Vacuum energy in the effective field theory of general relativity with a scalar field

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A consistency condition of general relativity as an effective field theory in Minkowskian background uniquely fixes the value of the cosmological constant. In two-loop calculations, including the interaction of gravitons with matter fields, it has been shown that this value of the cosmological constant leads to vanishing vacuum energy, under the assumption that the energy-momentum tensor of the gravitational field is given by the pseudotensor of Landau-Lifshitz's classic textbook. Here, we demonstrate that this result also holds when the self-interaction of a scalar field is taken into account. That is, our two-loop-order calculation suggests that, in an effective field theory of metric and scalar fields, one arrives at a consistent theory with massless gravitons if the cosmological constant is fixed from the condition of vanishing vacuum energy. Vice versa, imposing the consistency condition in Minkowskian background leads to a vanishing vacuum energy.

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It is widely accepted that at low energies the physics of fundamental interactions is adequately described by effective field theory (EFT) [1–3]. Gravitation can also be included in this formalism by considering the most general effective Lagrangian of metric and matter fields [4–8], which is invariant under all underlying symmetries including the gauge symmetry of massless spin-two particles [9]. It is well known that for nonvanishing values of the cosmological constant term Λ a quantum field theoretical treatment of general relativity with the metric field presented as the Minkowskian background plus the graviton field poses a problem due to the graviton propagator possessing a pole that corresponds to a massive ghost mode [9]. Setting Λ equal to zero does not improve the situation as radiative corrections lead to the reemergence of the problem with the massive ghost [10]. However, one can represent the cosmological constant as a power series in $\hbar$ with coefficients chosen to yield self-consistent EFT results to all orders in the loop expansion [10]. Thus, the consistency requirement of a perturbative EFT in flat Minkowski background uniquely fixes the cosmological constant as a function of other parameters of the effective Lagrangian. This also necessarily requires considering an EFT in a curved background field if any other value of the cosmological constant is assumed. In this case, the mass term of the graviton can be removed at classical level by imposing the equations of motion with respect to the background graviton field [11]. To the best of our knowledge, a systematic study beyond tree level has not been done due to the lack of an EFT on non-Minkowskian background.

In Refs. [12,13] it was found by performing two-loop calculations for gravitational interactions only that the vacuum energy vanishes exactly for the value of the cosmological constant corresponding to that of Ref. [10], i.e., for the value that guarantees the vanishing of the graviton mass and of the vacuum expectation value of the graviton field. Provided that this result holds to all orders, also when including interactions among matter fields, the uniquely fixed value of the cosmological constant yielding a self-consistent perturbative EFT on Minkowskian background could also be interpreted as a consequence of imposing the condition of vanishing vacuum energy.

In this work we consider a simple EFT of general relativity with metric and a scalar matter fields. We perform two-loop-order calculations for the value of the cosmological constant leading to a consistent EFT in order to see if the vacuum energy vanishes when self-interactions of the scalar field are taken into account. Notice that, while there does not exist a commonly accepted expression of the energy-momentum tensor for the gravitational field (see, e.g., Refs. [14–18]), Refs. [12,13] and also the current work use the definition of the energy-momentum pseudotensor (EMPT) and of the full four-momentum of the matter and gravitational fields as given in the classic textbook [19].

In the framework of the EFT of general relativity, the action is given via the most general effective Lagrangian of gravitational and matter fields, which is invariant under general coordinate transformations and other underlying symmetries of the considered model:

$$
\begin{aligned}
S &= \int d^4x\sqrt{-g}\{\mathcal{L}_{\rm gr}(g)+\mathcal{L}_{\rm m}(g,\psi)\} \\
&= \int d^4x\sqrt{-g}\left\{\frac{2}{\kappa^2}(R-2\Lambda)+\mathcal{L}_{\rm gr,ho}(g)+\mathcal{L}_{\rm m}(g,\psi)\right\} \\
&= S_{\rm gr}(g)+S_{\rm m}(g,\psi). \qquad (1)
\end{aligned}
$$

Here, $\kappa^2 = 32\pi G$, with Newton's constant $G = 6.70881\times 10^{-39}$ GeV^{-2}, Λ is the cosmological constant, ψ and $g^{\mu\nu}$ denote the matter and metric fields, respectively, $g = \det g^{\mu\nu}$, and R is the scalar curvature. An infinite number of gravitational self-interactions involving higher orders in derivatives are contained in $\mathcal{L}_{\rm gr,ho}(g)$ and $\mathcal{L}_{\rm m}(g,\psi)$ is the effective Lagrangian of the matter fields interacting with the metric and the vielbein tetrad fields. Experimental evidence suggests that the contributions of $\mathcal{L}_{\rm gr,ho}(g)$ and of nonrenormalizable interactions of $\mathcal{L}_{\rm m}(g,\psi)$ to physical quantities are heavily suppressed.

The action of the matter part of the model considered here includes an infinite number of terms, of which we show below only those contributing in our specific calculations:

$$
S_{\rm m} = \int d^4x\sqrt{-g}\left\{\frac{g^{\mu\nu}}{2}\partial_\mu H\partial_\nu H - \frac{m^2}{2}H^2+\frac{g}{3!}H^3+fH\right\}, \qquad (2)
$$

where H is the scalar field. The low-energy EFT is obtained by representing the metric field as the sum of the Minkowskian background and the quantum fields [20]

$$
\begin{aligned}
g_{\mu\nu} &= \eta_{\mu\nu}+\kappa h_{\mu\nu}, \\
g^{\mu\nu} &= \eta^{\mu\nu}-\kappa h^{\mu\nu}+\kappa^2 h^\mu_\lambda h^{\lambda\nu}-\kappa^3 h^\mu_\lambda h^\lambda_\sigma h^{\sigma\nu}+\cdots, \qquad (3)
\end{aligned}
$$

and we calculate physical quantities perturbatively by expanding in κ and other coupling constants treating them independently.

