# New constraints on CPT symmetry violation in charm mesons

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Small CPT-violating effects are admissible in several theories beyond the Standard Model of interactions. We analyze experimental data obtained with neutral flavored meson mixing and place bounds on the scale of such phenomena. The constraints on the CPT-violating parameter  $\zeta$  are derived by reinterpreting the LHCb measurement of the time-dependent asymmetry in the Cabbibo-favored  $D^0 \to K^-\pi^+$  and  $\overline{D^0} \to K^+\pi^-$  decay rates. The bounds provided in this paper are 2 orders of magnitude stricter compared to the results obtained by the FOCUS Collaboration. Among other constraints, we report bounds on the differences in the decay widths and masses of the neutral D meson flavor states at a 95% confidence level. The decay width difference is bounded by  $(-4.7 < \delta\Gamma_D < 2.1) \times 10^{-16}$  GeV. The mass difference lies within  $(-2.0 < \delta m_D < 2.0) \times 10^{-15}$  GeV for 95% of arg $(z)$  values. The paper includes an assessment of the influence of CP violation on the results and a discussion of the future prospects.

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#### I. INTRODUCTION

The CPT symmetry is defined as invariance under the joint transformation of charge conjugation C, spatial inversion  $P$ , and time reversal  $T$ . It is one of the few fundamental symmetries that seem to be exactly conserved by Nature. The profound significance of CPT symmetry is manifested in the CPT theorem (for a complete account, cf. Ref. [[1\]](#page-7-0)), which states that all quantum field theories (QFTs) formulated on the flat space-time and respecting locality, unitarity, and Lorentz invariance of interactions must conserve CPT symmetry. Hence, all QFTs describing fundamental particle interactions within the Standard Model (SM) must be CPT invariant.

The importance of *CPT* symmetry is due to its deep connections to the Poincaré invariance of QFT in flat spacetime. However, when gravitational effects are taken into account, global spacetime symmetries are not symmetries of QFT anymore, and relations between the CPT and Lorentz invariance might have to be revisited. Therefore, some extensions of QFT, incorporating gravity or strings, will also include violation of CPT at the scales of energy or space-time dimensions where quantum gravity (QG) is expected to be significant, i.e., close to the Planck scale. At energies presently available at high-energy accelerators, or even in cosmic rays, these effects are expected to be tiny,

and their observation requires significant advancements in detector sensitivity and accuracy.

Possible extensions of the SM can be studied in a model-independent framework based on the effective field theory, known as the Standard-Model Extension (SME) (cf. Refs. [[2](#page-7-1)–[4\]](#page-7-2) for review). The Lagrangian density in SME contains the CPT- and Lorentz-violating terms with coupling constants serving as control parameters contributing to several physical observables which values can be determined experimentally.

Many theoretical models of QG explicitly incorporate CPT violation (CPTV), either by abandoning some assumptions of the CPT theorem or by the inclusion of the specific model of gravity, such as, e.g., the space-time deformation in noncommutative QFT at the Planck scale, where physical interactions are dominated by QG [[5\]](#page-7-3). This motivates the construction of models connecting the dynamical or Lorentz-induced CPTV to many fundamental differences between matter and antimatter [[6\]](#page-7-4). For example, they predict observable effects in the differences of the particle and antiparticle lifetimes [[7](#page-7-5)] or in interference patterns of neutral mesons [\[8](#page-7-6)[,9](#page-7-7)]. An idea underlying these approaches is that CPTV revises the very concept of antiparticle and introduces terms with a wrong CPT parity to the two-particle states.

Apart from the explicit CPTV in fundamental interactions, there is another general mechanism of accomplishing this effect related to the emergence of the thermodynamic arrow of time in systems propagating in a dissipative environment. This phenomenon can be studied in the density matrix formalism by following the time evolution of the density matrix expressed by the Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) equation [\[10](#page-7-8)[,11](#page-7-9)], which differs from the

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usual von Neumann formula by an additional, dissipative term. The GKSL evolution scheme results in a decoherence of two- or many-particle quantum states which should be observable with a sensitive detector. Such an effect, however, does not mean by itself that elementary interactions leading to decoherence violate CPT. A deeper interest in this phenomenon is again connected to QG, as being a universal, irreducible background of micro-black-holes at the Planck scale. These ideas were proposed theoretically in Refs. [[12](#page-7-10),[13](#page-7-11)] and further developed in Refs. [\[14](#page-7-12)[,15](#page-7-13)] as a model suitable for a coherent pair of neutral, heavy-flavored mesons.

In fundamental interactions, these effects can be experimentally approached at least in two ways. We can either study CPTV in the classical framework or in the effective field theory SME approach [[2](#page-7-1),[3,](#page-7-14)[16](#page-7-15)]. In the classical method, the CPTV parameter is assumed to be a complex constant, whereas in the SME, it exhibits a momentum and sidereal time dependence. In the latter approach, we are searching for CPTV induced by Lorentz-invariance violation. An extensive overview of SME-based tests of CPT symmetry can be found in Ref. [[4\]](#page-7-2). In this paper, we will stay within the phenomenological and classical framework where the exact source of CPTV is not directly specified.

