Production of a $B\bar{B}$ bound state via $\Upsilon(4S)$ radiative decays

André L. M. Britto*

Centro de Ciências Exatas e Tecnológicas, Universidade Federal do Recôncavo da Bahia, R. Rui Barbosa, Cruz das Almas, 44380-000, Bahia, Brazil

Luciano M. Abreu†

Instituto de Física, Universidade Federal da Bahia, Campus Universitário de Ondina, 40170-115, Bahia, Brazil

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Motivated by recent theoretical predictions about the existence of a $B\bar{B}$ bound state [also denoted as $X(10550)$], in this work we estimate the production of the S-wave B^+B^- molecule via $\Upsilon(4S)$ radiative decays. In particular, we make use of effective Lagrangian approach and the compositeness condition to calculate the $X(10550)$ production rate via $\Upsilon(4S) \to \gamma X(10550)$ decays employing triangle diagrams. Our results show that the partial decay width of this reaction is of the order of 0.5–192 keV for a respective binding energy of 1–100 MeV, corresponding to a branching fraction of 10^{-5}–10^{-3}. These findings suggest that the existence of the $X(10550)$ might be checked via the analysis of the mentioned decay in present and future experiments.

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I. INTRODUCTION

In the last decades, several new hadrons have been observed [1], and many of them present unconventional properties that are incompatible with the quark-model predictions [2–5]. Concerning their underlying structure, they have been interpreted as different configurations, e.g., weakly bound hadron molecules, compact multiquark states, excited conventional hadrons, cusps engendered by kinematical singularities, glueballs, hybrids, etc., or even a superposition of some of them. The fact is that there is no universal, consensual, and compelling understanding on this point, and it remains a topic of intense discussion. In order to establish criteria of distinction among these interpretations, observables like the masses, decay widths, and production rates of these states have been studied both theoretically and experimentally. The most emblematic and famous example of the landscape described above is the $X(3872)$, the first observed exotic state in 2003 [1,6], with quantum numbers $I^G(J^{PC}) = 0^+(1^{++})$. Its intrinsic nature continues to be a matter of dispute, and the most explored configurations are the weakly bound state of open charm mesons $(D\bar{D}^* + \text{c.c.})$ and the $c\bar{c}q\bar{q}$ compact tetraquark [2–5].

With the observation of the $X(3872)$, a natural consequence in the scenario of meson molecule configuration was the investigation of the existence of its lightest partner, i.e., the $D\bar{D}$ state [also usually denoted as $X(3700)$ or $X(3720)$]. It was predicted in the context of the coupled channel unitary approach [7]. Afterward this $0^+(0^{++})$ state was studied from distinct perspectives, namely, the peak in the $D\bar{D}$ mass distribution of $e^+e^- \to J/\psi D\bar{D}$ reactions [8,9], via the pole structure of meson-meson interactions within the heavy meson effective theory [10–12], the peak in the $\eta\eta'$ mass distribution of the radiative decays of $\psi(3770), \psi(4040)$ and the process $e^+e^- \to J/\psi\eta\eta'$ [13], the peak in the $D^0\bar{D}^0$ mass distribution of the $\psi(3770) \to \gamma D^0\bar{D}^0$ decay [14], as a pole in the coupled $D\bar{D}, D_s\bar{D}_s$ scattering on the lattice [15], its production in $\gamma\gamma \to D\bar{D}$ reactions [16], its production in B decays [17], the peak in the $\eta\eta$ mass distribution of the $B^+ \to K^+\eta\eta$ decay [18], its production in $\gamma\gamma \to D^+D^-$ reactions seen in ultraperipheral heavy ion collisions [19], and so on. On experimental grounds, there are some searches reported in the literature. For example, the Belle and *BABAR* collaborations analyzed, respectively, the $e^+e^- \to J/\psi D\bar{D}$ and $e^+e^- \to D\bar{D}$ reactions [20–22], and although theoretical works claim that these data might be explained by the existence of the hidden charm scalar resonance [8,13,16,23], there is no consensus in favor of its unequivocal observation.

