Investigations of the weak decays of $D\bar{B}$ molecules

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The decays of exotic states discovered experimentally always proceed via the strong and electromagnetic interactions. Recently, a tetraquark state with the quark content $bc\bar{q}\,\bar{q}$ was predicted by lattice QCD simulations. It is below the mass threshold of $D\bar{B}$, which can only decay via the weak interaction. In this work, based on the decay mechanism of T_{cc} as a DD^* molecule, we propose that the decays of the $bc\bar{q}\,\bar{q}$ tertaquark state as a $D\bar{B}$ molecule proceed via the Cabibbo-favored weak decays of the $\bar{B}$ or D meson, accompanied by the tree-level decay modes and the triangle decay modes. Our results indicate that the branching fraction of the $D\bar{B}$ molecule decaying into $\pi^+K^-\bar{B}^0$ is sizable, which is a good channel to observe the $D\bar{B}$ molecule in future experiments.

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I. INTRODUCTION

Many new hadron states beyond mesons made of a pair of quark and antiquark and baryons made of three quarks in the conventional quark model, often named as exotic states, have been discovered in recent years. Their quark configurations in terms of quantum chromodynamics (QCD) can be either compact multiquark state, hybrid, hadroncharmonium, or hadronic molecule [1–15]. Among them, the hadronic molecular picture, where these states are composed by a pair of conventional hadrons, have been intensively discussed, motivating us to study the relevant hadron-hadron interactions [16–22] as well as explore the corresponding few-body hadronic molecules [23–26]. There exist some candidates of hadronic molecules, such as $D_{s0}^*(2317)$ and $D_{s1}(2460)$ as the DK and D^*K molecules, $X(3872)$ as the $\bar{D}^*D$ molecule, $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ as the $\bar{D}^{(*)}\Sigma_c$ molecules, T_{cc} as the D^*D molecule, and so on [15]. Up to now, the molecular interpretations for the exotic states have not been firmly established, but at least one can conclude that these states contain sizable molecular components in their wave functions.

The decay of a hadronic molecule is responsible for its width. According to the number of final states in the decay, the decay modes of hadronic molecules can contain two-body, three-body, or four-body, among which the former two are rather common in hadroic molecules. For the two-body decay mode of a hadronic molecule, the inelastic hadron-hadron potential is crucial to calculate its partial decay width. A typical example is that the $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$, as $\bar{D}^{(*)}\Sigma_c$ molecules, decaying into $J/\psi p$ and $\bar{D}^{(*)}\Lambda_c$ can be described by the one-boson exchange model [27–32] or effective field theories [33–36]. Due to the large uncertainties in the $\bar{D}^{(*)}\Sigma_c \to J/\psi p$ and $\bar{D}^{(*)}\Sigma_c \to \bar{D}^{(*)}\Lambda_c$ potentials, there are large uncertainties in their partial decay widths as well. As for the three-body decay mode, the decay of a hadronic molecule proceeds via the decay of either its constituent. A classical example is that the doubly charmed tetraquark T_{cc} as a D^*D bound states decaying into $DD\pi$ and $DD\gamma$ proceeds via the off-shell D^* meson decaying into $D\pi$ and $D\gamma$ [37–48]. Similarly, it is natural to expect that the $\bar{D}^{(*)}\Sigma_c$ molecules can decay into $\bar{D}^{(*)}\Lambda_c\pi$ via the off-shell Σ_c baryon decaying into $\Lambda_c\pi$ [49], while no significant signal is observed in a recent analysis of LHCb Collaboration [50].

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Due to the fact that the order of magnitude of weak decays is much smaller than those of strong and radiative decays, the weak decay of a hadronic molecule is always neglected. Moreover, since hadronic molecules are all observed via their strong or radiative decays, the weak decay of hadronic molecule is scarcely discussed in the literature. In Ref. [51], assuming the $D_{s0}^*(2317)$ as the DK molecule, Branz *et al.*, investigated the weak decays of $D_{s0}^*(2317) \to f_0(980)X$ ($X = \pi$, K, and ρ mesons) via the triangle diagram mechanism. In Ref. [52], the weak decay of the doubly strange dibaryon $\Lambda\Lambda$ into a pair of nucleons nn is studied via the weak decay of $\Lambda \to n\pi$. In this work, we focus on the weak decays of hadronic molecules, especially those hadronic molecules that can only decay weakly.

