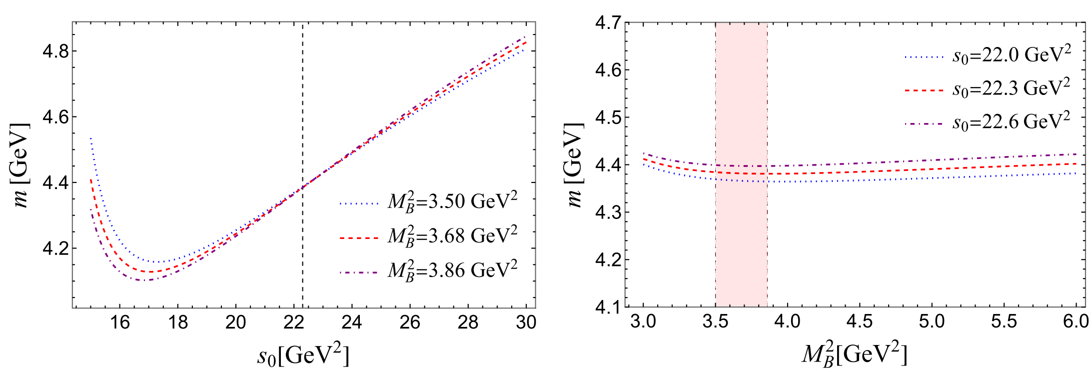

FIG. 3. Mass curves for the interpolating current $J_{1,2}(x)$ with $J^P = 1/2^-$ in c sector, as a function of the threshold s_0 (left) and the Borel mass M_B^2 (right). The band shows the working Borel window 3.50 GeV$^2 \leq M_B^2 \leq$ 3.86 GeV^2. In the left panel, the mass curves with different M_B^2 intersect at $s_0 = 22.3$ GeV^2, indicating that the mass has a minimum dependency on the Borel mass at $s_0 = 22.3$ GeV^2. In the right panel, the mass curves with different s_0 around 22.3 GeV^2 are stable in our working Borel window. The blue dotted, red dashed and purple dot-dashed curves on the left (right) figure are for Borel mass (threshold) $M_B^2 =$ 3.50 GeV^2, 3.68 GeV^2, 3.86 GeV^2 ($s_0 =$ 22.0 GeV^2, 22.3 GeV^2, 22.6 GeV^2), respectively.

TABLE I. The masses (the last second column) of various molecular structures extracted from the two-point correlation function based on the QCD sum rule approach. The errors are from the Borel windows, the threshold of higher states, the various condensates and the quark masses. The first five columns present the names of the currents, the corresponding molecular structure, the quantum number J^P, the Borel mass square and the threshold of the continuum s_0. The last column indicates the corresponding two-heavy-hadron thresholds.

