Tunneling away the relic neutrino asymmetry

Saarik Kalia [*]

School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA

(Received 20 May 2024; accepted 1 August 2024; published 3 September 2024)

The Earth acts as a matter potential for relic neutrinos which modifies their index of refraction from vacuum by $\delta \sim 10^{-8}$. It has been argued that the refractive effects from this potential should lead to a large $\mathcal{O}(\sqrt{\delta})$ neutrino-antineutrino asymmetry at the surface of the Earth. This result was computed by treating the Earth as flat. In this work, we revisit this calculation in the context of a perfectly spherical Earth. We demonstrate, both numerically and through analytic arguments, that the flat-Earth result is only recovered under the condition $\delta^{3/2}kR \gg 1$, where k is the typical momentum of the relic neutrinos and R is the radius of the Earth. This condition is required to prevent antineutrinos from tunneling into classically inaccessible trajectories below the Earth's surface and washing away the large asymmetry. As the physical parameters of the Earth do not satisfy this condition, we find that the asymmetry at the surface should only be $\mathcal{O}(\delta)$. While the asphericity of the Earth may serve as a loophole to our conclusions, we argue that it is still difficult to generate a large asymmetry even in the presence of local terrain.

DOI: 10.1103/PhysRevD.110.053001

I. INTRODUCTION

Standard cosmology predicts the existence of a cosmic neutrino background (CνB) of relic neutrinos produced in the early Universe, following a Fermi-Dirac distribution with temperature $T_\nu = 1.7 \times 10^{-4}$ eV [1,2]. While the detection of the CνB would lead to profound insights about the fundamental properties of neutrinos and early Universe cosmology [3,4], its direct detection via scattering/absorption is difficult because neutrino cross sections scale as $\mathcal{O}(G_F^2)$ [5,6]. Another proposal was put forth by Stodolsky to look for electron/nuclear spin precession due to the CνB [7]. While this effect is $\mathcal{O}(G_F)$, it is proportional to the neutrino-antineutrino asymmetry, which in vacuum is expected to be comparable to the observed baryon asymmetry of $\sim 10^{-9}$.

In Ref. [8], it was proposed that the neutrino-antineutrino asymmetry may be significantly enhanced near the surface of the Earth. This enhancement occurs because neutrinos/antineutrinos experience a potential

$$|U| \sim 10^{-14}\ \text{eV} \cdot \left(\frac{n_{\text{matter}}}{10^{22}\ \text{cm}^{-3}}\right) \tag{1}$$

in the presence of matter. For electron neutrinos and muon/tau antineutrinos, this potential is positive, while for electron antineutrinos and muon/tau neutrinos, it is negative. This leads to an index of refraction for neutrinos/antineutrinos inside the Earth which differs from vacuum by

$$\delta \equiv -\frac{m_\nu U}{k^2} \sim \pm 10^{-8} \cdot \left(\frac{n_{\text{matter}}}{10^{22}\ \text{cm}^{-3}}\right) \cdot \left(\frac{m_\nu}{0.1\ \text{eV}}\right), \tag{2}$$

where $k \sim T_\nu$ is the momentum of the neutrino and m_ν is the mass of the relevant neutrino species. Due to the different refractive indices for neutrinos vs antineutrinos, it is argued in Ref. [8] that the Earth induces a $\mathcal{O}(\sqrt{\delta}) \sim 10^{-4}$ fractional asymmetry near its surface. Importantly, the calculation in Ref. [8] treated the Earth as flat.

In this work, we recompute this asymmetry in the context of a perfectly spherical Earth. While similar computations have been performed in Refs. [9–11], the principal purpose of this work is to show that the large asymmetry observed in the flat-Earth calculation is only reproduced under the condition,

$$\delta^{3/2}kR \gg 1, \tag{3}$$

where R is the radius of the Earth. The physical parameters of the Earth have $\delta^{3/2}kR \sim 0.01$, and so do not satisfy Eq. (3). In this case, we show that a much smaller $\mathcal{O}(\delta)$ fractional asymmetry should be expected at the surface of the Earth. Our calculation treats the Earth as a perfect sphere, and so the presence of local terrain may provide a loophole to the above condition. In our concluding discussion, however, we comment that local terrain should typically make the condition more stringent rather than less.

[*]Contact author: kalias@umn.edu

This work is organized as follows. In Sec. II, we give a heuristic understanding, in terms of ray optics, of how the condition in Eq. (3) arises. In Sec. III, we perform a precise calculation of the neutrino-antineutrino asymmetry, under the assumption of a perfectly spherical earth of uniform density. As in Ref. [8], for this calculation, we consider only electron neutrinos/antineutrinos and treat the CνB as monochromatic and isotropic. In Sec. IV, we comment on how the asphericity of the Earth affects the interpretation of our results and contrast our work with Refs. [9–11]. We make all the code used in this work publicly available on Github [12].