The consequence of the CPT theorem is that the basic properties of a particle and its antiparticle such as masses, lifetimes, and magnetic momenta must be the same. This leads to experimental tests of CPT invariance in which one looks for tiny differences between matter and antimatter. Over the years, the searches for CPTV have been carried out with different experimental setups [[17](#page-7-16)–[23\]](#page-7-17), while new approaches have been developed [\[24](#page-7-18)–[29](#page-7-19)]. So far, no experimental evidence of CPTV has been found. Direct measurements are often experimentally difficult since strong and electromagnetic interactions dominate renormalized masses and lifetimes. Attractive possibilities are provided for CPT invariance tests in the neutral flavor mixing systems, such as neutrino and the neutral flavored meson mixing dominated by the second-order weak interactions. Various CPTV scenarios and experimental tests with flavored neutral mesons have been proposed [\[2](#page-7-1)[,7](#page-7-5),[8](#page-7-6),[24](#page-7-18)[,25](#page-7-20)[,30](#page-7-21)–[37](#page-7-22)].

In this paper, we provide new bounds for CPTV in decays of charmed mesons. These constraints were derived by reinterpreting experimental measurements of CP violation (CPV) and mixing in the neutral D meson system. More specifically, we reanalyzed the published LHCb results of the time-dependent asymmetry measurement in the Cabbibo-favored (CF)  $D^0 \rightarrow K^-\pi^+{}^1$  decays [[38](#page-7-23)]. The constraints reported in this paper supersede the current bounds obtained by the FOCUS Collaboration [\[17](#page-7-16)[,39\]](#page-7-24).

The rest of this paper is structured as follows. Section [II](#page-1-0) describes the mixing formalism for the neutral flavored mesons including the parametrization of the potential CPTV. Furthermore, the equations for the time-dependent asymmetry for uncorrelated neutral-flavored mesons are provided. Section [III](#page-3-0) includes a review of the existing experimental limits on CPTV. In Sec. [IV,](#page-4-0) the new bounds are presented, and the influence of the potential SMcompliant CPVeffect on the obtained bounds is considered. Section [V](#page-4-1) is dedicated to future prospects in the context of the ongoing data-taking campaign at the Large Hadron Collider. Finally, alternative analysis approaches such as the SME framework and interferometric measurements are briefly discussed.

## <span id="page-1-0"></span>II. NEUTRAL FLAVORED MESON TIME EVOLUTION FORMALISM

In this section, the formalism describing the time evolution of the neutral flavored mesons is introduced. It includes mixing effects and the parametrization of the CP, T, and CPT violation.

#### A. Neutral meson system

The neutral meson system is described by a linear combination of  $|P^0\rangle$  and  $|\overline{P^0}\rangle$  strong interaction eigenstates (flavor states). The state of the system can be represented by the ket  $|\Psi(t)\rangle$ . In the Weisskopf-Wigner approximation [[40](#page-7-25)], the evolution of this state is governed by an effective Hamiltonian  $H^{\text{eff}}$ , represented by a  $2 \times 2$ matrix, according to

$$
i\hbar \frac{d}{dt} |\Psi(t)\rangle = H^{\text{eff}} |\Psi(t)\rangle = \left(M - \frac{i\Gamma}{2}\right) |\Psi(t)\rangle, \quad (1)
$$

where  $\Gamma$  and M are Hermitian matrices. Here,  $\Gamma$  represents the exponential decay component, and  $M$  is the mass term. In this formalism, weak interactions are considered as a perturbation to the sum of the strong and electromagnetic parts of the Hamiltonian. The time evolution of flavor states is evaluated up to the second order in weak interactions, and  $H^{\text{eff}}$  is constructed to ensure the same evolution in the leading order of perturbation theory [\[41\]](#page-7-26). The physical propagating states are the eigenstates of the Hamiltonian  $H^{\text{eff}}$  and denoted as  $|P_{1,2}(t)\rangle$ , henceforth referred to as the mass states. They evolve in time as

$$
|P_{1,2}(t)\rangle = e^{-i\lambda_{1,2}t}|P_{1,2}\rangle, \tag{2}
$$

<span id="page-1-1"></span>where the complex parameters  $\lambda_{1,2}$  represent the eigenvalues of H<sup>eff</sup>, while  $|P_{1,2}\rangle \equiv |P_{1,2}(t=0)\rangle$ . Since  $|P_{1,2}\rangle$ have well-defined masses  $m_{1,2}$  and decay widths,  $\Gamma_{1,2}$ ,  $\lambda_{1,2}$ can be decomposed and represented as

$$
\lambda_{1,2} = m_{1,2} - i\Gamma_{1,2}/2. \tag{3}
$$

<sup>&</sup>lt;sup>1</sup>Throughout this paper, we assume the reaction is accompanied by its conjugate unless stated otherwise.