The energy-momentum tensor of the matter fields coupled to the gravitational field is given by

$$
T^{\mu\nu}_{\rm m}(g,\psi) = \frac{2}{\sqrt{-g}}\frac{\delta S_{\rm m}}{\delta g_{\mu\nu}}. \qquad (4)
$$

The energy-momentum tensor corresponding to Eq. (2) has the form

$$
T^{\mu\nu}_m = \partial_\mu H\partial_\nu H - g^{\mu\nu}\left\{\frac{g^{\alpha\beta}}{2}\partial_\alpha H\partial_\beta H - \frac{m^2}{2}H^2+\frac{g}{3!}H^3+fH\right\}. \qquad (5)
$$

The pseudotensor of the gravitational field (alone) is given as

$$
T^{\mu\nu}_{\rm gr}(g) = \frac{4}{\kappa^2}\Lambda g^{\mu\nu}+T^{\mu\nu}_{LL}(g), \qquad (6)
$$

where $T^{\mu\nu}_{LL}(g)$ is defined via [19]

$$
\begin{aligned}
(-g)T^{\mu\nu}_{LL}(g) = \frac{2}{\kappa^2}\Big(&\frac{1}{8}g^{\lambda\sigma}g^{\mu\nu}g_{\alpha\gamma}g_{\beta\delta}\mathfrak{g}^{\alpha\gamma}{}_{,\sigma}\,\mathfrak{g}^{\beta\delta}{}_{,\lambda} - \frac{1}{4}g^{\mu\lambda}g^{\nu\sigma}g_{\alpha,\gamma}g_{\beta\delta}\mathfrak{g}^{\alpha\gamma}{}_{,\sigma}\,\mathfrak{g}^{\beta\delta}{}_{,\lambda} - \frac{1}{4}g^{\lambda\sigma}g^{\mu\nu}g_{\beta\alpha}g_{\gamma\delta}\mathfrak{g}^{\alpha\gamma}{}_{,\sigma}\,\mathfrak{g}^{\beta\delta}{}_{,\lambda} \\
&+\frac{1}{2}g^{\mu\lambda}g^{\nu\sigma}g_{\beta\alpha}g_{\gamma\delta}\mathfrak{g}^{\alpha\gamma}{}_{,\sigma}\,\mathfrak{g}^{\beta\delta}{}_{,\lambda} + g^{\beta\alpha}g_{\lambda\sigma}\mathfrak{g}^{\nu\sigma}{}_{,\alpha}\,\mathfrak{g}^{\mu\lambda}{}_{,\beta} + \frac{1}{2}g^{\mu\nu}g_{\lambda\sigma}\mathfrak{g}^{\lambda\beta}{}_{,\alpha}\,\mathfrak{g}^{\alpha\sigma}{}_{,\beta} \\
&- g^{\mu\lambda}g_{\sigma\beta}\mathfrak{g}^{\nu\beta}{}_{,\alpha}\,\mathfrak{g}^{\sigma\alpha}{}_{,\lambda} - g^{\nu\lambda}g_{\sigma\beta}\mathfrak{g}^{\mu\beta}{}_{,\alpha}\,\mathfrak{g}^{\sigma\alpha}{}_{,\lambda} + \mathfrak{g}^{\lambda\sigma}{}_{,\sigma}\,\mathfrak{g}^{\mu\nu}{}_{,\lambda} - \mathfrak{g}^{\mu\lambda}{}_{,\lambda}\,\mathfrak{g}^{\nu\sigma}{}_{,\sigma}\Big), \qquad (7)
\end{aligned}
$$

with $\mathfrak{g}^{\mu\nu} = \sqrt{-g}g^{\mu\nu}$ and $\mathfrak{g}^{\mu\nu}{}_{,\lambda} = \partial\mathfrak{g}^{\mu\nu}/\partial x^\lambda$.

The conserved full four-momentum of the matter and the gravitational fields is defined via the full EMPT $T^{\mu\nu} = T^{\mu\nu}_{\rm m}(g,\psi)+T^{\mu\nu}_{\rm gr}(g)$ as [19]

$$
P^\mu = \int(-g)T^{\mu\nu}dS_\nu, \qquad (8)
$$

where the integration covers any hypersurface containing the whole three-dimensional space. Thus, for a vanishing

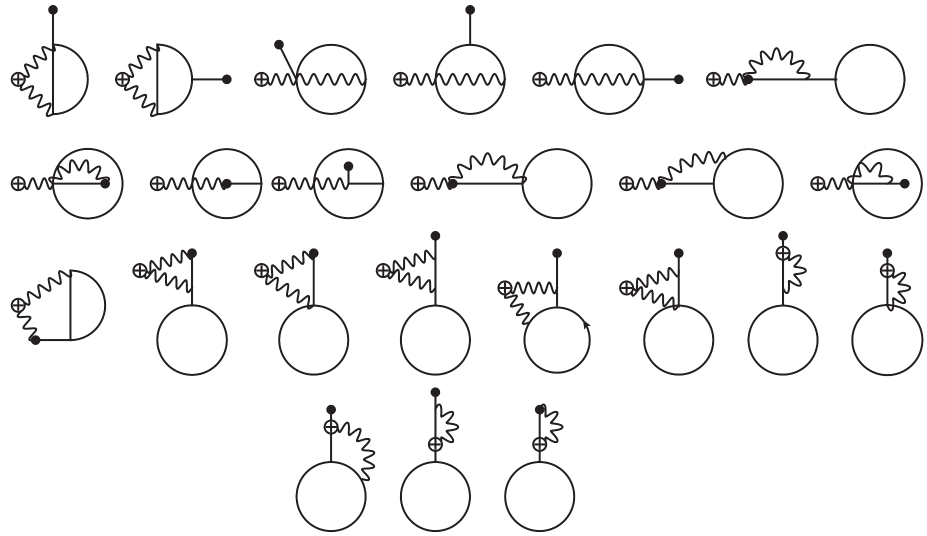

FIG. 1. Topologies of two-loop diagrams whose sums (but not the separate diagrams) lead to the same contribution to the cosmological constant for the graviton tadpole and the matrix element of $(-g)T^{\mu\nu}$. The wavy and solid lines correspond to gravitons and scalars, respectively. The filled dots stand for vertices generated by the fH term of the Lagrangian. The cross corresponds to either external graviton line or a $(-g)T^{\mu\nu}$ insertion.

vacuum expectation value of $(-g)T^{\mu\nu}$, the energy of the vacuum will be zero. This expectation value can be represented via the following path integral:

$$\langle 0|(-g)T^{\mu\nu}|0\rangle = \int \mathcal{D}g\mathcal{D}\psi(-g)[T^{\mu\nu}_{\rm gr}(g) + T^{\mu\nu}_{\rm m}(g,\psi)] \times \exp\left\{ i\int d^4x\sqrt{-g}[\mathcal{L}(g,\psi) + \mathcal{L}_{\rm GF}]\right\}, \tag{9}$$

where

$$\mathcal{L}_{\rm GF} = \xi\left(\partial_\nu h^{\mu\nu} - \frac{1}{2}\partial^\mu h^\nu_\nu\right)\left(\partial^\beta h_{\mu\beta} - \frac{1}{2}\partial_\mu h^\alpha_\alpha\right) \tag{10}$$

is the gauge-fixing term with parameter ξ and the integration measure includes the Faddeev-Popov determinant.

