Probing $\chi_{cJ}(J=0,1,2)$ decay into baryon and antibaryon with SU(3) flavor analysis

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With the available experimental results on $\chi_{cJ}(J=0,1,2)$ decays, we explore the decays of $\chi_{cJ} \to \mathcal{B}_8\bar{\mathcal{B}}_8$ and $\chi_{cJ} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ based on the SU(3) flavor symmetry. In addition, we give an extensive discussion on the possible contributions from the SU(3) breaking effect and $\Sigma^0-\Lambda$ mixing. With the decay amplitude determined from an effective Hamiltonian, we also present the expected branching fractions for those unobserved decay modes, e.g., $\chi_{cJ} \to n\bar{n}$ and $\chi_{cJ} \to \Lambda\bar{\Sigma}^0 + \bar{\Lambda}\Sigma^0$, which are expected to be observed at the BESIII experiment and the future planned Super Tau-Charm facilities. Our results provide valuable insights that will aid in advancing our understanding of the mechanisms and characteristics of χ_{cJ} decays.

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I. INTRODUCTION

Since the discovery of the J/ψ meson [1,2], research on charmonium $(c\bar{c})$ has gradually developed [3–5]. Understanding charmonium decays into baryon and antibaryon pairs could provide insights into various mechanisms, such as the interference between the strong and electromagnetic interactions, contributions from color octet and singlet states, the pQCD 12% rule in J/ψ and ψ' decays, and SU(3) flavor symmetry breaking effects. The χ_{cJ} mesons, being P-wave $c\bar{c}$ triple states with a spin parity J^{++}, have been the subject of study in the context of their decay into baryon pairs, for example, within the framework of perturbative QCD [6–8] and the quark-pair creation model [9]. Some χ_{cJ} meson decays into baryon pairs, such as $\chi_{c1,2} \to p\bar{p}$, have long presented a theoretical challenge [6]. Further studies of the χ_{cJ} decays could enrich our knowledge of the nature of these charmonium states.

With the development of heavy quark effective theory (HQET) [10–12], researchers have acquired effective approaches to depict the physics of hadronic decays that involve heavy quarks. However, the physics of decays involving charmed hadrons presents a challenge for traditional QCD-based methods, such as QCD factorization [13–15], perturbative QCD [16,17], and soft collinear effective theory [18–20], since the mass of the charm quark is not heavy enough to undergo significant heavy-quark expansion but not light enough for perturbation theory to be effective [21,22]. With the lack of reliable calculations, the symmetry analysis can provide very useful information about the decays, such as SU(3) flavor symmetry [23,24]. The SU(3) flavor symmetry approach, which is independent of the detailed dynamics, offers an opportunity to relate different decay modes.

Based on the SU(3) flavor symmetry methods, significant progress has been made in J/ψ and $\Psi(2S)$ decays [25–31]. However, there has been relatively little attention given to the χ_{cJ} mesons [32,33], which also belong to the charmonium family. In this work, we will study χ_{cJ} meson decay into a baryon-antibaryon pair via strong and electromagnetic interactions based on the SU(3) flavor symmetry method. We will first derive the amplitude relations for the $\chi_{cJ} \to \mathcal{B}\bar{\mathcal{B}}$ decays, taking into account the research methods used for J/ψ and $\Psi(2S)$ in Refs. [27,31]; we will then constrain relative nonperturbative parameters and obtain the branching ratio results for all $\chi_{cJ} \to \mathcal{B}_8\bar{\mathcal{B}}_8, \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$. Furthermore, we will analyze the addition of a mixing between Λ and Σ^0.

This paper is organized as follows: In Sec. II, the amplitude relations and predicted branching ratios of $\chi_{cJ} \to \mathcal{B}_8\bar{\mathcal{B}}_8$ are given under three different cases. In Sec. III, the amplitude relations and predicted branching ratios of $\chi_{cJ} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ are given. Our conclusion is presented in Sec. IV.

II. DECAYS $\chi_{cJ} \to \mathcal{B}_8\bar{\mathcal{B}}_8$

A. Amplitude relations

The light baryon octet under the SU(3) flavor symmetry of u, d, s quarks can be written as

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$$\mathcal{B}_8 = \begin{pmatrix} \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & \frac{-2\Lambda}{\sqrt{6}} \end{pmatrix}, \quad (1)$$

where Λ and Σ^0 denote the unmixed states, and the mixed states Λ' and Σ'^0 will be introduced later.

Since J/ψ, $\Psi(2S)$, χ_{c0}, χ_{c1}, and χ_{c2} are all SU(3) meson singlets, the final effective interaction Hamiltonian and amplitude relations governing their decays into octet baryon pairs are similar. According to the $J/\psi, \Psi(2S) \to \mathcal{B}_8\bar{\mathcal{B}}_8$ decays in Refs. [28,31], the final effective interaction Hamiltonian for $\chi_{cJ} \to \mathcal{B}_8\bar{\mathcal{B}}_8$ can be represented as

$$\begin{aligned}\mathcal{H}_{\text{eff}}(\chi_{cJ} \to \mathcal{B}_8\bar{\mathcal{B}}_8) = {} & g_0 \bar{\mathcal{B}}^i_j \mathcal{B}^j_i + g_m([\bar{\mathcal{B}}\mathcal{B}]_f)^3_3 + g'_m([\bar{\mathcal{B}}\mathcal{B}]_d)^3_3 \\ & + g_e([\bar{\mathcal{B}}\mathcal{B}]_f)^1_1 + g'_e([\bar{\mathcal{B}}\mathcal{B}]_d)^1_1, \end{aligned} \quad (2)$$

with

$$([\bar{\mathcal{B}}\mathcal{B}]_f)^i_j = \bar{\mathcal{B}}^i_k \mathcal{B}^k_j - \bar{\mathcal{B}}^k_j \mathcal{B}^i_k, \quad (3)$$

$$([\bar{\mathcal{B}}\mathcal{B}]_d)^i_j = \bar{\mathcal{B}}^i_k \mathcal{B}^k_j + \bar{\mathcal{B}}^k_j \mathcal{B}^i_k - \frac{2}{3}\delta^i_j(\bar{\mathcal{B}}^m_n \mathcal{B}^n_m), \quad (4)$$

where $\mathcal{B}^i_j$ denotes the matrix element of the baryon octet $\mathcal{B}_8$ with row i and column j, $g_{0,m,e}$ and $g'_{m,e}$ are nonperturbative coupling coefficients, and g_0 is the nonperturbative coefficient under the SU(3) flavor symmetry; two types of the SU(3) flavor breaking effects are considered—mass breaking g_m/g'_m and electromagnetic breaking g_e/g'_e. Further details can be found in Refs. [28,30,31]. Since these coefficients $g_{0,m,e}$ and $g'_{m,e}$ vary across different χ_{c0}, χ_{c1}, and χ_{c2} decays, we redefine them as $A_J = g_0$, $D_J = g'_e/3$, $F_J = -g_e$, $D'_J = -g'_m/3$, and $F'_J = g_m$. Using Eq. (2), the amplitudes of $\chi_{cJ} \to \mathcal{B}_8\bar{\mathcal{B}}_8$ are parametrized, and the results are listed in the first nine lines of Table I.

In addition, in the case of Λ and Σ^0 baryons, a small mixing angle α exists between the isoscalar state (Λ) and the neutral component of the isotriplet (Σ^0), owing to isospin violation. The mixing between Λ and Σ^0 baryons has been studied in many works, for example, in Refs. [34–37]. The physical mass eigenstates Λ' and Σ'^0 are

$$\begin{aligned}\Lambda' &= \Lambda\cos\alpha - \Sigma^0\sin\alpha, \\ \Sigma'^0 &= \Lambda\sin\alpha + \Sigma^0\cos\alpha. \end{aligned} \quad (5)$$

The mixing angle α is estimated using different approaches, for instance, the quark model [34,38,39], chiral perturbation theory [40], QCD sum rules in vacuum [41,42], QCD sum rules [43,44], and lattice QCD [45]. The mixing angle is predicted by these approaches as in the range of $[0.14, 2.0] \times 10^{-2}$ radians. After considering the $\Sigma^0 - \Lambda$ mixing, the parametrized amplitudes of $\chi_{cJ} \to \Sigma'^0\bar{\Sigma}'^0, \Lambda'\bar{\Lambda}'$, and $\Sigma'^0\bar{\Lambda}' + \bar{\Sigma}'^0\Lambda'$ decays are given in the last three lines of Table I.