# B. Parametrizations of CPT and CP violation

The mass states can be expressed as linear combinations of flavor states. In this representation, effects of the CP, T, and CPT violation can be naturally parametrized by introducing different, complex weights for the time-independent flavor eigenstates  $|P^0\rangle$  and  $|\overline{P^0}\rangle$  representing the particle and antiparticle [[41](#page-7-26)]:

<span id="page-2-0"></span>
$$
|P_1\rangle = N_1 e^{i\eta_1} \left\{ |P^0\rangle + \sqrt{\frac{1+z}{1-z}} \frac{q}{p} |\overline{P^0}\rangle \right\},
$$
  

$$
|P_2\rangle = N_2 e^{i\eta_2} \left\{ |P^0\rangle - \sqrt{\frac{1-z}{1+z}} \frac{q}{p} |\overline{P^0}\rangle \right\},
$$
 (4)

where  $N_{1,2} = \{1 + |\sqrt{\frac{1 \pm z}{1 \mp z}}\}$  $\sqrt{\frac{1 \pm z}{1 \pm z}}$  $\frac{p}{q}$ 2  $\int_{0}^{-1/2}$  are the normalization terms, while  $\eta_{12}$  are phases that can be chosen freely. In this formalism,  $p$  and  $q$  parameters control T violation, with  $|\frac{q}{p}| = 1$  if and only if T is preserved. The parameter z<br>controls CDTV with  $z = 0$  if and only if CDT is concerned. controls CPTV, with  $z = 0$  if and only if CPT is conserved. Both conditions need to be fulfilled for the CP symmetry to hold.

<span id="page-2-1"></span>The time evolution of the flavor eigenstates can be explicitly accounted for by using Eqs. [\(2\)](#page-1-1), [\(4\),](#page-2-0)

$$
|P^{0}(t)\rangle = (g_{+}(t) + zg_{-}(t))|P^{0}\rangle - \sqrt{1-z^{2}}\frac{q}{p}g_{-}(t)|\overline{P^{0}}\rangle,
$$
  

$$
|\overline{P^{0}}(t)\rangle = (g_{+}(t) - zg_{-}(t))|\overline{P^{0}}\rangle - \sqrt{1-z^{2}}\frac{p}{q}g_{-}(t)|P^{0}\rangle, \quad (5)
$$

where  $g_{\pm}(t) = \frac{1}{2} (e^{-im_2 t - \Gamma_2 t/2} \pm e^{-im_1 t - \Gamma_1 t/2})$ . The time-de-<br>pendent decay probabilities of the flavor eigenstates (5) to pendent decay probabilities of the flavor eigenstates [\(5\)](#page-2-1) to the final states  $|f\rangle$  or the C-coupled  $|\bar{f}\rangle$  can now be calculated as  $P_f(t) = |\langle f|T|P^0\rangle|^2$ ,  $\bar{P}_{\bar{f}}(t) = |\langle \bar{f}|T|\overline{P^0}\rangle|^2$ .

<span id="page-2-2"></span>Matrix elements of the effective Hamiltonian can be expressed using the  $p, q, z$  parameters [[35](#page-7-27)]:

$$
H^{\text{eff}} = \frac{1}{2} \Delta \lambda \left( \frac{\lambda/\Delta \lambda + z}{\sqrt{1 - z^2} p/q} - \frac{\sqrt{1 - z^2} q/p}{\lambda/\Delta \lambda - z} \right), \quad (6)
$$

where  $\lambda = \lambda_1 + \lambda_2$  and  $\Delta \lambda = \lambda_2 - \lambda_1$ .

As discussed in Refs. [[41](#page-7-26),[42](#page-7-28)], CPTVoccurs if and only if the diagonal elements of  $H^{\text{eff}}$  differ, whereas T violation requires that the moduli of the off-diagonal elements are different. Using Eq. [\(6\)](#page-2-2), one can parametrize these two conditions as

$$
H_{11}^{\text{eff}} - H_{22}^{\text{eff}} = z\Delta\lambda
$$
  

$$
|H_{12}^{\text{eff}}| - |H_{21}^{\text{eff}}| = \left| \frac{\Delta\lambda}{2} \sqrt{1 - z^2} \right| (|q/p| - |p/q|) \quad (7)
$$

and express the two parameters quantifying CPT and T violation, z and  $q/p$ , via elements of the effective <span id="page-2-5"></span>Hamiltonian  $H^{\text{eff}}$  and its eigenvalues<sup>2</sup>:

$$
z = \frac{\delta m - i\delta \Gamma/2}{\Delta \lambda},
$$
  
 
$$
q/p = \frac{i \Im(M_{12}) - \frac{1}{2} \Im(\Gamma_{12})}{\Delta \lambda},
$$
 (8)

<span id="page-2-6"></span>where  $\Delta m$  and  $\Delta \Gamma$  are mixing parameters defined as differences in mass and decay rates between the two mass states. They can be written in the dimensionless form, suitable for the analysis of  $D^0$  mesons, as

$$
x = \frac{\Delta m}{\Gamma}, \qquad y = \frac{\Delta \Gamma}{2\Gamma}, \tag{9}
$$

where  $\Delta m = m_2 - m_1$ ,  $\Delta \Gamma = \Gamma_2 - \Gamma_1$ , and  $\Gamma = \frac{\Gamma_1 + \Gamma_2}{2}$  is the average decay width. The introduced formalism applies to all average decay width. The introduced formalism applies to all neutral flavored pseudoscalar mesons. In particular, it can be used to describe the time evolution of the  $D^0$  meson system.