Current	Structure	J^P	$M_B^2(\mathrm{GeV}^2)$	$s_0(\mathrm{GeV}^2)$	Mass (GeV)	Threshold (MeV)
η_3	$\Xi_c^+ D^{*+}$	$\frac{1}{2}^-$	3.20–4.38	24.3	$4.50^{+0.05}_{-0.04}$	4477
$\eta_{4\mu}$	$\Xi_c^{\prime +} D^{*+}$	$\frac{3}{2}^-$	3.30–4.25	24.3	$4.55^{+0.05}_{-0.04}$	4588
η_6	$\Xi_c^{*+} D^{*+}$	$\frac{1}{2}^-$	3.00–4.75	24.3	$4.63^{+0.06}_{-0.05}$	4655
ξ_1	$\Xi_{cc}^{++} \bar{K}^0$	$\frac{1}{2}^-$	3.16–4.05	20.3	$4.20^{+0.05}_{-0.05}$	4120
$\xi_{2\mu}$	$\Xi_{cc}^{++} \bar{K}^{*0}$	$\frac{3}{2}^-$	3.12–4.23	24.3	$4.52^{+0.06}_{-0.05}$	4512
$\xi_{3\mu}$	$\Xi_{cc}^{*++} \bar{K}^0$	$\frac{3}{2}^-$	3.38–4.23	21.3	$4.28^{+0.05}_{-0.05}$	4192
ξ_4	$\Xi_{cc}^{*++} \bar{K}^{*0}$	$\frac{1}{2}^-$	3.00–3.76	22.3	$4.37^{+0.05}_{-0.05}$	4584
$\xi_{5\mu\nu}$	$\Xi_{cc}^{*++} \bar{K}^{*0}$	$\frac{5}{2}^-$	3.00–4.45	24.3	$4.47^{+0.05}_{-0.04}$	4584
ψ_1	$\Omega_{cc}^+ \pi^+$	$\frac{1}{2}^-$	3.18–4.25	21.3	$4.27^{+0.05}_{-0.05}$	3853
$\psi_{2\mu}$	$\Omega_{cc}^+ \rho^+$	$\frac{3}{2}^-$	3.16–3.63	24.3	$4.50^{+0.06}_{-0.06}$	4488
$\psi_{3\mu}$	$\Omega_{cc}^{*+} \pi^+$	$\frac{3}{2}^-$	3.04–4.41	22.3	$4.36^{+0.05}_{-0.05}$	3925
ψ_4	$\Omega_{cc}^{*+} \rho^+$	$\frac{1}{2}^-$	3.00–3.77	22.3	$4.37^{+0.05}_{-0.05}$	4560
$\psi_{5\mu\nu}$	$\Omega_{cc}^{*+} \rho^+$	$\frac{5}{2}^-$	3.20–4.63	25.3	$4.55^{+0.05}_{-0.05}$	4560

of s_0 and M_B^2. The hadron mass for this molecular pentaquark with $J^P = 1/2^-$ can be obtained as

$$m_{J_{1,2}} = 4.38^{+0.04}_{-0.05}\ \mathrm{GeV}, \tag{30}$$

where the errors come from the uncertainties of the threshold s_0, Borel mass M_B^2, quark masses and various QCD condensates. Performing the same numerical analysis to all interpolating currents in Eqs. (6) and (9)–(12), we collect their numerical results with stable sum rule analysis in Tables I and II.

TABLE II. The masses (the last column) of various compact diquark-diquark-antiquark structures extracted from the two-point correlation function based on the QCD sum rule approach. The errors are from the Borel windows, the threshold of higher states, the various condensates and the quark masses. The first five columns present the names of the currents, the corresponding diquark-diquark-antidiquark structure, the quantum number J^P, the Borel mass square, and the threshold of the continuum s_0. The notation $[cc]_{s_1}[us]_{s_2}\bar{d}$ or $[cu]_{s_1}[cs]_{s_2}\bar{d}$ denotes the diquark components with spin s_1 and s_2, respectively.

Current	Structure	J^P	$M_B^2(\mathrm{GeV}^2)$	$s_0(\mathrm{GeV}^2)$	Mass (GeV)
$J_{1,2}$	$[cc]_1[us]_1\bar{d}$	$\frac{1}{2}^-$	3.50–3.86	22.3	$4.38^{+0.04}_{-0.05}$
$J_{1,3}$	$[cc]_1[us]_0\bar{d}$	$\frac{1}{2}^-$	3.14–4.22	21.3	$4.29^{+0.05}_{-0.05}$
$J_{1,5\mu}$	$[cc]_1[us]_0\bar{d}$	$\frac{3}{2}^-$	3.50–3.82	21.3	$4.27^{+0.05}_{-0.05}$
$J_{1,8\mu}$	$[cc]_1[us]_1\bar{d}$	$\frac{3}{2}^-$	3.50–4.10	22.3	$4.38^{+0.05}_{-0.05}$
$J_{1,9\mu\nu}$	$[cc]_1[us]_1\bar{d}$	$\frac{5}{2}^-$	3.46–4.25	23.3	$4.43^{+0.05}_{-0.05}$
$J_{2,1}$	$[cu]_0[cs]_0\bar{d}$	$\frac{1}{2}^-$	3.00–4.05	21.3	$4.25^{+0.04}_{-0.04}$
$J_{2,2}$	$[cu]_1[cs]_1\bar{d}$	$\frac{1}{2}^-$	3.00–5.08	24.3	$4.57^{+0.05}_{-0.04}$
$J_{2,3}$	$[cu]_1[cs]_0\bar{d}$	$\frac{1}{2}^-$	3.78–4.28	23.3	$4.44^{+0.05}_{-0.04}$
$J_{2,4}$	$[cu]_0[cs]_1\bar{d}$	$\frac{1}{2}^-$	4.00–4.11	22.3	$4.35^{+0.05}_{-0.04}$
$J_{2,5\mu}$	$[cu]_1[cs]_0\bar{d}$	$\frac{3}{2}^-$	3.00–4.03	21.3	$4.27^{+0.05}_{-0.04}$
$J_{2,6\mu}$	$[cu]_0[cs]_1\bar{d}$	$\frac{3}{2}^-$	3.00–4.31	22.3	$4.31^{+0.04}_{-0.04}$
$J_{2,7\mu}$	$[cu]_0[cs]_0\bar{d}$	$\frac{3}{2}^-$	3.00–4.05	21.3	$4.25^{+0.04}_{-0.04}$