*Contact author: andrebritto@ufrb.edu.br
†Contact author: luciano.abreu@ufba.br

Thus, by invoking heavy-quark flavor symmetry, one can ask about the existence of molecular partners in the bottom sector. This natural hypothesis, combined with the observation of the so-called $Z_b(10610)$ and $Z_b(10650)$ states with quantum numbers $1^+(1^+)$ [24,25], has yielded a lot of attention to the hidden bottom meson molecules; see, for instance, the works [12,26–46]. In this context, an obvious case that has been explored was the heavy-quark flavor symmetry partner of the $X(3872)$, the $0^+(1^{++})$ state denoted as X_b with a possible molecular configuration $(B\bar{B}^* + \mathrm{c.c.})$. We refer the reader to the theoretical and experimental analyses in Refs. [12,38,45–57] that have investigated the similarities between these partner states. However, on the experimental side, no significant X_b signals have been observed yet [48–51]. As noticed in Ref. [54], at the current electron-positron colliders the direct observation of X_b in hadronic decays is not likely because of its quantum numbers and large mass. Indeed, the Belle (Belle-II) Collaboration has found no X_b evidence in the search for $X_b \to \omega\Upsilon(1S)$ [48,49]. In addition, analyses of the CMS and ATLAS experiments at the LHC based on samples of pp collisions at $\sqrt{s} = 8$ TeV have searched for the X_b decaying into $\Upsilon(1S)\pi^+\pi^-$, and no significant excess above the background was observed [50,51], which might indicate that the isospin is conserved in this bottomonium system. Therefore, other possible channels have been proposed, and due to its high mass, a logical expectation is the X_b production by means of the radiative decays of higher bottomonia. For example, in Refs. [53,54] the X_b production as a $(B\bar{B}^* + \mathrm{c.c.})$ molecule was estimated to be small in the processes $\Upsilon(5S, 6S) \to \gamma X_b$, with a branching fraction of about 10^{-7}. On the other hand, in Ref. [57] the X_b production via the radiative transition of $\Upsilon(10753)$ was estimated to have a branching fraction a factor of about 10^{-3}–10^{-2} higher than the former case, making it testable by future Belle-II experiments.

Thus, benefiting from the discussion above, one can also focus on the bottomonium counterpart of the $X(3700)$ state, i.e., the $0^+(0^{++})$ state, also denoted as $X(10550)$, with a possible molecular configuration $B\bar{B}$. We mention that Ref. [58] made use of an effective Lagrangian consistent with the heavy-quark and chiral symmetries and argued that the existence of a bound state in the $(D\bar{D}^* + \mathrm{c.c.})$ channel does not necessarily imply the existence of a bound state in the $D\bar{D}$ or $B\bar{B}$ channels (see also the analysis of the $DD, \bar{B}\bar{B}$ cases in Ref. [59]). In contrast, the meson-meson interaction was analyzed in Ref. [38] via a coupled channel unitary approach, combining the heavy-quark spin symmetry and the dynamics of the local hidden gauge, and a weakly $0^+(0^{++})$ $B\bar{B}$ bound state was found. In the sequence, other works have studied the possible existence and properties of the $X(10550)$; the reader can consult, for example, Refs. [12,45,55,60]. Interestingly, we remark that, differently from the $X(3700)$, analyses exploring the potential observation of the $X(10550)$ via decays are very scarce. To the best of our knowledge, the available studies, like [55,61,62], investigated hadronic transitions with final states carrying $B^{(*)}\bar{B}^{(*)}$ or conventional bottomonia but did not explore the existence of the $X(10550)$.