It is important to note that there exist three decay modes: purely strong, purely electromagnetic, and mixed strong-electromagnetic interactions in the studies of J/ψ and $\Psi(2S)$ decays [30,31]; the amplitude can be broken down into three parts: $A^{ggg}_{\mathcal{B}\bar{\mathcal{B}}}$, $A^{\gamma}_{\mathcal{B}\bar{\mathcal{B}}}$, and $A^{gg\gamma}_{\mathcal{B}\bar{\mathcal{B}}}$, corresponding to the three decay modes. The total amplitude $A_{\mathcal{B}\bar{\mathcal{B}}} = A^{ggg}_{\mathcal{B}\bar{\mathcal{B}}} + A^{\gamma}_{\mathcal{B}\bar{\mathcal{B}}} + A^{gg\gamma}_{\mathcal{B}\bar{\mathcal{B}}}$ consists of the purely strong part $A^{ggg}_{\mathcal{B}\bar{\mathcal{B}}} = (A, D', F')e^{i\varphi}$, the purely electromagnetic part $A^{\gamma}_{\mathcal{B}\bar{\mathcal{B}}} = (D, F)$, and the mixed strong-electromagnetic part $A^{gg\gamma}_{\mathcal{B}\bar{\mathcal{B}}} = A^{ggg}_{\mathcal{B}\bar{\mathcal{B}}}R$, where the phase φ represents the relative phase between the strong and electromagnetic amplitudes and the ratio R denotes the relationship between $A^{gg\gamma}_{\mathcal{B}\bar{\mathcal{B}}}$ and $A^{ggg}_{\mathcal{B}\bar{\mathcal{B}}}$. However, for the $\chi_{cJ} \to \mathcal{B}_8\bar{\mathcal{B}}_8$ decays, present experimental data are not enough to determine all seven coupling coefficients. Therefore, we focus on the dominant contributions of the purely strong and purely electromagnetic

TABLE I. Amplitudes for the $\chi_{cJ} \to \mathcal{B}_8\bar{\mathcal{B}}_8$ decays. $A_J = g_0$, $D_J = g'_e/3$, $F_J = -g_e$, $D'_J = -g'_m/3$, and $F'_J = g_m$.

Decay modes	Amplitudes
$\chi_{cJ} \to p\bar{p}$	$A_J + D_J + F_J - D'_J + F'_J$
$\chi_{cJ} \to n\bar{n}$	$A_J - 2D_J - D'_J + F'_J$
$\chi_{cJ} \to \Sigma^+\bar{\Sigma}^-$	$A_J + D_J + F_J + 2D'_J$
$\chi_{cJ} \to \Sigma^-\bar{\Sigma}^+$	$A_J + D_J - F_J + 2D'_J$
$\chi_{cJ} \to \Xi^0\bar{\Xi}^0$	$A_J - 2D_J - D'_J - F'_J$
$\chi_{cJ} \to \Xi^-\bar{\Xi}^+$	$A_J + D_J - F_J - D'_J - F'_J$
$\chi_{cJ} \to \Sigma^0\bar{\Sigma}^0$	$A_J + D_J + 2D'_J$
$\chi_{cJ} \to \Lambda\bar{\Lambda}$	$A_J - D_J - 2D'_J$
$\chi_{cJ} \to \Sigma^0\bar{\Lambda} + cc$	$\sqrt{3}D_J$
$\chi_{cJ} \to \Sigma'^0\bar{\Sigma}'^0$	$A_J + (\cos 2\alpha - \sqrt{3}/2\sin 2\alpha)D_J + (2\cos 2\alpha)D'_J$
$\chi_{cJ} \to \Lambda'\bar{\Lambda}'$	$A_J + (\sqrt{3}/2\sin 2\alpha - \cos 2\alpha)D_J - (2\cos 2\alpha)D'_J$
$\chi_{cJ} \to \Sigma'^0\bar{\Lambda}' + cc$	$(\sqrt{3}\cos 2\alpha + 2\sin 2\alpha)D_J + (4\sin 2\alpha)D'_J$

interactions, and neglect the weakest contribution part $A^{gg\gamma}_{\mathcal{B}\bar{\mathcal{B}}}$ in the hadron decay of charmonium [46]. This leads us to consider the amplitude as $A(\chi_{cJ}\to\mathcal{B}_8\bar{\mathcal{B}}_8)=(A_J,D'_J,F'_J)e^{i\varphi_J}+(D_J,F_J)$, which only includes six coupling coefficients in each χ_{c0}, χ_{c1}, or χ_{c2} decay.

In terms of the decay amplitude of $\chi_{cJ}\to\mathcal{B}_8\bar{\mathcal{B}}_8$ given in Table I, the branching ratio (B) can be written as

$$B(\chi_{cJ}\to\mathcal{B}_8\bar{\mathcal{B}}_8)=\frac{|\vec{p}|}{8\pi M^2_{\chi_{cJ}}\Gamma_{\chi_{cJ}}}|A(\chi_{cJ}\to\mathcal{B}_8\bar{\mathcal{B}}_8)|^2, \quad (6)$$

where $\Gamma_{\chi_{cJ}}$ is the width of the χ_{cJ} meson, and $|\vec{p}|\equiv\frac{\sqrt{[M^2_{\chi_{cJ}}-(m^2_{\mathcal{B}_8}+m^2_{\bar{\mathcal{B}}_8})][M^2_{\chi_{cJ}}-(m^2_{\mathcal{B}_8}-m^2_{\bar{\mathcal{B}}_8})]}}{2M_{\chi_{cJ}}}$.

B. Numerical results and discussion

The theoretical input parameters, such as the decay widths and the masses, and the experimental data within the 1σ and 2σ errors from PDG [46] will be used for the numerical analysis of the $\chi_{cJ}\to\mathcal{B}_8\bar{\mathcal{B}}_8$ decays.

The present experimental status on $\chi_{cJ}(J=0,1,2)$ decaying into baryon-antibaryon pairs is summarized in Tables II–IV, respectively. Even though the experimental results are still scarce and some of decay modes are not observed, we perform a global fit with the measured branching fractions to determine the coupling parameters. Then, relevant branching ratios that have not yet been measured are predicted by the constrained coupling coefficients. After using all relevant experimental constraints and the theoretical input parameters within the 1σ error, some χ_{cJ} decays fail to obtain results; i.e., as given in Tables II–IV, the predictions of $B(\chi_{c0}\to p\bar{p})$, $B(\chi_{c1}\to\Sigma^{(\prime)0}\bar{\Sigma}^{(\prime)0})$, $B(\chi_{c1}\to\Sigma^-\bar{\Sigma}^+)$, and $B(\chi_{c2}\to\Xi^-\bar{\Xi}^+)$ cannot explain current data within the 1σ error. Thus, we will analyze the results within both the 1σ and 2σ errors.

In the simplest case (fit I), only the SU(3) flavor symmetry contributions are considered, in which the SU(3) flavor breaking contribution and the $\Sigma^0-\Lambda$ mixing are ignored. There is only one coupling parameter, A_J, for the $\chi_{cJ}\to\mathcal{B}_8\bar{\mathcal{B}}_8$ decays. It is apparent that the range of $|A_J|$ can be directly inferred from experimental values and Eq. (6). However, the permissible values of A_J for each decay channel may differ significantly. We take $\chi_{c0}\to\mathcal{B}_8\bar{\mathcal{B}}_8$ as an example. The central values of $|A_0|$ are 0.022, 0.030, 0.035, 0.035, 0.036, 0.030, 0.037 from the experimental data of the branching ratios of $\chi_{c0}\to p\bar{p},\Lambda\bar{\Lambda},\Sigma^0\bar{\Sigma}^0,\Sigma^+\bar{\Sigma}^-,\Sigma^-\bar{\Sigma}^+,\Xi^0\bar{\Xi}^0,\Xi^-\bar{\Xi}^+$, respectively. Seven central values are in the same order of magnitude, but some values are obviously different. We cannot find one value of $|A_0|$ that satisfies all present relevant experiential data within the 1σ or 2σ error bars. Therefore, we only use $B(\chi_{cJ}\to p\bar{p})$ to constrain $|A_J|$; then, use the constrained $|A_J|$ to give the branching ratios of other processes. The results within the 1σ and 2σ errors are listed in the third and fourth columns of Tables II–IV, respectively. It becomes apparent that these SU(3) flavor symmetry predictions do not effectively align with the experimental values, and the SU(3) flavor breaking effects must be considered. These SU(3) flavor symmetry predictions can serve as benchmarks for gauging the magnitude of the breaking effect. Additionally, there are still two decay branching ratios for which no experimental data are available, and they are predicted by the constrained $|A_J|$ from the data of $B(\chi_{cJ}\to p\bar{p})$.