The hadronic molecules containing the quark content $\bar{Q}\,\bar{Q}\,qq$ are particularly good for studying weak decays of hadronic molecules, which are intensively studied since the doubly charmed tetraquark state T_{cc} is discovered. Very recently, lattice QCD simulations studied the $D\bar{B}$ interaction and found a bound state below the $D\bar{B}$ mass threshold [53,54], denoted as T_{cb}, which can only be discovered via the weak decay modes [55]. In this work, we take the contact-range effective field theory(EFT) to calculate the mass of the $D\bar{B}$ molecule, then adopt the tree-level and triangle diagram decays of the $D\bar{B}$ molecule to calculate its partial decay widths, which are helpful to experimentally search for the predicted $D\bar{B}$ molecule.

This work is organized as follows. We briefly introduce the weak decay mechanisms of the $D\bar{B}$ molecule and the effective Lagrangian approach in Sec. II. Numerical results and discussions are given in Sec. III, followed by a summary in the last section.

II. THEORETICAL FORMALISM

Following the decay mechanism of T_{cc} as a D^*D molecule [37–46], we propose that the hadronic molecule $D\bar{B}$ decay via the weak decays of the D or the $\bar{B}$ meson. Considering the efficiency in experimental measurements and focusing on the dominant branching fraction in the $D\bar{B}$ molecule decay, we select the Cabibbo favored weak decays $\bar{B} \to D^{(*)}\bar{D}_s$ and $D \to \pi K$ as the secondary decay. In Fig. 1, we illustrate the weak decays of the $D\bar{B}$ molecule via the tree diagram. Moreover, we describe the $D\bar{B}$ molecule decaying into the T_{cc} as a DD^* molecule via the triangle diagram mechanism as shown in Fig. 2, where $\bar{B}$ first weakly decays into $D^*\bar{D}_s$, and then the D^*D interaction dynamically generates the T_{cc}.

A. Effective Lagrangian

In this work, we employ the effective Lagrangian approach to calculate the weak decay widths. At first, we present the relevant Lagranian to be used in this work. The hadronic molecule couplings to the corresponding constituents are described by the following Lagrangians [38,56]

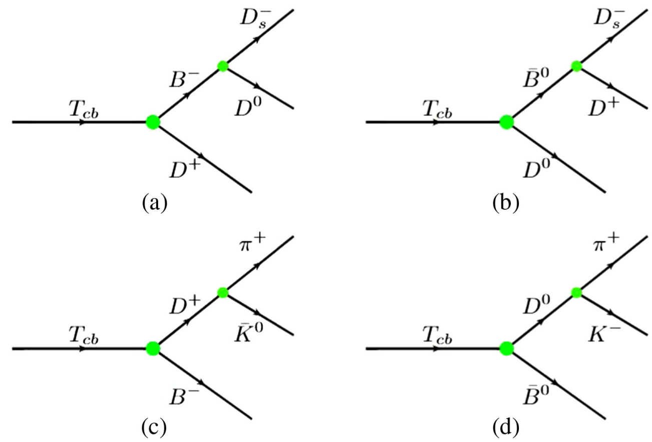

FIG. 1. Tree diagrams for the decays of $T_{cb} \to D^+D^0D_s^-$ (a),(b), $T_{cb} \to B^-\pi^+\bar{K}^0$ (c), and $T_{cb} \to \bar{B}^0\pi^+K^-$ (d).

$$\mathcal{L}_{T_{cb}D\bar{B}} = g_{T_{cb}D\bar{B}} T_{cb} D\bar{B},$$
$$\mathcal{L}_{T_{cc}DD^*} = g_{T_{cc}DD^*} T^\mu_{cc} D D^*_\mu, \tag{1}$$

where g with the specific subscript denotes the molecule's couplings to their constituents, which are estimated in the contact-range EFT approach.