According to Ref. [10] the cosmological constant Λ is uniquely fixed by the condition that the vacuum expectation value of the quantum field $h_{\mu\nu}$ vanishes, and this also removes the graviton mass from the dressed propagator as a result of a Ward identity [10]. To study the implications for the vacuum energy at two-loop order we represent Λ as

$$\Lambda = \sum_{i=0}^{\infty} \hbar^i \Lambda_i. \tag{11}$$

The vacuum expectation value of $(-g)T^{\mu\nu}$ at tree order vanishes for $\Lambda_0 = 0$, and this choice also removes the graviton mass from the propagator at tree level. It is a straightforward consequence of Eq. (4) that the same value of Λ_1 cancels the one-loop contribution to the vacuum expectation value of the graviton field $h_{\mu\nu}$ and the vacuum energy. Calculations taking into account only gravitational interaction have been done in Refs. [12,13].

In this work we consider the order-fg (that is, linear in f and linear in g) two-loop contributions to the vacuum expectation values of $(-g)T^{\mu\nu}$ and of the graviton field $h_{\mu\nu}$, and we find that the same value of Λ_2 leads to an exact cancellation of both of them. To show this, we first fix Λ_2 by imposing the condition that the corresponding two-loop order contribution to the expectation value of $h_{\mu\nu}$ vanishes and next by imposing an analogous condition on the corresponding contribution to the vacuum expectation value of $(-g)T^{\mu\nu}$. We subtract the obtained two values of Λ_2 from each other and denote the difference by R. It is a trivial consequence of Eq. (4) that diagrams with the same structure, where the vertices with external graviton and with $(-g)T^{\mu\nu}$ insertion couple only to scalar lines, give contributions to R which cancel each other diagram by diagram.[1] On the other hand, we find that individual diagrams contributing to a given topology in Fig. 1 yield different contributions to the cosmological constant Λ_2 in the two cases. However, considering the sums of the diagrams in Fig. 1 we find that the contributions of the two sets of diagrams [one for the vacuum expectation value of $(-g)T^{\mu\nu}$ and one for the graviton field $h_{\mu\nu}$] to R neatly cancel. The first nontrivial result is obtained at two-loop order. The relevant Feynman rules are included in the Appendix, and in calculations we used the *Mathematica*-based program FeynCalc [21,22].

[1]This is because due to Eq. (4) the Feynman rules of one graviton coupling to scalars and $(-g)T^{\mu\nu}$ coupling to the same number of scalars (and no gravitons) are the same, modulo an overall factor.

A self-consistent EFT should lead to finite physical quantities after renormalizing (an infinite number of) parameters of the effective Lagrangian. Therefore, it is mandatory that the unique value of the cosmological constant that defines the perturbative EFT of the Standard Model, coupled to gravitons on a Minkowskian flat background, leads to a finite expression of the energy (density) of the vacuum to all orders of perturbation theory. Based on the two-loop order results including the one of the current work, we expect that this finite value should be at least of three-loop order and thus very small. Moreover, it seems natural that an adequate theory formulated in the language of mathematics should assign zero energy to the vacuum state [23]. Therefore turning the argument around we expect that by demanding that the vacuum energy should be vanishing to all orders we obtain a self-consistent perturbative low-energy EFT of matter and gravitational fields on a flat Minkowskian background.

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APPENDIX

Below we give Feynman rules used in the calculation of the vacuum expectation values of the graviton field and the energy-momentum tensor.

1. Propagators

(i) Scalar propagator with momentum p:

$$\frac{i}{p^2-m^2+i\epsilon}; \tag{A1}$$

(ii) graviton propagator in D dimensions with Lorentz indices (μ,ν), (α,β) and momentum p:

$$\frac{i}{2}\frac{g^{\lambda\nu}g^{\mu\sigma}+g^{\lambda\mu}g^{\nu\sigma}-\frac{2g^{\lambda\sigma}g^{\mu\nu}}{D-2}}{p^2+i\epsilon} - \frac{i\xi}{2}\frac{p^\nu(p^\sigma g^{\lambda\mu}+p^\lambda g^{\mu\sigma})+p^\mu(p^\sigma g^{\lambda\nu}+p^\lambda g^{\nu\sigma})}{(p^2+i\epsilon)^2}. \tag{A2}$$

2. Vertices (all momenta in all vertices are incoming)

(i) One scalar:

$$if; \tag{A3}$$

(ii) three scalars:

$$ig; \tag{A4}$$

(iii) one graviton with indices (μ,ν):

$$-\frac{2i\Lambda g^{\mu\nu}}{\kappa}; \tag{A5}$$

(iv) one graviton with indices (μ,ν) and one scalar:

$$\frac{1}{2}if\kappa g^{\mu\nu}; \tag{A6}$$

(v) one graviton with indices (μ,ν) and two scalars with momenta p_1 and p_2:

$$\frac{1}{2}i\kappa(-g^{\mu\nu}(m^2+p_1\cdot p_2)+p_2^\mu p_1^\nu+p_1^\mu p_2^\nu); \tag{A7}$$

(vi) one graviton with indices (μ,ν) and three scalars:

$$\frac{1}{2}ig\kappa g^{\mu\nu}; \tag{A8}$$

(vii) two gravitons with indices (μ,ν) and (α,β) and one scalar:

$$-\frac{1}{4}if\kappa^2(g^{\alpha\nu}g^{\beta\mu}+g^{\alpha\mu}g^{\beta\nu}-g^{\alpha\beta}g^{\mu\nu}); \tag{A9}$$

(viii) two gravitons with indices (μ,ν) and (α,β) and two scalars with momenta p_1 and p_2:

$$\begin{aligned}-\frac{1}{4}i\kappa^2(&-m^2g^{\alpha\nu}g^{\beta\mu}-m^2g^{\alpha\mu}g^{\beta\nu}+m^2g^{\alpha\beta}g^{\mu\nu}\\&+p_1^\beta p_2^\nu g^{\alpha\mu}+p_1^\beta p_2^\mu g^{\alpha\nu}+p_1^\alpha p_2^\nu g^{\beta\mu}\\&+p_1^\nu(-p_2^\mu g^{\alpha\beta}+p_2^\beta g^{\alpha\mu}+p_2^\alpha g^{\beta\mu})+p_1^\alpha p_2^\mu g^{\beta\nu}\\&+p_1^\mu(-p_2^\nu g^{\alpha\beta}+p_2^\beta g^{\alpha\nu}+p_2^\alpha g^{\beta\nu})-p_2^\alpha p_1^\beta g^{\mu\nu}\\&-p_1^\alpha p_2^\beta g^{\mu\nu}-p_1\cdot p_2(g^{\alpha\nu}g^{\beta\mu}+g^{\alpha\mu}g^{\beta\nu}-g^{\alpha\beta}g^{\mu\nu}));\end{aligned} \tag{A10}$$

