Purely leptonic decays of heavy-flavored charged mesons

Kalpalata Dash,[1,*] P. C. Dash,[1] R. N. Panda,[1] Susmita Kar,[2] and N. Barik[3]

[1]*Department of Physics, Siksha 'O' Anusandhan Deemed to be University, Bhubaneswar-751030, India*
[2]*Department of Physics, Maharaja Sriram Chandra Bhanja Deo University, Baripada-757003, India*
[3]*Department of Physics, Utkal University, Bhubaneswar-751004, India*

(Received 13 May 2024; accepted 14 August 2024; published 10 September 2024)

We study the purely leptonic decays of heavy-flavored charged pseudoscalar (P) and vector (V) mesons ($D^{(*)+}_{(s)}$, $B^{(*)+}_{(c)}$) in the relativistic independent quark (RIQ) model based on an average flavor-independent confining potential in an equally mixed scalar-vector harmonic form. We first compute the mass spectra of the ground-state-mesons and fix the model parameters necessary for the present analysis. Using the meson wave functions derivable in the RIQ model, and model parameters so fixed from hadron spectroscopy, we predict the decay constants: $f_{P(V)}$, ratios of decay constants: f_V/f_P, f_{P_1}/f_{P_2}, f_{V_1}/f_{V_2}, and the branching fractions (BFs): $\mathcal{B}(P(V) \to l^+\nu_l)$, $l = e, \mu, \tau$, which agree with the available experimental data and other Standard Model (SM) predictions. For the unmeasured decay constants, especially in the purely leptonic decays of the charged vector mesons, our predictions could be tested in the upcoming Belle-II, SCTF, CEPC, FCC-ee, and LHCb experiments in the near future.

DOI: 10.1103/PhysRevD.110.053004

I. INTRODUCTION

In the last few decades, the heavy-flavored mesonic systems have attracted a great deal of attention as they provide important information on the determination of fundamental parameters of the Standard Model (SM). In general, the purely leptonic charged pseudoscalar (P) and vector (V) meson decays with the final lepton-neutrino pair, or lepton-lepton pair are considered as rare decays, which have simpler physics than hadronic decays. These decay rates are expressed in terms of the weak decay constants $f_{P(V)}$, which bear considerable theoretical and phenomenological importance as they govern the strength of leptonic and nonleptonic meson decays, determine the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements, and help in the description of the neutral $D - \bar{D}$ and $B - \bar{B}$ mixing process. The precise determination of these decay constants also helps us to test the unitarity of the quark mixing matrix and study CP violation in the SM [1].

The experimental measurements of decay constants for purely leptonic decays of heavy-flavored charged mesons have so far made limited progress. While the decay constants f_{D^+} [2–7], $f_{D_s^+}$ [4,5,8–15], $f_{B_u^+}$ [16–19] in the pseudoscalar sectors have so far been measured by CLEO-c, Belle, *BABAR*, BESIII Collaborations, $f_{B_c^+}$ is yet to be measured. In the vector meson sector, only $f_{D_s^{*+}}$ have been measured recently by BESIII Collaborations [20], whereas $f_{D^{*+}}$ and $f_{B^{*+}_{(C)}}$ have not been measured so far. The BFs for $D^+_{(s)} \to \mu^+\nu_\mu, \tau^+\nu_\tau$, and $B_u^+ \to \tau^+\nu_\tau$ have been precisely measured by CLEO, Belle, *BABAR*, and BESIII. For $D^+_{(s)} \to e^+\nu_e$ and $B_u^+ \to e^+\nu_e, \mu^+\nu_\mu$, only the upper bound of BFs are available now. This is because the BFs being proportional to m_l^2, these decay processes suffer from strong helicity suppression. In the vector meson sector, only the BFs of $D_s^{*+} \to e^+\nu_e$ has been measured recently by BESIII Collaboration [20]. From the currently available data statistics, it is expected that BFs for $D^{*+} \to e^+\nu_e$, $D^{*+}_{(s)} \to \mu^+\nu_\mu, \tau^+\nu_\tau$ could be carefully investigated at Belle-II, SCTF or STCF, CEPC, FCC-ee, LHCb future experiments.

The experimental information about b—flavored mesons are indeed scarce. The possibility of an experimental investigation on purely leptonic decays of B_u, B_u^*, B_c, B_c^* mesons discussed in [21] can be summarized as follows. Considering about 10^{13} Z bosons at FCC-ee [22] and BF $\mathcal{B}(Z \to b\bar{b}) = 12.03 \pm 0.21\%$ [23], and assuming the fragmentation fraction $f(b \to B_u^*) \sim 20\%$ [24], more than 4×10^{11} B_u^* events are expected, which can hopefully search for the $B_u^{*+} \to l^+\nu_l$ decays. In addition, the b—quark production cross section of about $\sigma(pp \to b\bar{b}X) \simeq$ 495 μb at the center-of-mass energy $\sqrt{s} = 13$ TeV is found at LHCb [25,26]. That is expected to yield more than

*Contact author: kalpalatadash982@gmail.com

5×10^{13} B_u^* events with a dataset of 300 fb^{-1} at LHCb and fragmentation fraction $f(b \to B_u^*) \sim 20\%$. This indicates that the $B_u^{*+} \to e^+\nu_e, \mu^+\nu_\mu, \tau^+\nu_\tau$ could be investigated at FCC-ee and LHCb future experiments. For experimental investigation of $B_c^{*+} \to l^+\nu_l$ decays, there should be at least more than 10^7 B_c^* events available. As of now, it is expected that more than 10^{12} Z bosons can be available at the future e^+e^- colliders of CEPC [27] and FCC-ee [22]. With BF $\mathcal{B}(Z \to b\bar{b}) = 12.03 \pm 0.21\%$ [23] and fragmentation fraction $f(b \to B_c^*) \sim 6 \times 10^{-4}$ [28–30], there will more than 10^8 B_c^* events to search for $B_c^{*+} \to e^+\nu_e, \mu^+\nu_\mu, \tau^+\nu_\tau$ decays. In addition, the B_c^* production cross sections at LHC are estimated to be about 100 nb for pp collisions at $\sqrt{s} = 13$ TeV [31], yielding more than 3×10^{10} B_c^* events corresponding to a dataset of 300 fb^{-1} at LHCb. Hence, the $B_c^{*+} \to e^+\nu_e, \mu^+\nu_\mu, \tau^+\nu_\tau$ decays are expected to be carefully measured at LHCb experiments in the future.

An important issue in flavor physics in recent years is to test lepton flavor universality by calculating the ratios of BFs: $\mathcal{R}_\mu^\tau$. For purely leptonic decays of charged mesons (PLDCMs), $D_{(s)}^+ \to \tau^+\nu_\tau$ to $D_{(s)}^+ \to \mu^+\nu_\mu$, the available experimental data [23] $(\mathcal{R}_\mu^\tau)^D = \frac{\mathcal{B}(D^+ \to \tau^+\nu_\tau)}{\mathcal{B}(D^+ \to \mu^+\nu_\mu)} = 3.21 \pm 0.73$, $(\mathcal{R}_\mu^\tau)^{D_s} = \frac{\mathcal{B}(D_s^+ \to \tau^+\nu_\tau)}{\mathcal{B}(D_s^+ \to \mu^+\nu_\mu)} = 9.82 \pm 0.40$ are consistent with SM expectations. Similar observables, such as $(R_\mu^\tau)^{D_{(s)}^*}$, $(R_\mu^\tau)^{B_{(c)}}$, and $(R_\mu^\tau)^{B_{(c)}^*}$, have not yet been measured.

The theoretical description of the PLDCMs requires a nonperturbative approach as the interactions at short distances are mediated by strong force. Within the SM, the PLDCMs are typically induced by the tree-level exchange of the gauge boson W, which subsequently decays to a charged lepton and a lepton neutrino. The decay amplitudes represented by the decay constants $f_{P(V)}$ of the PLDCMs are evaluated from the quark and antiquark wave functions at the origin, which cannot be computed from the first principle. Therefore, alternate routes, based on various theoretical and phenomenological approaches, have predicted these decays. Some of them are based on the nonrelativistic quark model (NRQM) [32–39], relativistic quark model (RQM) [36–38,40–44], Bethe-Salpeter (BS) [45–47] formalism, light front quark model (LFQM) [48–58], light cone quark model (LCQM) [59,60], light front holographic QCD (LFHQCD) [61,62], QCD sum rule (SR) [63–70], and lattice QCD (LQCD) [71–84] approaches. The predictions of the decay constants $f_{P(V)}$ and BFs in different theoretical and phenomenological approaches have been obtained in wide ranges.

In view of the potential prospects of increased data statistics in high luminosity experiments at Belle-II, SCTF, CEPC, FCC-ee and LHCb, which are likely to yield careful measurement of the yet unmeasured decay constants and BFs, several theoretical attempts, as cited in the literature, have been taken in studying the PLDCMs. This has inspired us to undertake the present study of the PLDCMs ($D_{(s)}^+$, $B_{(c)}^+$ and $D_{(s)}^{*+}$, $B_{(c)}^{*+}$) in the framework of our relativistic independent quark (RIQ) model. Wide-ranging properties of hadrons have been described in the earlier application of the RIQ model, which includes their static properties [85–89], namely the electromagnetic form factors, charge radii of pion and kaon, and pion decay constant and their decay properties [90–103], such as the weak radiative, rare radiative, weak leptonic, radiative leptonic, electromagnetic, and semileptonic and nonleptonic decays of c—and b—flavored mesons. Here, we would like to predict $f_{P(V)}$, BFs and (R_μ^τ)s and compare our results with the available experimental data and other theoretical model predictions. For the yet unmeasured decay modes, our predictions may be useful for experimental testing in the future.

In the present study, we consider that (1) the quark-antiquark pair inside the decaying meson-bound state annihilate to massive W- boson, which subsequently decays to a lepton pair $(l\bar{\nu}_l)$. Although, in principle, the decay can take place at any arbitrary momentum $\vec{k}$ of the parent meson, for the sake of simplicity, we consider the decay in its rest frame. We also believe that (2) there exists a strong correlation between the quark and antiquark momenta so as to have their total momentum identically zero in the decaying meson rest frame. Here, of course, an obvious difficulty arises in the context of the energy conservation at the hadron- boson vertex since the total kinetic energy of the annihilating quark-antiquark pair is not equal to the rest mass energy of the decaying meson. In the absence of any rigorous field-theoretic treatment of the quark-antiquark annihilation inside the meson, such difficulty arises as a common feature in all phenomenological model descriptions based on leading order calculation. This leads us to believe that the differential amount of energy is somehow made available to the virtual W- boson when quark-antiquark annihilation occurs with the disappearance of the meson-bound state.

The paper is organized as follows. In Sec. II, we present the theoretical framework that includes the description of (a) the invariant transition matrix element $\mathcal{M}_{\text{fi}}$ leading to the general expression of the decay widths $\Gamma(P(V) \to l^+\nu_l)$, (b) a brief account of the RIQ model conventions and the quark orbitals, and (c) the meson-bound state and extraction of decay constants $f_{P(V)}$ in terms of model quantities. Section III is devoted to the numerical results and discussion. Finally, Sec. IV encompasses our summary and conclusion.