Hence, considering the increasing interest in exotic hadron spectroscopy and the search for more possible exotic states via the estimation of relevant observables, as well as taking advantage of the similarities between the $0^+(0^{++})$ partner states in charmonium and bottomonium sectors discussed previously, in the present work we investigate the possible existence of the S-wave B^+B^- bound state [here we continue denoting this charged component as $X(10550)$] and propose a method to estimate its production via $\Upsilon(4S)$ radiative decays. In particular, we make use of the effective Lagrangian approach and the compositeness condition to calculate the $X(10550)$ production rate in $\Upsilon(4S) \to \gamma X(10550)$ decays employing triangle diagrams.

This work is organized as follows. We introduce the formalism to calculate the amplitude associated with the triangle mechanism for the $\Upsilon(4S) \to \gamma X(10550)$ decay in Sec. II. Results and discussions are given in Sec. III, followed by concluding remarks in Sec. IV.

II. FORMALISM

In what follows, we describe the effective formalism used to evaluate the production of the so-called exotic state $X(10550)$ via $\Upsilon(4S)$ radiative decays. Assuming that this bound state is a S-wave B^+B^- molecule (here denoted just as $B\bar{B}$) with quantum numbers $J^{PC} = 0^{++}$, its production at the hadron level via the mentioned reactions can be described using the triangle diagrams depicted in Fig. 1. To calculate the partial decay width of this reaction, we employ the effective Lagrangian approach.

We start by presenting the effective Lagrangian responsible for the interaction between the exotic state $X(10550)$, here associated with the field X, and the $B\bar{B}$ pair [63],

$$\mathcal{L}_{XB\bar{B}} = g_{XB\bar{B}} X(x) \int dy B(x + \omega_{\bar{B}B} y) \bar{B}(x - \omega_{B\bar{B}} y) \Phi(y), \tag{1}$$

where y is the relative Jacobi coordinate and $\omega_{ij} = \frac{m_i}{m_i + m_j}$; since $m_B = m_{\bar{B}}$, we employ $\omega_{\bar{B}B} = 1/2$ henceforth. $\Phi(y)$ is

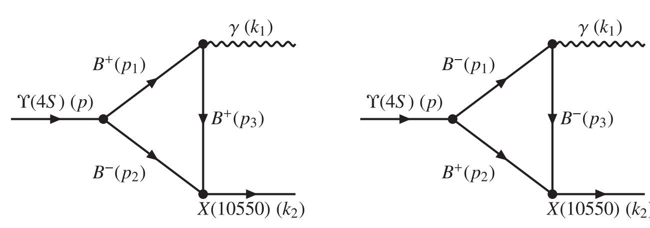

FIG. 1. Triangle Feynman diagrams for the radiative decays $\Upsilon(4S) \to \gamma X(10550)$ via the S-wave bottom meson loops. Particle labels and their momenta (in parentheses) are defined.

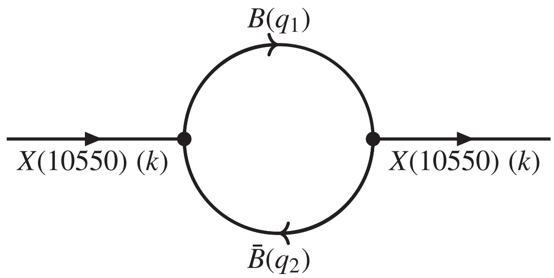

FIG. 2. Feynman diagram contributing to the self-energy $\Sigma(k^2)$ of the $X(10550)$ state.

the correlation function expressing the distribution of the two constituent hadrons in a molecule and also preventing the artificial growth of the amplitudes with energy. Its Fourier transform adopted here is the Gaussian function,

$$\tilde{\Phi}(p^2) = e^{-p_E^2/\Lambda^2}, \tag{2}$$

with p_E being the Euclidean Jacobi momentum and Λ a size parameter characterizing the distribution of the constituents inside the molecule.