As for the weak decays, the amplitudes of $\bar{B}(k_0) \to \bar{D}_s(q_1)D^{(*)}(q_2)$ and $D(k_0) \to \pi(q_1)K(q_2)$ have the following form [57]

$$\begin{aligned}
\mathcal{A}(\bar{B} \to \bar{D}_s D^*) &= \frac{G_F}{\sqrt{2}} V_{cb}V_{cs}a_1 f_{\bar{D}_s}\{-q_1\cdot\varepsilon(q_2) \\
&(m_{D^*}+m_{\bar{B}})A_1(q_1^2) + (k_0+q_2)\cdot\varepsilon(q_2)q_1\cdot(k_0+q_2) \\
&\frac{A_2(q_1^2)}{m_{D^*}+m_{\bar{B}}} + (k_0+q_2)\cdot\varepsilon(q_2)[(m_{D^*}+m_{\bar{B}})A_1(q_1^2) \\
&-(m_{\bar{B}}-m_{D^*})A_2(q_1^2) - 2m_{D^*}A_0(q_1^2)]\}, \\
\mathcal{A}(\bar{B} \to \bar{D}_s D) &= \frac{G_F}{\sqrt{2}} V_{cb}V_{cs}a_1 f_{\bar{D}_s}(m_{\bar{B}}^2 - m_D^2)F_0(q_1^2), \\
\mathcal{A}(D \to K\pi) &= \frac{G_F}{\sqrt{2}} V_{sc}V_{ud}a_1 f_\pi(m_D^2 - m_K^2)F_0(q_1^2),
\end{aligned} \tag{2}$$

where a_1 is the Wilson coefficient, $f_{\bar{D}_s}$ is the decay constant for the $\bar{D}_s$ meson, and ϵ_μ denotes the polarization vector of a vector particle. In this work, we take $G_F = 1.166 \times 10^{-5}$ GeV^{-2}, $V_{cb} = 0.0395$, $V_{cs} = 0.991$, and $f_{\bar{D}_s} = 250$ MeV [58,59]. The form factors of $F_0(t)$, $A_0(t)$,

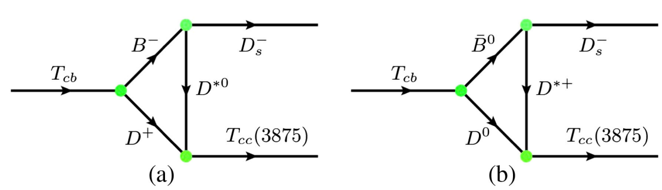

FIG. 2. Triangle diagrams for the decays of $T_{cb} \to T_{cc}(3875)\,D_s^-$ (a), (b).

TABLE I. Values of $F(0)$, a, b in the $\bar{B} \to D^{(*)}$ transition form factors [60].

	F_0	F_1	V	A_0	A_1	A_2
$F(0)^{B\to D^{(*)}}$	0.67	0.67	0.77	0.68	0.65	0.61
$a^{B\to D^{(*)}}$	0.63	1.22	1.25	1.21	0.60	1.12
$b^{B\to D^{(*)}}$	−0.01	0.36	0.38	0.36	0.00	0.31

$A_1(t)$, and $A_2(t)$ with $t \equiv q^2$ can be parametrized in the form of $F(t) = F(0)/[1 - a(t/m_B^2) + b(t^2/m_B^4)]$. The values of F_0, a, and b in the transition form factors of $\bar{B} \to D^{(*)}$ are taken from Ref. [60] and shown in Table I.

For the weak decays of $\bar{B} \to \bar{D}_s D$ and $D \to K\pi$, the particles in the tree decay modes are on-shell, resulting in the amplitudes of $\mathcal{B}(\bar{B} \to \bar{D}_s D)$ and $\mathcal{B}(D \to K\pi)$ to be the constants. Therefore, we further parametrize their Lagrangians as

$$\mathcal{L}_{\bar{B}\bar{D}_s D} = g_{\bar{B}\bar{D}_s D}\bar{B}\bar{D}_s D, \quad \mathcal{L}_{DK\pi} = g_{DK\pi} DK\pi, \tag{3}$$