Furthermore, we consider the dependence of the mass of the double heavy pentaquark on the mass of the heavy quark by varying the heavy quark mass to perform the sum rules analysis. In heavy quark spin symmetry, the mass of heavy hadron can be written as follows [58–60]:

$$m_{P_{QQ}} = 2m_Q + \bar{\Lambda} + \frac{\Delta m^2}{4m_Q} + O(1/m_Q^2), \tag{31}$$

where $\bar{\Lambda}$ denotes the contribution independent with heavy quark mass and spin. Δm^2 denotes the contribution from

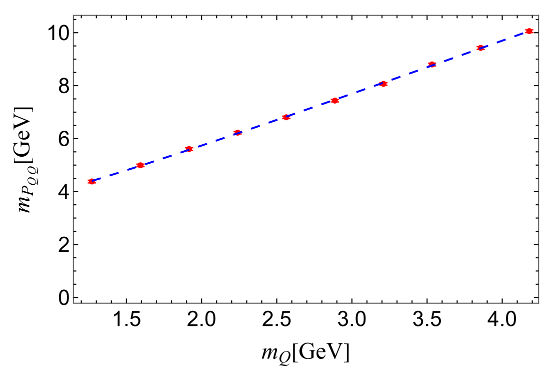

FIG. 4. The linear dependence of double heavy pentaquarks on the heavy quark mass m_Q. The red dots are the QCD sum rule results with errors from the Borel windows, the threshold of higher states, the various condensates and the quark masses. The blue dashed curve is the fitted result with the formula in Eq. (32). Here the current $J_{1,2}$ is presented as an illustration. The cases for other currents are also linear.

TABLE III. Fitting coefficients of Eq. (32) for molecular pentaquarks. The first, second, and third column show the currents, J^P and molecular structure for each pentaquark, respectively. The fourth and the fifth columns show present the fitted coefficients b and c. The sixth column shows the mass spectra for double bottom pentaquarks by replacing the charm quark mass by the bottom quark mass. The errors are from the Borel windows, the threshold of higher states, the various condensates and the quark masses. The last two columns list the mass and spin-parity of the HADS partners for the corresponding pentaquarks. While "–" denotes that there are not the HADS partner, as the two heavy quarks are not formed as a $\bar{3}$ diquark in color space.