(ix) two gravitons with indices (μ,ν) and (α,β) and three scalars:

$$-\frac{1}{4}ig\kappa^2(g^{\alpha\nu}g^{\beta\mu}+g^{\alpha\mu}g^{\beta\nu}-g^{\alpha\beta}g^{\mu\nu}); \tag{A11}$$

(x) three gravitons with (Lorentz indices, momentum) combinations (μ,ν,p_1), (α,β,p_2) and (λ,σ,p_3):

$$
\begin{aligned}
&\frac{1}{8}[\mathrm{hhh}(\{\mu,\nu,p_1\},\{\alpha,\beta,p_2\},\{\lambda,\sigma,p_3\})+\mathrm{hhh}(\{\mu,\nu,p_1\},\{\alpha,\beta,p_2\},\{\sigma,\lambda,p_3\})\\
&\quad+\mathrm{hhh}(\{\mu,\nu,p_1\},\{\beta,\alpha,p_2\},\{\lambda,\sigma,p_3\})+\mathrm{hhh}(\{\mu,\nu,p_1\},\{\beta,\alpha,p_2\},\{\sigma,\lambda,p_3\})\\
&\quad+\mathrm{hhh}(\{\nu,\mu,p_1\},\{\alpha,\beta,p_2\},\{\lambda,\sigma,p_3\})+\mathrm{hhh}(\{\nu,\mu,p_1\},\{\alpha,\beta,p_2\},\{\sigma,\lambda,p_3\})\\
&\quad+\mathrm{hhh}(\{\nu,\mu,p_1\},\{\beta,\alpha,p_2\},\{\lambda,\sigma,p_3\})+\mathrm{hhh}(\{\nu,\mu,p_1\},\{\beta,\alpha,p_2\},\{\sigma,\lambda,p_3\})],
\end{aligned}
\tag{A12}
$$