#### C. Time-dependent asymmetries

It can be shown that for studies of CPTV with the uncorrelated flavored neutral mesons the relevant observable is the time-dependent asymmetry, which can be constructed by combining the time-dependent decay probabilities in the following way [[43](#page-7-29),[44](#page-7-30)]:

$$
A_{CPT}(t) = \frac{\bar{P}_{\bar{f}}(t) - P_f(t)}{\bar{P}_{\bar{f}}(t) + P_f(t)}.
$$
 (10)

<span id="page-2-3"></span>If we restrict our attention to flavor-specific final states  $(P_{\bar{f}} = 0, \bar{P}_f = 0)$ , Eq. [\(11\)](#page-2-3) simplifies to

 $A_{\text{CPT}}(t)$ 

$$
=A_{\text{dir}}+\frac{2\Re(z)\sinh\Delta\Gamma t/2-2\Im(z)\sin\Delta mt}{(1+|z|^2)\cosh\Delta\Gamma t/2+(1-|z|^2)\cos\Delta mt},\quad(11)
$$

where all asymmetries, including the direct CPTV term  $A_{\text{dir}}$ , are assumed to be much smaller than 1. In this case, the measurement of the CPTV is independent of the CPV effects.

The oscillation in the neutral charm meson system is very slow compared to the  $D^0$  decay time  $\tau$ , since the corresponding mixing parameters x and y are of the order of  $~\sim 0.5\%$  [\[45\]](#page-7-31). This allows us to further simplify [\(11\)](#page-2-3) by approximating it to the leading order in these parameters  $(x\tau\Gamma \ll 1$  and  $y\tau\Gamma \ll 1$ ):

<span id="page-2-4"></span>
$$
A_{\rm CPT}(t) = A_{\rm dir} + (\Re(z) y - \Im(z) x) \Gamma t. \tag{12}
$$

 $\lambda^2 \delta m = m_{P^0} - m_{\overline{P^0}}$  and  $\delta \Gamma = \Gamma_{P^0} - \Gamma_{\overline{P}^0}$  represent the ferences of the diagonal elements of M and  $\Gamma$  matrices. differences of the diagonal elements of  $M$  and  $\Gamma$  matrices.

The CPTV term in Eq. [\(12\)](#page-2-4) corresponds to the slope of the line describing the dependence of the asymmetry on decay time.

# <span id="page-3-4"></span>D. Influence of CP violation on CPT asymmetry

Studies presented here are based on the two-hadron decays of  $D^0$  mesons:  $D^0 \rightarrow K^-\pi^+$  and its CPTconjugate  $\overline{D^0}$  →  $K^+\pi^-$ . Decaying mesons originate from the strongly decaying  $D^*$  mesons:  $D^{*+}(2010) \rightarrow D^0 \pi^+$ and  $D^{*-}(2010) \rightarrow \overline{D^0} \pi^-$ . Electric charge of the lowmomentum (so-called *soft*) pion indicates the initial flavor, while the final flavor is determined by the pion charge from the  $D^0$  decay. The soft and final pions are used for flavor tagging. When both tagging pions are of the same sign, the flavors of the initial and final states coincide. An asymmetry determined from such events is called the right-sign (RS) asymmetry and is equal to

<span id="page-3-1"></span>
$$
A_{\rm RS}(t) = \frac{N(D^0 \to K^- \pi^+)(t) - N(\overline{D^0} \to K^+ \pi^-)(t)}{N(D^0 \to K^- \pi^+)(t) + N(\overline{D^0} \to K^+ \pi^-)(t)}.
$$
 (13)

The  $A_{RS}$  asymmetry [\(13\)](#page-3-1) is dominated by the direct CF decays of  $D^0$  mesons, which makes it almost fully flavor specific. The only contamination is due to the cases where  $D^0$  mesons oscillate into  $\overline{D^0}$  and subsequently decay via a Doubly-Cabbibo suppressed (DCS) mode  $\overline{D^0} \rightarrow K^- \pi^+$ . However, the smallness of  $D^0 \rightarrow K^+ \pi^$ branching fraction,  $1.36 \times 10^{-4}$  for DCS as compared to  $3.947 \times 10^{-2}$  [[39\]](#page-7-24) for CF, combined with the slow mixing (the oscillation period is 3 orders of magnitude larger than the  $D^0$  decay time) makes this admixture so small that it is usually neglected in the analyses. Nevertheless, with the increasing experimental precision, these phenomena could become important. Indeed, the potential effects of mixing and time-dependent CPV in  $D^0 \rightarrow K^+\pi^-$  channel are discussed in detail in Refs. [[46,](#page-7-32)[47\]](#page-7-33) in the context of  $y_{CP}$  measurements.