where the couplings are determined by reproducing the experimental data. With the experimental branching fractions $\mathcal{B}(D^0 \to K^-\pi^+) = (3.947 \pm 0.030)\%$ and $\mathcal{B}(D^+ \to \bar{K}^0\pi^+) = (3.067 \pm 0.053)\%$ [61], we obtain the values for the couplings $g_{D^0K^-\pi^+} = 2.535 \times 10^{-6}$ GeV and $g_{D^+\bar{K}^0\pi^+} = 1.411 \times 10^{-6}$ GeV. Similarly, with the branching fractions of $\mathcal{B}(B^- \to D_s^- D^0) = (9.0 \pm 0.9) \times 10^{-3}$ and $\mathcal{B}(\bar{B}^0 \to D_s^- D^+) = (7.2 \pm 0.8) \times 10^{-3}$, we derive the couplings of $g_{B^-D_s^-D^0} = 1.182 \times 10^{-6}$ GeV and $g_{\bar{B}^0D_s^-D^+} = 1.098 \times 10^{-6}$ GeV. To further reduce the uncertainty of the weak decay vertex $\bar{B} \to \bar{D}_s D^*$, we take their experimental branching fractions of $\mathcal{B}(B^- \to D_s^- D^{*0}) = (8.2 \pm 1.7) \times 10^{-3}$ and $\mathcal{B}(\bar{B}^0 \to D_s^- D^{*+}) = (8.0 \pm 1.1) \times 10^{-3}$ to fix the effective Wilson coefficient a_1 as 0.93 and 0.96, respectively [57], a bit smaller than that of $a_1 = 1.07$ at the energy scale of m_c [62], which indicates that the factorization contribution plays the dominant role.

B. Contact-range effective field theory

In the following, we explain how to determine the molecule couplings to their constituents in the contact-range EFT approach. These couplings are estimated by solving the Lippmann-Schwinger equation,

$$T(\sqrt{s}) = [1 - VG(\sqrt{s})]^{-1}V, \tag{4}$$

where $G(\sqrt{s})$ is the loop function, and V is the hadron-hadron potential derived in the contact EFT approach. In the heavy quark limit, the contact potentials between a pair of heavy mesons are parametrized as [63,64]

$$V(I=0, DD^*) = C_a + C_b, \quad V(I=0, D^*D^*) = C_a + C_b, \quad V(I=0, D\bar{B}) = C_a \tag{5}$$

where C_a and C_b represent the spin-spin independent term and dependent term, which are determined by fitting to the mass of a hadronic molecule candidate.

To avoid the divergence of the loop function $G(\sqrt{s})$, one introduces a regulator of Gaussian form e^{-2q^2/Λ^2} in the integral as

$$G(\sqrt{s}) = \int \frac{d^3q}{(2\pi)^3} \frac{e^{-2q^2/\Lambda^2}}{\sqrt{s} - m_1 - m_2 - q^2/(2\mu_{12}) + i\varepsilon}, \tag{6}$$

where $\sqrt{s}$ is the total energy in the c.m.frame of m_1 and m_2, $\mu_{12} = m_1 m_2/(m_1 + m_2)$ is the reduced mass, and Λ is the momentum cutoff. Following our previous works [65], we take $\Lambda = 0.7$ GeV in the present work.

With the potentials given in Eq. (5), we search for poles in the vicinity of the $D\bar{B}$ and $D^{(*)}D^*$ mass thresholds and then determine the couplings from the residues of the corresponding poles,

$$g_i g_j = \lim_{\sqrt{s}\to\sqrt{s_0}} (\sqrt{s} - \sqrt{s_0}) T_{ij}(\sqrt{s}), \tag{7}$$

where g_i denotes the coupling of channel i to the dynamically generated state and $\sqrt{s_0}$ is the pole position.

C. Decay amplitudes

With the above relevant Lagrangians, the decay amplitudes of $T_{cb} \to D^+D^0D_s^-$, $T_{cb} \to B^-\pi^+K_s^0$, and $T_{cb} \to \bar{B}^0\pi^+K^-$ in Fig. 1 can be written as

$$i\mathcal{M}_{a,b} = g_{T_{cb}D\bar{B}} g_{\bar{B}\bar{D}_s D} \frac{1}{k_0^2 - m_{\bar{B}}^2}, \quad i\mathcal{M}_{c,d} = g_{T_{cb}D\bar{B}} g_{DK\pi} \frac{1}{k_0^2 - m_D^2}, \tag{8}$$

where k_0 is the momentum of the $\bar{B}$ meson or D meson.