II. HEURISTIC ARGUMENT

We begin by presenting a heuristic understanding of the full calculation presented in Sec. III. The relevant features of the argument are depicted in Fig. 1. In order to determine the asymmetry at the surface of the Earth, let us consider a point P located inside the Earth but close to the surface. If the asymmetry at this point is large, then by continuity, the asymmetry at the surface will be as well. The neutrino and antineutrino densities at P receive contributions from all of the rays passing through P.[1] For every antineutrino ray, there is a corresponding neutrino ray which travels the same path through the interior of the Earth; e.g., see the rays labeled 1a and 1b in Fig. 1. The contributions from these rays cancel, leading to no significant asymmetry.[2]

Consider instead a neutrino ray which traverses a chord very close to the surface of the Earth; e.g., ray 2b in Fig. 1. For such rays there exists no corresponding antineutrino ray. This is because such an antineutrino ray attempting to emerge from the interior of the Earth would experience total reflection, as it exits from a medium of higher index of refraction to one of a lower index. Conversely, this implies that classically no antineutrino ray originating from outside the Earth can reach this trajectory in the interior. These unpaired neutrino rays are the source of the large asymmetry described in Ref. [8]. These rays are unpaired so long as they form an angle $\theta < \theta_c \equiv \sqrt{2|\delta|}$ with the surface of the Earth (from the interior; see Fig. 1). All points P which are within a distance $\theta_c^2 R/2 = |\delta| R$ from the surface will receive a contribution from such rays, and so will exhibit a large asymmetry. By continuity, points just above the surface will exhibit this asymmetry as well.

The above argument only holds in the context of classical ray optics, where neutrino/antineutrinos travel on fixed paths determined by refraction. When quantum wave effects are included, antineutrinos have the ability to tunnel from a ray outside the Earth onto this classically forbidden trajectory inside the Earth; e.g., ray 2a in Fig. 1 may tunnel inside the Earth to cancel the contribution from ray 2b. If this tunneling occurs efficiently, the large asymmetry will be washed out. Conservation of energy and angular momentum relate the parameters of the external and internal trajectories. Suppose the external trajectory has a velocity v_1 and impact parameter r_1, while the internal trajectory has a velocity v_2 and impact parameter r_2. Then, we have

$$\frac{m_\nu v_1^2}{2} = \frac{m_\nu v_2^2}{2} - U \tag{4}$$

$$m_\nu v_1 r_1 = m_\nu v_2 r_2, \tag{5}$$

which implies

$$r_2 = \frac{r_1}{\sqrt{1 + \frac{2U}{m_\nu v_1^2}}} \approx (1 - \delta) r_1. \tag{6}$$

In other words, the ray must tunnel a distance roughly δR from its trajectory outside the Earth to its trajectory inside the Earth. As we will see in Sec. III, the typical distance that a ray can efficiently tunnel is $L_{\mathrm{tunnel}} \sim (kR)^{1/3}/k$. Therefore, when

$$L_{\mathrm{tunnel}} \ll \delta R \Rightarrow \delta^{3/2} kR \gg 1, \tag{7}$$

the antineutrino rays outside the Earth will be unable to tunnel in, and so the large asymmetry will persist.

III. SPHERICAL CALCULATION

Now we make the arguments of Sec. II more precise by performing a calculation of the neutrino-antineutrino asymmetry near the Earth's surface, under the assumption of a perfectly spherical Earth. The formalism introduced in this section will be similar to the formalisms applied in Refs. [9–11], but we will focus on determining the conditions under which the asymmetry exists. As mentioned in Sec. I, we will assume that the Earth is a perfect sphere of uniform density, and that the CνB is monochromatic (with momentum k) but isotropic.[3] We will also compute the time-averaged asymmetry, in which case, it

[1]For illustrative purposes, we phrase the argument in this section in terms of individual rays, but more properly, the detailed calculation presented in the next section will deal with angular modes. As all rays with the same impact parameter (with respect to the center of the Earth) possess the same angular momentum, this argument should be understood in a context where all rays with the same impact parameter are considered equivalent. For instance, the relative orientations of rays 2a and 2b in Fig. 1 are not particularly important. What is relevant are their impact parameters.

[2]In fact, due to their different incident angles outside the Earth, these rays possess slightly different normalizations which results in an $\mathcal{O}(\delta)$ asymmetry.

[3]We note that the CνB is not in fact isotropic in the rest frame of the Earth, due to their relative motion. As the thermal velocity of relic neutrinos $v_{\mathrm{th}} \sim T_\nu/m_\nu \sim 10^{-3} \cdot (m_\nu/0.1\ \mathrm{eV})$ may be comparable to this relative motion, this anisotropy may be significant, depending on the mass of the relevant neutrino species. In this work, we will neglect this potential anisotropy.