<span id="page-3-3"></span>In our case, the possible SM-compliant CPV would manifest as an additional linear time-dependent term in the asymmetry that could mimic the CPTV effect. Taking into account the DSC contribution, the asymmetry in Eq. [\(12\)](#page-2-4) becomes

$$
A_{\text{CPT}}(t) = A_{\text{dir}} + (\Re(z)y - \Im(z)x)\Gamma t - \frac{\sqrt{R_{\text{DCS}}}}{2}\sqrt{|1 - z^2|}
$$

$$
\times \left\{ \sin \phi[x \cos(\delta + \kappa) - y \sin(\delta + \kappa))] \right\}
$$

$$
\times \left( \left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) - \cos \phi[y \cos(\delta + \kappa)
$$

$$
+ x \sin(\delta + \kappa)) \left( \left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) \right\} \Gamma t, \tag{14}
$$

where  $\kappa = 0.5$  arg $(1 - z^2)$ ,  $R_{\text{DCS}} = |A_{\text{DCS}}/A_{\text{CF}}|^2$ , the weak phase difference  $\phi \equiv \arg(q/p)$ , and the strong phase difference  $\delta \equiv \arg(A_{CF}/A_{DCS})$ . Since both CF and DCS decays in the SM proceed via tree-level amplitudes dominated by a single weak phase, the direct CPV term can be neglected [[39](#page-7-24)]. Generally, the mixing and CPV effects in charm are strongly suppressed in the SM predictions; however, they are rather imprecise because of the difficulty in calculation of contributions dominated by the long-distance interactions [\[39\]](#page-7-24).

## <span id="page-3-0"></span>III. EXPERIMENTAL STUDY OF CPT VIOLATION IN CHARM DECAYS

The RS asymmetry  $A_{RS}$  [\(13\)](#page-3-1) is an experimental observable used in our studies. The first experimental search for the CPTV was performed by the FOCUS Collaboration [[17](#page-7-16)]. The charm mesons were produced in the interaction of the 180 GeV photons with the fixed target. The  $A_{RS}(t)$  is presented in Fig. [1](#page-3-2) (upper plot). Because of the limited event sample consisting of 35,000 event candidates, the estimated limits for the expression  $\Re(z) - \Im(z)$  provided rather loose bounds of the order  $\mathcal{O}(1)$  but until now represented the best CPT upper limit in the charm sector. The FOCUS analysis was also performed in the SME ([[35](#page-7-27)]) framework. Because of its abundance, the CF mode  $D^0 \rightarrow$  $K^-\pi^+$  was used as control channel by several analyses performed by the LHCb Collaboration [[48](#page-7-34)–[50\]](#page-7-35). In particular, the RS asymmetry was determined as a reference

<span id="page-3-2"></span>

FIG. 1. Experimental, time-dependent  $A_{RS}$  asymmetry of  $D^0 \rightarrow K^-\pi^+$  candidates determined by the FOCUS (top plot) and by the LHCb (bottom plot) Collaborations. In the bottom plot the result of the linear fit with the  $\chi^2$  per number of degrees of freedom  $(n.d.f.)$  equal to  $17/19$  is shown. The figures are adapted from Refs. [\[17,](#page-7-16)[38](#page-7-23)], respectively.

<span id="page-4-2"></span>

FIG. 2. Experimental limits on the  $CPT$ -violating z parameter at the 95% CL in the  $D^0 \rightarrow K^-\pi^+$  channel by the FOCUS Collaboration [\[17\]](#page-7-16) and deduced from the LHCb measurement [\[38\]](#page-7-23) denoted as red and green areas, respectively. The uncertainty originating from the possible SM-compliant CPV is shown as a blue area.

observable (see Fig. [1](#page-3-2) bottom plot) in the analysis of the asymmetries in the time-dependent rates of  $D^0 \to K^+K^$ and  $D^0 \rightarrow \pi^+\pi^-$  decays measured in a pp collision data sample collected with the LHCb detector [\[38\]](#page-7-23).

In this paper, we use the  $A_{RS}$  to extract bounds on the CPTV parameter z. Because of the abundance of the LHCb sample and the good control of systematic errors, we can significantly improve the accuracy of the CPT invariance.

## <span id="page-4-0"></span>IV. NEW BOUNDS ON CPT VIOLATION IN THE CHARM SECTOR

According to the model defined in Eq. [\(12\),](#page-2-4) the slope of the linear fit to the time-dependent RS asymmetry (Fig. [1](#page-3-2), bottom plot) can be identified as the  $y\Re(z) - x\Im(z)$  term. The value of the slope determined from the fit to data is equal to  $s = (-4 \pm 5 \pm 2) \times 10^{-5}$ , where the first uncertainty is statistical and the second is systematic [381]. The tainty is statistical and the second is systematic [[38](#page-7-23)]. The result is consistent with zero within one statistical standard deviation. The real and imaginary parts of  $z$  cannot be disentangled from this fit without further assumptions.