Current	J^P	Structure	b(GeV)	c (GeV^2)	$m_{P_{bbs}}$(GeV)	$m_{T_{c\bar{s}}}$(GeV)	$J^P(T_{c\bar{s}})$
η_3	$\frac{1}{2}^-$	$\Xi_c^+ D^{*+}$	1.65	0.40	$10.16^{+0.06}_{-0.05}$	–	–
$\eta_{4\mu}$	$\frac{3}{2}^-$	$\Xi_c^{\prime +} D^{*+}$	1.79	0.25	$10.31^{+0.06}_{-0.05}$	–	–
η_6	$\frac{1}{2}^-$	$\Xi_c^{*+} D^{*+}$	2.33	−0.35	$10.68^{+0.06}_{-0.05}$	–	–
ξ_1	$\frac{1}{2}^-$	$\Xi_{cc}^{++} \bar{K}^0$	1.48	0.24	$9.93^{0.06}_{0.05}$	2.94	0^+
$\xi_{2\mu}$	$\frac{3}{2}^-$	$\Xi_{cc}^{++} \bar{K}^{*0}$	1.64	0.44	$10.17^{0.06}_{0.06}$	3.26	1^+
$\xi_{3\mu}$	$\frac{3}{2}^-$	$\Xi_{cc}^{*++} \bar{K}^0$	1.46	0.38	$9.96^{+0.06}_{-0.05}$	3.03	1^+
ξ_4	$\frac{1}{2}^-$	$\Xi_{cc}^{*++} \bar{K}^{*0}$	1.04	1.07	$9.59^{+0.07}_{-0.07}$	3.16	0^+
$\xi_{5\mu\nu}$	$\frac{5}{2}^-$	$\Xi_{cc}^{*++} \bar{K}^{*0}$	1.63	0.38	$9.99^{+0.06}_{-0.05}$	3.20	$0^+, 1^+, 2^+$
ψ_1	$\frac{1}{2}^-$	$\Omega_{cc}^{*+} \pi^+$	1.48	0.30	$9.96^{+0.06}_{-0.05}$	2.99	0^+
$\psi_{2\mu}$	$\frac{3}{2}^-$	$\Omega_{cc}^{+} \rho^+$	1.57	0.51	$10.04^{+0.06}_{-0.06}$	3.25	1^+
$\psi_{3\mu}$	$\frac{3}{2}^-$	$\Omega_{cc}^{*+} \pi^+$	1.45	0.47	$9.96^{+0.06}_{-0.05}$	3.09	1^+
ψ_4	$\frac{1}{2}^-$	$\Omega_{cc}^{*+} \rho^+$	1.16	0.88	$9.71^{+0.06}_{-0.06}$	3.12	0^+
$\psi_{5\mu\nu}$	$\frac{5}{2}^-$	$\Omega_{cc}^{*+} \rho^+$	1.80	0.25	$10.14^{+0.07}_{-0.06}$	3.27	$0^+, 1^+, 2^+$

heavy quark spin symmetry breaking of the order $1/m_Q$. We choose ten testing points with masses equidistant from m_c to m_b, and fit our results using

$$m_{P_{QQ}} = 2m_Q + b + \frac{c}{m_Q}. \tag{32}$$

For example, we show the dependence of the pentaquark mass on the heavy quark mass from current $J_{1,2}$ in Fig. 4. We collect all the fitting parameters in Eq. (32) and the mass of the bottom partner for currents we dealt with in Tables III and IV.

V. DISCUSSION AND CONCLUSION

We investigated the mass spectra for $[\Xi_c^{(*)+} D^{(*)+}]$, $[\Xi_{cc}^{(*)+} \bar{K}^{(*)+}]$, $[\Omega_{cc}^{(*)+} \pi^+(\rho^+)]$ molecular pentaquark states and $[cu][cs]\bar{d}$, $[cc][us]\bar{d}$ compact pentaquark states in the framework of the QCD sum rule. We construct the interpolating pentaquark currents and calculate their

TABLE IV. The same as that of Table IV but for a compact pentaquark. The notation $[cc]_{s_1}[us]_{s_2}\bar{d}$, or $[cu]_{s_1}[cs]_{s_2}\bar{d}$ denotes the diquark components with spin s_1 and s_2, respectively.