After taking into account the SU(3) breaking effect and ignoring the $\Sigma^0-\Lambda$ mixing, a fit (fit II) is performed by employing the six free parameters $(A_J,D_J,F_J,D'_J,F'_J,\varphi_J)$ (we set $A_J\geq 0$). The fitted results with 2σ errors are summarized in the sixth columns of Tables II–IV, which satisfy all of the existing relevant experimental data. Comparing our predictions with corresponding experimental data, one can see that many of them are same, which means that both upper limits and lower limits of these data give effective constraints on the relevant parameters. If only the upper limit or lower limit of the prediction is the same as its experimental one, then only the upper limit or lower limit gives an effective constraint. One can similarly analyze other cases. For the $\chi_{c0}\to\mathcal{B}_8\bar{\mathcal{B}}_8$ decays given in the sixth column of Table II, one can see that the experimental upper and lower limits of $B(\chi_{c0}\to p\bar{p})$, $B(\chi_{c0}\to\Lambda\bar{\Lambda})$, $B(\chi_{c0}\to\Sigma^0\bar{\Sigma}^0)$, $B(\chi_{c0}\to\Sigma^-\bar{\Sigma}^+)$, and $B(\chi_{c0}\to\Xi^-\bar{\Xi}^+)$, the experimental lower limit of $B(\chi_{c0}\to\Sigma^+\bar{\Sigma}^-)$, as well as the experimental upper limit of $B(\chi_{c0}\to\Xi^0\bar{\Xi}^0)$ give the effective constraints on the parameters (A_0,D_0,F_0,D'_0,F'_0). Note that $B(\chi_{c0}\to n\bar{n})$ is predicted to be at the order of 10^{-4}. Due to the weak constraint on $|D_0|$, the predicted $B(\chi_{c0}\to\Lambda\bar{\Sigma}^0+\mathrm{cc})$ has the form $a\pm a$, and the measurement of $B(\chi_{c0}\to\Lambda\bar{\Sigma}^0+\mathrm{cc})$ in the future may give a better constraint on $|D_0|$. For the $\chi_{c1}\to\mathcal{B}_8\bar{\mathcal{B}}_8$ decays given in the sixth column of Table III, one can see that all measured branching ratios give the effective constraints on (A_1,D_1,F_1,D'_1,F'_1). Both $B(\chi_{c1}\to n\bar{n})$ and $B(\chi_{c1}\to\Lambda\bar{\Sigma}^0+\mathrm{cc})$ are predicted in the form of $a\pm a$, and the prediction of $B(\chi_{c1}\to n\bar{n})$ may be quite larger than its SU(3) flavor symmetry prediction. Because of the presence of another numerical range for D'_1 and F'_1, $B(\chi_{c1}\to n\bar{n})$ displays another quite larger branching ratio. As for the $\chi_{c2}\to\mathcal{B}_8\bar{\mathcal{B}}_8$ decays given in the sixth column of Table IV, all measured branching ratios, except the lower limit of $B(\chi_{c2}\to\Xi^0\bar{\Xi}^0)$, give effective constraints on (A_2,D_2,F_2,D'_2,F'_2). It can be observed that the phase angles φ_J are not evidently constrained by current experimental data, primarily due to the limited availability of experimental data and the lack of additional parameter constraints. Moreover, some errors of the constrained A_J,D_J,F_J,D'_J,F'_J are large. The results of $\chi_{cJ}\to\mathcal{B}_8\bar{\mathcal{B}}_8$ with 1σ errors are listed in the fifth columns of Tables II–IV. For the $\chi_{c0}\to\mathcal{B}_8\bar{\mathcal{B}}_8$ decays, the data of $B(\chi_{c0}\to p\bar{p})$ are not used to constrain the relevant parameters or give the

TABLE II. The allowed amplitude parameters and branching ratios (in units of 10^{-4}) of $\chi_{c0} \to \mathcal{B}_8\bar{\mathcal{B}}_8$ decays. Fit I: Only the SU(3) flavor symmetry contribution is considered. Fit II: Both the SU(3) flavor symmetry and the SU(3) flavor breaking contributions are considered, and the $\Sigma^0 - \Lambda$ mixing is not considered. Fit III: All the SU(3) flavor symmetry, the SU(3) flavor breaking contributions, and the $\Sigma^0 - \Lambda$ mixing are considered, and $\alpha = (0.015 \pm 0.001)$ radians [36].

	Exp. data (1σ)	Predictions with fit I (1σ)	Predictions with fit I (2σ)	Predictions with fit II (1σ)	Predictions with fit II (2σ)	Predictions with fit III (1σ)	Predictions with fit III (2σ)	Predictions with fit II′ (1σ)	Predictions with fit II′ (2σ)	Predictions with fit III′ (1σ)	Predictions with fit III′ (2σ)
$B(\chi_{c0} \to p\bar{p})$	2.21 ± 0.08	2.21 ± 0.08[a]	2.21 ± 0.16[a]	4.48 ± 1.81[b]	2.21 ± 0.16	4.42 ± 1.75[b]	2.21 ± 0.16	4.41 ± 1.68[b]	2.21 ± 0.16	4.39 ± 1.67[b]	2.21 ± 0.16
$B(\chi_{c0} \to n\bar{n})$	⋯	2.21 ± 0.08	2.21 ± 0.16	3.86 ± 1.98	3.77 ± 2.40	3.96 ± 1.89	3.77 ± 2.37	3.31 ± 1.34	3.98 ± 2.00	3.32 ± 1.33	3.97 ± 1.99
$B(\chi_{c0} \to \Lambda^{(\prime)}\bar{\Lambda}^{(\prime)})$	3.59 ± 0.15	2.00 ± 0.07	2.00 ± 0.14	3.59 ± 0.15	3.59 ± 0.30	3.59 ± 0.15	3.59 ± 0.30	3.59 ± 0.15	3.59 ± 0.30	3.59 ± 0.15	3.59 ± 0.30
$B(\chi_{c0} \to \Sigma^{(\prime)0}\bar{\Sigma}^{(\prime)0})$	4.68 ± 0.32	1.89 ± 0.07	1.89 ± 0.14	4.68 ± 0.32	4.68 ± 0.64	4.68 ± 0.32	4.68 ± 0.64	4.68 ± 0.32	4.67 ± 0.63	4.68 ± 0.32	4.66 ± 0.62
$B(\chi_{c0} \to \Sigma^+\bar{\Sigma}^-)$	4.6 ± 0.8	1.89 ± 0.07	1.89 ± 0.14	4.60 ± 0.80	3.92 ± 0.92	4.60 ± 0.80	3.92 ± 0.92	4.60 ± 0.80	3.91 ± 0.91	4.57 ± 0.77	3.94 ± 0.94
$B(\chi_{c0} \to \Sigma^-\bar{\Sigma}^+)$	5.1 ± 0.5	1.89 ± 0.07	1.89 ± 0.14	5.10 ± 0.50	5.10 ± 1.00	5.10 ± 0.50	5.10 ± 1.00	5.10 ± 0.50	5.10 ± 1.00	5.10 ± 0.50	5.10 ± 1.00
$B(\chi_{c0} \to \Xi^0\bar{\Xi}^0)$	3.1 ± 0.8	1.69 ± 0.06	1.69 ± 0.12	3.10 ± 0.80	4.03 ± 0.67	3.10 ± 0.80	4.03 ± 0.67	3.10 ± 0.80	4.04 ± 0.66	3.10 ± 0.80	4.05 ± 0.65
$B(\chi_{c0} \to \Xi^-\bar{\Xi}^+)$	4.8 ± 0.7	1.67 ± 0.06	1.67 ± 0.12	4.80 ± 0.70	4.80 ± 1.40	4.80 ± 0.70	4.80 ± 1.40	4.80 ± 0.70	4.80 ± 1.40	4.80 ± 0.70	4.80 ± 1.40
$B(\chi_{c0} \to \Lambda^{(\prime)}\bar{\Sigma}^{(\prime)0} + \mathrm{cc})$	⋯	⋯	⋯	1.05 ± 1.05	0.37 ± 0.37	1.00 ± 1.00	0.80 ± 0.80	0.25 ± 0.25	0.13 ± 0.13	0.24 ± 0.24	0.26 ± 0.26
A_0 (10^{-2} GeV)	⋯	2.21 ± 0.10	2.21 ± 0.20	3.14 ± 0.25	3.14 ± 0.33	3.13 ± 0.26	3.13 ± 0.34	3.20 ± 0.19	3.17 ± 0.29	3.20 ± 0.19	3.17 ± 0.29
D_0 (10^{-3} GeV)	⋯	⋯	⋯	-0.10 ± 13.36	0.20 ± 7.70	-0.11 ± 13.17	-0.01 ± 7.91	0.04 ± 6.60	-0.10 ± 4.74	0.05 ± 6.58	-0.10 ± 4.74
F_0 (10^{-3} GeV)	⋯	⋯	⋯	0.08 ± 16.34	0.32 ± 13.92	0.08 ± 16.34	-0.16 ± 13.44	0.00 ± 6.63	-0.09 ± 6.69	0.00 ± 6.63	-0.09 ± 6.69
D_0' (10^{-3} GeV)	⋯	⋯	⋯	0.82 ± 0.92	1.67 ± 1.70	0.82 ± 0.92	1.64 ± 1.74	0.83 ± 0.93	1.74 ± 1.74	0.83 ± 0.93	1.73 ± 1.75
F_0' (10^{-3} GeV)	⋯	⋯	⋯	-2.25 ± 4.25	-4.44 ± 5.28	-1.90 ± 3.89	-4.61 ± 5.45	-2.67 ± 3.67	-2.95 ± 3.89	-2.67 ± 3.67	-2.95 ± 3.89
φ_0 (radian)	⋯	⋯	⋯	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$
α (radian)	⋯	⋯	⋯	0	0	0.015 ± 0.001	0.015 ± 0.002	0	0	0.015 ± 0.001	0.015 ± 0.002

[a]Only the data are used to determine the relevant parameter $|A_J|$.
[b]The experimental upper limit is not used to determine the relevant parameters.