II. THEORETICAL FRAMEWORK

A. Invariant transition matrix amplitude and decay width: $\Gamma(P(V) \to l^+\nu_l)$

The leptons being free from the strong interaction, the effective Hamiltonian [21,104,105] for PLDCMs could be written as the product of quark current and leptonic current in the form:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} [\bar{q}_1 \gamma_\mu (1-\gamma_5) q_2][\bar{l}\gamma^\mu (1-\gamma_5)\nu_l] + \text{H.c.}, \quad (1)$$

where the contribution of the W-bosons is embodied in the Fermi coupling constant G_F, and $V_{q_1 q_2}$ is the CKM matrix element between the constituent quarks of the decaying mesons. The decay amplitude is written as

$$\begin{aligned}\mathcal{M}_{\rm fi} &= \langle \bar{l}\nu_l | \mathcal{H}_{\rm eff} | P(V)\rangle \\ &= \frac{G_F}{\sqrt{2}} V_{q_1 q_2} \langle \bar{l}\nu_l | \bar{l}\gamma^\mu (1-\gamma_5)\nu_l | 0\rangle \\ &\quad \times \langle 0 | \bar{q}_1 \gamma_\mu (1-\gamma_5) q_2 | P(V)\rangle. \end{aligned} \quad (2)$$

The leptonic part of decay amplitude can be calculated reliably with perturbative theory. The hadronic matrix element (HME) interpolating the diquark current between the decaying meson $P(V)$ and the vacuum states can be expressed in terms of a nonperturbative parameter, the decay constant $f_{P(V)}$ in the form:

$$\langle 0 | \bar{q}_1(0)\gamma_\mu q_2(0) | P(\vec{k})\rangle = 0, \quad (3)$$

$$\langle 0 | \bar{q}_1(0)\gamma_\mu \gamma_5 q_2(0) | P(\vec{k})\rangle = i f_P k_\mu, \quad (4)$$

$$\langle 0 | \bar{q}_1(0)\gamma_\mu q_2(0) | V(\vec{k},\epsilon)\rangle = f_V m_V \epsilon_\mu, \quad (5)$$

$$\langle 0 | \bar{q}_1(0)\gamma_\mu \gamma_5 q_2(0) | V(\vec{k},\epsilon)\rangle = 0. \quad (6)$$

Here, $m_{P(V)}, \vec{k}$ are the mass, three momentum of decaying meson, and ϵ_μ is the polarization vector of the decaying vector meson (V). It is straightforward to find that the spacelike component of the HME for the purely leptonic decay of charged pseudoscalar mesons (PLDCPMs) and its timelike component for the purely leptonic decay of charged vector mesons (PLDCVMs) are zero. With the corresponding nonvanishing parts of HME, the invariant transition amplitude squared $|\mathcal{M}_{\rm fi}|^2$ is expressed in terms of hadronic (H^P_{00}, H^V_{ij}) and leptonic tensor (L^{00}_P, L^{ij}_V). For PLDCPMs,

$$H^P_{00} = f_P^2 m_P^2 \quad (7)$$

$$L^{00}_P = 8(\vec{k}_l \, \vec{k}_\nu + E_{k_l} E_{k_\nu}), \quad (8)$$

and for PLDCVMs,

$$H^V_{ij} = f_V^2 m_V^2 \epsilon_i \epsilon_j^\dagger, \quad (9)$$

$$L^{ij}_V = 8\left(k_l^i k_\nu^j - k_{l\alpha} k_\nu^\alpha g^{ij} + k_l^j k_\nu^i - i\epsilon^{ij\alpha\beta} k_{l\alpha} k_{\nu\beta}\right). \quad (10)$$

Then decay width Γ in the decaying meson rest frame is calculated from the generic expression:

$$\begin{aligned}\Gamma &= \frac{1}{(2\pi)^2} \int \frac{d\vec{k}_l d\vec{k}_\nu}{2 m_{P(V)} 2E_{k_l} 2E_{k_\nu}} \delta^{(4)}\left(k_l + k_\nu - \hat{O} m_{P(V)}\right) \\ &\quad \times \sum |\mathcal{M}_{\rm fi}|^2. \end{aligned} \quad (11)$$

Here, the operator $\hat{O}$ symbolically denotes $\hat{O} m_{P(V)} = (m_{P(V)}, 0, 0, 0)$. We use two frames of reference to evaluate the contribution of Lorentz invariant pieces: hadronic and leptonic tensors. For the sake of simplicity, we evaluate the contribution of the leptonic tensor in the lepton rest frame and that of the hadronic tensor in the decaying meson rest frame to finally obtain the decay widths in the form:

$$\Gamma(P \to l^+ \nu_l) = \frac{G_F^2}{8\pi} |V_{q_1 q_2}|^2 f_P^2 m_P {m_l}^2 \left(1 - \frac{m_l^2}{m_P^2}\right)^2, \quad (12)$$

$$\Gamma(V \to l^+ \nu_l) = \frac{G_F^2}{12\pi} |V_{q_1 q_2}|^2 m_V^3 f_V^2 \left(1 - \frac{m_l^2}{m_V^2}\right)^2 \left(1 + \frac{m_l^2}{2m_V^2}\right). \quad (13)$$

B. The RIQ model conventions and quark orbitals

In the RIQ model, a meson is picturized as a color-singlet assembly of a quark and an antiquark independently confined by an effective and average flavor-independent potential in the form:

$$U(r) = \frac{1}{2}(1 + \gamma^0) V(r),$$

where $V(r) = (ar^2 + V_0)$ with $a > 0$, is believed to provide zeroth order constituent quark dynamics inside the meson-bound state. Here, (a, V_0) are the potential parameters, and r is the relative distance between constituent quark and antiquark inside the meson core. The potential $U(r)$, taken in equally mixed scalar-vector harmonic form, phenomenologically represents the confining interaction, expected to be generated by a nonperturbative multigluon mechanism. The quark-gluon interaction at short distance originating from the one-gluon exchange and the quark-pion interaction required in the nonstrange flavor sector to preserve chiral symmetry is presumed here to be residual interactions compared to the dominant confining interaction. Although these residual interactions, treated perturbatively in the model, are crucial in determining the mass splitting in hadron spectroscopy, their role in hadronic decay processes is considered less significant. Therefore, to a first approximation, it is believed that the zeroth-order quark dynamics inside the meson core generated by the confining part of the interaction, phenomenologically represented by $U(r)$, can provide an adequate description of the purely leptonic decays of heavy-flavored charged pseudoscalar and vector mesons.

With the interaction potential $U(r)$, put into the zeroth-order quark lagrangian density $\mathcal{L}_q^0$, the ensuing Dirac equation admits static solutions of all possible positive and negative energy eigen modes as

$$\psi_\xi^{(+)}(\vec{r}) = \begin{pmatrix} \frac{ig_\xi(r)}{r} \\ \frac{\vec{\sigma}.\hat{r}f_\xi(r)}{r} \end{pmatrix} \chi_{ljm_j}(\hat{r}),$$

$$\psi_\xi^{(-)}(\vec{r}) = \begin{pmatrix} \frac{i(\vec{\sigma}.\hat{r})f_\xi(r)}{r} \\ \frac{g_\xi(r)}{r} \end{pmatrix} \tilde{\chi}_{ljm_j}(\hat{r}), \tag{14}$$

where $\xi = (nlj)$ represents a set of Dirac quantum numbers specifying the eigenmodes. $\chi_{ljm_j}(\hat{r})$ and $\tilde{\chi}_{ljm_j}(\hat{r})$ are the spin angular parts given by

$$\chi_{ljm_j}(\hat{r}) = \sum_{m_l,m_s} < lm_l \frac{1}{2} m_s | jm_j > Y_l^{m_l}(\hat{r}) \chi_{\frac{1}{2}}^{m_s},$$

$$\tilde{\chi}_{ljm_j}(\hat{r}) = (-1)^{j+m_j-l} \chi_{lj-m_j}(\hat{r}). \tag{15}$$

With the quark binding energy parameter E_q and quark mass parameter m_q, written in the form $E'_q = (E_q - V_0/2)$, $m'_q = (m_q + V_0/2)$, and $\omega_q = E'_q + m'_q$, one can obtain solutions to the radial equation for $g_\xi(r)$ and $f_\xi(r)$ in the form:

$$g_{nl} = \mathcal{N}_{nl} \left(\frac{r}{r_{nl}}\right)^{l+1} \exp(-r^2/2r_{nl}^2) L_{n-1}^{l+1/2}(r^2/r_{nl}^2),$$

$$f_{nl} = \frac{\mathcal{N}_{nl}}{r_{nl}\omega_q} \left(\frac{r}{r_{nl}}\right)^{l} \exp(-r^2/2r_{nl}^2) \times \left[\left(n + l - \frac{1}{2}\right) L_{n-1}^{l-1/2}(r^2/r_{nl}^2) + n L_n^{l-1/2}(r^2/r_{nl}^2)\right], \tag{16}$$

where $r_{nl} = (a\omega_q)^{-1/4}$ is a state-independent length parameter, $\mathcal{N}_{nl}$ is an overall normalization constant given by

$$\mathcal{N}_{nl}^2 = \frac{4\Gamma(n)}{\Gamma(n + l + 1/2)} \frac{(\omega_q/r_{nl})}{(3E'_q + m'_q)}, \tag{17}$$

and $L_{n-1}^{l+1/2}(r^2/r_{nl}^2)$ are associated Laguerre polynomials. With the radial solutions taken in the form (16), an independent quark bound-state condition is obtained in the RIQ model in the form of a cubic equation:

$$\sqrt{(\omega_q/a)}(E'_q - m'_q) = (4n + 2l - 1). \tag{18}$$

C. Meson-bound state and the decay constant

In the relativistic independent particle picture of this model, the relativistic constituent quark and antiquark are thought to move independently inside the meson-bound state. The meson decay in fact takes place in the momentum eigenstate of the decaying meson. Therefore, in the description of any decay process in this model, we take a wave-packet representation of the meson-bound state with appropriate momentum distribution among the constituent quark and antiquark in their corresponding $SU(6)$-spin flavor configuration. Here, the transition probability amplitude for the weak leptonic decay, calculated from the appropriate Feynman diagram, can be expressed as the free quark-antiquark pair annihilation integrated over an effective momentum distribution function $\mathcal{G}(\vec{p}_{q_1}, \vec{p}_{q_2})$. In this model, we take $\mathcal{G}(\vec{p}_{q_1}, \vec{p}_{q_2}) = \sqrt{G_{q_1}(\vec{p}_{q_1}) G_{q_2}(\vec{p}_{q_2})}$ in the light of the ansatz of Margolis and Mendel [106] in their bag model description of the meson-bound state. Here, $G_{q_1}(\vec{p}_{q_1})$ and $\tilde{G}_{q_2}(\vec{p}_{q_2})$, the momentum probability amplitude of the constituent quark q_1 and antiquark $\overline{q_2}$, respectively, are obtained via momentum space projection of the corresponding static solutions (quark orbitals) (14). For the ground state mesons ($n = 1, l = 0$), we find

$$G_{q_1}(\vec{p}_{q_1}) = \frac{i\pi\mathcal{N}_{q_1}}{2\alpha_{q_1}\omega_{q_1}} \sqrt{\frac{(E_{p_{q_1}} + m_{q_1})}{E_{p_{q_1}}}} (E_{p_{q_1}} + E_{q_1}) \times \exp\left(-\frac{\vec{p}_{q_1}{}^2}{4\alpha_{q_1}}\right), \tag{19}$$

$$\tilde{G}_{q_2}(\vec{p}_{q_2}) = -\frac{i\pi\mathcal{N}_{q_2}}{2\alpha_{q_2}\omega_{q_2}} \sqrt{\frac{(E_{p_{q_2}} + m_{q_2})}{E_{p_{q_2}}}} (E_{p_{q_2}} + E_{q_2}) \times \exp\left(-\frac{\vec{p}_{q_2}^2}{4\alpha_{q_2}}\right), \tag{20}$$

where $\alpha_{q_{1,2}} = \sqrt{a\omega_{q_{1,2}}}/2$. With the effective momentum distribution function $\mathcal{G}(\vec{p}_{q_1}, \vec{p}_{q_2})$, so obtained from model dynamics, the wave-packet representation of meson-bound state at definite momentum $\vec{k}$ and spin projection $S_{P(V)}$ is taken in the form [90–103]:

$$|P(V)(\vec{k}, S_{P(V)})\rangle = \hat{\Lambda}(\vec{k}, S_{P(V)}) |(\vec{p}_{q_1}, \lambda_{q_1}); (\vec{p}_{q_2}, \lambda_{q_2})\rangle = \hat{\Lambda}(\vec{k}, S_{P(V)}) \hat{b}_{q_1}^\dagger(\vec{p}_{q_1}, \lambda_{q_1}) \hat{\tilde{b}}_{q_2}^\dagger(\vec{p}_{q_2}, \lambda_{q_2}) |0\rangle. \tag{21}$$