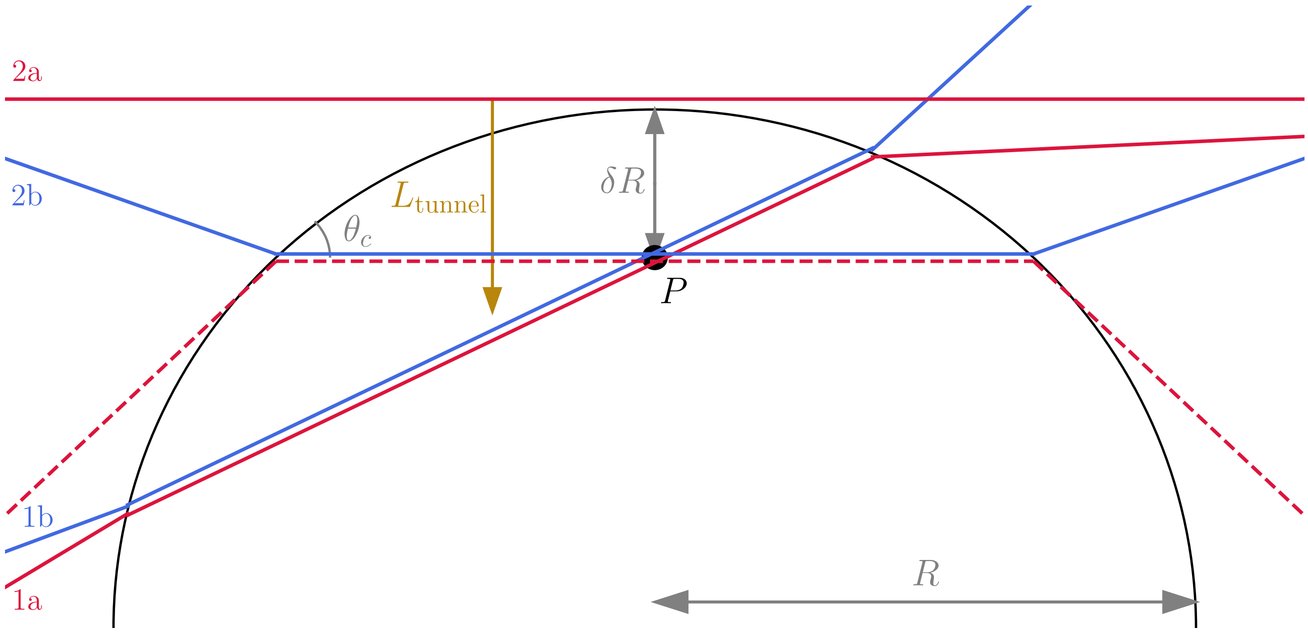

FIG. 1. Diagram of the heuristic argument presented in Sec. II (not to scale). The asymmetry at a point P receives contributions from all rays passing through it. Blue rays represent neutrino paths, while red rays represent antineutrino paths. Rays that arrive at large angles, such as those labeled 1a and 1b, traverse the interior of the Earth in pairs, contributing a small asymmetry at P. However, neutrino rays that form angles $\theta < \theta_c$ with the surface of the Earth, such as ray 2b, have no corresponding antineutrino ray, as antineutrinos cannot classically access this path in the interior of the Earth. This can lead to a large asymmetry for all points P within δR of the surface. Quantum mechanically, however, the ray 2a can tunnel to this classically inaccessible trajectory, thereby washing out the asymmetry. This can occur when its characteristic tunneling range $L_{\text{tunnel}} \sim (kR)^{1/3}/k$ [see Eq. (30)] exceeds the required distance δR that it must tunnel, or in other words, when $\delta^{3/2} kR \ll 1$. See text for more details.

suffices to solve for the steady-state neutrino/antineutrino wave functions. In order to determine these wave functions at a given point for an isotropic background, it is sufficient to compute the solution for a single incoming plane wave and average this solution over a sphere of constant radius. More precisely, given an incoming plane wave from direction $\hat{\boldsymbol{n}}$, let $\psi_{\hat{\boldsymbol{n}}}(r, \Omega)$ denote the resulting wave function in the vicinity of the Earth. By symmetry, it is simple to see that $\psi_{\hat{\boldsymbol{n}}}(r, \hat{z}) = \psi_{\hat{z}}(r, \hat{\boldsymbol{n}}')$, where $\hat{\boldsymbol{n}}'$ is $\hat{\boldsymbol{n}}$ rotated by π around the z-axis. The total neutrino/antineutrino density is given by the average of $|\psi_{\hat{\boldsymbol{n}}}|^2$ over all $\hat{\boldsymbol{n}}$, so we can write

$$\frac{n_{\nu,\bar{\nu}}(r)}{n_0} = \frac{1}{4\pi} \int d\hat{\boldsymbol{n}} |\psi_{\hat{\boldsymbol{n}}}(r, \hat{z})|^2 \tag{8}$$

$$= \frac{1}{4\pi} \int d\Omega |\psi_{\hat{z}}(r, \Omega)|^2, \tag{9}$$

where $n_{\nu,\bar{\nu}}$ represents the neutrino/antineutrino density, and n_0 denotes their density in the absence of the Earth. We see then that it is sufficient to compute only the solution $\psi_{\hat{z}}$ for a plane wave incoming from the $\hat{z}$-direction. Henceforth we will simply denote this solution by ψ.