where

$$
\begin{aligned}
&\mathrm{hhh}(\{\mu,\nu,p_1\},\{\alpha,\beta,p_2\},\{\lambda,\sigma,p_3\})=-\frac{1}{4}i\kappa(p_1^\alpha p_2^\beta g^{\lambda\sigma}g^{\mu\nu}+p_2^\beta p_3^\alpha g^{\lambda\sigma}g^{\mu\nu}+2\Lambda g^{\alpha\beta}g^{\lambda\sigma}g^{\mu\nu}\\
&\quad+2g^{\alpha\beta}g^{\lambda\sigma}(p_1\cdot p_2+p_1\cdot p_3+p_2\cdot p_3)g^{\mu\nu}+2g^{\alpha\beta}g^{\lambda\sigma}(p_1^2+p_2^2+p_3^2)g^{\mu\nu}\\
&\quad+g^{\alpha\beta}(p_1^\nu(p_2^\mu+p_3^\mu)g^{\lambda\sigma}+(p_1^\lambda+p_2^\lambda)p_3^\sigma g^{\mu\nu})+4(p_1^\sigma p_3^\nu g^{\alpha\beta}g^{\lambda\mu}+p_1^\beta p_2^\nu g^{\alpha\mu}g^{\lambda\sigma}+p_2^\sigma p_3^\beta g^{\alpha\lambda}g^{\mu\nu})\\
&\quad+4((p_1^\sigma p_2^\nu+p_2^\sigma p_3^\nu)g^{\alpha\beta}g^{\lambda\mu}+(p_2^\nu p_3^\beta+p_1^\beta p_3^\nu)g^{\alpha\mu}g^{\lambda\sigma}+(p_1^\beta p_2^\sigma+p_1^\sigma p_3^\beta)g^{\alpha\lambda}g^{\mu\nu})\\
&\quad-2((p_1^\nu p_2^\sigma+p_2^\nu p_3^\sigma)g^{\alpha\beta}g^{\lambda\mu}+(p_1^\nu p_3^\beta+p_2^\beta p_3^\nu)g^{\alpha\mu}g^{\lambda\sigma}+(p_1^\sigma p_2^\beta+p_1^\beta p_3^\sigma)g^{\alpha\lambda}g^{\mu\nu})\\
&\quad+2((p_1^\nu p_1^\sigma+p_3^\nu p_3^\sigma)g^{\alpha\beta}g^{\lambda\mu}+(p_1^\beta p_1^\nu+p_2^\beta p_2^\nu)g^{\alpha\mu}g^{\lambda\sigma}+(p_2^\beta p_2^\sigma+p_3^\beta p_3^\sigma)g^{\alpha\lambda}g^{\mu\nu})\\
&\quad-2(p_1^\mu p_1^\nu g^{\alpha\beta}g^{\lambda\sigma}+(p_3^\lambda p_3^\sigma g^{\alpha\beta}+p_2^\alpha p_2^\beta g^{\lambda\sigma})g^{\mu\nu})-2(p_1^\beta p_3^\alpha g^{\lambda\sigma}g^{\mu\nu}+p_1^\alpha p_3^\beta g^{\lambda\sigma}g^{\mu\nu}\\
&\quad+g^{\alpha\beta}((p_2^\nu p_3^\mu+p_2^\mu p_3^\nu)g^{\lambda\sigma}+(p_1^\sigma p_2^\lambda+p_1^\lambda p_2^\sigma)g^{\mu\nu}))-4(p_1^\alpha p_1^\beta g^{\lambda\sigma}g^{\mu\nu}+p_3^\alpha p_3^\beta g^{\lambda\sigma}g^{\mu\nu}\\
&\quad+g^{\alpha\beta}((p_2^\mu p_2^\nu+p_3^\mu p_3^\nu)g^{\lambda\sigma}+(p_1^\lambda p_1^\sigma+p_2^\lambda p_2^\sigma)g^{\mu\nu}))-5(p_2^\alpha(p_1^\beta+p_3^\beta)g^{\lambda\sigma}g^{\mu\nu}+g^{\alpha\beta}(p_1^\mu(p_2^\nu+p_3^\nu)g^{\lambda\sigma}\\
&\quad+(p_1^\sigma+p_2^\sigma)p_3^\lambda g^{\mu\nu}))+2(p_3^\mu p_3^\sigma g^{\alpha\beta}g^{\lambda\nu}+p_1^\alpha p_1^\nu g^{\beta\mu}g^{\lambda\sigma}+p_3^\alpha p_3^\sigma g^{\beta\lambda}g^{\mu\nu}+p_2^\beta(p_2^\mu g^{\alpha\nu}g^{\lambda\sigma}+p_2^\lambda g^{\alpha\sigma}g^{\mu\nu})\\
&\quad+p_1^\lambda p_1^\nu g^{\alpha\beta}g^{\mu\sigma})+2(p_2^\sigma p_3^\mu g^{\alpha\beta}g^{\lambda\nu}+p_2^\mu p_3^\beta g^{\alpha\nu}g^{\lambda\sigma}+p_1^\alpha p_3^\nu g^{\beta\mu}g^{\lambda\sigma}+p_1^\beta p_2^\lambda g^{\alpha\sigma}g^{\mu\nu}+p_1^\sigma p_3^\alpha g^{\beta\lambda}g^{\mu\nu}+p_1^\lambda p_2^\nu g^{\alpha\beta}g^{\mu\sigma})\\
&\quad+4(p_1^\sigma p_3^\mu g^{\alpha\beta}g^{\lambda\nu}+p_1^\beta p_2^\mu g^{\alpha\nu}g^{\lambda\sigma}+p_1^\alpha p_2^\nu g^{\beta\mu}g^{\lambda\sigma}+p_2^\lambda p_3^\beta g^{\alpha\sigma}g^{\mu\nu}+p_2^\sigma p_3^\alpha g^{\beta\lambda}g^{\mu\nu}+p_1^\lambda p_3^\nu g^{\alpha\beta}g^{\mu\sigma})\\
&\quad-4(p_1^\beta p_3^\nu g^{\alpha\sigma}g^{\lambda\mu}+p_1^\sigma(p_2^\nu g^{\alpha\mu}g^{\beta\lambda}+p_3^\beta g^{\alpha\nu}g^{\lambda\mu})+p_2^\sigma(p_3^\nu g^{\alpha\lambda}g^{\beta\mu}+p_1^\beta g^{\alpha\mu}g^{\lambda\nu})+p_2^\nu p_3^\beta g^{\alpha\lambda}g^{\mu\sigma})\\
&\quad-6(p_2^\nu p_3^\beta g^{\alpha\sigma}g^{\lambda\mu}+p_2^\sigma(p_3^\nu g^{\alpha\mu}g^{\beta\lambda}+p_1^\beta g^{\alpha\nu}g^{\lambda\mu})+p_1^\sigma(p_2^\nu g^{\alpha\lambda}g^{\beta\mu}+p_3^\beta g^{\alpha\mu}g^{\lambda\nu})+p_1^\beta p_3^\nu g^{\alpha\lambda}g^{\mu\sigma})\\
&\quad+8\Lambda(g^{\alpha\sigma}g^{\beta\mu}g^{\lambda\nu}+g^{\alpha\nu}g^{\beta\lambda}g^{\mu\sigma})-2(p_2^\nu p_3^\mu g^{\alpha\sigma}g^{\beta\lambda}+p_2^\mu p_3^\nu g^{\alpha\sigma}g^{\beta\lambda}+p_1^\sigma p_2^\lambda g^{\alpha\nu}g^{\beta\mu}+p_1^\lambda p_2^\sigma g^{\alpha\nu}g^{\beta\mu}\\
&\quad+p_1^\beta p_3^\alpha g^{\lambda\nu}g^{\mu\sigma}+p_1^\alpha p_3^\beta g^{\lambda\nu}g^{\mu\sigma})-2(p_1^\sigma p_3^\alpha g^{\beta\mu}g^{\lambda\nu}+g^{\alpha\sigma}(p_1^\lambda p_2^\nu g^{\beta\mu}+p_2^\mu p_3^\beta g^{\lambda\nu})+p_1^\alpha p_3^\nu g^{\beta\lambda}g^{\mu\sigma}\\