Similarly, we express the decay amplitudes of $T_{cb} \to T_{cc}D_s^-$ of Fig. 2

$$i\mathcal{M}_{2a,2b} = g_{T_{cb}D\bar{B}} g_{T_{cc}DD^*} \int \frac{d^4q}{(2\pi)^4} \frac{1}{k_0^2 - m_{\bar{B}}^2} \times \frac{1}{q^2 - m_D^2} \frac{-g^{\mu\nu} + \frac{q_2^\mu q_2^\nu}{q_2^2}}{q_2^2 - m_{D^*}^2} \mathcal{A}_\nu(\bar{B} \to \bar{D}_s D^*) \varepsilon_\mu(p_1), \tag{9}$$

where q and p_1 represent the momenta for D meson and T_{cc} state, and ε_μ represent the polarization vector for the state of spin $S = 1$.

With the weak amplitudes of $T_{cb} \to T_{cc}D_s$, one can further calculate the corresponding partial decay widths as

$$\Gamma = \frac{1}{2J+1}\frac{1}{8\pi}\frac{|\vec{p}|}{m_{T_{cb}}^2}|\bar{\mathcal{M}}|^2, \tag{10}$$

where J is the total angular momentum of the initial state T_{cb}, the overline indicates the sum over the polarization vectors of final states, and $|\vec{p}|$ is the momentum of either final state in the rest frame of T_{cb}.

As for the three-body decay, the partial decay widths of $T_{cb} \to D\bar{D}\bar{D}_s$ and $T_{cb} \to \bar{B}\pi\bar{K}$ as a function of m_{12}^2 and m_{23}^2 read

$$\Gamma = \frac{1}{(2\pi)^3}\frac{1}{2J+1}\int\int \frac{\overline{|\mathcal{M}|^2}}{32 m_{T_{cb}}^3} dm_{12}^2 dm_{23}^2, \tag{11}$$

with m_{12} the invariant mass of $\bar{D}_s D$ or $\pi\bar{K}$ and m_{23} the invariant mass of $\bar{D}D$ or $\bar{B}\bar{K}$ for the $T_{cb} \to D\bar{D}\bar{D}_s$ and $T_{cb} \to \bar{B}\pi\bar{K}$ decays, respectively.

III. NUMERICAL RESULTS AND DISCUSSION

In Table II, we collect the masses and quantum numbers of the mesons relevant to the present work. At first, we take the contact range EFT to analyse the likely bound states with the quark content of $QQ\bar{q}\,\bar{q}$. Identifying the T_{cc} as a bound state of DD^*, we obtain the value of $C_a + C_b = -27.26\ \mathrm{GeV}^{-2}$ for a cutoff $\Lambda = 0.7$ GeV, and then predict a bound state below the D^*D^* mass threshold 1.58 MeV, in agreement with the predictions of Refs. [39,41,66–68], which is the heavy quark spin symmetry partner of the D^*D molecule. Since the two-body decay mode of the D^*D^* molecule is allowed, the width of the D^*D^* molecule is larger than that of the D^*D molecule by two orders of magnitude [66]. As for the $D\bar{B}$ system, we turn to the light meson saturation mechanism to determine the values of C_a and C_b. Following Refs. [64,69], C_a and C_b can be written as

$$C_a^{\mathrm{sat}} \propto -\frac{g_\sigma^2}{m_\sigma^2} + \frac{g_v^2}{m_v^2}(1 + \vec{\tau}_1\cdot\vec{\tau}_2),$$
$$C_b^{\mathrm{sat}} \propto \frac{f_v^2}{4M^2}(1 + \vec{\tau}_1\cdot\vec{\tau}_2), \tag{12}$$

TABLE II. Masses and quantum numbers of mesons relevant to the present work [70].