Using the value of s, one gets a linear dependence of the imaginary and real parts of z as  $\Im(z) = y \Re(z)/x - s/x$  in the  $(\Re(z), \Im(z))$  plane. The determined constraint region at 95% confidence level (CL), is presented in Fig. [2](#page-4-2) as a green area, where the statistical and systematic uncertainties of all quantities were added in quadrature. The previous strongest bound established by FOCUS Collaboration is denoted as the red area, and the possible SM-compliant CPV is shown in blue. The hypothetical influence of the CP violation is discussed in the next section. The values of  $x$  and  $y$  mixing parameters are taken from the HFLAV group [[45](#page-7-31)]. The

<span id="page-4-3"></span>TABLE I. Parameters entering the CPV term in Eq. [\(14\)](#page-3-3) taken from the HFLAV fit [[45](#page-7-31)].

| $x(\%)$ | $v(\%)$   | $\delta$ ( $\degree$ ) | $R_{\rm DCS}$ (%) | q/p | $\phi$ (°) |
|---------|---|------------------------|-------------------|-----|------------|
|         | $0.407_{-0.044}^{+0.044}$ $0.645_{-0.023}^{+0.024}$ $11.4_{-3.8}^{+3.5}$ $0.344_{-0.002}^{+0.002}$ $0.994_{-0.015}^{+0.016}$ $-2.6_{-1.2}^{+1.1}$ |                        |                   |     |            |

uncertainties of the mixing parameters are taken into account by assuming the Gaussian prior with the standard deviation set to the parameter uncertainties. We note that the production and detection asymmetries enter as a timeindependent additive factor that does not affect the final slope [\[38\]](#page-7-23).

The parameters needed to estimate the effect of CPV are taken from the HFLAV fits [\[45\]](#page-7-31) and gathered in Table [I](#page-4-3). Inspecting the values of the  $\phi$  and  $|q/p|$ , one sees again that the expected CP effect is tiny but the uncertainties are rather large. Assuming that the CPTV is small, i.e.,  $|z|^2 \ll 1$ , and taking into account the HFLAV parameters<br>with their uncertainties, we estimated the 95% CL interval with their uncertainties, we estimated the 95% CL interval for the CP-related term proportional to  $\sqrt{R_{\text{DCS}}}$  in Eq. [\(14\)](#page-3-3) to be  $(-1.0 \pm 2.0) \times 10^{-5}$ .<br>This result is consistent

This result is consistent with the one determined in Ref. [[38](#page-7-23)]. The CP-related term in the  $(\Re(z), \Im(z))$  space is represented as a blue band in Fig. [2.](#page-4-2) At the current experimental accuracy level, the CPV term is still small; however, it can become significant in future analyses with higher statistics. On the other hand, with more collected data from LHCb, Belle-2, and BES-III, CPV bounds may be tightened by the independent measurements.

#### V. DISCUSSION AND PROSPECTS

<span id="page-4-1"></span>The constraints on CPTV restrict the space of the possible values of particle-antiparticle mass and decay width differences,  $\delta m$  and  $\delta \Gamma$ , respectively via Eq. [\(8\)](#page-2-5). The slope of the RS asymmetry is directly connected to  $\delta\Gamma$  (see Appendix). This relationship can be used to constrain the decay width difference at 95% CL:  $(-4.7 < \delta \Gamma < 2.1) \times$  $10^{-16}$  GeV. In contrast, without further assumptions,  $\delta m$ bounds cannot be deduced from the linear term of asymmetry. For instance,  $\delta m$  can be expressed as a function of phase  $\theta \equiv \arg(z)$  [cf. Eq. [\(A8\)\]](#page-6-0):

<span id="page-4-4"></span>
$$
\delta m(\theta) = s \times \frac{x \cos(\theta) + y \sin(\theta)}{y \cos(\theta) - x \sin(\theta)}.
$$
 (15)

This dependence leads to  $\delta m$  confidence intervals corresponding to the 95% CL shown as a green area in Fig. [3](#page-5-0). For the two angles where the linear term of asymmetry vanishes,  $\delta m(\theta)$  is undefined. The singular points in Fig. [3](#page-5-0) are marked with red dashed lines. Outside this neighborhood, the estimated  $\delta m$  is bounded by values of the order of  $10^{-15} - 10^{-16}$  GeV. More specifically, this mass difference lies within  $(-2.0 < \delta m < 2.0) \times 10^{-15}$  GeV excluding 5%

<span id="page-5-0"></span>

FIG. 3. Distribution of the mass difference  $\delta m$  parameter as a function of z phase [\(15\)](#page-4-4). The green area denotes the 95% CL. Red dashed lines show the arg $(z)$  values for which  $\delta m$  is undefined since the linear asymmetry term vanishes.

of arg(z) corresponding to the largest  $|\delta m|$  values. In particular, the assumption that  $\Re(z) = 0$  results in the following constraint:  $(-1.6 < \delta m < 3.5) \times 10^{-16}$  GeV.