Let us decompose this wave function into angular modes as

$$\psi(r, \Omega) = \sum_{\ell=0}^{\infty} \psi_\ell(r) Y_{\ell 0}(\Omega), \tag{10}$$

where we only require $m = 0$ modes by azimuthal symmetry. The Schrödinger equation for the radial part ψ_ℓ of the wave function becomes

$$\partial_r^2 \psi_\ell + \frac{2}{r} \partial_r \psi_\ell + \left(k^2 - 2mU \cdot \Theta(R - r) - \frac{\ell(\ell+1)}{r^2} \right) \psi_\ell = 0, \tag{11}$$

The solution to Eq. (11) is simply a combination of spherical Bessel functions, with momentum k for $r > R$ and momentum

$$k' \equiv \sqrt{k^2 - 2mU} = k\sqrt{1 + 2\delta} \tag{12}$$

for $r < R$. The corresponding boundary conditions for Eq. (11) should be that the only incoming modes are that of the incoming plane wave, and that the wave function is regular at the origin. An incoming plane wave from the $\hat{z}$-direction can be decomposed in terms of spherical harmonics as

$$e^{ikz} = \sum_{\ell=0}^{\infty} i^{\ell}\sqrt{4\pi(2\ell+1)}\,j_{\ell}(kr)Y_{\ell 0}(\Omega), \tag{13}$$

where j_{ℓ} is the spherical Bessel function of the first kind. The full solution for the wavefunction in and around the Earth should therefore be

$$\psi_{\ell}(r) = \begin{cases} A_{\ell}j_{\ell}(kr) + B_{\ell}h_{\ell}^{(1)}(kr), & r > R \\ C_{\ell}j_{\ell}(k'r), & r < R, \end{cases} \tag{14}$$

$$A_{\ell} \equiv i^{\ell}\sqrt{4\pi(2\ell+1)}, \tag{15}$$

where the spherical Hankel function $h_{\ell}^{(1)}$ of the first kind corresponds to an outgoing wave.

Continuity of ψ_{ℓ} and its gradient enforce

$$A_{\ell}j_{\ell}(kR) + B_{\ell}h_{\ell}^{(1)}(kR) = C_{\ell}j_{\ell}(k'R), \tag{16}$$

$$kA_{\ell}j'_{\ell}(kR) + kB_{\ell}h_{\ell}^{(1)\prime}(kR) = k'C_{\ell}j'_{\ell}(k'R). \tag{17}$$

These can be solved numerically for sufficiently large ℓ to determine the full solution ψ. From Eq. (9), we see that the number density is then given by

$$\frac{n_{\nu,\bar{\nu}}(r)}{n_0} = \frac{1}{4\pi}\sum_{\ell=0}^{\infty}|\psi_{\ell}(r)|^2 \tag{18}$$

$$= \begin{cases} \frac{1}{4\pi}\sum_{\ell}\left|A_{\ell}j_{\ell}(kr) + B_{\ell}h_{\ell}^{(1)}(kr)\right|^2, & r > R \\ \frac{1}{4\pi}\sum_{\ell}|C_{\ell}j_{\ell}(k'r)|^2, & r < R. \end{cases} \tag{19}$$

Using $\delta < 0$ to compute the above solution will give the neutrino density n_{ν}, while using $\delta > 0$ will give the antineutrino density $n_{\bar{\nu}}$. By taking the difference of Eq. (19) between these two cases, we can compute the fractional neutrino asymmetry $\Delta = (n_{\nu} - n_{\bar{\nu}})/n_0$.

The sums in Eq. (19) converge only after $\ell \gtrsim kR$, so evaluating them for the physical parameters of the Earth ($kR \sim 9 \times 10^9$) is computationally challenging. Reference [9] claims to have achieved this with the use of unspecified asymptotic Bessel function expansions. In this work, we instead opt to understand the relevant behavior by evaluating the solution for smaller parameter values. In Fig. 2, we compute the asymmetry $\Delta(r)$ as a function of radius, for $kR = 3 \times 10^4$ and two choices of $\delta = 10^{-2}$ and $\delta = 10^{-3}$. A number of important features can be seen even with these parameter values. In both cases, the asymmetry begins at $\Delta = -2\delta$ deep within the Earth and eventually reaches $\Delta = 0$ outside the Earth. In the case of small δ, the profile transitions directly between these regimes, leading to an asymmetry of $\Delta = -\delta$ at the surface $r = R$.[4] In the case of large δ, however, the asymmetry becomes large and positive in the region $(1-\delta)R < r \leq R$, leading to an asymmetry larger than $\mathcal{O}(\delta)$ at $r = R$.[5] In Fig. 3, we show the asymmetry at the Earth's surface $\Delta(r = R)$ for various values of kR and δ. This plot clearly demonstrates that a large asymmetry at the surface is only achieved when $\delta^{3/2}kR \gg 1$. Below, we justify why this is the case.