Here, $|(\vec{p}_{q_1}, \lambda_{q_1}); (\vec{p}_{q_2}, \lambda_{q_2})\rangle$ is the Fock-space representation of the bound quark and antiquark in their color-singlet configuration with respective momentum and spin: $(\vec{p}_{q_1}, \lambda_{q_1})$ and $(\vec{p}_{q_2}, \lambda_{q_2})$. $\hat{b}_{q_1}^\dagger(\vec{p}_{q_1}, \lambda_{q_1})$ and $\hat{\tilde{b}}_{q_2}^\dagger(\vec{p}_{q_2}, \lambda_{q_2})$ are the quark and antiquark creation operators, and $\hat{\Lambda}(\vec{k}, S_{P(V)})$ is a baglike operator taken here in the integral form:

$$\hat{\Lambda}(\vec{k}, S_{P(V)}) = \frac{\sqrt{3}}{\sqrt{N_{P(V)}(\vec{k})}} \sum_{\lambda_{q_1},\lambda_{q_2}} \zeta^{P(V)}_{q_1,q_2} \int d\vec{p}_{q_1} d\vec{p}_{q_2} \delta^{(3)} \times (\vec{p}_{q_1} + \vec{p}_{q_2} - \vec{k}) \mathcal{G}_{P(V)}(\vec{p}_{q_1}, \vec{p}_{q_2}), \tag{22}$$

where $\sqrt{3}$ is the effective color factor, and $\zeta^{P(V)}_{q_1,q_2}$ is the $SU(6)$ spin-flavor coefficients for the meson state $|P(V)(\vec{k}, S_{P(V)})\rangle$. We impose the normalization condition $\langle P(V)(\vec{k}')|P(V)(\vec{k})\rangle = (2\pi)^3 2E_k \delta^{(3)}(\vec{k} - \vec{k}')$ and obtain the meson state normalization $N_{P(V)}(\vec{k})$ in the integral form:

$$N_{P(V)}(\vec{k}) = \frac{1}{(2\pi)^3 2E_k} \int d\vec{p}_{q_1} |\mathcal{G}_{P(V)}(\vec{p}_{q_1}, \vec{p}_{q_2})|^2 = \frac{\bar{N}_{P(V)}(\vec{k})}{(2\pi)^3 2E_k}. \tag{23}$$

Using the wave-packet representation of the meson-bound state (21–23), it is straightforward to calculate the decay constants f_P and f_V from Eqs. (4) and (5), respectively, in the form:

$$f_P = \frac{2\sqrt{3}}{\sqrt{(2\pi)^3 m_P \bar{N}_P(0)}} \int \frac{d\vec{p}_{q_1}}{\sqrt{2E_{p_{q_1}} 2E_{-p_{q_1}}}} \mathcal{G}_P(\vec{p}_{q_1}, -\vec{p}_{q_1}) \times \left[\frac{|\vec{p}_{q_1}|^2 - (E_{p_{q_1}} + m_{q_1})(E_{-p_{q_1}} + m_{q_2})}{\sqrt{(E_{p_{q_1}} + m_{q_1})(E_{-p_{q_1}} + m_{q_2})}} \right], \tag{24}$$

and

$$f_V = \frac{2\sqrt{3}}{\sqrt{(2\pi)^3 m_V \bar{N}_V(0)}} \int \frac{d\vec{p}_{q_1}}{\sqrt{2E_{p_{q_1}} 2E_{-p_{q_1}}}} \mathcal{G}_V(\vec{p}_{q_1}, -\vec{p}_{q_1}) \times \left[\frac{|\vec{p}_{q_1}|^2 + 3(E_{p_{q_1}} + m_{q_1})(E_{-p_{q_1}} + m_{q_2})}{3\sqrt{(E_{p_{q_1}} + m_{q_1})(E_{-p_{q_1}} + m_{q_2})}} \right]. \tag{25}$$

III. NUMERICAL RESULTS AND DISCUSSION

In this section, we calculate the decay constants $f_{P(V)}$, BFs, and ratio of BFs: ($\mathcal{R}^{\tau}_{\mu}$) for purely leptonic decays of charged pseudoscalar and vector mesons: ($D^+_{(s)}, B^+_{(c)}$; $D^{*+}_{(s)}, B^{*+}_{(c)}$). In our numerical calculation, we use the input parameters: the quark masses ($m_u = m_d$, m_s, m_c, m_b) and potential parameters (V_0, a). We first fixed the input parameters here from hadron spectroscopy by generating the ground state hyperfine mass splitting of the heavy-flavored mesons: (D^{*+}, D^+), (D^{*+}_s, D^+_s), (B^{*+}_u, B^+_u), (B^{*+}_c, B^+_c), (B^{*0}_s, B^0_s), (J/ψ, η_c), and (Υ, η_b). For this, we appropriately take into account the correction due to the residual interactions such as the quark-gluon interaction at short distance originating from the one gluon exchange along with the correction arising out of the spurious center-of-mass motion as per Refs. [88,89]. The correction due to the quark-pion interaction, required in the nonstrange light flavor sector to preserve the chiral symmetry at $SU(2) \times SU(2)$ level, is not considered in the heavy flavor sector here. Our input parameters, so fixed from hadron spectroscopy, are shown in Table I.

TABLE I. The quark masses m_q (in GeV) and potential parameters: V_0 (in GeV) and a (in GeV^3).

$m_u = m_d$	m_s	m_c	m_b	a	V_0
0.26	0.49	1.64	4.92	0.023	−0.307

The resulting ground-state meson masses in the RIQ model are cited in Table II in good comparison with the available experimental data. The phenomenological input parameters such as the Fermi coupling constant G_F, lifetime τ of pseudoscalar mesons ($D_{(s)}, B_{(c)}$), total decay width $\Gamma^{\text{total}}_{D^*}$, and the precise global fit values of CKM parameters are taken from PDG [23] and listed in Table III. However, in the absence of the observed mass of B^*_c-meson, we use our model predicted value $m_{B^*_c} = 6.2909$ GeV (Table II).

Before calculating numerically, the physical quantities of interest (the decay constants $f_{P(V)}$ and corresponding BFs), we study the behavior of radial quark momentum distribution amplitude $|\vec{p}_q|\mathcal{G}_{P(V)}(\vec{p}_q, -\vec{p}_q)$ for decaying meson-ground-state $|P(V)\rangle$, over the allowed physical

TABLE II. Hyperfine splitting of the ground-state heavy flavored mesons with the quark-gluon coupling constant $\alpha_s = 0.37$.

	Spin-averaged mass (MeV)		Meson mass (MeV)	
Meson	Theory	Experiment	Theory	Experiment
$D^{*\pm}$	1954.93	1975.07	1979.95	2010.26
$D^{\pm}$			1889.83	1869.50
$D^{*\pm}_s$	2067.33	2076.40	2090.38	2112.20
$D^{\pm}_s$			1998.18	1969.0
$B^{*\pm}_u$	5290.04	5313.35	5300.11	5324.71
$B^{\pm}_u$			5239.04	5279.25
$B^{*\pm}_c$	6288.43	⋯	6290.90	⋯
$B^{\pm}_c$			6264.46	6274.47
B^{*0}_s	5385.39	5403.58	5395.02	5415.80
B^0_s			5360.59	5366.91
J/ψ	3037.68	3068.65	3051.55	3096.90
η_c			3012.16	2983.90
Υ	9443.46	9444.98	9447.06	9460.40
η_b			9421.95	9398.70

TABLE III. Numerical inputs.

Parameter	Value	Unit	References
G_F	1.1663788×10^{-5}	GeV^{-2}	[23]
$\lvert V_{cd}\rvert$	0.22486 ± 0.00067	···	[23]
$\lvert V_{cs}\rvert$	0.97349 ± 0.00016	···	[23]
$\lvert V_{ub}\rvert$	0.00369 ± 0.00011	···	[23]
$\lvert V_{cb}\rvert$	$0.04182^{+0.00085}_{-0.00074}$	···	[23]
τ_{D^+}	1033 ± 5	fs	[23]
$\tau_{D_s^+}$	504 ± 4	fs	[23]
$\tau_{B_u^+}$	1.638 ± 0.004	ps	[23]
$\tau_{B_c^+}$	0.51 ± 0.009	ps	[23]
$\Gamma^{\mathrm{total}}_{D^{*+}}$	83.4 ± 1.8	keV	[23]

range of the quark momentum $|\vec{p}_q|$. The behaviors of $|\vec{p}_q|\mathcal{G}_{P(V)}(\vec{p}_q, -\vec{p}_q)$, shown in Fig. 1, indicate sharper peaks for (B_u, B_u^*) and (B_c, B_c^*) than those for (D, D^*) and (D_s, D_s^*) meson states. This is due to the large mass difference between b and c quarks. The quark momentum distribution amplitude for (D_s, D_s^*), (B_c, B_c^*) are also found to have comparatively sharper peaks than (D, D^*), (B_u, B_u^*), respectively. This is because of the dominant contributions of heavier quark masses m_s and m_c as compared to that of m_d and m_u, respectively. As expected from heavy quark symmetry, the difference between the behavior of $|\vec{p}_q|\mathcal{G}_{P(V)}(\vec{p}_q, -\vec{p}_q)$ of the heavier pseudoscalar (B_u, B_c) and vector (B_u^*, B_c^*) mesons gets reduced in contrast to that of comparatively lighter pseudoscalar (D, D_s) and vector (D^*, D_s^*) meson in the charm sector.