Current	J^P	Structure	b (GeV)	c (GeV^2)	$m_{P_{bbs}}$ (GeV)	$m_{T_{c\bar{s}}}$ (GeV)	$J^P(T_{c\bar{s}})$
$J_{1,2}$	$\frac{1}{2}^-$	$[cc]_1[us]_1\bar{d}$	1.61	0.30	$10.06^{+0.06}_{-0.05}$	3.11	$0^+, 1^+$
$J_{1,3}$	$\frac{1}{2}^-$	$[cc]_1[us]_0\bar{d}$	1.52	0.29	$10.00^{+0.06}_{-0.05}$	3.02	0^+
$J_{1,5\mu}$	$\frac{3}{2}^-$	$[cc]_1[us]_0\bar{d}$	1.53	0.17	$9.97^{+0.06}_{-0.05}$	2.94	1^+
$J_{1,8\mu}$	$\frac{3}{2}^-$	$[cc]_1[us]_1\bar{d}$	1.61	0.30	$10.06^{+0.06}_{-0.05}$	3.11	1^+
$J_{1,9\mu\nu}$	$\frac{5}{2}^-$	$[cc]_1[us]_1\bar{d}$	1.66	0.29	$10.10^{+0.06}_{-0.06}$	3.15	2^+
$J_{2,1}$	$\frac{1}{2}^-$	$[cu]_0[cs]_0\bar{d}$	1.79	−0.16	$10.21^{+0.07}_{-0.06}$	…	…
$J_{2,2}$	$\frac{1}{2}^-$	$[cu]_1[cs]_1\bar{d}$	1.87	0.14	$10.33^{+0.07}_{-0.06}$	…	…
$J_{2,3}$	$\frac{1}{2}^-$	$[cu]_1[cs]_0\bar{d}$	1.74	0.07	$10.18^{+0.07}_{-0.06}$	…	…
$J_{2,4}$	$\frac{1}{2}^-$	$[cu]_0[cs]_1\bar{d}$	1.72	0.08	$10.17^{+0.06}_{-0.06}$	…	…
$J_{2,5\mu}$	$\frac{3}{2}^-$	$[cu]_1[cs]_0\bar{d}$	1.65	0.07	$10.08^{+0.06}_{-0.06}$	…	…
$J_{2,6\mu}$	$\frac{3}{2}^-$	$[cu]_0[cs]_1\bar{d}$	1.61	0.08	$10.08^{+0.06}_{-0.06}$	…	…
$J_{2,7\mu}$	$\frac{3}{2}^-$	$[cu]_0[cs]_0\bar{d}$	1.79	−0.16	$10.21^{+0.07}_{-0.06}$	…	…

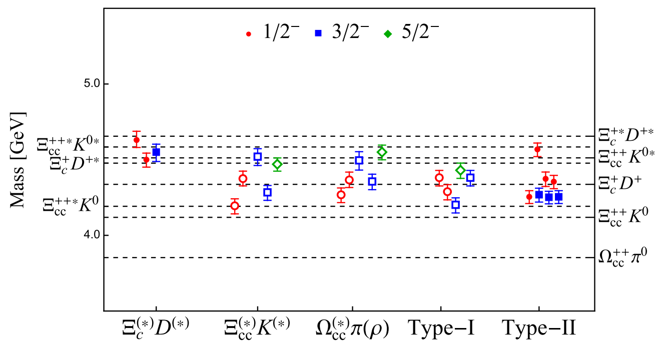

FIG. 5. The mass spectrum of strange double charm pentaquark for the interpolating currents, $\Xi_c D$, $\Xi_{cc} K$, $\Omega_{cc}\pi$, Type-I and Type-II currents in order. The red circles, blue boxes, red rhomboids are $J^P = \frac{1}{2}^-$, $\frac{3}{2}^-$ and $\frac{5}{2}^-$, respectively. The errors are from the Borel windows, the threshold of higher states, the various condensates and the quark masses. The hollow points represent the HADS partner of single charm tetraquark. The eight horizontal dashed lines are the double charm two-hadron thresholds.

two-point correlation functions including perturbative term and various condensate terms. With appropriate Borel mass and threshold, we obtain stable sum rule for some currents and extract the corresponding mass spectra listed in Tables I and II, as well as those in Fig. 5. In Fig. 5, the two-hadron thresholds are also plotted to illustrate whether the pentaquarks are stable or not. Here the Ξ_{cc} and Ω_{cc} masses are taken from the lattice calculation in Refs. [61]. We also list their possible strong decay mode in Table V, where we denote the negative parity baryon state H with spin-J as $H(J^-)$. The production for double charm pentaquark P_{cc} has already been discussed in Ref. [29,31]. The production mechanism of strange double charm pentaquark P_{ccs} is similar, i.e., via the weak decay of double heavy baryon Ξ_{bc} or triply charmed baryon Ω_{ccc}.