&\quad+g^{\alpha\nu}(p_2^\sigma p_3^\mu g^{\beta\lambda}+p_1^\beta p_2^\lambda g^{\mu\sigma}))+2(p_2^\nu p_3^\sigma g^{\alpha\mu}g^{\beta\lambda}+p_1^\sigma p_2^\beta g^{\alpha\nu}g^{\lambda\mu}+p_2^\beta p_3^\nu g^{\alpha\sigma}g^{\lambda\mu}+p_1^\beta p_3^\sigma g^{\alpha\mu}g^{\lambda\nu}\\
&\quad+p_1^\nu g^{\alpha\lambda}(p_2^\sigma g^{\beta\mu}+p_3^\beta g^{\mu\sigma}))+2(p_1^\mu(p_2^\sigma g^{\alpha\beta}g^{\lambda\nu}+p_3^\beta g^{\alpha\nu}g^{\lambda\sigma})+p_2^\alpha(p_3^\nu g^{\beta\mu}g^{\lambda\sigma}+p_1^\sigma g^{\beta\lambda}g^{\mu\nu})\\
&\quad+p_3^\lambda(p_1^\beta g^{\alpha\sigma}g^{\mu\nu}+p_2^\nu g^{\alpha\beta}g^{\mu\sigma}))-2(p_1^\mu(p_3^\sigma g^{\alpha\beta}g^{\lambda\nu}+p_2^\beta g^{\alpha\nu}g^{\lambda\sigma})+(p_2^\beta p_3^\lambda g^{\alpha\sigma}+p_2^\alpha p_3^\sigma g^{\beta\lambda})g^{\mu\nu}\\
&\quad+p_1^\nu(p_2^\alpha g^{\beta\mu}g^{\lambda\sigma}+p_3^\lambda g^{\alpha\beta}g^{\mu\sigma}))+2(p_1^\mu(p_1^\sigma g^{\alpha\beta}g^{\lambda\nu}+p_1^\beta g^{\alpha\nu}g^{\lambda\sigma})+p_2^\alpha(p_2^\nu g^{\beta\mu}g^{\lambda\sigma}+p_2^\sigma g^{\beta\lambda}g^{\mu\nu})\\
&\quad+p_3^\lambda(p_3^\beta g^{\alpha\sigma}g^{\mu\nu}+p_3^\nu g^{\alpha\beta}g^{\mu\sigma}))-4(p_1^\sigma p_3^\nu(g^{\alpha\mu}g^{\beta\lambda}+g^{\alpha\lambda}g^{\beta\mu})+p_2^\sigma p_3^\beta(g^{\alpha\nu}g^{\lambda\mu}+g^{\alpha\mu}g^{\lambda\nu})\\
&\quad+p_1^\beta p_2^\nu(g^{\alpha\sigma}g^{\lambda\mu}+g^{\alpha\lambda}g^{\mu\sigma}))-4(p_2^\nu p_2^\sigma(g^{\alpha\mu}g^{\beta\lambda}+g^{\alpha\lambda}g^{\beta\mu})+p_1^\beta p_1^\sigma(g^{\alpha\nu}g^{\lambda\mu}+g^{\alpha\mu}g^{\lambda\nu})\\
&\quad+p_3^\beta p_3^\nu(g^{\alpha\sigma}g^{\lambda\mu}+g^{\alpha\lambda}g^{\mu\sigma}))+2(p_2^\mu p_3^\sigma g^{\alpha\nu}g^{\beta\lambda}+p_2^\beta p_3^\mu g^{\alpha\sigma}g^{\lambda\nu}+p_1^\alpha p_3^\sigma g^{\beta\mu}g^{\lambda\nu}+p_1^\lambda p_2^\beta g^{\alpha\nu}g^{\mu\sigma}\\
&\quad+p_1^\nu(p_2^\lambda g^{\alpha\sigma}g^{\beta\mu}+p_3^\alpha g^{\beta\lambda}g^{\mu\sigma}))+16(p_3^\alpha p_3^\mu g^{\beta\nu}g^{\lambda\sigma}+p_1^\alpha p_1^\lambda g^{\beta\sigma}g^{\mu\nu}+p_2^\lambda p_2^\mu g^{\alpha\beta}g^{\nu\sigma})-8(p_1^\alpha p_2^\mu g^{\beta\nu}g^{\lambda\sigma}\\
&\quad+p_2^\lambda p_3^\alpha g^{\beta\sigma}g^{\mu\nu}+p_1^\lambda p_3^\mu g^{\alpha\beta}g^{\nu\sigma})+2(p_1^\alpha p_1^\mu g^{\beta\nu}g^{\lambda\sigma}+p_3^\alpha p_3^\lambda g^{\beta\sigma}g^{\mu\nu}+p_2^\alpha(p_2^\mu g^{\beta\nu}g^{\lambda\sigma}+p_2^\lambda g^{\beta\sigma}g^{\mu\nu})\\
&\quad+p_1^\lambda p_1^\mu g^{\alpha\beta}g^{\nu\sigma}+p_3^\lambda p_3^\mu g^{\alpha\beta}g^{\nu\sigma})+2(p_3^\alpha(p_2^\mu g^{\beta\nu}g^{\lambda\sigma}+p_1^\lambda g^{\beta\sigma}g^{\mu\nu})+p_1^\alpha(p_3^\mu g^{\beta\nu}g^{\lambda\sigma}+p_2^\lambda g^{\beta\sigma}g^{\mu\nu})\\
&\quad+(p_1^\lambda p_2^\mu+p_2^\lambda p_3^\mu)g^{\alpha\beta}g^{\nu\sigma})-4(p_2^\mu p_3^\lambda g^{\alpha\sigma}g^{\beta\nu}+p_1^\alpha p_3^\lambda g^{\mu\sigma}g^{\beta\nu}+p_1^\mu g^{\beta\sigma}(p_2^\lambda g^{\alpha\nu}+p_3^\alpha g^{\lambda\nu})+p_2^\alpha p_3^\mu g^{\beta\lambda}g^{\nu\sigma}\\
&\quad+p_1^\lambda p_2^\alpha g^{\beta\mu}g^{\nu\sigma})-8(p_1^\lambda p_1^\mu g^{\alpha\nu}g^{\beta\sigma}+p_1^\alpha p_1^\mu g^{\lambda\nu}g^{\beta\sigma}+p_3^\lambda g^{\beta\nu}(p_3^\mu g^{\alpha\sigma}+p_3^\alpha g^{\mu\sigma})+p_2^\alpha p_2^\mu g^{\beta\lambda}g^{\nu\sigma}+p_2^\alpha p_2^\lambda g^{\beta\mu}g^{\nu\sigma})\\
&\quad+4(p_3^\lambda p_3^\sigma g^{\alpha\mu}g^{\beta\nu}+p_1^\mu p_1^\nu g^{\alpha\lambda}g^{\beta\sigma}+p_2^\alpha p_2^\beta g^{\lambda\mu}g^{\nu\sigma})-2(p_1^\lambda p_3^\sigma g^{\alpha\mu}g^{\beta\nu}+p_2^\lambda p_3^\sigma g^{\alpha\mu}g^{\beta\nu}+p_1^\nu p_2^\mu g^{\alpha\lambda}g^{\beta\sigma}\\
&\quad+p_1^\nu p_3^\mu g^{\alpha\lambda}g^{\beta\sigma}+p_1^\alpha p_2^\beta g^{\lambda\mu}g^{\nu\sigma}+p_2^\beta p_3^\alpha g^{\lambda\mu}g^{\nu\sigma})+8(p_1^\sigma p_2^\lambda g^{\alpha\mu}g^{\beta\nu}+p_1^\lambda p_2^\sigma g^{\alpha\mu}g^{\beta\nu}+p_2^\nu p_3^\mu g^{\alpha\lambda}g^{\beta\sigma}+p_2^\mu p_3^\nu g^{\alpha\lambda}g^{\beta\sigma}
\end{aligned}
$$