TABLE III. Allowed amplitude parameters and branching ratios (in units of 10^{-5}) of $\chi_{c1} \to \mathcal{B}_8\bar{\mathcal{B}}_8$ decays.

	Exp. data (1σ)	Predictions with fit I (1σ)	Predictions with fit I (2σ)	Predictions with fit II (1σ)	Predictions with fit II (2σ)	Predictions with fit III (1σ)	Predictions with fit III (2σ)	Predictions with fit II′ (1σ)	Predictions with fit II′ (2σ)	Predictions with fit III′ (1σ)	Predictions with fit III′ (2σ)
$B(\chi_{c1} \to p\bar{p})$	7.60 ± 0.34	7.60 ± 0.34[a]	7.60 ± 0.68[a]	7.60 ± 0.34	7.60 ± 0.68	7.60 ± 0.34	7.60 ± 0.68	7.56 ± 0.30	7.60 ± 0.68	7.60 ± 0.33	7.60 ± 0.68
$B(\chi_{c1} \to n\bar{n})$	⋯	7.60 ± 0.34	7.60 ± 0.68	6.35 ± 5.68 or 54.86 ± 9.85	7.40 ± 7.40 or 56.06 ± 24.14	6.14 ± 5.88 or 55.01 ± 8.79	7.57 ± 7.57 or 52.66 ± 23.67	5.20 ± 0.55	5.75 ± 1.90	5.13 ± 0.69	5.79 ± 1.81
$B(\chi_{c1} \to \Lambda^{(\prime)}\bar{\Lambda}^{(\prime)})$	12.7 ± 0.8	6.94 ± 0.31	6.94 ± 0.62	12.70 ± 0.80	12.70 ± 1.60	12.70 ± 0.80	12.70 ± 1.60	13.24 ± 0.26	12.99 ± 1.31	13.19 ± 0.31	12.99 ± 1.31
$B(\chi_{c1} \to \Sigma^{(\prime)0}\bar{\Sigma}^{(\prime)0})$	4.2 ± 0.6	6.60 ± 0.30	6.60 ± 0.59	4.20 ± 0.60	4.20 ± 1.20	4.20 ± 0.60	4.20 ± 1.20	2.53 ± 0.28[b]	4.20 ± 1.20	2.57 ± 0.35[b]	4.18 ± 1.18
$B(\chi_{c1} \to \Sigma^+\bar{\Sigma}^-)$	3.6 ± 0.7	6.59 ± 0.30	6.59 ± 0.59	3.60 ± 0.70	3.60 ± 1.40	3.60 ± 0.70	3.60 ± 1.40	3.12 ± 0.22	3.71 ± 1.29	3.20 ± 0.30	3.71 ± 1.29
$B(\chi_{c1} \to \Sigma^-\bar{\Sigma}^+)$	5.7 ± 1.5	6.59 ± 0.30	6.59 ± 0.59	5.70 ± 1.50	5.70 ± 3.00	5.70 ± 1.50	5.70 ± 3.00	2.13 ± 0.45[b]	4.33 ± 1.63	2.32 ± 0.64[b]	4.33 ± 1.63
$B(\chi_{c1} \to \Xi^0\bar{\Xi}^0)$	< 6	5.96 ± 0.27	5.96 ± 0.53	3.96 ± 2.04	3.52 ± 2.48	3.93 ± 2.07	3.52 ± 2.48	3.20 ± 0.28	3.77 ± 1.21	3.27 ± 0.36	3.76 ± 1.12
$B(\chi_{c1} \to \Xi^-\bar{\Xi}^+)$	8.0 ± 2.1	5.92 ± 0.27	5.92 ± 0.53	8.00 ± 2.09	8.00 ± 4.20	8.00 ± 2.09	8.00 ± 4.20	6.18 ± 0.28	5.19 ± 1.39	6.18 ± 0.28	5.23 ± 1.43
$B(\chi_{c1} \to \Lambda^{(\prime)}\bar{\Sigma}^{(\prime)0} + \mathrm{cc})$	⋯	⋯	⋯	5.15 ± 4.88	6.34 ± 6.34	4.76 ± 4.36	11.84 ± 11.83	0.11 ± 0.02	0.06 ± 0.06	0.16 ± 0.03	0.18 ± 0.18
A_1 (10^{-3} GeV)	⋯	3.65 ± 0.17	3.65 ± 0.33	1.07 ± 0.22	1.09 ± 0.43	1.06 ± 0.23	1.09 ± 0.43	1.41 ± 0.08	1.17 ± 0.28	1.40 ± 0.09	1.17 ± 0.28
D_1 (10^{-3} GeV)	⋯	⋯	⋯	0.02 ± 2.58	0.01 ± 2.92	0.01 ± 2.50	0.01 ± 2.92	0.00 ± 0.29	0.00 ± 0.28	0.00 ± 0.29	0.00 ± 0.28
F_1 (10^{-3} GeV)	⋯	⋯	⋯	0.01 ± 1.97	0.03 ± 2.67	-0.02 ± 2.00	0.03 ± 2.67	0.00 ± 0.29	0.00 ± 0.28	0.00 ± 0.29	0.00 ± 0.28
D_1' (10^{-3} GeV)	⋯	⋯	⋯	-2.12 ± 0.35 or -0.85 ± 0.14	-2.09 ± 0.58 or -0.82 ± 0.41	-2.12 ± 0.34 or -0.93 ± 0.07	-2.08 ± 0.57 or -0.80 ± 0.42	-1.96 ± 0.08	-2.06 ± 0.25	-1.97 ± 0.09	-2.07 ± 0.25
F_1' (10^{-3} GeV)	⋯	⋯	⋯	0.11 ± 1.13 or 3.82 ± 0.89	-0.42 ± 1.67 or 3.47 ± 1.25	-0.34 ± 1.13 or 3.93 ± 0.78	-0.42 ± 1.67 or 3.45 ± 1.26	0.16 ± 0.12	0.06 ± 0.22	0.13 ± 0.15	0.06 ± 0.22
φ_1 (radian)	⋯	⋯	⋯	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$
α (radian)	⋯	⋯	⋯	0	0	0.015 ± 0.001	0.015 ± 0.002	0	0	0.015 ± 0.001	0.015 ± 0.002

[a]Only the data are used to determine the relevant parameter $|A_J|$.
[b]The experimental lower limit is not used to determine the relevant parameters.

TABLE IV. Allowed amplitude parameters and branching ratios (in units of 10^{-5}) of $\chi_{c2} \to \mathcal{B}_8\bar{\mathcal{B}}_8$ decays.