We then study the behavior of decay constants $f_{P(V)}$ with the change of potential parameters (V_0, a) values over a chosen range. The behavior of decay constants over $\pm 10\%$ variation of the potential parameter values are shown in Fig. 2. In our attempt to assess the sensitivity of input parameters in the present calculation, we include the systematic errors in the analysis, obtained both from the $\pm 10\%$ variation of the potential parameters (V_0, a) for fixed quark masses (m_q) and $\pm 10\%$ variation of the quark masses for fixed values of potential parameters. Our predicted decay constants in MeV are found to be: $f_{D^+} = 219.58^{+10.72+8.76}_{-11.49-9.33}$, $f_{D^{*+}} = 256.09^{+7.49+12.45}_{-7.79-13.03}$, $f_{D_s} = 253.50^{+13.12+9.46}_{-14.03-10.06}$, $f_{D_s^{*+}} = 285.97^{+9.92+12.75}_{-10.38-13.37}$, $f_{B_u^+} = 161.34^{+7.42+5.8}_{-7.81-6.14}$, $f_{B_u^{*+}} = 172.61^{+4.9+8.54}_{-5.06-8.93}$, $f_{B_c^+} = 249.50^{+10.68+9.29}_{-11.45-9.85}$, $f_{B_c^{*+}} = 258.66^{+9.85+10.1}_{-10.47-10.68}$. The first errors in our predictions come from the $\pm 10\%$ variation of quark masses for fixed potential parameters, and the second errors

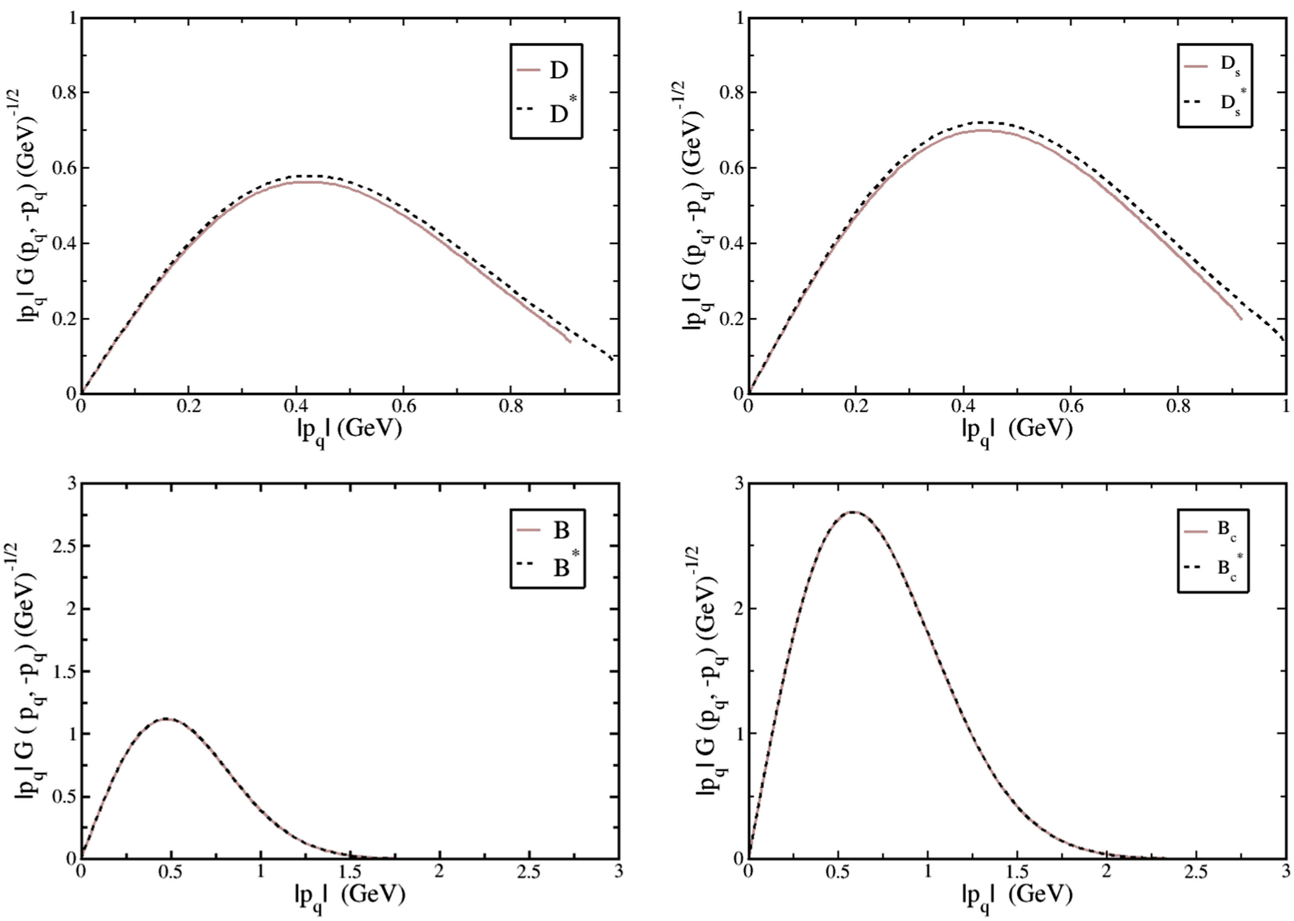

FIG. 1. Radial quark momentum distribution amplitude for heavy pseudoscalar meson (solid line) and vector meson (dotted line).

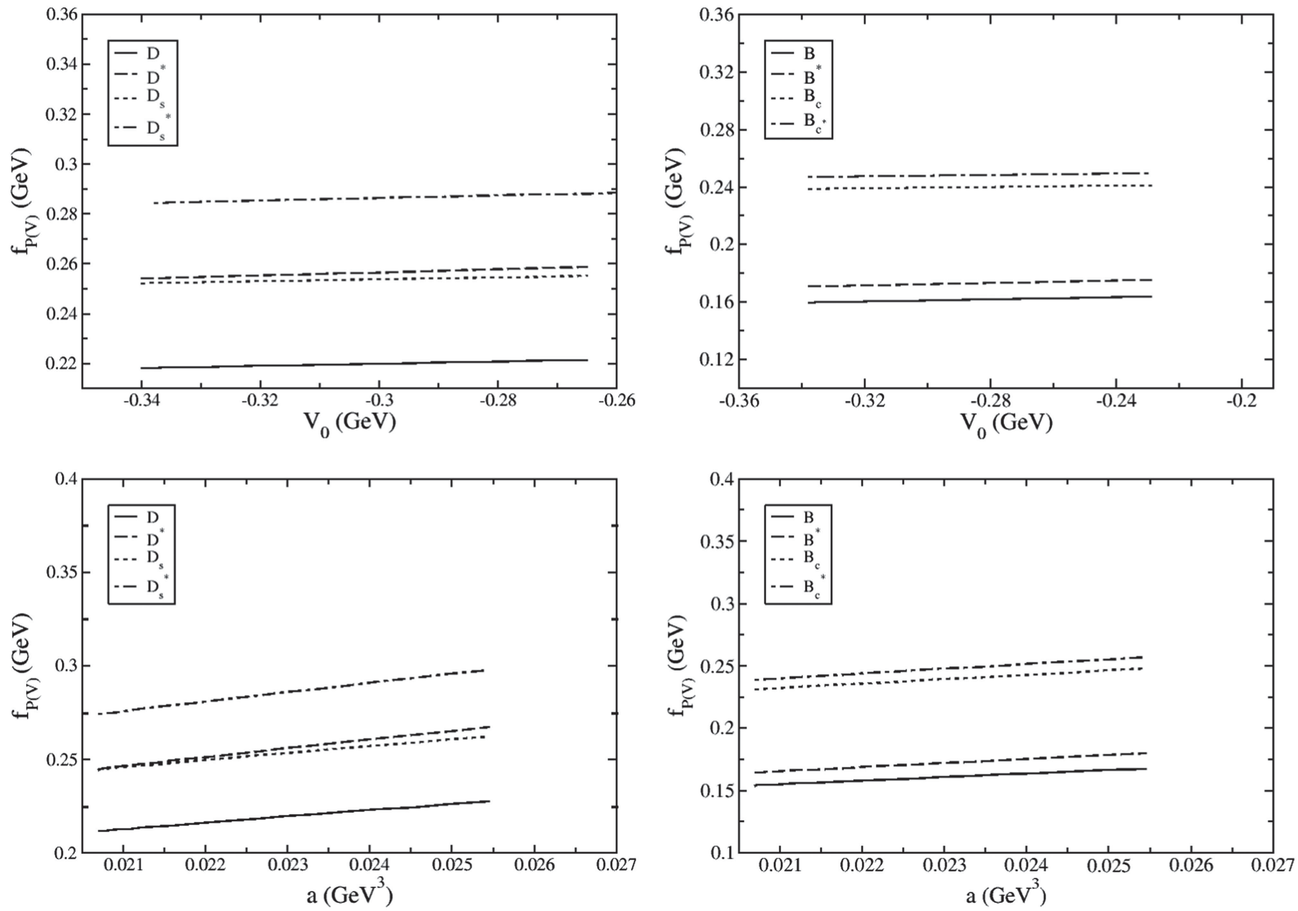

FIG. 2. Dependence of decay constants $f_{P(V)}$ on the potential parameters (V_0, a).

come from the ±10% variation of potential parameters for fixed quark masses. As one can see, our predictions for PLDCVMs are more sensitive to the variation of potential parameters than those of quark masses, whereas the reverse is found to be true for PLDCPMs.

We also predict the uncertainties in the decay constant values over ±10% simultaneous variations of all the input parameters: the quark masses m_q and potential parameters (V_0, a). Our predicted decay constants with corresponding uncertainties are cited in Tables IV and V, compared to the available experimental data and other SM predictions. Our predictions on f_{D^+}, $f_{D_s^+}$, $f_{D_s^{*+}}$, as cited in Table IV, agree with the corresponding observed data within the experimental limits and compare well with the results based on NRQM [35,39], BS [45,46], LFQM [50,57], LFHQCD [62], QCD SR [63,64], and LQCD [72,74,75] calculations. Our predicted $f_{D^{*+}}$ also compares well with that of LFQM Lin [49], LFQM [57], LCQM [56], QCD SR [63,64], and the LQCD [72,74,75] calculations. The results for the decay constants, $f_{B_u}, f_{B_u^*}, f_{B_c}, f_{B_c^*}$, obtained in the present study, are shown in Table V. We find that our result $f_{B_u^+} = 161.34^{+13.43}_{-13.70}$ MeV not only agrees with the observed data [107] within the experimental limit but also compares well with that of BS [45], LFQM [49,57], LQCD [72] calculation. In the absence of observed data in B_u^{*+} sector, we find that our prediction $f_{B_u^{*+}} = 172.61^{+13.56}_{-13.84}$ MeV compares well with those of LFQM [57] and the LQCD [72]. In the B_c sector, however, the experimental measurements have been too scarce. Not many theoretical attempts have been made in this sector to adequately address the issue. We, therefore, compare our prediction on $f_{B_c^+}$, $f_{B_c^{*+}}$ with a few SM predictions available in the literature. Although our result $f_{B_c^+} = 249.50^{+20.34}_{-20.85}$ MeV is in reasonable agreement with QCD SR [108], our prediction $f_{B_c^{*+}} = 258.66^{+20.27}_{-20.80}$ MeV appears somewhat underestimated compared to those obtained in BS [45], LFQM Lin [50], LCDA [59], and QCD SR [109] approaches. Note that both the valence quarks of $B_c^{(*)}$ mesons being heavy, their Compton wavelengths $\sim 1/m_{b,c}$ are much shorter than typical hadron size. Our prediction in this sector, showing $f_{B_c} \simeq f_{B_c^*}$, tallies with the result expected from spin-flavor symmetry in the heavy quark limit.

The predictions on ratios f_V/f_P, f_{P_1}/f_{P_2}, and f_{V_1}/f_{V_2} are important as they are sensitive to the difference between

TABLE IV. Predicted decay constants for (D, D_s, D^*, D_s^*) mesons (in MeV) compared to available experimental data and other theoretical predictions.