$$
\begin{aligned}
&+p_1^\beta p_3^\alpha g^{\lambda\mu} g^{\nu\sigma} + p_1^\alpha p_3^\beta g^{\lambda\mu} g^{\nu\sigma}) + 8(p_1^\lambda p_1^\sigma g^{\alpha\mu} g^{\beta\nu} + p_2^\lambda p_2^\sigma g^{\alpha\mu} g^{\beta\nu} + p_2^\mu p_2^\nu g^{\alpha\lambda} g^{\beta\sigma} + p_3^\mu p_3^\nu g^{\alpha\lambda} g^{\beta\sigma} + p_1^\alpha p_1^\beta g^{\lambda\mu} g^{\nu\sigma} \\
&+p_3^\alpha p_3^\beta g^{\lambda\mu} g^{\nu\sigma}) + 10((p_1^\sigma + p_2^\sigma) p_3^\lambda g^{\alpha\mu} g^{\beta\nu} + p_1^\mu (p_2^\nu + p_3^\nu) g^{\alpha\lambda} g^{\beta\sigma} + p_2^\alpha (p_1^\beta + p_3^\beta) g^{\lambda\mu} g^{\nu\sigma}) \\
&- 4\Lambda(g^{\alpha\mu} g^{\beta\nu} g^{\lambda\sigma} + g^{\alpha\lambda} g^{\beta\sigma} g^{\mu\nu} + g^{\alpha\beta} g^{\lambda\mu} g^{\nu\sigma}) + 4(p_2^\alpha (p_3^\sigma g^{\beta\nu} + p_1^\nu g^{\beta\sigma}) g^{\lambda\mu} + p_1^\mu g^{\alpha\lambda} (p_3^\sigma g^{\beta\nu} + p_2^\beta g^{\nu\sigma}) \\
&+p_3^\lambda g^{\alpha\mu} (p_1^\nu g^{\beta\sigma} + p_2^\beta g^{\nu\sigma})) - 8((p_3^\alpha p_3^\sigma g^{\beta\nu} + p_1^\alpha p_1^\nu g^{\beta\sigma}) g^{\lambda\mu} + g^{\alpha\mu} (p_1^\lambda p_1^\nu g^{\beta\sigma} + p_2^\beta p_2^\lambda g^{\nu\sigma}) \\
&+g^{\alpha\lambda} (p_3^\mu p_3^\sigma g^{\beta\nu} + p_2^\beta p_2^\mu g^{\nu\sigma})) - 10(p_2^\alpha (p_1^\sigma g^{\beta\nu} + p_3^\nu g^{\beta\sigma}) g^{\lambda\mu} + p_3^\lambda g^{\alpha\mu} (p_2^\nu g^{\beta\sigma} + p_1^\beta g^{\nu\sigma}) \\
&+p_1^\mu g^{\alpha\lambda} (p_2^\sigma g^{\beta\nu} + p_3^\beta g^{\nu\sigma})) - 4((p_2^\sigma p_3^\alpha g^{\beta\nu} + p_1^\alpha p_2^\nu g^{\beta\sigma}) g^{\lambda\mu} + g^{\alpha\lambda} (p_1^\sigma p_3^\mu g^{\beta\nu} + p_1^\beta p_2^\mu g^{\nu\sigma}) \\
&+g^{\alpha\mu} (p_1^\lambda p_3^\nu g^{\beta\sigma} + p_2^\lambda p_3^\beta g^{\nu\sigma})) - 4((p_1^\sigma p_3^\alpha g^{\beta\nu} + p_1^\alpha p_3^\nu g^{\beta\sigma}) g^{\lambda\mu} + g^{\alpha\mu} (p_1^\lambda p_2^\nu g^{\beta\sigma} + p_1^\beta p_2^\lambda g^{\nu\sigma}) \\
&+g^{\alpha\lambda} (p_2^\sigma p_3^\mu g^{\beta\nu} + p_2^\mu p_3^\beta g^{\nu\sigma})) + 10((p_1^\alpha p_2^\sigma g^{\beta\nu} + p_2^\nu p_3^\alpha g^{\beta\sigma}) g^{\lambda\mu} + g^{\alpha\mu} (p_2^\lambda p_3^\nu g^{\beta\sigma} + p_1^\lambda p_3^\beta g^{\nu\sigma}) \\
&+g^{\alpha\lambda} (p_1^\sigma p_2^\mu g^{\beta\nu} + p_1^\beta p_3^\mu g^{\nu\sigma})) + 8(p_2^\alpha (p_3^\mu g^{\beta\nu} g^{\lambda\sigma} + p_1^\lambda g^{\beta\sigma} g^{\mu\nu}) + p_1^\mu (p_3^\alpha g^{\beta\nu} g^{\lambda\sigma} + p_2^\lambda g^{\alpha\beta} g^{\nu\sigma}) \\
&+p_3^\lambda (p_1^\alpha g^{\beta\sigma} g^{\mu\nu} + p_2^\mu g^{\alpha\beta} g^{\nu\sigma})) + 12(p_2^\alpha p_3^\lambda g^{\beta\sigma} g^{\mu\nu} + p_1^\mu (p_2^\alpha g^{\beta\nu} g^{\lambda\sigma} + p_3^\lambda g^{\alpha\beta} g^{\nu\sigma})) - 6(p_2^\mu p_3^\lambda g^{\alpha\nu} g^{\beta\sigma} \\
&+p_2^\alpha p_3^\mu g^{\lambda\nu} g^{\beta\sigma} + p_1^\lambda p_2^\alpha g^{\beta\nu} g^{\mu\sigma} + p_1^\alpha p_3^\lambda g^{\beta\mu} g^{\nu\sigma} + p_1^\mu (p_2^\lambda g^{\alpha\sigma} g^{\beta\nu} + p_3^\alpha g^{\beta\lambda} g^{\nu\sigma})) - 4(p_2^\lambda p_2^\mu (g^{\alpha\sigma} g^{\beta\nu} + g^{\alpha\nu} g^{\beta\sigma}) \\
&+p_3^\alpha p_3^\mu (g^{\beta\sigma} g^{\lambda\nu} + g^{\beta\lambda} g^{\nu\sigma}) + p_1^\alpha p_1^\lambda (g^{\beta\nu} g^{\mu\sigma} + g^{\beta\mu} g^{\nu\sigma})) + 4(p_1^\lambda p_3^\mu (g^{\alpha\sigma} g^{\beta\nu} + g^{\alpha\nu} g^{\beta\sigma}) \\
&+p_1^\alpha p_2^\mu (g^{\beta\sigma} g^{\lambda\nu} + g^{\beta\lambda} g^{\nu\sigma}) + p_2^\lambda p_3^\alpha (g^{\beta\nu} g^{\mu\sigma} + g^{\beta\mu} g^{\nu\sigma})) - 4(p_1^\alpha p_3^\mu g^{\beta\sigma} g^{\lambda\nu} + p_1^\lambda (p_2^\mu g^{\alpha\nu} g^{\beta\sigma} + p_3^\alpha g^{\beta\nu} g^{\mu\sigma}) \\
&+p_2^\mu p_3^\alpha g^{\beta\lambda} g^{\nu\sigma} + p_2^\lambda (p_3^\mu g^{\alpha\sigma} g^{\beta\nu} + p_1^\alpha g^{\beta\mu} g^{\nu\sigma})) + 2(p_2^\mu p_3^\alpha g^{\beta\sigma} g^{\lambda\nu} + p_2^\lambda (p_3^\mu g^{\alpha\nu} g^{\beta\sigma} + p_1^\alpha g^{\beta\nu} g^{\mu\sigma}) + p_1^\alpha p_3^\mu g^{\beta\lambda} g^{\nu\sigma} \\
&+p_1^\lambda (p_2^\mu g^{\alpha\sigma} g^{\beta\nu} + p_3^\alpha g^{\beta\mu} g^{\nu\sigma})) - 12(p_2^\alpha p_3^\lambda (g^{\beta\nu} g^{\mu\sigma} + g^{\beta\mu} g^{\nu\sigma}) + p_1^\mu (p_3^\lambda (g^{\alpha\sigma} g^{\beta\nu} + g^{\alpha\nu} g^{\beta\sigma}) \\
&+p_2^\alpha (g^{\beta\sigma} g^{\lambda\nu} + g^{\beta\lambda} g^{\nu\sigma}))) + 4(g^{\alpha\sigma} g^{\beta\nu} g^{\lambda\mu} p_1 \cdot p_2 + g^{\alpha\mu} g^{\beta\lambda} g^{\nu\sigma} p_1 \cdot p_3 + g^{\alpha\lambda} (g^{\beta\nu} g^{\mu\sigma} p_1 \cdot p_2 + g^{\beta\mu} g^{\nu\sigma} p_1 \cdot p_3) \\
&+g^{\alpha\nu} g^{\beta\sigma} g^{\lambda\mu} p_2 \cdot p_3 + g^{\alpha\mu} g^{\beta\sigma} g^{\lambda\nu} p_2 \cdot p_3) + 2(g^{\alpha\nu} g^{\beta\mu} g^{\lambda\sigma} p_1 \cdot p_2 + g^{\alpha\beta} g^{\lambda\nu} g^{\mu\sigma} p_1 \cdot p_3 + g^{\alpha\sigma} g^{\beta\lambda} g^{\mu\nu} p_2 \cdot p_3) \\
&- 8(g^{\alpha\mu} g^{\beta\nu} g^{\lambda\sigma} p_1 \cdot p_2 + g^{\alpha\beta} g^{\lambda\mu} g^{\nu\sigma} p_1 \cdot p_3 + g^{\alpha\lambda} g^{\beta\sigma} g^{\mu\nu} p_2 \cdot p_3) + 8(g^{\alpha\mu} (g^{\beta\sigma} g^{\lambda\nu} \\
&+g^{\beta\lambda} g^{\nu\sigma}) p_1 \cdot p_2 + g^{\alpha\sigma} g^{\beta\nu} g^{\lambda\mu} p_1 \cdot p_3 + g^{\alpha\nu} g^{\beta\sigma} g^{\lambda\mu} p_1 \cdot p_3 + g^{\alpha\lambda} g^{\beta\nu} g^{\mu\sigma} p_2 \cdot p_3 + g^{\alpha\lambda} g^{\beta\mu} g^{\nu\sigma} p_2 \cdot p_3) \\
&- 4(g^{\alpha\lambda} g^{\beta\sigma} g^{\mu\nu} (p_1 \cdot p_2 + p_1 \cdot p_3) + g^{\alpha\beta} g^{\lambda\mu} g^{\nu\sigma} (p_1 \cdot p_2 + p_2 \cdot p_3) + g^{\alpha\mu} g^{\beta\nu} g^{\lambda\sigma} (p_1 \cdot p_3 + p_2 \cdot p_3)) \\
&- 4(g^{\alpha\lambda} g^{\beta\sigma} g^{\mu\nu} p_1^2 + g^{\alpha\beta} g^{\lambda\mu} g^{\nu\sigma} p_2^2 + g^{\alpha\mu} g^{\beta\nu} g^{\lambda\sigma} p_3^2) + 8(g^{\alpha\nu} g^{\beta\sigma} g^{\lambda\mu} p_1^2 + g^{\alpha\lambda} g^{\beta\mu} g^{\nu\sigma} p_2^2 + g^{\alpha\sigma} g^{\beta\nu} g^{\lambda\mu} p_3^2 \\
&+g^{\alpha\lambda} g^{\beta\nu} g^{\mu\sigma} p_3^2 + g^{\alpha\mu} (g^{\beta\sigma} g^{\lambda\nu} p_1^2 + g^{\beta\lambda} g^{\nu\sigma} p_2^2)) - 4(g^{\alpha\mu} g^{\beta\nu} g^{\lambda\sigma} (p_1^2 + p_2^2) \\
&+g^{\alpha\beta} g^{\lambda\mu} g^{\nu\sigma} (p_1^2 + p_3^2) + g^{\alpha\lambda} g^{\beta\sigma} g^{\mu\nu} (p_2^2 + p_3^2))); \qquad \text{(A13)}
\end{aligned}
$$