	Exp. data (1σ)	Predictions with fit I (1σ)	Predictions with fit I (2σ)	Predictions with fit II (1σ)	Predictions with fit II (2σ)	Predictions with fit III (1σ)	Predictions with fit III (2σ)	Predictions with fit II′ (1σ)	Predictions with fit II′ (2σ)	Predictions with fit III′ (1σ)	Predictions with fit III′ (2σ)
$B(\chi_{c2} \to p\bar{p})$	7.33 ± 0.33	7.33 ± 0.33[a]	7.33 ± 0.66[a]	7.33 ± 0.33	7.33 ± 0.66	7.33 ± 0.33	7.33 ± 0.66	7.33 ± 0.33	7.33 ± 0.66	7.33 ± 0.33	7.33 ± 0.66
$B(\chi_{c2} \to n\bar{n})$	…	7.33 ± 0.33	7.33 ± 0.66	5.64 ± 5.57	7.31 ± 7.31	5.72 ± 5.49	7.31 ± 7.31	8.52 ± 3.75	8.26 ± 4.04	8.57 ± 3.78	8.23 ± 3.79
$B(\chi_{c2} \to \Lambda^{(\prime)}\bar{\Lambda}^{(\prime)})$	18.3 ± 1.6	6.72 ± 0.30	6.72 ± 0.60	18.30 ± 1.60	18.30 ± 3.20	18.30 ± 1.60	18.30 ± 3.20	18.30 ± 1.60	18.32 ± 3.18	18.30 ± 1.60	18.35 ± 3.15
$B(\chi_{c2} \to \Sigma^{(\prime)0}\bar{\Sigma}^{(\prime)0})$	3.7 ± 0.6	6.40 ± 0.29	6.40 ± 0.58	3.70 ± 0.60	3.70 ± 1.20	3.70 ± 0.60	3.70 ± 1.20	3.70 ± 0.60	3.70 ± 1.20	3.70 ± 0.60	3.69 ± 1.19
$B(\chi_{c2} \to \Sigma^+\bar{\Sigma}^-)$	3.4 ± 0.7	6.40 ± 0.29	6.40 ± 0.58	3.40 ± 0.70	3.40 ± 1.40	3.40 ± 0.70	3.40 ± 1.40	3.40 ± 0.70	3.51 ± 1.29	3.40 ± 0.70	3.51 ± 1.29
$B(\chi_{c2} \to \Sigma^-\bar{\Sigma}^+)$	4.4 ± 1.8	6.40 ± 0.29	6.40 ± 0.58	4.40 ± 1.79	4.52 ± 3.48	4.40 ± 1.79	4.57 ± 3.43	3.87 ± 1.27	3.38 ± 1.66	3.86 ± 1.26	3.36 ± 1.67
$B(\chi_{c2} \to \Xi^0\bar{\Xi}^0)$	< 10	5.81 ± 0.26	5.81 ± 0.52	7.46 ± 2.54	6.36 ± 3.64	7.69 ± 2.31	6.52 ± 3.48	7.43 ± 2.51	7.03 ± 2.97	7.31 ± 2.39	7.11 ± 2.89
$B(\chi_{c2} \to \Xi^-\bar{\Xi}^+)$	14.2 ± 3.2	5.77 ± 0.26	5.77 ± 0.52	14.20 ± 3.20	14.20 ± 6.40	14.20 ± 3.20	14.20 ± 6.40	6.84 ± 3.48[b]	9.93 ± 2.13	6.78 ± 3.53[b]	9.93 ± 2.13
$B(\chi_{c2} \to \Lambda^{(\prime)}\bar{\Sigma}^{(\prime)0} + cc)$	…	…	…	2.66 ± 2.59	3.65 ± 3.65	2.66 ± 2.56	7.25 ± 7.25	0.10 ± 0.10	0.12 ± 0.12	0.11 ± 0.11	0.27 ± 0.27
A_2 (10^{-3} GeV)	…	5.51 ± 0.25	5.52 ± 0.50	2.53 ± 0.46	2.54 ± 0.84	2.53 ± 0.47	2.54 ± 0.84	2.46 ± 0.42	2.69 ± 0.64	2.46 ± 0.41	2.69 ± 0.63
D_2 (10^{-3} GeV)	…	…	…	0.03 ± 2.79	-0.09 ± 3.39	0.03 ± 2.79	-0.09 ± 3.39	0.00 ± 0.56	0.00 ± 0.61	0.00 ± 0.56	0.00 ± 0.61
F_2 (10^{-3} GeV)	…	…	…	-0.06 ± 2.99	0.01 ± 4.15	-0.06 ± 2.99	0.01 ± 4.15	0.00 ± 0.55	0.01 ± 0.65	0.00 ± 0.55	0.01 ± 0.64
D_2' (10^{-3} GeV)	…	…	…	-3.56 ± 0.49	-3.49 ± 0.87	-3.57 ± 0.50	-3.51 ± 0.89	-3.23 ± 0.34	-3.37 ± 0.41	-3.23 ± 0.34	-3.39 ± 0.40
F_2' (10^{-3} GeV)	…	…	…	-1.55 ± 1.60	-1.67 ± 2.44	-1.52 ± 1.57	-1.67 ± 2.44	-0.13 ± 0.43	-0.21 ± 0.44	-0.13 ± 0.43	-0.21 ± 0.43
φ_2 (radian)	…	…	…	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$
α (radian)	…	…	…	0	0	0.015 ± 0.001	0.015 ± 0.002	0	0	0.015 ± 0.001	0.015 ± 0.002

[a]Only the data are used to determine the relevant parameter $|A_J|$.
[b]The experimental lower limit is not used to determine the relevant parameters.

branching ratio prediction since they cannot be explained by other data. One can see that the predicted $B(\chi_{c0} \to p\bar{p})$ is much larger than its experimental upper limit, and all errors of the parameter $(A_J, D_J, F_J, D_J', F_J')$ are still very large.

For the analysis including both SU(3) breaking effects and $\Sigma^0 - \Lambda$ mixing, we also performed an alternative fit (fit III). The mixing angle $\alpha = (0.015 \pm 0.001)$ radians are taken from the theoretical calculation by using PDG input masses [36]. All allowed coupling parameter spaces and the branching ratios are recalculated. The results with 1σ and 2σ errors are listed in the seventh and eighth columns of Tables II–IV, respectively. Since the mixing angle α is quite small, the allowed parameter spaces and the branching ratio predictions are consistent with those of fit II.

From the results in Tables II–IV, one can see that some absolute values of the constrained D_J, F_J, D_J', or F_J' are not far less than that for A_J, which means the SU(3) flavor breaking effects (both charge breaking D_J, F_J and mass breaking D_J', F_J') are not constrained much for $\chi_{cJ} \to \mathcal{B}_8\bar{\mathcal{B}}_8$ decays by present experimental measurements. Referring to charmonium $J/\psi, \Psi(2S) \to \mathcal{B}_8\bar{\mathcal{B}}_8$ decays in Refs. [27,31], the maximum value of the constrained $|D/A|, |F/A|, |D'/A|$, and $|F'/A|$ is 17.92%. Next, we assume that the ratios $|D_J/A_J|, |F_J/A_J|, |D_J'/A_J|$, and $|F_J'/A_J|$ are less than or equal to 20% to obtain our numerical results; then, the fits II and III are renamed fits II′ and III′.

The results of the $\chi_{c0} \to \mathcal{B}_8\bar{\mathcal{B}}_8$ decays within the 1σ and 2σ errors are listed in the last four columns of Table II. One can see that the predicted $B(\chi_{c0} \to p\bar{p})$ with fits II′ and III′ are similar to the ones with fits II and III; nevertheless, the predicted $B(\chi_{c0} \to \Lambda^{(\prime)}\bar{\Sigma}^{(\prime)0} + \mathrm{cc})$ with fits II′ and III′ have smaller allowed spaces.

As for the $\chi_{c1,2} \to \mathcal{B}_8\bar{\mathcal{B}}_8$ decays, the results are not obtained after assuming all $|D_J/A_J|, |F_J/A_J|, |D_J'/A_J|$, and $|F_J'/A_J|$ are less than or equal to 20%. The difficulty lies in $|D_J'/A_J| \leq 20\%$: If we remove the limit of $|D_J'/A_J| \leq 20\%$, the results within the 2σ errors can be obtained; nevertheless, $|D_{1,2}'|$ are really large. The results of the $\chi_{c1,2} \to \mathcal{B}_8\bar{\mathcal{B}}_8$ decays within the 2σ errors are listed in the 10th and 12th columns of Tables III and IV. One can find that the branching ratio predictions with fits II′ and III′, especially for $B(\chi_{c0} \to n\bar{n})$ and $B(\chi_{c0} \to \Lambda\bar{\Sigma}^0 + \mathrm{cc})$, are more accurate than the ones with fits II and III. When using the experimental constraints and theoretical inputs within 1σ error, we fail to obtain the results. The main reason is the experimental constraints of $B(\chi_{c1} \to \Sigma^{(\prime)0}\bar{\Sigma}^{(\prime)0})$, $B(\chi_{c1} \to \Xi^-\bar{\Xi}^+)$, and $B(\chi_{c2} \to \Xi^-\bar{\Xi}^+)$ cannot be satisfied together with other experimental constraints. If we ignore the lower limits of $B(\chi_{c1} \to \Sigma^{(\prime)0}\bar{\Sigma}^{(\prime)0})$, $B(\chi_{c1} \to \Xi^-\bar{\Xi}^+)$, and $B(\chi_{c2} \to \Xi^-\bar{\Xi}^+)$, the results can be obtained within 1σ error, as given in the 9th and 11th columns of Tables III and IV.