Reference	f_{D^+}	$f_{D^{*+}}$	$f_{D_s^+}$	$f_{D_s^{*+}}$
This work	$219.58^{+19.93}_{-20.26}$	$256.09^{+20.14}_{-20.59}$	$253.50^{+23.0}_{-23.41}$	$285.97^{+22.91}_{-23.42}$
Experiment [20]	...	...	...	$213.6^{+61.0}_{-45.8_{\text{stat.}}} \pm 43.9_{\text{syst.}}$
Experiment [23]	$203.8 \pm 4.7 \pm 0.6 \pm 1.4$	...	$250.1 \pm 2.2 \pm 0.04 \pm 1.8$	...
LFQM Lin [49]	197	239	233	274
LFQM HO [49]	180	212	218	252
LFQM Lin [50]	208	230	231	260
LFQM [57]	$197^{+19+0.2}_{-20-1.0}$	230^{+29-5}_{-28+6}	$219^{+21-0.2}_{-22-0.8}$	253^{+31-6}_{-31+6}
LCQM [56]	209	260	237	291
LFHQCD [61]	199	...	216	...
LFHQCD [62]	$214.2^{+7.6}_{-7.8}$	...	$253.5^{+7.6}_{-7.8}$	...
QCD SR [63]	201^{+12}_{-13}	242^{+20}_{-12}	238^{+13}_{-23}	293^{+19}_{-14}
QCD SR [64]	208 ± 10	263 ± 21	240 ± 10	308 ± 21
LQCD [72]	$206 \pm 4^{+17}_{-10}$	234 ± 26	$229 \pm 3^{+23}_{-12}$	254 ± 17
LQCD [74]	207.4 ± 3.8	223.5 ± 8.4	247.2 ± 4.1	268.8 ± 6.6
LQCD [75]	213 ± 5	234 ± 6	249 ± 7	274 ± 7
LQCD [84]	211.9 ± 1.1	...	249 ± 1.2	...
BS [45]	230 ± 25	340 ± 23	248 ± 27	375 ± 24
BS [46]	238	...	241	...
RQM [38]	234	310	268	315
RQM [40]	271 ± 14	327 ± 13	309 ± 15	362 ± 15
NRQM [34]	368.8	353.8	394.8	382.1
NRQM [35]	220	290	250	310
NRQM [39]	228	...	273	...

TABLE V. Predicted decay constants for (B_u, B_c, B_u^*, B_c^*) mesons (in MeV) compared to available experimental data and other theoretical predictions.

Reference	$f_{B_u^+}$	$f_{B_u^{*+}}$	Reference	$f_{B_c^+}$	$f_{B_c^{*+}}$
This work	$161.34^{+13.43}_{-13.70}$	$172.61^{+13.56}_{-13.84}$	This work	$249.50^{+20.34}_{-20.85}$	$258.66^{+20.27}_{-20.80}$
Experiment [107]	$188 \pm 17 \pm 18$	...	LFQM Lin [50]	389^{+16}_{-3}	391^{-5}_{+4}
LFQM Lin [49]	171	186	LCDA [59]	360	387
LFQM HO [49]	161	173	QCD SR [108]	270 ± 30	300 ± 30
LFQM Lin [50]	181	188	QCD SR [109]	371 ± 17	442 ± 44
LFQM [57]	163^{+21-4}_{-20+4}	172^{-23+6}_{+24-6}	BS [45]	...	418 ± 24
LCQM [56]	193	211			
LFHQCD [61]	194	...			
LFHQCD [62]	$191.7^{+7.9}_{-6.5}$	...			
LQCD [72]	$195 \pm 6^{+24}_{-23}$	190 ± 28			
QCD SR [63]	207^{+17}_{-9}	210^{+10}_{-12}			
QCD SR [64]	194 ± 15	213 ± 18			
BS [45]	196 ± 29	238 ± 28			
BS [46]	193	...			
RQM [38]	189	219			
RQM [40]	231 ± 9	252 ± 10			
NRQM [34]	235.9	234.7			
NRQM [35]	147	196			
NRQM [39]	149	...			

TABLE VI. Predicted ratios of decay constants for (D, D_s, D^*, D_s^*) mesons compared with the available experimental data and other theoretical predictions.

Reference	$f_{D^{*+}}/f_{D^+}$	$f_{D_s^{*+}}/f_{D_s^+}$	$f_{D_s^+}/f_{D^+}$	$f_{D_s^{*+}}/f_{D^{*+}}$
This work	$1.166^{+0.140}_{-0.143}$	$1.128^{+0.137}_{-0.139}$	$1.154^{+0.148}_{-0.151}$	$1.117^{+0.125}_{-0.128}$
Experiment [23]	...	...	$1.228 \pm 0.03 \pm 0.004 \pm 0.009$	...
LFQM Lin [49]	1.21	1.18	1.18	1.15
LFQM HO [49]	1.18	1.16	1.21	1.19
LFQM Lin [50]	1.11	1.13	1.11	1.13
LFQM [57]	$1.17^{+0.03-0.03}_{-0.03+0.04}$	$1.16^{+0.03-0.03}_{-0.03+0.03}$	$1.11^{-0.001-0.002}_{+0.001+0.002}$	$1.10^{-0.003-0.002}_{-0.001-0.003}$
LCQM [56]	1.24	1.23	1.13	1.12
LFHQCD [61]	...	...	1.09	...
LFHQCD [62]	...	...	$1.184^{+0.054}_{-0.052}$	...
QCD SR [63]	$1.20^{+0.13}_{-0.07}$	$1.21^{+0.13}_{-0.05}$	$1.18^{+0.04}_{-0.05}$	1.21 ± 0.05
QCD SR [64]	...	...	1.15 ± 0.06	...
LQCD [72]	...	...	$1.11 \pm 0.1^{+1}_{-1}$	...
LQCD [74]	1.078 ± 0.036	1.087 ± 0.020	...	...
LQCD [75]	1.10 ± 0.03	1.10 ± 0.04	1.16 ± 0.03	1.17 ± 0.03
BS [45]	...	...	1.08 ± 0.01	1.10 ± 0.06
BS [46]	...	...	1.01	...
RQM [38]	1.32	1.18	1.15	1.02
RQM [40]	1.21 ± 0.02	1.17 ± 0.02	1.14 ± 0.01	...
NRQM [34]	0.96	0.97	1.07	1.08
NRQM [35]	1.32	1.24	1.14	1.07
NRQM [39]	...	...	1.20	...

vector (V) & pseudoscalar (P), pseudoscalar (P_1) & pseudoscalar (P_2) and vector (V_1) & vector (V_2) wave functions. Our predictions on these ratios are cited in Tables VI and VII. As shown in Table VI, our result $f_{D^{*+}}/f_{D^+} = 1.166^{+0.140}_{-0.143}$ compares well with that of RQM [40], LFQM HO [49], LFQM [57], and QCD SR [63] calculations. The predicted ratio $f_{D_s^{*+}}/f_{D_s^+} = 1.128^{+0.137}_{-0.139}$ exactly matches with 1.13 of LFQM Lin [50] and also agrees well with many other SM predictions including those of RQM [40], LFQM [49,57], QCD SR [63], and LQCD [75] calculations. The ratio $f_{D_s^+}/f_{D^+} = 1.154^{+0.148}_{-0.151}$, obtained in the present study is not only comparable to the observed data [23] but also consistent with other SM predictions from RQM [38,40], LFHQCD [62], and the QCD SR [63,64]. Our result $f_{D_s^{*+}}/f_{D^{*+}} = 1.117^{+0.125}_{-0.128}$ also agrees well with the results of LFQM Lin [50] and LCQM [56]. As can be seen from Table VII, our predicted ratio $f_{B_u^{*+}}/f_{B_u^+} = 1.069^{+0.122}_{-0.125}$ agrees well with that of LFQM Lin [50], LFQM Lin [49], and LFQM [57] and is comparable to that of RQM [40], QM Lin [50]. In B_c^* sector, our predicted $f_{B_c^{*+}}/f_{B_c^+}$ is also found in reasonable agreement with those of LFQM Lin [50], QCD SR [108,109]. Finally, we predict $f_{B_c^{*+}}/f_{B_u^{*+}} = 1.498^{+0.166}_{-0.170}$ and $f_{B_c^+}/f_{B_u^+} = 1.546^{+0.180}_{-0.184}$, which can be verified in the upcoming experimental measurements and compared with future SM predictions in this sector.

TABLE VII. Predicted ratios of decay constants for (B_u, B_c, B_u^*, B_c^*) mesons compared with the available experimental data and other theoretical predictions.

Reference	$f_{B_u^{*+}}/f_{B_u^+}$	Reference	$f_{B_c^{*+}}/f_{B_c^+}$
This work	$1.069^{+0.122}_{-0.125}$	This work	$1.037^{-0.120}_{+0.117}$
LFQM Lin [49]	1.09	LFQM Lin [50]	$1.005^{-0.052}_{+0.018}$
LFQM HO [49]	1.07	LCDA [59]	1.08
LFQM Lin [50]	1.04	QCD SR [108]	$1.11^{+0.1}_{-0.015}$
LFQM [57]	$1.06^{+0.010-0.011}_{-0.013+0.011}$	QCD SR [109]	$1.19^{+0.061}_{-0.067}$
LCQM [56]	1.09		
QCD SR [63]	$1.02^{+0.02}_{-0.09}$		
RQM [38]	1.16		
RQM [40]	1.09 ± 0.01		
NRQM [34]	0.99		
NRQM [35]	1.33		

Taking into account our predicted decay constants, f_P and f_V and other relevant phenomenological input parameters shown in Table III, the decay rates for PLDCPMs $(D^+_{(s)}, B^+_{(c)})$ and PLDCVMs $(D^{*+}_{(s)}, B^{*+}_{(c)})$ are calculated in a straightforward manner from Eqs. (12) and (13), respectively. Thereafter, the BFs are calculated using the observed lifetimes (τ_P) of the pseudoscalar mesons $(D^+_{(s)}, B_{(c)})$ and the total decay width $\Gamma^{\rm total}_{D_d^{*+}}$. In the absence of observed

TABLE VIII. The predicted BFs $\mathcal{B}(P \to l^+\nu_l)$ in comparison with the observed value and other theoretical predictions.