Meson	$I(J^P)$	M (MeV)	Meson	$I(J^P)$	M (MeV)
π^0	$1(0^-)$	134.977	$\pi^\pm$	$1(0^-)$	139.570
K^0	$\frac{1}{2}(0^-)$	497.611	$K^\pm$	$\frac{1}{2}(0^-)$	493.677
D^0	$\frac{1}{2}(0^-)$	1864.84	$D^\pm$	$\frac{1}{2}(0^-)$	1869.66
D^{*0}	$\frac{1}{2}(1^-)$	2006.85	$D^{*\pm}$	$\frac{1}{2}(1^-)$	2010.26
$D_s^\pm$	$0(0^-)$	1968.35	T_{cc}	$0(1^+)$	3874.74
$B^\pm$	$\frac{1}{2}(0^-)$	5279.34	B^0	$\frac{1}{2}(0^-)$	5279.66

where $m_\sigma = 600$ MeV, $m_v = 780$ MeV, $g_\sigma = 3.4$, $g_v = 2.6$, and $f_v = \kappa\cdot 2.6$ with $\kappa = 2.3$ and $M = 940$ MeV [63]. The product of $\tau_1\cdot\tau_2$ is -3 for isospin $I = 0$. Thus the ratio of C_b to C_a is determined as 0.25, and we further fix the values of $C_a = -21.84\ \mathrm{GeV}^{-2}$ and $C_b = -5.42\ \mathrm{GeV}^{-2}$. With the obtained value of C_a, we obtain a weakly bound state below the $D\bar{B}$ mass threshold 2.61 MeV, consistent with the lattice QCD simulation [53]. Finally, with the poles generated by the $D\bar{B}$ and DD^* interactions, we derive the couplings of $g_{T_{cb}D\bar{B}} = 15.16$ GeV and $g_{T_{cc}DD^*} = 6.59$ GeV. In the isospin limit, we obtain the hadronic molecules couplings to the channels in particle basis, i.e., $\frac{1}{\sqrt{2}}g_{T_{cb}D\bar{B}} = g_{T_{cb}D^+B^-} = g_{T_{cb}D^0\bar{B}^0}$ and $\frac{1}{\sqrt{2}}g_{T_{cc}DD^*} = g_{T_{cc}D^+D^{*0}} = g_{T_{cc}D^0D^{*+}}$.

In this work, we take the heavy quark limit to derive the heavy meson- heavy meson potentials, to which we assign a 15% uncertainty [71,72]. To show the impact of the heavy quark symmetry breaking on the T_{cb} mass, we vary its mass from 7142 to 7146 MeV. In Fig. 3(a), we present the partial decay widths $T_{cb} \to D_s^- D^0 D^+$, $T_{cb} \to \pi^+\bar{K}^0 B^-$, and $T_{cb} \to \pi^+ K^- \bar{B}^0$ as a function of the T_{cb} mass. The results show that the partial decay width of $T_{cb} \to D_s^- D^0 D^+$ varies from 6.16×10^{-15} GeV to 2.90×10^{-15} GeV, and the partial decay widths $T_{cb} \to \pi^+\bar{K}^0 B^-$ and $T_{cb} \to \pi^+ K^-\bar{B}^0$ are in the range of $(11.43\sim 5.22)\times 10^{-15}$ GeV and $(4.61\sim 1.77)\times 10^{-14}$ GeV, respectively. In Fig. 3(b), we show the partial decay widths of $T_{cb} \to T_{cc}^+ D_s^-$ as a function of the T_{cb} mass, which varies from 4.42×10^{-16} to 1.42×10^{-16} GeV. Because the coupling $g_{T_{cb}D\bar{B}}$ decreases as the T_{cb} mass increases, its partial decay widths decrease as well.