III. $\chi_{cJ} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ DECAYS

A. Amplitude relations

The light baryon decuplet under the SU(3) flavor symmetry of u, d, s quarks can be written as

$$\mathcal{B}_{10} = \frac{1}{\sqrt{3}}\left(\begin{pmatrix} \sqrt{3}\Delta^{++} & \Delta^{+} & \Sigma^{*+} \\ \Delta^{+} & \Delta^{0} & \frac{\Sigma^{*0}}{\sqrt{2}} \\ \Sigma^{*+} & \frac{\Sigma^{*0}}{\sqrt{2}} & \Xi^{*0} \end{pmatrix}, \begin{pmatrix} \Delta^{+} & \Delta^{0} & \frac{\Sigma^{*0}}{\sqrt{2}} \\ \Delta^{0} & \sqrt{3}\Delta^{-} & \Sigma^{*-} \\ \frac{\Sigma^{*0}}{\sqrt{2}} & \Sigma^{*-} & \Xi^{*-} \end{pmatrix}, \begin{pmatrix} \Sigma^{*+} & \frac{\Sigma^{*0}}{\sqrt{2}} & \Xi^{*0} \\ \frac{\Sigma^{*0}}{\sqrt{2}} & \Sigma^{*-} & \Xi^{*-} \\ \Xi^{*0} & \Xi^{*-} & \sqrt{3}\Omega^{-} \end{pmatrix}\right). \tag{7}$$

According to the studies in Refs. [27,28], the effective interaction Hamiltonian for the $\chi_{cJ} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ decays can be represented as

$$\mathcal{H}_{\rm eff}(\chi_{cJ} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}) = \hat{g}_0 \mathcal{B}_{10}^{ijk}\bar{\mathcal{B}}_{10}^{ijk} + \hat{g}_m H_3^3 + \hat{g}_e H_1^1, \tag{8}$$

with

$$H_3^3 = \mathcal{B}_{10}^{3jk}\bar{\mathcal{B}}_{10}^{3jk} - \frac{1}{3}(\mathcal{B}_{10}^{ijk}\bar{\mathcal{B}}_{10}^{ijk}), \tag{9}$$

$$H_1^1 = \mathcal{B}_{10}^{1jk}\bar{\mathcal{B}}_{10}^{1jk} - \frac{1}{3}(\mathcal{B}_{10}^{ijk}\bar{\mathcal{B}}_{10}^{ijk}), \tag{10}$$

where i is the number of matrices, and j and k are rows and columns, respectively. Note that $\hat{g}_{0,m,e}$ are nonperturbative coupling coefficients for the $\chi_{cJ} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ decays: $\hat{g}_0$ is from the SU(3) flavor symmetry, and $\hat{g}_{m,e}$ are from the SU(3) flavor breaking (mass breaking $\hat{g}_m$ and electromagnetic breaking $\hat{g}_e$). By using Eq. (8), the amplitudes of the $\chi_{cJ} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ decays are obtained, and they are listed in Table V.

TABLE V. Amplitudes for the $\chi_{cJ} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ decays. $\hat{A}_J = \hat{g}_0$, $\hat{D}_J = \hat{g}_e/3$, and $\hat{D}'_J = \hat{g}_m/3$.

Decay modes	Amplitudes	Decay modes	Amplitudes
$\chi_{cJ} \to \Delta^{++}\bar{\Delta}^{--}$	$\hat{A}_J + 2\hat{D}_J - \hat{D}'_J$	$\chi_{cJ} \to \Sigma^{*0}\bar{\Sigma}^{*0}$	$\hat{A}_J$
$\chi_{cJ} \to \Delta^{+}\bar{\Delta}^{-}$	$\hat{A}_J + \hat{D}_J - \hat{D}'_J$	$\chi_{cJ} \to \Sigma^{*-}\bar{\Sigma}^{*+}$	$\hat{A}_J - \hat{D}_J$
$\chi_{cJ} \to \Delta^{0}\bar{\Delta}^{0}$	$\hat{A}_J - \hat{D}'_J$	$\chi_{cJ} \to \Xi^{*0}\bar{\Xi}^{*0}$	$\hat{A}_J + \hat{D}'_J$
$\chi_{cJ} \to \Delta^{-}\bar{\Delta}^{+}$	$\hat{A}_J - \hat{D}_J - \hat{D}'_J$	$\chi_{cJ} \to \Xi^{*-}\bar{\Xi}^{*+}$	$\hat{A}_J - \hat{D}_J + \hat{D}'_J$
$\chi_{cJ} \to \Sigma^{*+}\bar{\Sigma}^{*-}$	$\hat{A}_J + \hat{D}_J$	$\chi_{cJ} \to \Omega^{-}\bar{\Omega}^{+}$	$\hat{A}_J - \hat{D}_J + 2\hat{D}'_J$

B. Numerical results and discussion

The theoretical input parameters and the experimental data within 2σ and 1σ errors from PDG [46] are used for the numerical analysis of $\chi_{cJ} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ decays.

As for the $\chi_{cJ} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ decays, there are only four free parameters: $(\hat{A}_J, \hat{D}_J, \hat{D}'_J, \hat{\varphi}_J)$. Three measured decay modes exist in the $\chi_{c0} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ decays, along with one experimental data point and two experimental upper limits in the $\chi_{c1} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ or $\chi_{c2} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ decays. Note that $B(\chi_{cJ} \to \Sigma^{*\pm}\bar{\Sigma}^{*\mp})$ from PDG [46] and $B(\chi_{cJ} \to \Omega^{-}\bar{\Omega}^{+})$ from the BESIII Collaboration [47] are used for our numerical

TABLE VI. Allowed amplitude parameters and branching ratios (in units of 10^{-4}) of the $\chi_{c0} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ decays. $^{\sharp}$ The experimental data are not used in the S_1 case.

	Exp. data	Predictions with fit S_1 (1σ)	Predictions with fit S_1 (2σ)	Predictions with fit S_2 (1σ)	Predictions with fit S_2 (2σ)	Predictions with fit S'_2 (1σ)	Predictions with fit S'_2 (2σ)
$B(\chi_{c0} \to \Delta^{++}\bar{\Delta}^{--})$	$\cdots$	1.21 ± 0.22	1.35 ± 0.29	5.06 ± 3.96	5.69 ± 5.51	3.43 ± 2.34	4.34 ± 4.16
$B(\chi_{c0} \to \Delta^{+}\bar{\Delta}^{-})$	$\cdots$	1.21 ± 0.22	1.35 ± 0.29	4.79 ± 4.79	5.85 ± 5.85	2.87 ± 1.45	3.57 ± 3.20
$B(\chi_{c0} \to \Delta^{0}\bar{\Delta}^{0})$	$\cdots$	1.21 ± 0.22	1.35 ± 0.29	5.22 ± 5.21	6.33 ± 6.33	3.11 ± 1.59	3.63 ± 3.10
$B(\chi_{c0} \to \Delta^{-}\bar{\Delta}^{+})$	$\cdots$	1.21 ± 0.22	1.35 ± 0.29	6.89 ± 4.72	7.37 ± 6.52	3.92 ± 1.75	4.59 ± 3.75
$B(\chi_{c0} \to \Sigma^{*+}\bar{\Sigma}^{*-})$	1.60 ± 1.20	$1.02 \pm 0.18^{\sharp}$	1.14 ± 0.24	1.60 ± 0.60	1.60 ± 1.20	1.60 ± 0.60	1.60 ± 1.20
$B(\chi_{c0} \to \Sigma^{*0}\bar{\Sigma}^{*0})$	$\cdots$	1.02 ± 0.18	1.15 ± 0.24	1.27 ± 1.27	1.60 ± 1.60	1.80 ± 0.78	1.87 ± 1.35
$B(\chi_{c0} \to \Sigma^{*-}\bar{\Sigma}^{*+})$	2.30 ± 1.40	$1.02 \pm 0.18^{\sharp}$	1.14 ± 0.24	2.30 ± 0.70	2.30 ± 1.40	2.30 ± 0.70	2.30 ± 1.40
$B(\chi_{c0} \to \Xi^{*0}\bar{\Xi}^{*0})$	$\cdots$	0.77 ± 0.14	0.86 ± 0.18	0.62 ± 0.62	0.75 ± 0.75	0.87 ± 0.38	0.92 ± 0.59
$B(\chi_{c0} \to \Xi^{*-}\bar{\Xi}^{*+})$	$\cdots$	0.77 ± 0.14	0.86 ± 0.18	0.75 ± 0.74	0.88 ± 0.88	1.19 ± 0.30	1.19 ± 0.61
$B(\chi_{c0} \to \Omega^{-}\bar{\Omega}^{+})$	0.351 ± 0.122	0.351 ± 0.061	0.390 ± 0.083	0.351 ± 0.061	0.351 ± 1.22	0.351 ± 0.061	0.351 ± 0.122
$\hat{A}_0$ $(10^{-2}$ GeV)	$\cdots$	1.80 ± 0.21	1.90 ± 0.30	1.50 ± 1.40	1.70 ± 1.60	2.35 ± 0.58	2.29 ± 1.03
$\hat{D}_0$ $(10^{-2}$ GeV)	$\cdots$	$\cdots$	$\cdots$	0.00 ± 2.90	0.00 ± 2.99	0.00 ± 1.37	0.02 ± 1.57
$\hat{D}'_0$ $(10^{-2}$ GeV)	$\cdots$	$\cdots$	$\cdots$	-0.23 ± 2.30	-0.23 ± 2.58	-0.53 ± 0.37	-0.66 ± 0.86

TABLE VII. Allowed amplitude parameters and branching ratios (in units of 10^{-5}) of the $\chi_{c1} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ decays.