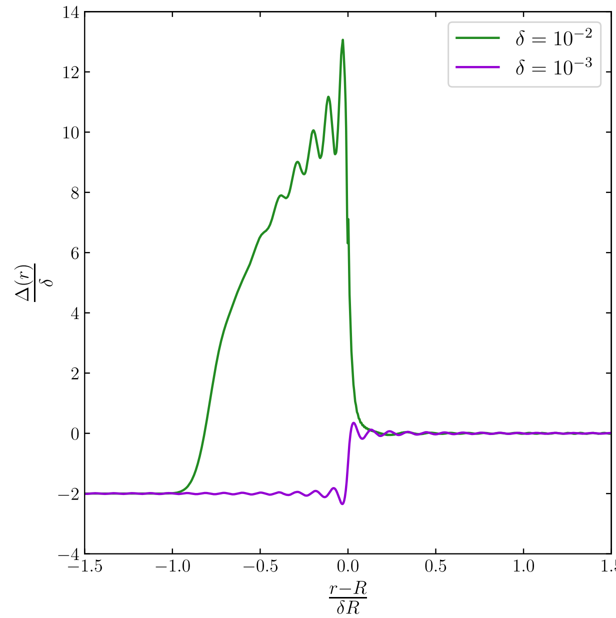

FIG. 2. Plot of the neutrino-antineutrino asymmetry $\Delta(r)$ (normalized by δ) near the surface of the Earth, for $kR = 3 \times 10^4$ and two different values of δ. [To compute these profiles, we sum up to $\ell = 3.2 \times 10^4$ in Eq. (19).] In both cases, we see $\Delta = -2\delta$ for $r < (1-\delta)R$ and $\Delta = 0$ well above the surface of the Earth. In the $\delta = 10^{-2}$ case, however, we see that the asymmetry becomes large and positive for $(1-\delta)R < r \leq R$.

First, let us derive the $\Delta = -2\delta$ asymmetry for $r < (1-\delta)R$. It is straightforward to solve Eqs. (16) and (17) to find

$$C_{\ell} = \frac{iA_{\ell}k^{-2}R^{-2}}{h_{\ell}^{(1)\prime}(kR)j_{\ell}(k'R) - \sqrt{1+2\delta}\cdot h_{\ell}^{(1)}(kR)j'_{\ell}(k'R)}. \tag{20}$$

Now the spherical Bessel and Hankel functions satisfy

[4] The profile exhibits oscillations near the surface of the Earth, but these will be smoothed out after averaging over k.

[5] The exact shape and size of this large asymmetry can depend on kR and δ. In Ref. [8], it is argued that this asymmetry is $\mathcal{O}(\sqrt{\delta})$ and extends for a distance $\lambda_c \equiv 2\pi/\theta_c k$ above the surface of the Earth. We find similar results in our numerical computations, but as the purpose of this work is to understand when this large asymmetry exists, we choose not to focus on how large it is or how far it extends.

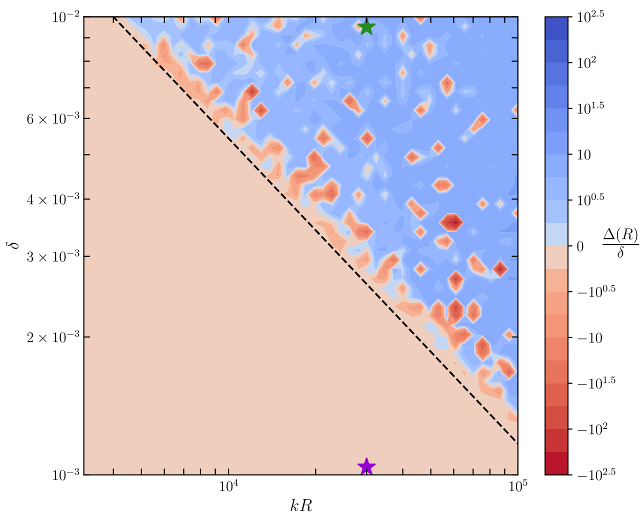

FIG. 3. Contour plot of the asymmetry at the surface of the Earth $\Delta(r = R)$ (again normalized by δ) for various values of kR and δ. The dashed black line denotes $\delta^{3/2}kR = 4$, and the green and purple stars correspond to the parameter values used in Fig. 2. Note that large asymmetries are only obtained above the line. Below the line, $\Delta(R) = -\delta$ for all parameter values. The physical parameters of the Earth have $\delta^{3/2}kR \sim 0.01$ and so fall well below the black line.