In our calculation, the main uncertainty comes from the couplings of vertices in the Feynman diagrams. For the couplings between molecules and their constituents, i.e., $g_{T_{cb}}$ and $g_{T_{cc}}$, the variation of cutoff in the form factors of scattering amplitude T in Eq. (4), varying from 0.7 to 2 GeV, lead to the couplings $g_{T_{cb}}$ and $g_{T_{cb}}$ decreasing from 15.16 to 13.44 GeV and from 6.59 to 6.17 GeV, respectively, resulting in around 10% uncertainty. The error of experimental branching fractions of weak decays also bring about 10% uncertainties for the couplings of weak vertices as shown in Ref. [57]. Therefore, we estimate the uncertainties for the partial decay widths originating from the uncertainties of these couplings via a Monte Carlo sampling within their 1σ intervals. In Table III, we show the partial decay widths and corresponding branching fractions of T_{cb} at a mass of 7144 MeV.[1] One can see that the error of two-body decays are larger than those of three-body decays because

[1]Since the widths of heavy hadrons are dominantly responsible by the weak decay of heavy flavor quarks, the widths of doubly heavy tetraquark states are expected to be similar to those of doubly heavy baryons. The life time of Ξ_{bb} and Ξ_{bc} are predicted to be around 0.8 and 0.28 ps [73], and therefore the life time of T_{bc} is taken as 0.3 ps or 2.2×10^{-9} MeV in this work.

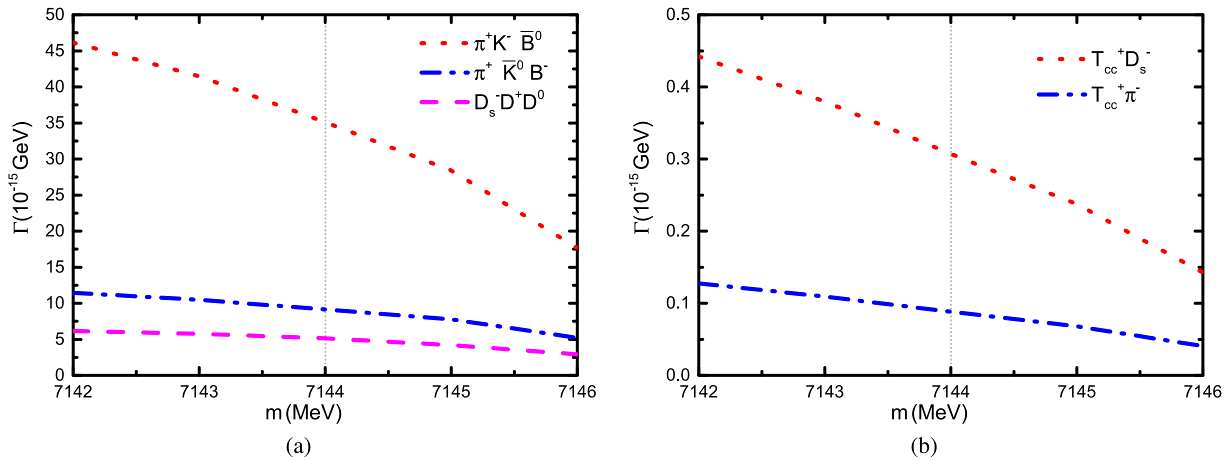

FIG. 3. Partial decay widths $T_{cb} \to D_s^- D^0 D^+$, $T_{cb} \to \pi^+ \bar{K}^0 B^-$, and $T_{cb} \to \pi^+ K^- \bar{B}^0$ in the tree diagrams as a function of T_{cb} mass (a), and widths of partial decays $T_{cb} \to T_{cc}^+ D_s^-$ and $T_{cb} \to T_{cc}^+ \pi^-$ in the triangle diagrams as a function of T_{cb} mass (b).