$\mathcal{B}(P \to l^+\nu_l)$	This work	[33]	[35]	[39]	[111]	Experiment [23]
$\mathcal{B}(D^+ \to e^+\nu_e)$	$(10.109^{+2.052}_{-1.871}) \times 10^{-9}$	17.7×10^{-9}	9.84×10^{-9}	11.3×10^{-9}	$(8.6 \pm 0.5) \times 10^{-9}$	$< 8.8 \times 10^{-6}$
$\mathcal{B}(D^+ \to \mu^+\nu_\mu)$	$(4.295^{+0.872}_{-0.795}) \times 10^{-4}$	7.54×10^{-4}	4.29×10^{-4}	4.77×10^{-4}	$(3.6 \pm 0.2) \times 10^{-4}$	$(3.74 \pm 0.17) \times 10^{-4}$
$\mathcal{B}(D^+ \to \tau^+\nu_\tau)$	$(11.419^{+2.394}_{-2.169}) \times 10^{-4}$	17.9×10^{-4}	10.55×10^{-4}	20.3×10^{-4}	$(9.6 \pm 0.6) \times 10^{-4}$	$(12 \pm 2.7) \times 10^{-4}$
$\mathcal{B}(P \to l^+\nu_l)$	This work	[33]	[35]	[39]	[111]	Experiment [23]
$\mathcal{B}(D_s^+ \to e^+\nu_e)$	$(1.298^{+0.260}_{-0.238}) \times 10^{-7}$	1.82×10^{-7}	1.163×10^{-7}	1.63×10^{-7}	$(1.3 \pm 0.1) \times 10^{-7}$	$< 8.3 \times 10^{-5}$
$\mathcal{B}(D_s^+ \to \mu^+\nu_\mu)$	$(5.517^{+1.105}_{-1.013}) \times 10^{-3}$	7.74×10^{-3}	5.078×10^{-3}	6.9×10^{-3}	$(5.5 \pm 0.5) \times 10^{-3}$	$(5.43 \pm 0.15) \times 10^{-3}$
$\mathcal{B}(D_s^+ \to \tau^+\nu_\tau)$	$(5.408^{+1.158}_{-1.044}) \times 10^{-2}$	8.2×10^{-2}	0.4451×10^{-3}	6.49×10^{-2}	$(5.4 \pm 0.5) \times 10^{-2}$	$(5.32 \pm 0.11) \times 10^{-2}$
$\mathcal{B}(P \to l^+\nu_l)$	This work	[35]	[39]	[111]	[113]	Experiment [23]
$\mathcal{B}(B_u^+ \to e^+\nu_e)$	$(6.582^{+1.629}_{-1.407}) \times 10^{-12}$	6.162×10^{-12}	6.22×10^{-12}	$(8.4 \pm 0.4) \times 10^{-12}$	8.64×10^{-12}	$< 9.8 \times 10^{-7}$
$\mathcal{B}(B_u^+ \to \mu^+\nu_\mu)$	$(2.812^{+0.696}_{-0.601}) \times 10^{-7}$	2.705×10^{-7}	2.63×10^{-7}	$(3.5 \pm 0.3) \times 10^{-7}$	0.37×10^{-7}	$< 8.6 \times 10^{-7}$
$\mathcal{B}(B_u^+ \to \tau^+\nu_\tau)$	$(6.257^{+1.550}_{-1.338}) \times 10^{-5}$	6.088×10^{-5}	5.9×10^{-5}	$(8.0 \pm 0.4) \times 10^{-5}$	8.2×10^{-5}	$(10.9 \pm 2.4) \times 10^{-5}$
$\mathcal{B}(P \to l^+\nu_l)$	This work	[111]	[112]			
$\mathcal{B}(B_c^+ \to e^+\nu_e)$	$(0.748^{+0.182}_{-0.150}) \times 10^{-9}$	$(2.2 \pm 0.2) \times 10^{-9}$	$(2.24 \pm 0.24) \times 10^{-9}$			
$\mathcal{B}(B_c^+ \to \mu^+\nu_\mu)$	$(3.197^{+0.765}_{-0.652}) \times 10^{-5}$	$(9.2 \pm 0.9) \times 10^{-5}$	$(9.6 \pm 1.0) \times 10^{-5}$			
$\mathcal{B}(B_c^+ \to \tau^+\nu_\tau)$	$(0.765^{+0.183}_{-0.156}) \times 10^{-2}$	$(2.2 \pm 0.2) \times 10^{-2}$	$(2.29 \pm 0.24) \times 10^{-2}$			

data on the total decay widths for the decaying vector meson, D_s^{*+}, we take $\Gamma^{\text{total}}_{D_s^{*+}} = 0.07 \pm 0.028$ keV from the LQCD [76] calculation. However, for B_u^* and B_c^*, the isospin violating decay modes, $B_u^* \to B_u\pi$ and $B_c^* \to B_c\pi$ are explicitly forbidden as $m_{B_u^*} - m_{B_u} \simeq 45$ MeV $< m_\pi$ and $m_{B_c^*} - m_{B_c} \simeq 16$ MeV $< m_\pi$. In this sector, the electromagnetic radiative transitions, $B_u^* \to B_u\gamma$ and $B_c^* \to B_c\gamma$, should be dominant decay modes. Therefore, in the present calculation, we take the assumption that $\Gamma^{\text{total}}_{B_u^*} \simeq \Gamma(B_u^* \to B_u\gamma) = 372 \pm 56$ eV and $\Gamma^{\text{total}}_{B_c^*} \simeq \Gamma(B_c^* \to B_c\gamma) = 33 \pm 5$ eV from the covariant confined quark model (CCQM) [110] calculations.

Our predicted BFs for the PLDCPMs and PLDCVMs are listed in Tables VIII and IX, respectively, in comparison

TABLE IX. The predicted BFs $\mathcal{B}(V \to l^+\nu_l)$ in comparison with the observed value and other theoretical predictions.

$\mathcal{B}(V \to l^+\nu_l)$	This work	[21]	[111]	
$\mathcal{B}(D^{*+} \to e^+\nu_e)$	$(11.655^{+1.700}_{-1.762}) \times 10^{-10}$	$(9.5^{+2.9}_{-2.4}) \times 10^{-10}$	$(11 \pm 1) \times 10^{-10}$	
$\mathcal{B}(D^{*+} \to \mu^+\nu_\mu)$	$(11.607^{+1.693}_{-1.755}) \times 10^{-10}$	$(9.5^{+2.9}_{-2.4}) \times 10^{-10}$	$(11 \pm 1) \times 10^{-10}$	
$\mathcal{B}(D^{*+} \to \tau^+\nu_\tau)$	$(0.775^{+0.113}_{-0.117}) \times 10^{-10}$	$(0.6 \pm 0.2) \times 10^{-10}$	$(0.72 \pm 0.08) \times 10^{-10}$	
$\mathcal{B}(V \to l^+\nu_l)$	This work	[21]	[111]	Experiment [20]
$\mathcal{B}(D_s^{*+} \to e^+\nu_e)$	$(3.765^{-0.625}_{+1.519}) \times 10^{-5}$	$(6.7 \pm 0.4) \times 10^{-6}$	$(3.1 \pm 0.4) \times 10^{-6}$	$(2.1^{+1.2}_{-0.9_{stat.}} \pm 0.2_{syst.}) \times 10^{-5}$
$\mathcal{B}(D_s^{*+} \to \mu^+\nu_\mu)$	$(3.751^{-0.622}_{+1.513}) \times 10^{-5}$	$(6.7 \pm 0.4) \times 10^{-6}$	$(3.1 \pm 0.4) \times 10^{-6}$	
$\mathcal{B}(D_s^{*+} \to \tau^+\nu_\tau)$	$(0.436^{-0.071}_{+0.175}) \times 10^{-5}$	$(0.78 \pm 0.04) \times 10^{-6}$	$(0.36 \pm 0.04) \times 10^{-6}$	
$\mathcal{B}(V \to l^+\nu_l)$	This work	[21]	[111]	[112]
$\mathcal{B}(B_u^{*+} \to e^+\nu_e)$	$(5.942^{+0.429}_{-0.372}) \times 10^{-10}$	$(3.0 \pm 0.4) \times 10^{-10}$	$(6.4 \pm 2.6) \times 10^{-11}$	$(9.0 \pm 2.5) \times 10^{-10}$
$\mathcal{B}(B_u^{*+} \to \mu^+\nu_\mu)$	$(5.938^{+0.429}_{-0.372}) \times 10^{-10}$	$(3.0 \pm 0.4) \times 10^{-10}$	$(6.4 \pm 2.6) \times 10^{-11}$	$(9.0 \pm 2.5) \times 10^{-10}$
$\mathcal{B}(B_u^{*+} \to \tau^+\nu_\tau)$	$(4.953^{+0.358}_{-0.310}) \times 10^{-10}$	$(2.5 \pm 0.4) \times 10^{-10}$	$(5.4 \pm 2.2) \times 10^{-11}$	$(7.5 \pm 2.1) \times 10^{-10}$
$\mathcal{B}(V \to l^+\nu_l)$	This work	[21]	[111]	
$\mathcal{B}(B_c^{*+} \to e^+\nu_e)$	$(3.186^{+0.164}_{-0.123}) \times 10^{-6}$	$3.8^{+0.4}_{-0.3} \times 10^{-6}$	$(4.3 \pm 0.4) \times 10^{-6}$	
$\mathcal{B}(B_c^{*+} \to \mu^+\nu_\mu)$	$(3.184^{+0.164}_{-0.123}) \times 10^{-6}$	$3.8^{+0.4}_{-0.3} \times 10^{-6}$	$(4.3 \pm 0.4) \times 10^{-6}$	
$\mathcal{B}(B_c^{*+} \to \tau^+\nu_\tau)$	$(2.805^{+0.144}_{-0.107}) \times 10^{-6}$	$3.3^{+0.4}_{-0.3} \times 10^{-6}$	$(3.8 \pm 0.4) \times 10^{-6}$	

with the available observed data and other SM model predictions. While calculating the uncertainties in our predictions for BFs, we take into account the uncertainties of the relevant physical quantities such as the decaying meson masses $m_{P(V)}$, lepton masses m_l ($l = e,\ \mu,\ \tau$), lifetimes τ_P, total decay widths $\Gamma_V^{\rm total}$, CKM parameters $|V_{q_1 q_2}|$, and those of our predicted decay constants $f_{P(V)}$, shown in Tables IV and V. Our predictions on the BFs for PLDCPMs: $\mathcal{B}(D^+_{(s)} \to l^+\nu_l)$, $\mathcal{B}(B^+_u \to e^+\nu_e)$, and $\mathcal{B}(B^+_u \to \mu^+\nu_\mu)$ listed in Table VIII, are obtained well within the experimental limit and in reasonable agreement with other theoretical results of Refs. [35,39,111]. Our prediction on $\mathcal{B}(B^+_u \to \tau^+\nu_\tau)$ is also comparable to the observed data within the experimental uncertainties and the theoretical predictions of [35,39]. We predict the BFs for B_c meson decay in the same order of magnitude, although in terms of their absolute values, our results are underestimated as compared to the available theoretical results of [111,112]. As one can see from Table IX, our predicted $\mathcal{B}(D^{*+}_s \to e^+\nu_e)$ agree with the recently observed data from BESIII Collaboration within the experimental uncertainties and compares well with the prediction of Ref. [111]. In the PLDCVMs, our predictions (Table IX) on $\mathcal{B}(D^{*+} \to l^+\nu_l)$, $\mathcal{B}(B^{*+}_u \to l^+\nu_l)$, $\mathcal{B}(B^{*+}_c \to l^+\nu_l)$, ($l = e,\ \mu,\ \tau$), and $\mathcal{B}(D^{*+}_s \to l^+\nu_l)$, ($l = \mu,\ \tau$), are in good agreement with those of Refs. [21,111], Ref. [111], Ref. [21], and Ref. [111], respectively.

Finally, we calculate the observables: $(\mathcal{R}^\tau_\mu)^{D^+} = \frac{\mathcal{B}(D^+\to\tau^+\nu_\tau)}{\mathcal{B}(D^+\to\mu^+\nu_\mu)}$ and $(\mathcal{R}^\tau_\mu)^{D^+_s} = \frac{\mathcal{B}(D^+_s\to\tau^+\nu_\tau)}{\mathcal{B}(D^+_s\to\mu^+\nu_\mu)}$. These observables are significant as the CKM matrix elements do not contribute in their evaluation. The uncertainties due to the model calculation and those of the CKM parameter values are canceled in the evaluation of $(\mathcal{R}^\tau_\mu)^{D^+}$ and $(\mathcal{R}^\tau_\mu)^{D^+_s}$. Therefore, contrary to other observables, these ratios $(\mathcal{R}^\tau_\mu)^{D^+_{(s)}}$ provide an essential test of the phenomenological model used in the description of the PLDCPMs and PLDCVMs. Our predictions $(\mathcal{R}^\tau_\mu)^{D^+} = 2.66^{+0.78}_{-0.71}$ and $(\mathcal{R}^\tau_\mu)^{D^+_s} = 9.80^{+2.87}_{-2.61}$ are consistent with the corresponding observed data: 3.21 ± 0.73 and 9.82 ± 0.40, respectively, from PDG [23].