The coupling constant $g_{XB\bar{B}}$ can be estimated through the compositeness condition [63–67]. Accordingly, $g_{XB\bar{B}}$ is determined from the fact that the renormalization constant of the wave function associated with the composite state $X(10550)$ should be set equal to zero, i.e.,

$$Z_X = 1 - \frac{d\Sigma(k^2)}{dk^2}\bigg|_{k^2=m_X^2} = 0, \tag{3}$$

where $\Sigma(k^2)$ is the $X(10550)$ self-energy, represented by the diagram in Fig. 2. Here we define $m_X = m_B + m_{\bar{B}} - E_B$, with m_X and E_B being, respectively, the mass and binding energy characterizing the state $X(10550)$. The condition shown in Eq. (3) means that the physical state is uniquely described by a bound state of its constituents, and as a consequence its mass and wave function must be renormalized due to the interaction of the X with its constituents (see Refs. [63–67] for a more detailed discussion).

After the use of some mathematical manipulations, the Schwinger parametrization technique, and Gaussian integration, the X self-energy $\Sigma(k^2)$ represented in Fig. 2 can be expressed in the form [68]

$$i\Sigma(k^2) = i\frac{g^2}{16\pi^2}\int d\alpha d\beta \frac{1}{z^2}\exp\left\{-\frac{1}{\Lambda^2}\left[\frac{-k^2}{2} + m_B^2\alpha + \left(-k^2 + m_{\bar{B}}^2\right)\beta + \frac{\Delta^2}{4z}\right]\right\}, \tag{4}$$

where $z = \alpha + \beta + 2$ and $\Delta = -2k(1+\beta)$, with α, β being the Schwinger parameters. Thus, according to Eq. (3), the coupling constant $g_{XB\bar{B}}$ can be given by

$$g_{XB\bar{B}} = \left\{\frac{1}{16\pi^2\Lambda^2}\int d\alpha d\beta \frac{1}{z^2}\left(\beta + \frac{1}{2} - \frac{(1+\beta)^2}{\alpha+\beta+2}\right)\exp\left[\frac{m_X^2}{\Lambda^2}\left(\beta + \frac{1}{2} - \frac{(1+\beta)^2}{\alpha+\beta+2}\right)\right]\exp\left[-\frac{m_B^2}{\Lambda^2}(\alpha+\beta)\right]\right\}^{-\frac{1}{2}}. \tag{5}$$

Beyond the effective Lagrangian given in Eq. (1), to calculate the two-body decay via the triangle diagram shown in Fig. 1 one needs the effective Lagrangians responsible for the other vertices. The interactions between the $\Upsilon(4S)$ and bottomed mesons are described by the following effective Lagrangian:

$$\mathcal{L}_{\Upsilon B\bar{B}} = -g_{\Upsilon B\bar{B}}\Upsilon_\mu(B\partial^\mu\bar{B} - \bar{B}\partial^\mu B), \tag{6}$$

where Υ_μ is the vector field associated with the $\Upsilon(4S)$. The coupling constant $g_{\Upsilon B\bar{B}}$ can be determined from the central values of the $\Upsilon(4S)$ total decay width, $\Gamma_{\Upsilon(4S)} = 20.5$ MeV, and the branching ratio of the decay mode $\Upsilon(4S) \to B^+B^-$, $\mathcal{B}_{\Upsilon(4S)\to B^+B^-} = \Gamma_{\Upsilon(4S)\to B^+B^-}/\Gamma_{\Upsilon(4S)} = 51.4\%$ [1]. Therefore, we can relate the partial decay width $\Gamma_{\Upsilon(4S)\to B^+B^-}$ to $g_{\Upsilon B\bar{B}}$ through the expression

$$\Gamma_{\Upsilon(4S)\to B^+B^-} = \frac{1}{24\pi}\frac{|\vec{p}_B|^3}{m_\Upsilon^2}|g_{\Upsilon B\bar{B}}|^2, \tag{7}$$

where $|\vec{p}_B| = \lambda^{1/2}(m_\Upsilon^2, m_B^2, m_{\bar{B}}^2)/(2m_\Upsilon)$ is the magnitude of the three-momentum of the B meson in the rest frame of Υ, with $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$ being the Källen function. With the use of the experimental decay width and masses of the particles involved in this reaction reported in [1], we get $g_{\Upsilon B\bar{B}} \sim 24$.