$$|h_\ell^{(1)}(x)|^2 \approx |h_\ell^{(1)\prime}(x)|^2 \approx j_\ell(x)^2 + j_\ell'(x)^2 \approx \frac{1}{x^2}, \tag{21}$$

$$\langle j_\ell(x)^2\rangle \approx \frac{1}{2x^2}, \tag{22}$$

for $x > \ell$, where $\langle\cdot\rangle$ indicates an average over ℓ. With these approximations, Eq. (20) becomes

$$\langle |C_\ell|^2\rangle \approx 4\pi(2\ell+1)(1+\delta), \tag{23}$$

for $\ell < kR$ and $\delta \ll 1$. Plugging Eq. (23) into Eq. (19) gives

$$\frac{n_{\nu,\bar{\nu}}(r)}{n_0} \approx (1+\delta)\sum_{\ell=0}^{\infty}(2\ell+1)j_\ell(k'r)^2 = 1+\delta. \tag{24}$$

Subtracting the neutrino and antineutrino cases gives the asymmetry $\Delta = -2\delta$. The sum in Eq. (24) is dominated by $\ell < k'r$, because as we will see momentarily, a mode with angular momentum ℓ does not penetrate to radii smaller than ℓ/k'. Therefore, in order to apply the approximation in Eq. (23), we require $k'r < kR$. In other words, this asymmetry only holds for $r < (1-\delta)R$.

In order to understand the behavior for $(1-\delta)R < r \leq R$, let us reexpress Eq. (11) in terms of $\varphi_\ell = r\psi_\ell$ as

$$-\frac{1}{2m}\partial_r^2\varphi_\ell + V_{\mathrm{eff}}(r)\varphi_\ell = \frac{k^2}{2m}\varphi_\ell, \tag{25}$$

$$V_{\mathrm{eff}}(r) \equiv \frac{\ell(\ell+1)}{2mr^2} + U\cdot\Theta(R-r). \tag{26}$$

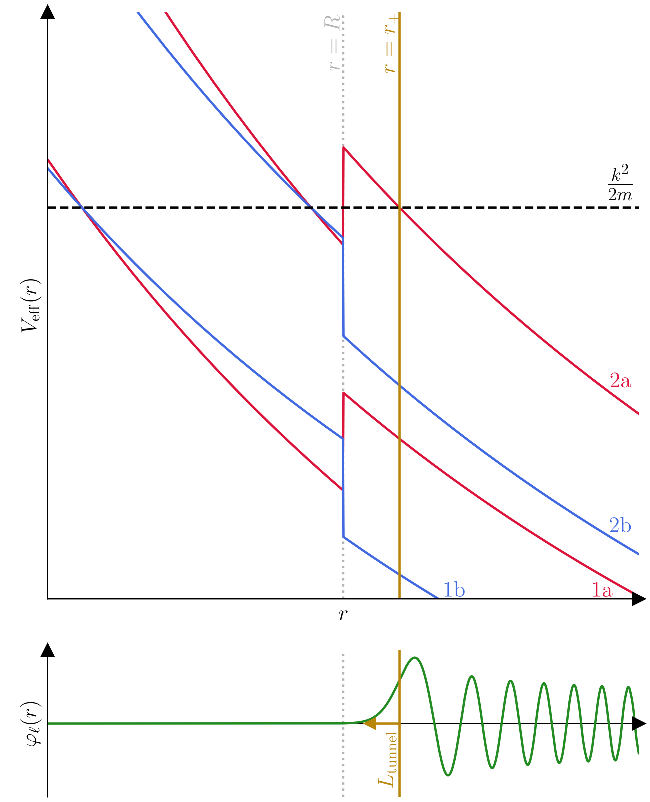

FIG. 4. Schematic diagram of several effective potentials V_{eff} near the surface of the Earth. The potentials for neutrino modes are shown in blue, while the potentials for antineutrino modes are shown in red, and the potentials are labeled to correspond schematically to the rays shown in Fig. 1. The dashed black line denotes the energy of the incoming wave. The point where a given potential meets the dashed line is its classical "turning point" r_0, and the potentials have been paired so that they share the same turning point inside the Earth. Potentials 1a and 1b have low (but slightly different) values of ℓ, and so their turning point lies deep within the Earth. Neutrino potential 2b, on the other hand, has a turning point $(1-\delta)R < r_0 < R$. The corresponding antineutrino potential 2a exhibits a second turning point r_+ outside the Earth, so that an incident wave will not be able to penetrate into the Earth classically. Quantum mechanically, however, the wave may tunnel through the barrier of potential 2a to reach the interior of the Earth. In the lower plot, we show an incoming wave incident on potential 2a. The characteristic length scale over which it decays once entering the barrier is L_{tunnel} [see Eqs. (27)–(30)]. In the case shown here, $L_{\mathrm{tunnel}} < r_+ - R$, so that the wave cannot tunnel into the Earth. For the physical parameters of the Earth, however, $L_{\mathrm{tunnel}} > r_+ - R$, so that the wave can enter and cancel the contribution of mode 2b, leading to no large asymmetry.