IV. SUMMARY AND CONLUSION

In the present work, we study the purely leptonic decays of heavy-flavored charged pseudoscalar and vector ($D^{(*)+}_{(s)}, B^{(*)+}_{(c)}$) mesons in the framework of the relativistic independent quark (RIQ) model based on an average flavor-independent confining potential in the scalar-vector harmonic form. Using the meson wave functions derivable in the RIQ model, we first compute the mass spectra in reasonable agreement with the observed data for the masses of the ground state pseudoscalar ($J^P = 0^-$) and vector ($J^P = 1^-$) mesons. With the model parameters, quark masses m_q and potential parameters (V_0, a), as fixed from the hadron spectroscopy, we predict the decay constants: f_P and f_V, for purely leptonic decays of charged pseudoscalar (P) and vector (V) mesons, respectively.

To study the sensitivity of the input parameters in our predictions of decay constants f_P and f_V, we include the systematic errors in our analysis both from the $\pm10\%$ variation of the potential parameters (V_0, a) for fixed quark masses m_q and $\pm10\%$ variation of quark masses m_q for fixed values of potential parameters (V_0, a). We find that our predicted decay constants for the PLDCVMs are more sensitive to the variation of potential parameters with fixed quark masses. However, our predicted decay constants for the PLDCPMs are found to be more sensitive to the variation of quark mass value with fixed potential parameters.

Our predictions on f_{D^+}, $f_{D^+_s}$, and $f_{D^{*+}_s}$ not only agree with the corresponding observed data within their experimental limits but compare well with several SM predictions. Our result for f^{*+}_D is also in good comparison with the results of LFQM Lin, LFQM, QCD SR, and LQCD calculations. The decay constant $f_{B^+_u}$, predicted in the present calculation, lies well within the experimental limit. In the present study, our predictions on $f_{B^+_u}$ and $f_{B^{*+}_u}$ compare well with those of BS, LFQM, LQCD and LFQM, LQCD, respectively. In the absence of the observed data in B^{*+}_c sector, we compare our predicted decay constant with a few theoretical results available in the literature. Our result for $f_{B^+_c}$ is comparable to that of QCD SR. However, our prediction on $f_{B^{*+}_c}$ appears somewhat underestimated compared to those of BS, LFQM Lin, LCDA, and QCD SR calculations.

We also calculate the ratios, f_V/f_P, f_{P_1}/f_{P_2}, f_{V_1}/f_{V_2}, which are sensitive to the difference between the vector (V) and pseudoscalar (P), pseudoscalar (P_1) and pseudoscalar (P_2) and vector (V_1) and vector (V_2) wave functions, respectively. While our predicted $f_{D^{*+}_s}/f_{D^+_s}$ matches exactly with the LFQM Lin prediction, our results for $f_{D^{*+}_s}/f_{D^+_s}$ and $f_{D^{*+}}/f_{D^+}$ agree with those of theoretical predictions based on RQM, LFQM, QCD SR, and LQCD calculations. We find $f_{D^+_s}/f_{D^+}$ in good comparison with the observed data. Our predicted $f_{D^+_s}/f_{D^+}$ is also consistent with other SM predictions including those obtained from the RQM, LFHQCD, and QCD SR approaches, while our result for $f_{D^{*+}_s}/f_{D^{*+}}$ compares well with the finding of LFQM Lin, and LFQM calculations. In the b-flavored meson sector, our result for $f_{B^{*+}_u}/f_{B^+_u}$ is comparable to the predictions of LFQM Lin, LFQM, and that for $f_{B^{*+}_c}/f_{B^+_c}$ is in reasonable agreement with the results of LFQM Lin and QCD SR calculation. Our predictions on $f_{B^{*+}_c}/f_{B^{*+}_u}$ and $f_{B^+_c}/f_{B^+_u}$ can be verified in the upcoming experimental measurement and compared with future SM predictions.

Our predicted BFs for PLDCPMs $\mathcal{B}(D^+_{(s)} \to l^+\nu_l)$, $\mathcal{B}(B^+_{(c)} \to l^+\nu_l)$, $l = e, \mu, \tau$ are found to lie within experimental limits and also in reasonable agreement with theoretical results based on the NRQM, RQM and LFQM calculations. In the PLDCVMs, our prediction of $\mathcal{B}(D_s^{*+} \to e^+\nu_e)$ agrees with the recently observed data from BESIII Collaboration. For other modes such as $D^{*+} \to l^+\nu_l$, $B^{*+}_{(c)} \to l^+\nu_l$, our predicted BFs find good agreement with those obtained from LFQM and LQCD calculations. Finally, we predict the ratios of BFs, which are important observables that provide an essential test of the phenomenological model used in the study of PLDCMs. Our predictions, $(\mathcal{R}^\tau_\mu)^{D^+} = 2.66^{+0.78}_{-0.71}$ and $(\mathcal{R}^\tau_\mu)^{D_s^+} = 9.80^{+2.87}_{-2.61}$, are consistent with the corresponding observed data.

With the potential prospects of precision measurements, high data statistics, and improved analytical tools available in the high-energy experimental frontiers, careful measurement of yet unmeasured decay constants $f_{P(V)}$ and BFs might lead to predictions even close to the accessible limits of ongoing experiments and their upgrades at Belle-II, SCTF or STCF, CEPC, FCC-ee, and LHCb. The predicted decay constants for the vector mesons are helpful in studying the radiative decays in the charm meson sector, for example, $D_s^* \to D_s\gamma$. In $B_c^{(*)}$ meson sector, where experimental data are scant, the predicted decay constants of the $B_c^{(*)}$ mesons can be used to study the possible effect of new physics in the $b \to c$ channels. In heavy-flavor sector, the PLDCMs thus provide a fascinating area of experimental and theoretical research in the future.

ACKNOWLEDGMENTS

The library and computational facilities provided by authorities of Siksha 'O' Anusandhan Deemed to be University, Bhubaneswar, 751030, India are duly acknowledged

[1] M. Blanke, CERN Yellow Rep. School Proc. **71**, 1705 (2017).
[2] M. Artuso, C. Boulahouache, S. Blusk, J. Butt, O. Dorjkhaidav, J. Li, N. Menaa, R. Mountain, R. Nandakumar, K. Randrianarivony *et al.*, Phys. Rev. Lett. **95**, 251801 (2005).
[3] B. Eisenstein, I. Karliner, S. Mehrabyan, N. Lowrey, M. Selen, E. White, J. Wiss, R. Mitchell, M. Shepherd, D. Besson *et al.*, Phys. Rev. D **78**, 052003 (2008).
[4] M. Artuso, S. Blusk, J. Butt, S. Khalil, J. Li, N. Menaa, R. Mountain, S. Nisar, K. Randrianarivony, R. Sia *et al.*, Phys. Rev. Lett. **99**, 071802 (2007).
[5] J. Alexander, D. Cassel, J. Duboscq, R. Ehrlich, L. Fields, R. Galik, L. Gibbons, R. Gray, S. Gray, D. Hartill *et al.*, Phys. Rev. D **79**, 052001 (2009).
[6] M. Ablikim, M. Achasov, X. Ai, O. Albayrak, D. Ambrose, F. An, Q. An, J. Bai, R. B. Ferroli, Y. Ban *et al.*, Phys. Rev. D **89**, 051104 (2014).
[7] M. Ablikim, M. N. Achasov, P. Adlarson, S. Ahmed, M. Albrecht, M. Alekseev, A. Amoroso, F. An, Q. An, Y. Bai *et al.*, Phys. Rev. Lett. **123**, 211802 (2019).
[8] K. Ecklund, W. Love, V. Savinov, A. Lopez, H. Mendez, J. Ramirez, J. Ge, D. H. Miller, I. Shipsey, B. Xin *et al.*, Phys. Rev. Lett. **100**, 161801 (2008).
[9] P. Onyisi, J. Rosner, J. Alexander, D. Cassel, J. Duboscq, R. Ehrlich, L. Fields, R. Galik, L. Gibbons, R. Gray *et al.*, Phys. Rev. D **79**, 052002 (2009).
[10] A. Zupanc, I. Adachi, H. Aihara, K. Arinstein, D. M. Asner, T. Aushev, A. Bakich, A. Bala, B. Bhuyan, G. Bonvicini *et al.*, J. High Energy Phys. 09 (2013) 139.
[11] P. del Amo Sanchez, J. Lees, V. Poireau, E. Prencipe, V. Tisserand, J. G. Tico, E. Grauges, M. Martinelli, A. Palano, M. Pappagallo *et al.*, Phys. Rev. D **82**, 091103 (2010).
[12] M. Ablikim, M. N. Achasov, X. Ai, O. Albayrak, M. Albrecht, D. Ambrose, A. Amoroso, F. An, Q. An, J. Bai *et al.*, Phys. Rev. D **94**, 072004 (2016).
[13] M. Ablikim, M. Achasov, P. Adlarson, S. Ahmed, M. Albrecht, R. Aliberti, A. Amoroso, M. An, Q. An, X. Bai *et al.*, Phys. Rev. D **104**, 052009 (2021).
[14] M. Ablikim, M. N. Achasov, P. Adlarson, S. Ahmed, M. Albrecht, R. Aliberti, A. Amoroso, M. An, Q. An, X. Bai *et al.*, Phys. Rev. Lett. **127**, 171801 (2021).
[15] M. Ablikim, M. N. Achasov, P. Adlarson, S. Ahmed, M. Albrecht, R. Aliberti, A. Amoroso, M. An, Q. An, X. Bai *et al.*, Phys. Rev. D **104**, 032001 (2021).
[16] K. Hara, Y. Horii, T. Iijima, I. Adachi, H. Aihara, D. Asner, T. Aushev, T. Aziz, A. Bakich, M. Barrett *et al.*, Phys. Rev. Lett. **110**, 131801 (2013).
[17] B. Kronenbitter, M. Heck, P. Goldenzweig, T. Kuhr, A. Abdesselam, I. Adachi, H. Aihara, S. Al Said, K. Arinstein, D. Asner *et al.*, Phys. Rev. D **92**, 051102 (2015).
[18] J. Lees, V. Poireau, V. Tisserand, J. G. Tico, E. Grauges, A. Palano, G. Eigen, B. Stugu, D. N. Brown, L. Kerth *et al.*, Phys. Rev. D **88**, 031102 (2013).
[19] B. Aubert, Y. Karyotakis, J. Lees, V. Poireau, E. Prencipe, X. Prudent, V. Tisserand, J. G. Tico, E. Grauges, M. Martinelli *et al.*, Phys. Rev. D **81**, 051101 (2010).
[20] M. Ablikim, M. Achasov, P. Adlarson, R. Aliberti, A. Amoroso, M. An, Q. An, Y. Bai, O. Bakina, I. Balossino *et al.*, Phys. Rev. Lett. **131**, 141802 (2023).
[21] Y. Yang, Z. Li, K. Li, J. Huang, and J. Sun, Eur. Phys. J. C **81**, 1110 (2021).
[22] A. Abada, M. Abbrescia, S. S. AbdusSalam, I. Abdyukhanov, J. A. Fernandez, A. Abramov, M. Aburaia, A. Acar, P. Adzic, P. Agrawal *et al.*, Eur. Phys. J. C **79**, 474 (2019).