	Exp. data	Predictions with fit S_1 (1σ)	Predictions with fit S_1 (2σ)	Predictions with fit S_2 (1σ)	Predictions with fit S_2 (2σ)	Predictions with fit S_2' (1σ)	Predictions with fit S_2' (2σ)
$B(\chi_{c1} \to \Delta^{++}\bar{\Delta}^{--})$	⋯	3.50 ± 0.60	3.50 ± 1.19	<33.88	<34.99	<20.46	<20.76
$B(\chi_{c1} \to \Delta^{+}\bar{\Delta}^{-})$	⋯	3.50 ± 0.60	3.50 ± 1.19	<28.26	<29.47	<12.85	<13.04
$B(\chi_{c1} \to \Delta^{0}\bar{\Delta}^{0})$	⋯	3.50 ± 0.60	3.50 ± 1.19	<24.22	<24.45	<9.85	<10.15
$B(\chi_{c1} \to \Delta^{-}\bar{\Delta}^{+})$	⋯	3.50 ± 0.60	3.50 ± 1.19	<21.07	<21.61	<7.74	<8.32
$B(\chi_{c1} \to \Sigma^{*+}\bar{\Sigma}^{*-})$	<9.00	3.01 ± 0.51	3.02 ± 1.02	<9.00	<9.00	<9.00	<9.00
$B(\chi_{c1} \to \Sigma^{*0}\bar{\Sigma}^{*0})$	⋯	3.02 ± 0.51	3.02 ± 1.02	<6.65	<6.74	<6.79	<6.74
$B(\chi_{c1} \to \Sigma^{*-}\bar{\Sigma}^{*+})$	<5.00	3.01 ± 0.51	3.02 ± 1.02	<5.00	<5.00	<5.00	<5.00
$B(\chi_{c1} \to \Xi^{*0}\bar{\Xi}^{*0})$	⋯	2.40 ± 0.41	2.40 ± 0.81	<4.68	<4.84	<4.68	<4.92
$B(\chi_{c1} \to \Xi^{*-}\bar{\Xi}^{*+})$	⋯	2.38 ± 0.40	2.38 ± 0.81	<3.32	<3.52	<3.34	<3.53
$B(\chi_{c1} \to \Omega^{-}\bar{\Omega}^{+})$	1.49 ± 0.50	1.49 ± 0.50	1.49 ± 0.50	1.49 ± 0.25	1.49 ± 0.50	1.49 ± 0.25	1.49 ± 0.50
$\hat{A}_1$ $(10^{-3}$ GeV)	⋯	2.69 ± 0.29	2.68 ± 0.59	2.03 ± 2.03	2.11 ± 2.11	2.55 ± 1.57	2.51 ± 1.65
$\hat{D}_1$ $(10^{-3}$ GeV)	⋯	⋯	⋯	-0.01 ± 4.04	0.02 ± 4.17	0.00 ± 1.95	0.02 ± 1.93
$\hat{D}_1'$ $(10^{-3}$ GeV)	⋯	⋯	⋯	-0.04 ± 3.16	-0.10 ± 3.29	0.21 ± 0.82	0.15 ± 0.94

TABLE VIII. Allowed amplitude parameters and branching ratios (in units of 10^{-5}) of the $\chi_{c2} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ decays.

	Exp. data	Predictions with fit S_1 (1σ)	Predictions with fit S_1 (2σ)	Predictions with fit S_2 (1σ)	Predictions with fit S_2 (2σ)	Predictions with fit S_2' (1σ)	Predictions with fit S_2' (2σ)
$B(\chi_{c2} \to \Delta^{++}\bar{\Delta}^{--})$	⋯	9.08 ± 0.14	8.76 ± 0.46	<64.30	<65.04	<32.01	<29.79
$B(\chi_{c2} \to \Delta^{+}\bar{\Delta}^{-})$	⋯	9.08 ± 0.14	8.76 ± 0.46	<51.89	<52.23	<18.40	<18.07
$B(\chi_{c2} \to \Delta^{0}\bar{\Delta}^{0})$	⋯	9.08 ± 0.14	8.76 ± 0.46	<43.71	<43.39	<13.44	<13.07
$B(\chi_{c2} \to \Delta^{-}\bar{\Delta}^{+})$	⋯	9.08 ± 0.14	8.76 ± 0.46	<37.82	<37.90	<9.32	<9.58
$B(\chi_{c2} \to \Sigma^{*+}\bar{\Sigma}^{*-})$	<16.00	7.89 ± 0.11	7.61 ± 0.39	<16.00	<16.00	<16.00	<16.00
$B(\chi_{c2} \to \Sigma^{*0}\bar{\Sigma}^{*0})$	⋯	7.90 ± 0.11	7.63 ± 0.39	<10.91	<11.18	<11.59	<11.23
$B(\chi_{c2} \to \Sigma^{*-}\bar{\Sigma}^{*+})$	<8.00	7.89 ± 0.11	7.61 ± 0.39	<8.00	<8.00	<8.00	<8.00
$B(\chi_{c2} \to \Xi^{*0}\bar{\Xi}^{*0})$	⋯	6.39 ± 0.09	6.17 ± 0.32	<9.73	<9.85	<9.79	<9.95
$B(\chi_{c2} \to \Xi^{*-}\bar{\Xi}^{*+})$	⋯	6.35 ± 0.09	6.13 ± 0.32	<6.78	<6.99	<6.78	<7.01
$B(\chi_{c2} \to \Omega^{-}\bar{\Omega}^{+})$	4.52 ± 0.60	4.28 ± 0.06	4.13 ± 0.21	4.52 ± 0.30	4.52 ± 0.60	4.52 ± 0.30	4.52 ± 0.60
A_2 $(10^{-3}$ GeV)	⋯	6.66 ± 0.19	6.54 ± 0.46	3.95 ± 3.95	4.09 ± 4.09	5.32 ± 2.83	5.35 ± 2.78
D_2 $(10^{-3}$ GeV)	⋯	⋯	⋯	-0.03 ± 7.85	0.02 ± 8.17	0.08 ± 3.87	0.03 ± 3.72
D_2' $(10^{-3}$ GeV)	⋯	⋯	⋯	-0.24 ± 6.74	-0.05 ± 7.10	1.22 ± 1.26	1.18 ± 1.32

analysis. The experimental data within 2σ error are listed in the second columns of Tables VI–VIII.

The method of obtaining numerical results is similar to the ones for the $\chi_{cJ} \to \mathcal{B}_8\bar{\mathcal{B}}_8$ decays. Two kinds of fits are considered for the $\chi_{cJ} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ numerical results. In fit S_1, only the SU(3) flavor symmetry contributions are considered; i.e., the coupling coefficients $\hat{A}_J$ are free parameters, and $\hat{D}_J = \hat{D}_J' = 0$. In fit S_2, both the SU(3) flavor symmetry and the SU(3) flavor breaking contributions are considered. We set $\hat{A}_J \geq 0$, and the other three parameters are free parameters.

The results within 2σ errors with fit S_1 and fit S_2 are listed in the fourth and sixth columns of Tables VI–VIII. From the results with fit S_2, it can be seen that the channels of χ_{c0} have significantly different branching ratios, which are attributed to the sufficient ability of modes $\Sigma^{*+}\bar{\Sigma}^{*-}$, $\Sigma^{*-}\bar{\Sigma}^{*+}$, and $\Omega^{-}\bar{\Omega}^{+}$ to reflect the contributions of various amplitude parameters. However, the same situation did not occur in the χ_{c1} and χ_{c2} decays, the branching ratios of the majority of decay channels for which we can only provide upper limits, because there are only experimental upper limits of $B(\chi_{c2,3} \to \Sigma^{*\pm}\bar{\Sigma}^{*\mp})$. Despite this, the results demonstrate the potential of SU(3) flavor analysis in investigating $\chi_{cJ} \to B_{10}\bar{B}_{10}$, allowing for the extraction of information from limited data through correlations between modes. Notably, considering theoretical input parameters and experimental data within 2σ errors, all relevant experimental data can be accounted for both fit S_1 and fit S_2. However, the results with fit S_2 exhibit significantly large errors in branching ratio predictions. The phase angles $\hat{\varphi}_J$ remain unconstrained by current experimental data and are thus omitted from the tables.

From the results with fit S_2, one can see that some absolute values of the constrained $\hat{D}_J, \hat{D}'_J$ are not far less than that for $\hat{A}_J$. Referring to charmonium $J/\psi, \Psi(2S) \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ decays in Ref. [27], the maximum value of the constrained $|\hat{D}/\hat{A}|$ and $|\hat{D}'/\hat{A}|$ is 52.30%. Next, we assume that the ratios $|\hat{D}_J/\hat{A}_J|, |\hat{D}'_J/\hat{A}_J|$ are less than or equal to 50% to obtain our numerical results, renamed fit S'_2. The results with fit S'_2 within 2σ errors are listed in the last columns of Tables VI–VIII. One can see that the predictions are more accurate than the ones with fit S_2; nevertheless, the errors are still large due to few experimental measurements.

In addition, an analysis of the results in the 1σ error range has been conducted and is also documented in Tables VI–VIII for the $\chi_{c0} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$, $\chi_{c1} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$, and $\chi_{c2} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ decays, respectively. For the $\chi_{c0} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ decays with fit S_1, three experimental branching ratios cannot be explained at the same time if only considering the SU(3) flavor symmetry effects; thus, we only use $B(\chi_{c0} \to \Omega^-\bar{\Omega}^+)$ to constrain $\hat{A}_0$ and to predict other branching ratios, and one can see that the predicted $B(\chi_{c0} \to \Sigma^{*+}\bar{\Sigma}^{*-})$ lies in the experimental data within 1σ error. Nevertheless, the predicted $B(\chi_{c0} \to \Sigma^{*-}\bar{\Sigma}^{*+})$ exceeds the experimental data within 1σ error. Comparing these predictions with those within the 2σ error range, the predictions within the 1σ error range exhibit higher accuracy, yet their errors remain substantial. It is anticipated that further measurements of these decays will facilitate more precise predictions.