The masses of strange double charm pentaquarks for the molecular and compact pentaquark currents are listed in Tables I and II, respectively. In the two tables, only the quantum numbers $J^P = 1/2^-, 3/2^-, 5/2^-$ are considered, as these quantum numbers can be achieved by the considered two-hadron channel in S-waves. The mass region for these three quantum numbers are 4.2–4.6 GeV, 4.2–4.5 GeV, and 4.4–4.5 GeV, respectively. One should notice that the masses of the currents $\Xi_c^+ D^{*+}$, $\Xi_c^{\prime +} D^{*+}$, $\Xi_c^{*+} D^{*+}$, $\Xi_{cc}^{*++}\bar{K}^{*0}$, and $\Omega_{cc}^{*+}\rho^+$ are below their corresponding threshold, indicating that these strange double charm pentaquarks could be stable against their strong decay channels. On the contrary, those for the $\Xi_{cc}^{++}\bar{K}^0$, $\Xi_{cc}^{++}\bar{K}^{*0}$, $\Xi_{cc}^{*++}\bar{K}^0$, $\Omega_{cc}^+\rho^+$, and $\Omega_{cc}^{*+}\pi^+$ channels are around or above their corresponding thresholds, indicating that they could be broader states and not easy to be detected experimentally. As we acknowledge the fact that the interpolating currents are not independent and do not necessarily directly correspond to the physical structure, the above statements about the internal structure of the pentaquark need further theoretical investigations. In comparison with other works in the market, we plot Fig. 6. In the figure, we also plot the thresholds of the potential strong decay channels for each quantum number. We mainly compare our results with those of Refs. [24,27,28,31,32]. In Ref. [32], the authors employ a color-magnetic interaction in Schrödinger equation and obtain the mass spectra by variational method. Their masses, namely the mass for $J^P = 1/2^-, 3/2^-, 5/2^-$ are around 4.0–4.8 GeV, 4.1–4.8 GeV, and 4.7 GeV respectively, are higher than ours. There are two reasons. One is that they mainly focus on the mass splitting of the pentaquark states and the accurate values need further dynamical calculations, as claimed by the authors in their work. Thus, the wider range of mass in Ref. [32] is probably due to the overlooked dynamical calculation. Another reason is that they use the variational method, which is well-known to only give the upper limit of a given state. Similarly, in Ref. [34], the authors analyze the pentaquark $q^4\bar{Q}$ system in a constituent quark model based on the chromomagnetic interaction in both the SU(3) flavor symmetric and SU(3) flavor broken case. They find that the $\Xi_{cc}\bar{K}$ could be stable pentaquark state. In Ref. [27], the authors study the double-heavy pentaquarks in nonrelativistic constituent quark model by solving the multibody Schrödinger equation including the color Coulomb

TABLE V. Possible decay mode for double charm pentaquark.

J^P	S-wave	P-wave
$1/2^-$	$\Xi_c^{(\prime)} D^{(*)}$, $\Xi_{cc}\bar{K}^{(*)}$, $\Omega_{cc}\pi/\rho$	$\Xi_{cc}\bar{K}_0^*$, $\Xi_{cc}(1/2^-)\bar{K}$, $\Xi_{cc}^*\bar{K}_0^*$, $\Xi_{cc}^*(3/2^-)\bar{K}$, $\Omega_{cc}(1/2^-, 3/2^-)\pi$
$3/2^-$	$\Xi_c D^*$, $\Xi_c^* D$, $\Xi_{cc}\bar{K}^*$, $\Xi_{cc}^*\bar{K}$, $\Omega_{cc}\rho$, $\Omega_{cc}^*\pi$	$\Xi_{cc}\bar{K}_0^*$, $\Xi_{cc}(1/2^-, 3/2)\bar{K}$, $\Omega_{cc}(1/2^-, 3/2^-)\pi$
$5/2^-$	$\cdots$	$\cdots$