The last vertex we need is associated with the interaction involving the pseudoscalar bottomed mesons and the photon, which is governed by the following Lagrangian coming from the usual scalar quantum electrodynamics:

$$\mathcal{L}_{\gamma\bar{B}B} = -ieA_\mu(\bar{B}\partial^\mu B - \partial^\mu\bar{B}B) + e^2A^\mu A_\mu\bar{B}B. \tag{8}$$

Thus, making use of the vertices discussed above, the amplitude of the radiative $\Upsilon(4S) \to X(10550)\gamma$ decay given in Fig. 1 can be written as

$$\mathcal{M}=-2ig_{XB\bar{B}}g_{\Upsilon B\bar{B}}e\epsilon_{\mu}^{(\gamma)}(k_1)\epsilon_{\nu}^{(\Upsilon)}(p)\int\frac{d^4q}{(2\pi)^4}\frac{(2q-p)^{\nu}(2q-k_1)^{\mu}}{[(p-q)^2-m_B^2+i\varepsilon][(q-k_1)^2-m_{\bar{B}}^2+i\varepsilon][q^2-m_{\bar{B}}^2+i\varepsilon]}\Phi\left(\frac{p+k_1}{2}-q\right), \quad (9)$$

where the factor 2 on the right-hand side of the first line comes from the fact that the two diagrams in Fig. 1 give equal contributions. Then, following [14,69], $\mathcal{M}$ in Eq. (9) can be expressed concisely as

$$\mathcal{M}=-i\,\epsilon_{\mu}^{(\gamma)}(k_1)\,\epsilon_{\nu}^{(\Upsilon)}(p)\big[a\,g^{\mu\nu}+b\,k_1^{\mu}k_1^{\nu}+c\,k_1^{\mu}p^{\nu} + d\,k_1^{\nu}p^{\mu}+e\,p^{\mu}p^{\nu}\big]. \quad (10)$$

By employing the transversality conditions $\epsilon_{\mu}^{(\gamma)}(k_1)k_1^{\mu}=0$ and $\epsilon_{\nu}^{(\Upsilon)}(p)p^{\nu}=0$, only the terms carrying the coefficients a and d need to be calculated. Besides, invoking the Ward identity, which is equivalent to replacing $\epsilon_{\mu}^{(\gamma)}(k_1)$ with $k_{1\mu}$ in Eq. (10) and requiring $k_{1\mu}[a\,g^{\mu\nu}+d\,k_1^{\nu}p^{\mu}]=0$ [14], we have the relationship $a=-d\,k_1\cdot p$. In addition, considering the Coulomb gauge, i.e., $\epsilon_0^{(\gamma)}=0$ and $\epsilon_i^{(\gamma)}(k_1)k_1^i=0$, in the $\Upsilon(4S)$ rest frame the term $\epsilon_i^{(\gamma)}(k_1)\epsilon_j^{(\Upsilon)}(p)\,d\,k_1^j\,p^i$ vanishes. As a consequence, the amplitude takes the simplified form

$$\mathcal{M}=i\,\epsilon_{\mu}^{(\gamma)}(k_1)\,\epsilon^{(\Upsilon)\mu}(p)\,d\,(k_1\cdot p). \quad (11)$$

The d coefficient is obtained from Eq. (9) and can be written using the Schwinger parametrization as [14]

$$d=-ig_{XB\bar{B}}g_{\Upsilon B\bar{B}}e\frac{1}{16\pi^2\Lambda^2}\times\int d\alpha d\beta d\gamma\frac{1}{\tilde{z}^2}\exp\left\{\frac{1}{\Lambda^2}\left[\left(-(\alpha+\beta+\gamma)\times\left(m_B^2-\frac{k_2^2}{4}\right)+k_1k_2\gamma\right)-\frac{\tilde{\Delta}^2}{4\tilde{z}}\right]\right\}, \quad (12)$$

with $\tilde{z}=(\alpha+\beta+\gamma+1)$ and $\tilde{\Delta}=k_2(-\alpha+\beta+\gamma)+2k_1\gamma$.