The new constraints in the charm sector can be juxtaposed with the current experimental limits on CPTV in other neutral flavored meson systems [[39](#page-7-24)]. The experimental precision to  $\delta m$  and  $\delta \Gamma$  is determined by the interplay of the relative values of the decay timescale and the mixing parameters, which differ considerably between meson families as discussed in Sec. [III.](#page-3-0) The strange sector is the most favorable from the point of view of CPTV studies. Indeed, the smallness of the denominator value in Eq. [\(8\)](#page-2-5) for the neutral kaons strongly amplifies sensitivity to  $\delta m$  and  $\delta \Gamma$ and results in a precision of about  $10^{-18}$  GeV [\[51\]](#page-7-36). The comparison between 95% CL limits for the decay width differences is shown in Fig. [4](#page-5-1). The constraint on  $\delta\Gamma$  in the  $D^0$ system, deduced in this article, reaches  $10^{-16}$  GeV. In the case of the bottom and bottom-strange systems, no direct  $\delta\Gamma$ measurements are available, and their values are obtained from the combination of  $\Re(z)$  and  $\Im(z)$  measured by Belle,  $BABAR$ , and LHCb collaborations. The result for  $B$  is of the order of ~10<sup>-14</sup> GeV [[52](#page-7-37),[53](#page-7-38)], while for  $B_s$ , it reaches the value at the level of  $\sim$ 10<sup>-12</sup> GeV [\[54\]](#page-7-39). The value of CPTV coupling can, in principle, vary depending on the meson quark content and therefore can be different for each flavor family [[43](#page-7-29)].

In the ongoing data-taking period at the Large Hadron Collider, the collected statistics of  $D^0 \to K^-\pi^+$  candidates should reach  $3 \times 10^9$  [\[55,](#page-7-40)[56\]](#page-7-41). Naive extrapolation of the statistical uncertainty for the charm CPT bounds gives a scale-down factor of 2.45. At that moment, the statistical uncertainty would reach the systematic error level. The bulk part of the latter is statistics dependent (cf. Ref. [[38](#page-7-23)]) and, hopefully, could be further reduced. The uncertainty of the DCS component, connected to CPV, should also be

<span id="page-5-1"></span>

FIG. 4. Current experimental bounds at 95% CL on particleantiparticle decay width difference  $\delta \Gamma$ . The right-hand plot is the same as the left-hand one but on a smaller scale, with the FOCUS result removed. In the case of  $D$  and  $K$  the direct measurement results are available while for B the limits are obtained from  $\Re(z)$ and  $\Im(z)$  Particle Data Group (PDG) averages according to the formula  $\delta \Gamma = 2\Gamma[\Re(z)y - \Im(z)x]$ . The bounds for  $B_s$  are much broader and are not shown.

further constrained. However, as discussed in Sec. [II D](#page-3-4), the precision of the future CPT measurement in the charm sector is affected by the possible SM-compliant CPVeffects, which would mimic the CPT behavior and must be taken into account in future analyses. In the SME framework, the dependence on particle boost and sidereal time is expected to be independent of the CPV phenomenon [[35](#page-7-27)]. This fact allows for the removal of the aforementioned artificial CPVinduced effects. A detailed discussion concerning the prospects of tests of CPT invariance within the SME framework lies outside of the scope of this article and will be presented in a separate review.

The interferometric studies of the coherent  $D^0$  pairs time evolution are an attractive alternative to the CPT tests with the uncorrelated mesons [\[2](#page-7-1)[,36,](#page-7-42)[57\]](#page-7-43). Application of the filtering method based on the final-state flavor and CP tagging [[58](#page-7-44)–[61](#page-7-45)] would provide a way to separately test CP, T, and CPT symmetries in a manner analogous to the measurements performed with B and K mesons  $[21,53,62]$  $[21,53,62]$  $[21,53,62]$  $[21,53,62]$ . However, in experiments such as BES-III [\[63\]](#page-7-48) where the quantum coherent  $D^0$  pairs are copiously produced in the  $\psi(3770)$  decay, lifetime analysis measurements are very challenging, due to low momenta in the laboratory frame of the  $D^0$  mesons produced in the symmetric  $e^+e^-$  collisions. Hence,  $D^0$  time-of-flight distances are impossible to measure. Such types of investigations will be feasible in future facilities operating at the asymmetric  $e^+e^-$  colliders such as the Super  $\tau$ -Charm facility [\[64\]](#page-7-49). Another interesting idea of quantum-correlated measurement is discussed in Ref. [[65](#page-7-50)] where it is proposed to exploit the quantumcorrelated  $D^0$  pairs produced in  $\chi_{c1}(3872) \rightarrow D^0 \overline{D^0} \pi^0$ decay. In this case, the meson pair can be described by a symmetric, i.e.,  $C = 1$ , wave function, in contrast to the asymmetric  $C = -1$  function describing the decay products of  $\psi(3770)$ . Analogously to the filtering method, a set of

transition conjugate decay modes is proposed. It was suggested that the analysis would be feasible in the facilities operating today, such as LHCb or Belle-II.