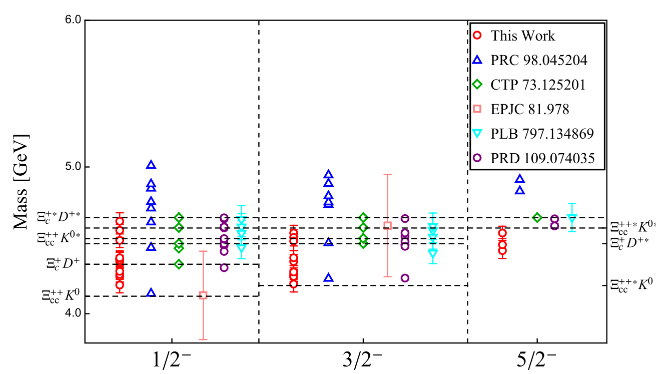

FIG. 6. Mass spectra of double charm pentaquark with $J^P = 1/2^-, 3/2^-, 5/2^-$, in comparison with other works. The blue upside triangles, green diamonds, pink squares, cyan downside triangles, and the purple circles are the results from Ref. [32], Ref. [24] (cutoff $\Lambda = 0.5$ GeV), Ref. [31], Ref. [27], and Ref. [28], respectively. The red circles are our results. The horizontal dashed lines represents the corresponding two-hadrons thresholds for given quantum numbers.

interaction and spin-dependent interaction. They conclude that the mass is about 4.4–4.5 GeV for compact pentaquark. Such results are slightly higher than our results, which may possibly due to the overlooked linear confinement potential and induce two free parameters β and the distance between the two heavy quarks R. The authors suggest that the optimal value of $R = 1$ fm for double heavy pentaquark, which may cause a higher mass spectra. Another potential reason is that they use variational method as discussed above. In Ref. [31], the authors employ the double heavy diquark-triquark model including the interaction between triquark and heavy diquark $[cc]_{\bar{3}}$ and the interaction in the light diquark $[qq']_{\bar{3}}$. They obtain the mass for $cc\bar{n}sn$ states with $J^P = 1/2^-$ and $3/2^-$ as 4.1 ± 0.3 GeV and 4.6 ± 0.3 GeV, which is consistent with ours. In Ref. [24], the authors study the heavy-heavy hadronic molecules by solving the single channel Bethe-Salpeter equation with the interactions following the heavy quark spin symmetry, including the interactions from light vector meson exchange. Their results illustrate that $D^{(*)}\Xi_c^{('*)}$ system can be bound easily, which is consistent with our results. In Ref. [28], the authors study the molecular pentaquarks by solving the Lippmann-Schwinger equations with near-threshold effective potentials. Their results suggest that $\Xi_c D^{(')*}$, $\Xi_c^* D^*$, and $\Xi_{cc}^* \bar{K}^*$ systems could possibly bound as molecular states, which is consistent with our results.

It is interesting to find that we obtain almost degenerate mass for all the currents with $J^P = 5/2^-$, as shown in Fig. 6. This is because that all these currents contain spin-1 cc diquark, leaving potential small splitting from light quarks. In addition, one can also see that the mass is far below the thresholds of corresponding strong decay channels and can be viewed as a stable and narrow state. As the result, we consider the existence of this state is a very solid conclusion of our work. The best observed channel is its semileptonic decay to double charm baryon, i.e., Ξ_{cc}^{*++} and Ω_{cc}^{*+}.