Finally, with all of the ingredients discussed above, the partial decay width for the $\Upsilon(4S)$ radiative decay producing the exotic state $X(10550)$ reads

$$\Gamma_{\Upsilon(4S)\to\gamma X(10550)}=\frac{1}{8\pi}\frac{|\vec{k}_1|}{m_{\Upsilon}^2}\sum\bar{\sum}|\mathcal{M}|^2, \quad (13)$$

where $|\vec{k}_1|$ is the magnitude of the three-momentum of the photon in the rest frame of $\Upsilon(4S)$ and $\sum\bar{\sum}$ represents the sum over the polarizations of the final state and average over the polarizations of the $\Upsilon(4S)$.

III. RESULTS

First, we present the estimation for the coupling constant $g_{XB\bar{B}}$ related to the vertex involving the $X(10550)$ state and its meson components $B,\bar{B}$. As in other works (e.g., Refs. [68,70]), we use $\Lambda=1$ GeV. However, to take into account the uncertainties inherent in the approach we show the results within the range 0.9–1.1Λ. From Eq. (5), one can see that $g_{XB\bar{B}}$ is dependent on the mass of the bound state and therefore on its binding energy. Noticing that in the literature (see, for example, Refs. [12,38]) this state is predicted with a distinct E_B in relation to the $B\bar{B}$ threshold, in Fig. 3 we plot $g_{XB\bar{B}}$ obtained from the solution of Eq. (5) as a function of the binding energy. We consider the range $E_B\sim 1$–100 MeV, corresponding to the bound state with mass $m_X\sim 10\,558$–$10\,458$ MeV. It can be seen that $g_{XB\bar{B}}$ acquires a bigger magnitude with increasing E_B; in other words, it grows as m_X decreases. In Table I we show explicitly the central values for the $g_{XB\bar{B}}$ for some specific binding energies. Interestingly, it should be mentioned that Ref. [38] found a $B\bar{B}^{(I=0)}$ bound state using a coupled channel unitary approach that combines the heavy-quark spin symmetry and the local hidden gauge formalism. This reference made use of cutoffs that yield masses of the bound state in the same range as those considered here, as well as the values of the coupling at the same order as those for $g_{XB\bar{B}}$ shown in Table I. But a direct comparison is not possible because [38] worked with potentials in the isospin basis, differently from our situation [71].

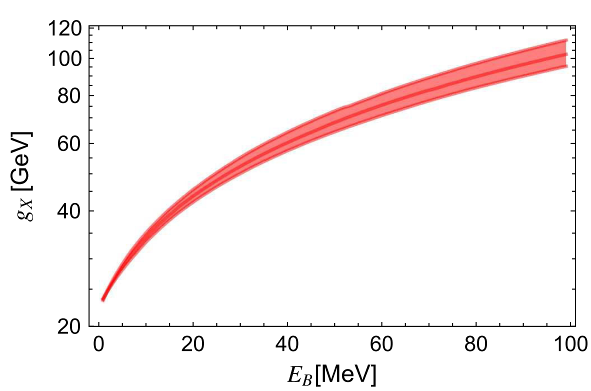

FIG. 3. Coupling constant $g_{XB\bar{B}}$ related to the molecular $X(10550)$ state and its meson components $B,\bar{B}$ as a function of the binding energy E_B. The band denotes the uncertainties coming from the values of the size parameter in the range 0.9–1.1Λ.