## VI. SUMMARY

We have established new constraints on CPT violation in neutral charm meson mixing through a reinterpretation of the RS time-dependent experimental asymmetry in decay rates for  $D^0 \to K^-\pi^+$  and  $\overline{D^0} \to K^+\pi^-$ . The determined 95% CL limit  $(-14.6 < \Re(z)y - \Im(z)x < 6.6) \times 10^{-5}$  is 2 orders of magnitude stricter compared to the previous bounds set by FOCUS. This result corresponds to precision of the order of  $10^{-15} - 10^{-16}$  GeV in terms of particleantiparticle mass and decay time differences. By taking the mixing parameters  $x$  and  $y$  from HFLAV, one finds a 95% CL limit for the decay width difference equal to  $(-4.7 < \delta \Gamma < 2.1) \times 10^{-16}$  GeV. The bounds on the mass difference cannot be extracted without additional assumptions. For example, assuming  $\Re(z) = 0$ , one finds  $(-1.6 <$  $\delta m < 3.5 \times 10^{-16}$  GeV at 95% CL. These results represent the most stringent constraint on CPTV in the charm sector. The potential presence of SM-compliant CPV terms and their uncertainty can limit the sensitivity of future phenomenological CPT analyses using larger statistical samples. The analysis in the SME framework should be less susceptible to these effects.

## APPENDIX: RELATION BETWEEN THE CPT LINEAR TERMS AND δm, δΓ PARAMETERS

<span id="page-6-1"></span>Let us consider the linear terms of the RS asymmetry corresponding to the slope in Eq. [\(12\)](#page-2-4)  $s = \text{Re}(z)y -$ Im $(z)x$ . We derive the expression on the  $\delta m$  and  $\delta \Gamma$ parameters. The  $CPT$ -violating parameter z can be expressed as

$$
z = \frac{\delta m - i\delta \Gamma/2}{\Delta \lambda} = \frac{\delta m - i\delta \Gamma/2}{\Delta m - \Delta \Gamma/2},
$$
 (A1)

<span id="page-6-2"></span>Combining Eqs. [\(A1\)](#page-6-1) and [\(9\),](#page-2-6) we obtain the following relation between z and  $\delta m$  as well as  $\delta \Gamma$ :

$$
z = \frac{\delta m - i\delta \Gamma/2}{\Gamma(x - iy)} = \frac{(x + iy)(\delta m - i\delta \Gamma/2)}{\Gamma(x^2 + y^2)}.
$$
 (A2)

<span id="page-6-3"></span>Taking the real and imaginary values of expression [\(A2\)](#page-6-2), we obtain

$$
Re(z) = \frac{x\delta m + y\delta\Gamma/2}{\Gamma(x^2 + y^2)},
$$
 (A3)

$$
\operatorname{Im}(z) = \frac{y\delta m - x\delta\Gamma/2}{\Gamma(x^2 + y^2)}.
$$
 (A4)

<span id="page-6-4"></span>Let us write the expression  $\text{Re}(z)y - \text{Im}(z)x$  using Eqs.  $(A3)$  and  $(A4)$ :

$$
s = \text{Re}(z)y - \text{Im}(z)x
$$
  
= 
$$
\frac{(yx\delta m + y^2 \delta \Gamma/2)}{\Gamma(x^2 + y^2)} - \frac{(yx\delta m - x^2 \delta \Gamma/2)}{\Gamma(x^2 + y^2)} = \frac{\delta \Gamma}{2\Gamma}.
$$
 (A5)

The slope of the asymmetry is directly related to the decay width difference  $\frac{\delta \Gamma}{2\Gamma}$ .

To express the relation for  $\delta m$ , an additional assumption, for instance, about the z phase  $\theta \equiv \arg(z)$  is needed. Let us start with the case Re $(z) = 0 \leftrightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$ . This leads to the following expression: following expression:

$$
Re(z) = 0 \Rightarrow \frac{\delta m}{\Gamma} = -\frac{y}{x} \frac{\delta \Gamma}{2\Gamma} = -\frac{y}{x} s.
$$
 (A6)

<span id="page-6-5"></span>If  $Re(z) \neq 0$ , then using Eqs. [\(A3\)](#page-6-3) and [\(A4\)](#page-6-4), we can express  $tan(\theta)$  as

$$
\tan(\theta) = \frac{\text{Im}(z)}{\text{Re}(z)} = \frac{y\delta m - x\delta\Gamma/2}{x\delta m + y\delta\Gamma/2}.
$$
 (A7)

<span id="page-6-0"></span>Using the Eq.  $(A7)$ ,  $\delta m$  can be expressed:

$$
\frac{\delta m}{\Gamma}(\theta) = \frac{\delta \Gamma}{2\Gamma} \times \frac{x + y \tan(\theta)}{y - x \tan(\theta)} = s \times \frac{x + y \tan(\theta)}{y - x \tan(\theta)}.
$$
 (A8)

Equation [\(A8\)](#page-6-0) has an undefined value for  $y = \tan(\theta)x$ , which corresponds to the situation where the linear term vanishes, and the higher order terms of the RS asymmetry must be taken into account. This is a case of the SME framework [\[43\]](#page-7-29).

Finally, for the particular case of  $\theta = 0$ ,  $\pi \leftrightarrow \text{Im}(z) = 0$ , we recover the solution:

$$
\frac{\delta m}{\Gamma} = s \times \frac{x}{y}.\tag{A9}
$$

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