Equation (25) has the exact form of the Schrodinger equation with an effective potential V_{eff}. Therefore, we can understand neutrino/antineutrino propagation near/inside the Earth in terms of an incoming wave scattering off this potential. Several examples of V_{eff} are shown in Fig. 4. The angular modes shown in Fig. 4 correspond schematically to the rays shown in Fig. 1. For instance, the curve labeled 1a in Fig. 4 shows the potential for an antineutrino mode with low enough angular momentum to penetrate deep into the Earth. Classically, an incoming wave which is incident on this potential will be reflected at the "turning point" $r_0 \approx \ell/k'$ of the potential. (In the ray optics picture, this corresponds to the impact parameter of the ray.) The wave will therefore not contribute to the asymmetry at radii $r < r_0$. The curve labeled 1b in Fig. 4 corresponds to a neutrino ray with the same classical turning point. Just as in Fig. 1, the angular modes 1a and 1b contribute to the asymmetry at exactly the same set of points. Thus, their contributions will cancel, leading to the small $\mathcal{O}(\delta)$ asymmetry derived above.

Consider instead the angular mode 2b with turning point $(1-\delta)R < r_0 < R$. The corresponding antineutrino mode 2a exhibits two turning points: $r_- \approx \ell/k'$ inside the Earth (at the same location as r_0 for mode 2b), and $r_+ \approx \ell/k$ outside the Earth. Classically, a wave incident on this potential will be reflected at r_+, and so will never reach the interior of the Earth. In this way, the contribution from neutrino mode 2b inside the Earth is left uncanceled. This results in the large asymmetry which can be observed for $(1-\delta)R < r \leq R$, in some cases.

Note that quantum mechanically, however, an incident wave may be able to tunnel through the external barrier of antineutrino mode 2a into the interior of the Earth. The lower plot in Fig. 4 shows in green a wave incident on the potential of mode 2a. The wave begins to damp once it reaches r_+, with some damping length scale L_{tunnel}. If L_{tunnel} exceeds the distance $r_+ - R$, then the wave will tunnel into the interior of the Earth without significant damping. Consequently, it will be able to reach r_- and cancel the contribution from neutrino mode 2b. It is simple to estimate L_{tunnel} in the Wentzel-Kramers-Brillouin (WKB) approximation. As the wave propagates from r_+ to $r_+ - L$, the number of e-folds it damps is given by

$$\int_{r_+-L}^{r_+} \sqrt{2m\left(V_{\mathrm{eff}}(r) - \frac{k^2}{2m}\right)} dr \tag{27}$$

$$\approx \int_{r_+-L}^{r_+} \sqrt{(r_+ - r)\cdot\frac{2\ell(\ell+1)}{r_+^3}} dr \tag{28}$$

$$= \frac{2}{3}\sqrt{L^3\cdot\frac{2\ell(\ell+1)}{r_+^3}}. \tag{29}$$

By setting Eq. (29) equal to 1, we find that the tunneling range is roughly

$$L_{\mathrm{tunnel}} \sim \frac{r_+}{\ell^{2/3}} \sim \frac{(kR)^{1/3}}{k}. \tag{30}$$

If we are interested in the asymmetry at the surface of the Earth, we should consider the mode with $r_- \approx R$, in which case $r_+ - R \approx \delta R$. Therefore, the asymmetry at the surface will be washed out by this tunneling effect when $L_{\mathrm{tunnel}} \gg \delta R$, or in other words, when $\delta^{3/2} kR \ll 1$. This justifies the behavior seen in Fig. 3.

IV. DISCUSSION

In this work, we have demonstrated that the large neutrino-antineutrino asymmetry derived in Ref. [8], under the assumption of a flat Earth, is only reproduced in the spherical calculation when Eq. (3) is satisfied. As the Earth is not large/dense enough to satisfy this condition, we showed that the asymmetry at the surface of the Earth is instead only $\mathcal{O}(\delta)$. Importantly, the calculation of Sec. III assumed that the Earth is perfectly spherical. The physical values of the relevant length scales in this computation are $L_{\mathrm{tunnel}} \sim 1$ m and $\delta R \sim 1$ cm. The Earth is certainly far from a perfect sphere on these length scales, and so the presence of local terrain can affect the conclusions of this work.

We argue, however, that deviations from sphericity should typically make it more difficult to induce a large asymmetry. To see why, let us return to the heuristic argument presented in Sec. II. A large asymmetry occurs when there exists a large region underneath the surface of the Earth which is inaccessible to glancing antineutrino rays. In particular, the extent of this region must be larger than the tunneling range L_{tunnel} derived in Eqs. (27)–(30). Note that this derivation only relied on the effective potential outside the Earth, and so L_{tunnel} should not be affected by local deformations of the Earth.[6] The effect of local terrain should instead be to modify the extent of the inaccessible region. The inaccessible region observed in the perfectly spherical case is a consequence of the smoothness of the Earth's surface, which limits the possible entry angles available to antineutrino rays. As the surface becomes more inhomogeneous, a larger variety of entry angles become available, allowing the antineutrinos to access a larger region below the Earth's surface. We therefore argue that the presence of local terrain will typically act to shrink the inaccesible region, implying that a large asymmetry will still be washed out by the tunneling argument presented in this work.