TABLE I. Central values of the coupling constant $g_{XB\bar{B}}$ related to the vertex involving the molecular $X(10550)$ state and its meson components B and $\bar{B}$ (considering $\Lambda = 1$ GeV), the decay width $\Gamma_{\Upsilon(4S)\to\gamma X(10550)}$, and the branching ratio $\mathcal{B}_{\Upsilon(4S)\to\gamma X(10550)}$ for some values of the binding energy E_B.

E_B [MeV]	$g_{XB\bar{B}}$ [GeV]	$\Gamma_{\Upsilon(4S)\to\gamma X(10550)}$ [keV]	$\mathcal{B}_{\Upsilon(4S)\to\gamma X(10550)}$
5	28.50	1.55	7.55×10^{-5}
10	33.96	2.59	1.25×10^{-4}
25	47.94	6.90	3.37×10^{-4}
50	67.87	19.87	9.70×10^{-4}
75	85.85	42.34	2.06×10^{-3}
100	102.73	76.79	3.75×10^{-3}

Now we present the results for the partial decay width of the radiative decay $\Upsilon(4S)\to\gamma X(10550)$, defined in Eq. (13), as a function of the binding energy. They are shown in Fig. 4. It can be seen that with increasing E_B, the radiative decay width increases. This behavior comes essentially from the dependence of $g_{XB\bar{B}}$ on the binding energy previously discussed. In Table I we show explicitly the central values for the decay width $\Gamma_{\Upsilon(4S)\to\gamma X(10550)}$ and the branching ratio for some specific binding energies. In particular, assuming the $X(10550)$ molecule with a binding energy of 1–100 MeV, in consonance with the range considered in Ref. [38], i.e., $m_X \sim 10\,558$–$10\,458$ MeV, the radiative decay width predicted is

$$\Gamma_{\Upsilon(4S)\to\gamma X(10550)} \sim 0.5\text{–}192 \text{ keV}, \tag{14}$$

which engenders a branching ratio of the order $\mathcal{B}_{\Upsilon(4S)\to\gamma X(10550)} \sim 10^{-5}$–$10^{-3}$. This result indicates a relatively large radiative width, suggesting a promising hunt for $X(10550)$ via the $\Upsilon(4S)\to\gamma X(10550)$ decay in updated Belle II experiments. This is the main finding of the present study.

It is also worth remarking that the prediction of the radiative decay width predicted in Eq. (14) is (taking into account the uncertainties) of the same order as that reported in Ref. [57] for the production of the X_b, the heavy-quark flavor symmetry counterpart of the $X(3782)$ in the bottomonium sector, via radiative transition of the $\Upsilon(10753)$, seen as an $S-D$ mixed state of the $\Upsilon(4S)$ and $\Upsilon_1(3^3D_1S)$, within a distinct framework based on a nonrelativistic effective field theory. Hence, these two findings corroborate the viewpoint that the radiative decays of Υ states might be an interesting ground for the study of new exotic states in the bottomonia sector.

Our final comment is devoted to the consideration of more decay channels. In principle, one might ask about calculations for other decay channels and therefore the ratios between them, which would be of greater utility to experimentalists. However, as pointed out in the Introduction, the $B^{(*)}\bar{B}^{(*)}$-molecule production from the decays of higher-mass states is not an easy task because of its large mass. Being more concrete, assuming $m_{X(10550)}\sim 10\,558$–$10\,458$ MeV, the mass of the state $\Upsilon(4S)$ is not large enough to provide enough phase space for a hadronic decay into the $X(10550)$ and a vector meson like the ρ, ω mesons. One can wonder how to circumvent this limitation, and a natural choice is to consider the decay of a higher excited bottomonium state. First, it should be noticed that the decay mode $\Upsilon(4S)\to B^+B^-$ provides a much more relevant contribution to the triangle mechanism than the other higher bottomonia, as can be seen from the branching ratio $\mathcal{B}_{\Upsilon(4S)\to B^+B^-}$ reported in [1] and in the previous section. Second, and more importantly, the highest bottomonium state according to [1] is the $\Upsilon(11020)$, which has a mass of 11 000 MeV; consequently, its decays like $\Upsilon(11020)\to \rho X(10550), \omega X(10550)$ are naturally suppressed due to the lack of available phase space. Thus, considering these arguments, we have restricted ourselves to the logical proposition of searching for the $B\bar{B}$ molecule via $\Upsilon(4S)$ radiative decays. In this regard, we believe that the results summarized in Eq. (14) and Table I give a valuable and useful prediction for the experimental collaborations.