We consider the dependency of the pentaquark mass P_{QQ} on the heavy quark mass m_Q and use Eq. (32) to fit our results. The fitting parameters and the predicted double bottom pentaquarks masses are listed in Tables III and IV. The masses of double bottom pentaquarks are around 9.6–10.6 GeV, 10.0–10.2 GeV, and 10.0 GeV for $J^P = 1/2^-, 3/2^-, 5/2^-$, respectively. Except that the two states with $J^P = 1/2^-$ are higher than the lowest two-hadron threshold $\Omega_{bb}\pi$ (10.33 GeV), all the other double bottom pentaquark states are lower than their corresponding lowest two-hadron thresholds and can be viewed as narrow states. Here the masses of double bottom baryons are taken from the result of lattice QCD in Ref. [62]. From the two tables, one can also see the heavy quark spin symmetry emerging in the spectrum. As we know, spin interaction of heavy quarks does not occur at the leading order in the $\Lambda_{\rm QCD}/m_Q$ expansion, which makes the masses of pentaquarks with the same light quark spin are degenerate. Such symmetry can be seen in our results, namely $J_{1,2}$, $J_{1,8\mu}$, and $J_{1,9\mu\nu}$ with spin-1 $[us]$ diquark component give a degenerate mass at 4.38 GeV. Such behavior can also be seen in the molecular structure currents ξ_1, $\xi_{3\mu}$ in the $\Xi_{cc}^{++}\bar{K}^0$ structure and ψ_4, $\psi_{5\mu\nu}$ in the $\Omega_{cc}^{+}\rho^+$ structure. Comparing Eq. (32) with Eq. (31), the parameter b in Eq. (32) should be the parameter $\bar{\Lambda}$ in Eq. (31) and is independent of heavy quark mass and spin. This feature can be reflected by the currents $(\psi_1, \psi_{3\mu})$, $(J_{1,3}, J_{1,5\mu})$, and $(J_{1,2}, J_{1,8\mu}, J_{1,9\mu\nu})$. They have the same light quark spin structure, leaving almost the same parameter b.

As we discussed in Sec. I, the double charm pentaquark $ccus\bar{d}$ should be the HADS partner of the singly charm tetraquark $us\bar{d}\,\bar{c}$, we can derive the corresponding mass spectra of singly charm tetraquark $T_{c\bar{s}}$ through Eq. (31) by replacing $2m_Q$ to m_Q, and we list these corresponding spectra and their spin-parity in Tables III and IV. The mass and spin-parity of HADS partner for current ξ_1 is consistent with the recently discovered $T_{c\bar{s}}(2900)$, which indicates that $T_{c\bar{s}}(2900)$ could be a molecular tetraquark with spin-0 $[s\bar{d}]$ meson component. Furthermore, current ($J_{1,2}$, $J_{1,8\mu}$, $J_{1,9\mu\nu}$) could be the HADS partner triplet for tetraquark $[us]_1[\bar{c}\,\bar{d}]_1$ with mass about 3.1 GeV, current ($J_{1,3}$, $J_{1,5\mu}$) or (ξ_1, $\xi_{3\mu}$) could be the HADS partner doublet for tetraquark $T_{c\bar{s}}(2900)$. With the spectra and decay modes in this work, we hope that these double charm and double bottom pentaquarks could be discovered by the LHCb, BelleII, CMS, and RHIC Collaborations and so on in the near future.

VI. SUMMARY

Motivated by the observation of the $T_{c\bar{s}}(2900)$, we study the mass spectrum of its HADS counter parts, i.e., strange double charm pentaquarks. By constructing currents in the molecular and compact pentaquark pictures, we extract the corresponding mass spectra for quantum numbers $J^P = 1/2^-, 3/2^-, 5/2^-$. The masses for the former two quantum numbers are within the energy region 4.2–4.6 GeV and 4.2–4.5 GeV, respectively. The masses of the three currents (two molecular currents and one compact pentquark current) of the quantum number $J^P = 5/2^-$ are almost degenerate and locate around 4.5 GeV. It is below the threshold of its two-hadron strong decay channel and can be viewed as a narrow state, making it easily to be measured in experiment. The best observed channel is the semileptonic decay to double charm baryon. The corresponding strange double bottom pentaquarks are locate at 9.6–10.6 GeV, 10.0–10.2 GeV, and 10.0–10.1 GeV for the above mentioned three quantum numbers. This kind of study is useful for the further measurements of strange double charm and double bottom pentaquarks.

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