there exist three couplings in the triangle diagrams but two couplings in the tree-diagrams. Our results indicate that, identifying T_{cb} as a bound state of $D\bar{B}$, the decay mode of $T_{cb} \to \pi^+ K^- \bar{B}^0$ is the largest, and therefore we suggest to experimentally search for the $D\bar{B}$ molecule in the $\pi^+ K^- \bar{B}^0$ mass distribution. Very recently, Ali *et al.*, estimated the weak decay widths of $T_{cb} \to T_{cc} X$ ($X = \pi^-$, ρ^-, and a_1^- mesons) to be the order of 10^{-15} GeV [74], where the doubly heavy tetraquark states are assumed as diquark-diquark states. Such a decay mode can be produced in our mechanism. As shown in Fig. 2, we replace the weak decay vertices $B^- \to D^{*0} D_s^-$ and $\bar{B}^0 \to D^{*+} D_s^-$ in the triangle diagrams by those of $B^- \to D^{*0} \pi^-$ and $\bar{B}^0 \to D^{*+} \pi^-$. Using the similar approach, we calculate the decay width of $T_{cb} \to T_{cc}^+ \pi^-$ in the range of $(1.24 \sim 0.41) \times 10^{-16}$ GeV as shown in Fig. 3(b), which is smaller than that of Ref. [74] by one order of magnitude. Considering the dominant decays of $T_{cc}^+ \to D^+ D^0 \pi^0$ and $T_{cc}^+ \to D^0 D^0 \pi^+$, the T_{cb} can also be observed in the channels of $D^+ D^0 \pi^0 D_s^-$, $D^0 D^0 \pi^+ D_s^-$, $D^+ D^0 \pi^0 \pi^-$, and $D^0 D^0 \pi^+ \pi^-$.

IV. SUMMARY AND DISCUSSION

Since $X(3872)$ was discovered by the Belle Collaboration in 2003, many exotic states have been discovered in the experiments. Since most of them lie to the mass thresholds of a pair of conventional hadrons, the hadronic molecules are expected to explain their internal structure. Within the molecular picture, the decay modes of exotic states are always including the strong decays and radiative decays. Very recently, the molecule composed by $\bar{B}$ meson and D meson is only allowed to proceed via the weak decays.

TABLE III. Partial decay widths and corresponding branching fractions of the $D\bar{B}$ molecule for a mass of 7144 MeV.

Decay mode	Width (GeV)	Branching fraction (%)
$T_{cb} \to D_s^- D^0 D^+$	$(5.13 \pm 0.74) \times 10^{-15}$	0.20 ± 0.03
$T_{cb} \to \pi^+ \bar{K}^0 B^-$	$(9.15 \pm 1.51) \times 10^{-15}$	0.40 ± 0.07
$T_{cb} \to \pi^+ K^- \bar{B}^0$	$(3.51 \pm 0.58) \times 10^{-14}$	2.0 ± 0.3
$T_{cb} \to T_{cc}^+ D_s^-$	$(3.07 \pm 0.62) \times 10^{-16}$	0.0150 ± 0.0025
$T_{cb} \to T_{cc}^+ \pi^-$	$(0.88 \pm 0.18) \times 10^{-16}$	0.0040 ± 0.0008

In this work, we assume that the $D^{(*)}D^{(*)}$ system is related to the $D^{(*)}\bar{B}^{(*)}$ system in the heavy quark symmetry. Their contact range potentials are parametrized by two parameters C_a and C_b. By reproducing the mass of T_{cc}, we determine the sum of C_a and C_b, and then fully determine the values of C_a and C_b in terms of the ratio of C_a to C_b estimated by the light meson saturation approach. With the obtained C_a and C_b, we predict the mass of $\bar{B}D$ molecule as 7144 MeV, consistent with the Lattice QCD simulations.

Based on the decay mechanism of DD^* molecule, we propose that the $D\bar{B}$ molecule decay via the weak decays of $\bar{B}$ meson or D meson, i.e., $T_{cb} \to D_s^- D^0 D^+$ and $T_{cb} \to \pi^+ \bar{K}^0 B^- / T_{cb} \to \pi^+ K^- \bar{B}^0$. Moreover, considering the final state interactions, we propose the decays of $T_{cb} \to T_{cc}^+ D_s^-$ and $T_{cb} \to T_{cc}^+ \pi^-$ proceeding via the triangle diagram mechanism. Using the effective Lagrangian approach, we calculate the partial decay widths and corresponding branching fractions of $D\bar{B}$ molecule as shown in Table III. Our calculation for the decay $T_{cb} \to T_{cc}^+ \pi^-$ is smaller than that of assuming the doubly heavy tetraquark states as the compact tetraquark states by one order of magnitude, which is an obvious signal to discriminate the nature of doubly heavy tetraquark states. From our calculations, we strongly suggest to experimental colleague to search for the $D\bar{B}$ molecule in the $\pi^+ K^- \bar{B}^0$ mass distribution. We hope that the present work can simulate more studies on the weak decays of exotic states.

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