Notwithstanding, one can also think about analogous discussions in other sectors of the spectrum to test the idea of comparison with other decay modes. For instance, some equivalent mechanisms in the light sector would be seen in terms of the ρ, ω decays into a $\gamma f_0(500)$ final state, as shown in the diagrams in Fig. 5. The couplings of the

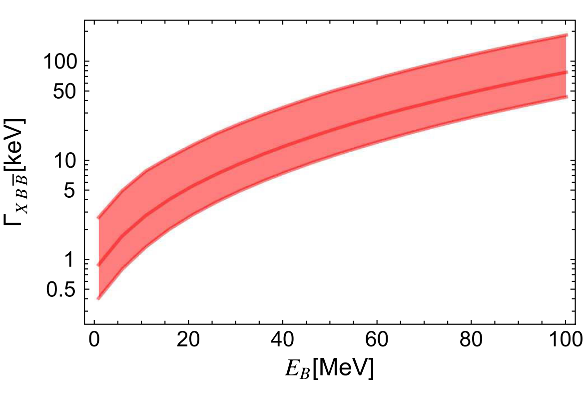

FIG. 4. Partial decay width $\Gamma_{\Upsilon(4S)\to\gamma X(10550)}$ as a function of the binding energy E_B. The band denotes the uncertainties coming from the values of the size parameter in the range 0.9–1.1Λ.

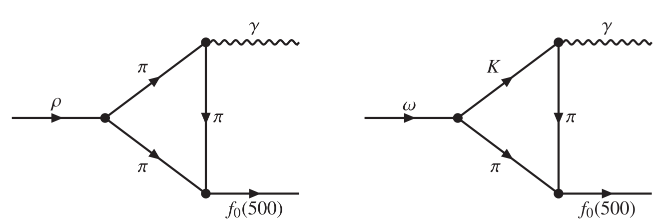

FIG. 5. Feynman diagrams for some equivalent mechanisms to those shown in Fig. 1 but in the light sector, like the ρ, ω decays into a $\gamma f_0(500)$ final state.

$f_0(500)/\sigma$ state to the channels $\pi\pi$ and $K\pi$ present in these diagrams have already been studied in the literature; see, for example, [72]. Unfortunately, there are no data for these decays. Should data be provided in the future, it would be worth studying these decays to substantiate the ideas exposed in this work.

IV. CONCLUDING REMARKS

In this work, we have proposed the search for the $X(10550)$ state, assumed as an S-wave $(0^{++})B^+B^-$ molecule, via $\Upsilon(4S)$ radiative decays. In this sense, the $X(10550)$ production rate for the $\Upsilon(4S) \to \gamma X(10550)$ process, described by triangle diagrams, has been evaluated by making use of an effective Lagrangian approach and the compositeness condition. The partial decay width of this reaction has been estimated to be of the order of 0.5–192 keV for a respective binding energy range of 1–100 MeV, corresponding to a branching fraction of 10^{-5}–10^{-3}. It is sufficiently large to check the existence of the $X(10550)$ via the mentioned channel in present and future experiments. We hope that this finding may stimulate experimental initiatives in this direction. In that regard, an eventual observation of the $X(10550)$ might represent another relevant piece in the puzzle of exotic hadron spectroscopy and help us to shed more light on the intrinsic nature of the partner states related by the heavy-quark symmetry.

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