IV. CONCLUSION

The study of charmonium decays into baryon-antibaryon pairs provides a powerful tool for investigating many topics in quantum chromodynamics. We have performed an analysis of the $\chi_{c0,1,2} \to \mathcal{B}_8\bar{\mathcal{B}}_8$ and $\mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ decays by using SU(3) flavor symmetry, which has also been employed for the J/ψ and $\Psi(2S)$ decays [27,31]. We first constrain the relevant coupling coefficients and then give the predictions of not-measured or not-well-measured branching ratios. The results within both 2σ and 1σ errors are analyzed in different cases.

For the $\chi_{c0,1,2} \to \mathcal{B}_8\bar{\mathcal{B}}_8$ decays, many decay modes are presently measured. However, we discovered that the SU(3) flavor symmetry effects alone cannot explain all experimental data simultaneously. By incorporating SU(3) flavor breaking effects, we successfully accounted for all experimental data within 2σ error bounds. Notably, within a 1σ error range, $B(\chi_{c0} \to p\bar{p})$ remains unexplained alongside other data. We also investigated $\Sigma^0 - \Lambda$ mixing with a specified mixing angle $\alpha = (0.015 \pm 0.001)$ radians. In addition, assuming the breaking couplings (D_J, F_J, D'_J, F'_J) are weaker than the SU(3) symmetry coupling A_J, i.e., $|D_J/A_J| \le 20\%, |F_J/A_J| \le 20\%$, $|D'_J/A_J| \le 20\%$, and $|F'_J/A_J| \le 20\%$, the results have been reanalyzed. We have found that all experimental data can be explained within 2σ errors; nevertheless, the experimental upper limit of $B(\chi_{c0} \to p\bar{p})$ and the lower limits of $B(\chi_{c1} \to \Sigma^{(')0}\bar{\Sigma}^{(')0})$, $B(\chi_{c1} \to \Sigma^-\bar{\Sigma}^+)$, and $B(\chi_{c1} \to \Sigma^-\bar{\Sigma}^+)$ cannot be explained together with other data within 1σ error.

For the $\chi_{c0,1,2} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ decays, the experimental data are less, and we can only give rough results. All experimental data can be explained in both the SU(3) symmetry and breaking cases within 2σ errors. Within 1σ error, $B(\chi_{c0} \to \Sigma^{*-}\bar{\Sigma}^{*+})$ cannot be explained together with other data under the SU(3) flavor symmetry. The predictions of $B(\chi_{c0,1,2} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10})$, which have not been measured or well measured, have been given.

At present, a data sample of 2.7 billion ψ events [48] was collected with the BESIII detector; a detailed analysis of χ_{cJ} decaying into baryon-antibaryon decays could be performed, making it possible to test the SU(3) symmetry and help understand their decay mechanisms. With the extremely high luminosity experiment of the Super Tau-Charm facility [49,50], we can expect that a continuous flow of interesting experimental results includes the precision measurement of branching fractions and likely the investigation of the polarizations of baryons in χ_{cJ} decays.

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[1] J. J. Aubert *et al.* (E598 Collaboration), Phys. Rev. Lett. **33**, 1404 (1974).
[2] J. E. Augustin *et al.* (SLAC-SP-017 Collaboration), Phys. Rev. Lett. **33**, 1406 (1974).
[3] E. Eichten, K. Gottfried, T. Kinoshita, J. B. Kogut, K. D. Lane, and T. M. Yan, Phys. Rev. Lett. **34**, 369 (1975); **36**, 1276(E) (1976).
[4] S. W. Herb *et al.* (E288 Collaboration), Phys. Rev. Lett. **39**, 252 (1977).
[5] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T. M. Yan, Phys. Rev. D **17**, 3090 (1978); **21**, 313(E) (1980).
[6] S. M. H. Wong, Eur. Phys. J. C **14**, 643 (2000).
[7] X. H. Liu and Q. Zhao, J. Phys. G **38**, 035007 (2011).
[8] S. J. Brodsky and G. P. Lepage, Phys. Rev. D **24**, 2848 (1981).
[9] R. G. Ping, B. S. Zou, and H. C. Chiang, Eur. Phys. J. A **23**, 129 (2004).
[10] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989).
[11] M. Neubert, Phys. Rep. **245**, 259 (1994).
[12] I. I. Y. Bigi, N. G. Uraltsev, and A. I. Vainshtein, Phys. Lett. B **293**, 430 (1992); **297**, 477(E) (1992).
[13] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999).
[14] J. C. Collins, D. E. Soper, and G. F. Sterman, Adv. Ser. Dir. High Energy Phys. **5**, 1 (1989).
[15] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Nucl. Phys. **B606**, 245 (2001).
[16] H. Georgi and H. D. Politzer, Phys. Rev. D **9**, 416 (1974).
[17] Y. L. Dokshitzer, Sov. Phys. JETP **46**, 641 (1977).
[18] C. W. Bauer, S. Fleming, and M. E. Luke, Phys. Rev. D **63**, 014006 (2000).
[19] T. Becher, A. Broggio, and A. Ferroglia, Lect. Notes Phys. **896**, 1 (2015).
[20] C. W. Bauer, S. Fleming, D. Pirjol, and I. W. Stewart, Phys. Rev. D **63**, 114020 (2001).
[21] H. Y. Cheng and C. W. Chiang, Phys. Rev. D **81**, 074021 (2010).
[22] H. Y. Cheng and C. W. Chiang, Phys. Rev. D **85**, 034036 (2012); **85**, 079903(E) (2012).
[23] M. Gell-Mann, The eightfold way: A theory of strong interaction symmetry, Technical Report, 1961, 10.1142/9789812836854_0006.
[24] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).
[25] D. H. Wei, J. Phys. G **36**, 115006 (2009).
[26] X. H. Mo, Phys. Rev. D **109**, 036036 (2024).
[27] X. H. Mo, P. Wang, and J. Y. Zhang, Phys. Rev. D **107**, 094009 (2023).
[28] X. H. Mo and J. Y. Zhang, Phys. Lett. B **826**, 136927 (2022).
[29] L. Kopke and N. Wermes, Phys. Rep. **174**, 67 (1989).
[30] R. Baldini Ferroli, A. Mangoni, S. Pacetti, and K. Zhu, Phys. Lett. B **799**, 135041 (2019).
[31] R. B. Ferroli, A. Mangoni, S. Pacetti, and K. Zhu, Phys. Rev. D **103**, 016005 (2021).
[32] R. G. Ping and H. Q. Jiang, Commun. Theor. Phys. **41**, 252 (2004).
[33] H. Q. Zhou, R. G. Ping, and B. S. Zou, Phys. Lett. B **611**, 123 (2005).
[34] R. H. Dalitz and F. Von Hippel, Phys. Lett. **10**, 153 (1964).
[35] G. Karl, Phys. Lett. B **328**, 149 (1994); **341**, 449(E) (1995).
[36] A. Gal, Phys. Rev. D **92**, 018501 (2015).
[37] C. Q. Geng, C. W. Liu, and T. H. Tsai, Phys. Rev. D **101**, 054005 (2020).
[38] A. Gal and F. Scheck, Nucl. Phys. **B2**, 110 (1967).
[39] N. Isgur, Phys. Rev. D **21**, 779 (1980); **23**, 817(E) (1981).
[40] J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).
[41] N. Yagisawa, T. Hatsuda, and A. Hayashigaki, Nucl. Phys. **A699**, 665 (2002).
[42] S. L. Zhu, W. Y. P. Hwang, and Z. s. Yang, Phys. Rev. D **57**, 1524 (1998).
[43] M. Radici, Nucl. Phys. **A699**, 144 (2002).
[44] T. M. Aliev, T. Barakat, and M. Savcı, J. High Energy Phys. 08 (2016) 068.
[45] R. Horsley, J. Najjar, Y. Nakamura, H. Perlt, D. Pleiter, P. E. L. Rakow, G. Schierholz, A. Schiller, H. Stüben, and J. M. Zanotti (QCDSF-UKQCD Collaboration), Phys. Rev. D **91**, 074512 (2015).
[46] R. L. Workman *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).
[47] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **107**, 092004 (2023).
[48] M. Ablikim *et al.* (BESIII Collaboration), Chin. Phys. C **48**, 093001 (2024).
[49] A. E. Bondar, Phys. At. Nucl. **76**, 1072 (2013).
[50] H. P. Peng, Y. H. Zheng, and X. R. Zhou, Physics **49**, 513 (2020).