Before concluding, we contrast this work with Refs. [9–11], which performed similar computations to the one outlined in Sec. III. As mentioned previously, Ref. [9] computed the asymmetry at the surface of the Earth for the physical parameter values of the Earth ($kR = 9\times10^9$ and $\delta = 3\times10^{-8}$). In order to make the computation numerically tractable, they utilized some unspecified Bessel function approximations, so we cannot directly check their result, but they do obtain an asymmetry $\Delta = -\delta$, and so their results

[6]The R appearing in Eq. (30) arises from the use of spherical coordinates, with the center of the Earth as our origin, rather than the presence of the Earth itself.

agree with the findings of this work. In contrast to Ref. [9], this work demonstrates that Eq. (3) is necessary in order for the flat-Earth result to hold.

Reference [10] utilizes a different approach to compute the neutrino-antineutrino asymmetry. Rather than treating the CνB as monochromatic, they account for its full Fermi-Dirac distribution and instead appeal to thermal arguments to demonstrate that the asymmetry should be $\mathcal{O}(\delta)$. In particular, their arguments imply that the asymmetry should be $\mathcal{O}(\delta)$, even when Eq. (3) is satisfied. Figure 3 clearly demonstrates that this is not the case for a monochromatic background, as most parameter values above the black line exhibit an asymmetry larger than $\mathcal{O}(\delta)$. However, because there are some values which exhibit positive asymmetries and some which exhibit negative asymmetries, it is possible that after averaging over neutrino momentum, the total asymmetry is reduced to $\mathcal{O}(\delta)$. These large negative asymmetries are simple to understand from Fig. 1. When Eq. (3) is satisfied, ray 2a has a low probability to tunnel to the dashed red line, and so for most parameter values, we will observe an underdensity of antineutrinos. However, for precisely the same reason that it is difficult for this ray to tunnel into the Earth, it is also difficult for it to tunnel *out of* the Earth. That is, if ray 2a does tunnel in, it will find itself in a long-lived bound state (sometimes called a "whispering gallery mode" [13]), and so it can instead contribute a large antineutrino *overdensity*. As can be seen in Fig. 3, the negative-asymmetry resonances corresponding to these bound states can be quite narrow, and so integrating over neutrino momentum precisely requires very good resolution in momentum. We therefore do not attempt to verify the claims of Ref. [10], but instead simply present Eq. (3) as a different argument for why the asymmetry should be small. We do note though that the arguments in Ref. [10] do not rely on the particular geometry of the Earth, and so apply even when the asphericity of the Earth is accounted for.

Finally, Appendix C of Ref. [11] also performs a number of numerical computations related to the problem considered in this work. Reference [11] simulates scattering of a thermal background, in both $2+1$ and $3+1$ dimensions. They find $\mathcal{O}(\delta)$ effects which are consistent with the thermal arguments advanced in Ref. [10].

ACKNOWLEDGMENTS

We thank Asimina Arvanitaki for numerous fruitful discussions related to this work, including discussions regarding the condition in Eq. (3). We also thank Savas Dimopoulos, Marios Galanis, Guo-yuan Huang, Zhen Liu, and Ken Van Tilburg for other discussions on this topic. S. K. is supported by the U.S. Department of Energy, Office of Science, National Quantum Information Science Research Centers, Superconducting Quantum Materials and Systems Center (SQMS) under Contract No. DE-AC02-07CH11359.

[1] E. Kolb and M. Turner, *The Early Universe* (Westview Press, Boulder, CO, 1994).
[2] D. Baumann, *Cosmology* (Cambridge University Press, Cambridge, England, 2022).
[3] C. Yanagisawa, Looking for cosmic neutrino background, Front. Phys. **2**, 30 (2014).
[4] D. Scott, The cosmic neutrino background, arXiv:2402.16243.
[5] S. Weinberg, Universal neutrino degeneracy, Phys. Rev. **128**, 1457 (1962).
[6] E. Baracchini, M. G. Betti, M. Biasotti, A. Bosca, F. Calle, J. Carabe-Lopez *et al.*, Ptolemy: A proposal for thermal relic detection of massive neutrinos and directional detection of mev dark matter, arXiv:1808.01892.
[7] L. Stodolsky, Speculations on detection of the "neutrino sea," Phys. Rev. Lett. **34**, 110 (1975).
[8] A. Arvanitaki and S. Dimopoulos, Cosmic neutrino background on the surface of the earth, Phys. Rev. D **108**, 043517 (2023).
[9] G. Huang, Neutrino-antineutrino asymmetry of cνb on the surface of the round earth, arXiv:2401.07347.
[10] A. Gruzinov and M. Mirbabayi, The density of relic neutrinos near the surface of earth, arXiv:2403.03152.
[11] K. V. Tilburg, Wake forces in a background of quadratically coupled mediators, Phys. Rev. D **109**, 096036 (2024).
[12] https://github.com/skalia618/CnuB-Asymmetry.
[13] https://en.wikipedia.org/wiki/Whispering-gallery_wave.