[23] R. Workman, V. Burkert, V. Crede, E. Klempt, U. Thoma, L. Tiator, K. Agashe, G. Aielli, B. Allanach *et al.*, Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).
[24] J. Abdallah, P. Abreu, W. Adam, P. Adzic, T. Albrecht, T. Alderweireld, R. Alemany-Fernandez, T. Allmendinger, P. Allport, U. Amaldi *et al.*, Phys. Lett. B **576**, 29 (2003).
[25] R. Aaij, B. Adeva, M. Adinolfi, A. Affolder, Z. Ajaltouni, S. Akar, J. Albrecht, F. Alessio, M. Alexander, S. Ali *et al.*, J. High Energy Phys. 09 (2015) 084.
[26] R. Aaij, B. Adeva, M. Adinolfi, A. Affolder, Z. Ajaltouni, S. Akar, J. Albrecht, F. Alessio, M. Alexander, S. Ali *et al.*, J. High Energy Phys. 08 (2018) 174.
[27] X.-K. Dong, L.-L. Wang, and C.-Z. Yuan, Chin. Phys. C **42**, 043002 (2018).
[28] G. Boroun, S. Zarrin, and S. Dadfar, Nucl. Phys. **A953**, 21 (2016).
[29] D. Yang and W. Zhang, Chin. Phys. C **43**, 083101 (2019).
[30] X.-C. Zheng, C.-H. Chang, T.-F. Feng, and X.-G. Wu, Phys. Rev. D **100**, 034004 (2019).
[31] G. Chen, C.-H. Chang, and X.-G. Wu, Phys. Rev. D **97**, 114022 (2018).
[32] C. Albertus, E. Hernández, J. Nieves, and J. M. Verde-Velasco, Phys. Rev. D **71**, 113006 (2005).
[33] B. Yazarloo and H. Mehraban, Europhys. Lett. **116**, 31004 (2016).
[34] B. Yazarloo and H. Mehraban, Eur. Phys. J. Plus **132**, 1 (2017).
[35] M. Abu-Shady, E. Khokha *et al.*, Adv. High Energy Phys. **2018**, 01 (2018).
[36] D. Ebert, R. Faustov, and V. Galkin, Mod. Phys. Lett. A **17**, 803 (2002).
[37] D. Ebert, R. Faustov, and V. Galkin, Phys. Rev. D **67**, 014027 (2003).
[38] D. Ebert, R. Faustov, and V. Galkin, Phys. Lett. B **635**, 93 (2006).
[39] H. Hassanabadi, M. Ghafourian, and S. Rahmani, Few-Body Syst. **57**, 249 (2016).
[40] D. S. Hwang and G.-H. Kim, Phys. Rev. D **55**, 6944 (1997).
[41] N. Soni and J. Pandya, Phys. Rev. D **96**, 016017 (2017).
[42] N. Soni and J. Pandya, Phys. Rev. D **99**, 059901(E) (2019).
[43] H.-K. Sun and M.-Z. Yang, Phys. Rev. D **95**, 113001 (2017).
[44] H.-K. Sun and M.-Z. Yang, Phys. Rev. D **99**, 093002 (2019).
[45] G.-L. Wang, Phys. Lett. B **633**, 492 (2006).
[46] Z.-G. Wang, W.-M. Yang, and S.-L. Wan, Nucl. Phys. **A744**, 156 (2004).
[47] G. Cvetič, C. S. Kim, G.-L. Wang, and W. Namgung, Phys. Lett. B **596**, 84 (2004).
[48] H.-M. Choi and C.-R. Ji, Phys. Rev. D **75**, 034019 (2007).
[49] H.-M. Choi, Phys. Rev. D **75**, 073016 (2007).
[50] H.-M. Choi, C.-R. Ji, Z. Li, and H.-Y. Ryu, Phys. Rev. C **92**, 055203 (2015).
[51] Q. Chang, X.-N. Li, X.-Q. Li, and F. Su, Chin. Phys. C **42**, 073102 (2018).
[52] Q. Chang, X.-N. Li, X.-Q. Li, F. Su, and Y.-D. Yang, Phys. Rev. D **98**, 114018 (2018).
[53] R. Verma, J. Phys. G **39**, 025005 (2012).
[54] C.-W. Hwang, Phys. Rev. D **81**, 054022 (2010).
[55] C.-Q. Geng, C.-C. Lih, and C. Xia, Eur. Phys. J. C **76**, 1 (2016).
[56] N. Dhiman and H. Dahiya, Eur. Phys. J. Plus **133**, 134 (2018).
[57] N. Dhiman, H. Dahiya, C.-R. Ji, and H.-M. Choi, Phys. Rev. D **100**, 014026 (2019).
[58] H.-Y. Cheng and X.-W. Kang, Eur. Phys. J. C **77**, 1 (2017).
[59] C.-W. Hwang, Phys. Rev. D **81**, 114024 (2010).
[60] N. Dhiman and H. Dahiya, Eur. Phys. J. Plus **133**, 134 (2018).
[61] H. G. Dosch, G. F. de Téramond, and S. J. Brodsky, Phys. Rev. D **95**, 034016 (2017).
[62] Q. Chang, S. Xu, and L. Chen, Nucl. Phys. **B921**, 454 (2017).
[63] P. Gelhausen, A. Khodjamirian, A. A. Pivovarov, and D. Rosenthal, Phys. Rev. D **88**, 014015 (2013).
[64] Z.-G. Wang, Eur. Phys. J. C **75**, 1 (2015).
[65] W. Lucha, D. Melikhov, and S. Simula, Phys. Lett. B **735**, 12 (2014).
[66] S. Narison, Int. J. Mod. Phys. A **30**, 1550116 (2015).
[67] S. Narison, Nucl. Part. Phys. Proc. **270–272**, 143 (2016).
[68] S. Narison, Phys. Lett. B **718**, 1321 (2013).
[69] M. J. Baker, J. Bordes, C. A. Dominguez, J. Penarrocha, and K. Schilcher, J. High Energy Phys. 07 (2014) 034.
[70] W. Lucha, D. Melikhov, and S. Simula, Phys. Rev. D **91**, 116009 (2015).
[71] D. Becirevic, P. Boucaud, J. Leroy, V. Lubicz, G. Martinelli, F. Mescia, and F. Rapuano, Phys. Rev. D **60**, 074501 (1999).
[72] K. Bowler, L. Del Debbio, J. Flynn, G. N. Lacagnina, V. I. Lesk, C. M. Maynard, and D. G. Richards, Nucl. Phys. **B619**, 507 (2001).
[73] D. Bečirević, V. Lubicz, F. Sanfilippo, S. Simula, and C. Tarantino, J. High Energy Phys. 02 (2012) 042.
[74] V. Lubicz, A. Melis, and S. Simula, Phys. Rev. D **96**, 034524 (2017).
[75] Y. Chen, W.-F. Chiu, M. Gong, Z. Liu, and Y. Ma, Chin. Phys. C **45**, 023109 (2021).
[76] G. Donald, C. Davies, J. Koponen, and G. Lepage, Phys. Rev. Lett. **112**, 212002 (2014).
[77] H. Na, C. J. Monahan, C. T. Davies, R. Horgan, G. P. Lepage, and J. Shigemitsu, Phys. Rev. D **86**, 034506 (2012).
[78] C. Davies, C. McNeile, E. Follana, G. P. Lepage, H. Na, and J. Shigemitsu, Phys. Rev. D **82**, 114504 (2010).
[79] C. McNeile, C. Davies, E. Follana, K. Hornbostel, and G. P. Lepage, Phys. Rev. D **85**, 031503 (2012).
[80] A. Bazavov *et al.* (Fermilab Lattice and MILC Collaborations), Phys. Rev. D **85**, 114506 (2012).
[81] R. Dowdall, C. Davies, R. Horgan, C. Monahan, and J. Shigemitsu (HPQCD Collaboration), Phys. Rev. Lett. **110**, 222003 (2013).
[82] J. Dingfelder and T. Mannel, Rev. Mod. Phys. **88**, 035008 (2016).
[83] X. Zuo, M. Fedele, C. Helsens, D. Hill, S. Iguro, and M. Klute, Eur. Phys. J. C **84**, 1 (2024).
[84] Y. Aoki, T. Blum, G. Colangelo, S. Collins, M. D. Morte, P. Dimopoulos, S. Dürr, X. Feng, H. Fukaya, M. Golterman *et al.*, Eur. Phys. J. C **82**, 869 (2022).
[85] N. Barik and B. K. Dash, Phys. Rev. D **34**, 2092 (1986).

[86] M. Priyadarsini, P. C. Dash, S. Kar, S. P. Patra, and N. Barik, Phys. Rev. D **94**, 113011 (2016).
[87] N. Barik, S. Kar, S. Naimuddin, and P. C. Dash, Phys. Rev. D **59**, 037301 (1998).
[88] N. Barik, B. K. Dash, and P. C. Dash, Pramana J. Phys. **29**, 543 (1987).
[89] N. Barik and B. K. Dash, Phys. Rev. D **33**, 1925 (1986).
[90] N. Barik and P. C. Dash, Mod. Phys. Lett. A **10**, 103 (1995).
[91] N. Barik, S. Kar, and P. C. Dash, Phys. Rev. D **57**, 405 (1998).
[92] N. Barik and P. C. Dash, Phys. Rev. D **47**, 2788 (1993).
[93] N. Barik, S. Naimuddin, P. C. Dash, and S. Kar, Phys. Rev. D **77**, 014038 (2008).
[94] N. Barik, S. Naimuddin, P. C. Dash, and S. Kar, Phys. Rev. D **78**, 114030 (2008).
[95] N. Barik, S. Naimuddin, and P. C. Dash, Int. J. Mod. Phys. A **24**, 2335 (2009).
[96] S. Patnaik, P. C. Dash, S. Kar, S. P. Patra, and N. Barik, Phys. Rev. D **96**, 116010 (2017).
[97] S. Patnaik, P. C. Dash, S. Kar, and N. Barik, Phys. Rev. D **97**, 056025 (2018).
[98] N. Barik, S. Naimuddin, P. C. Dash, and S. Kar, Phys. Rev. D **80**, 074005 (2009).
[99] S. Patnaik, L. Nayak, P. C. Dash, S. Kar, and N. Barik, Eur. Phys. J. Plus **135**, 1 (2020).
[100] L. Nayak, S. Patnaik, P. C. Dash, S. Kar, and N. Barik, Phys. Rev. D **104**, 036012 (2021).
[101] L. Nayak, P. C. Dash, S. Kar, and N. Barik, Eur. Phys. J. C **82**, 750 (2022).
[102] L. Nayak, P. C. Dash, S. Kar, and N. Barik, Phys. Rev. D **105**, 053007 (2022).
[103] K. Dash, P. C. Dash, R. N. Panda, L. Nayak, S. Kar, and N. Barik, Eur. Phys. J. C **83**, 1163 (2023).
[104] P. Ball, V. M. Braun, and A. Lenz, J. High Energy Phys. 11 (2006) 004.
[105] P. Ball and G. W. Jones, J. High Energy Phys. 03 (2007) 069.
[106] B. Margolis and R. Mendel, Phys. Rev. D **28**, 468 (1983).
[107] R. Workman, V. Burkert, V. Crede, E. Klempt, U. Thoma, L. Tiator, K. Agashe, G. Aielli, B. Allanach *et al.* (Particle Data Group), Chin. Phys. C **40**, 100001 (2016).
[108] T. Aliev, T. Barakat, and S. Bilmis, Nucl. Phys. **B947**, 114726 (2019).
[109] S. Narison, Phys. Lett. B **807**, 135522 (2020).
[110] M. A. Ivanov, Z. Tyulemissov, and A. Tyulemissova, Phys. Rev. D **107**, 013009 (2023).
[111] B.-B. Zhou, J.-J. Sun, and Y.-J. Zhang, Commun. Theor. Phys. **67**, 655 (2017).
[112] H.-K. Sun and M.-Z. Yang, Phys. Rev. D **99**, 093002 (2019).
[113] V. Kher, N. Devlani, and A. K. Rai, Chin. Phys. C **41**, 093101 (2017).