(xi) energy-momentum tensor with indices (μ, ν) and one scalar:

$$-f g^{\mu\nu}; \qquad \text{(A14)}$$

(xii) energy-momentum tensor with indices (μ, ν) and two scalars with momenta p_1 and p_2:

$$g^{\mu\nu}(m^2 + p_1 \cdot p_2) - p_2^\mu p_1^\nu - p_1^\mu p_2^\nu; \qquad \text{(A15)}$$

(xiii) energy-momentum tensor with indices (μ, ν) and three scalars:

$$-g g^{\mu\nu}; \qquad \text{(A16)}$$

(xiv) energy-momentum tensor with indices (μ, ν), graviton with indices (α, β) and one scalar:

$$\frac{1}{2} f \kappa (g^{\alpha\nu} g^{\beta\mu} + g^{\alpha\mu} g^{\beta\nu} - 2 g^{\alpha\beta} g^{\mu\nu}); \qquad \text{(A17)}$$

(xv) energy-momentum tensor with indices (μ, ν), graviton with indices (α, β) and two scalars with momenta p_1 and p_2:

$$
\begin{aligned}
&\frac{1}{2}\kappa(m^2(-g^{\alpha\nu}) g^{\beta\mu} - m^2 g^{\alpha\mu} g^{\beta\nu} + 2m^2 g^{\alpha\beta} g^{\mu\nu} \\
&\quad + p_1^\beta p_2^\nu g^{\alpha\mu} + p_1^\beta p_2^\mu g^{\alpha\nu} + p_1^\alpha p_2^\nu g^{\beta\mu} \\
&\quad + p_1^\nu(-2 p_2^\mu g^{\alpha\beta} + p_2^\beta g^{\alpha\mu} + p_2^\alpha g^{\beta\mu}) + p_1^\alpha p_2^\mu g^{\beta\nu} \\
&\quad + p_1^\mu(-2 p_2^\nu g^{\alpha\beta} + p_2^\beta g^{\alpha\nu} + p_2^\alpha g^{\beta\nu}) - p_2^\alpha p_1^\beta g^{\mu\nu} \\
&\quad - p_1^\alpha p_2^\beta g^{\mu\nu} - p_1 \cdot p_2 g^{\alpha\nu} g^{\beta\mu} - p_1 \cdot p_2 g^{\alpha\mu} g^{\beta\nu} \\
&\quad + 2 p_1 \cdot p_2 g^{\alpha\beta} g^{\mu\nu}); \qquad \text{(A18)}
\end{aligned}
$$

(xvi) energy-momentum tensor with indices (μ, ν), graviton with indices (α, β) and three scalars:

$$\frac{1}{2} g \kappa (g^{\alpha\nu} g^{\beta\mu} + g^{\alpha\mu} g^{\beta\nu} - 2 g^{\alpha\beta} g^{\mu\nu}); \qquad \text{(A19)}$$

(xvii) energy-momentum tensor with indices (μ,ν) and two gravitons with (Lorentz indices, momentum) combinations (λ,σ,p_1) and (α,β,p_2):

$$
\begin{aligned}
&\frac{1}{8}(-4p_1^\sigma p_2^\lambda g^{\alpha\nu}g^{\beta\mu} - 4p_1^\lambda p_2^\sigma g^{\alpha\nu}g^{\beta\mu} + 4p_1^\sigma p_2^\alpha g^{\lambda\nu}g^{\beta\mu} - 2p_1^\alpha p_2^\nu g^{\lambda\sigma}g^{\beta\mu} - 2p_1^\alpha p_2^\nu g^{\sigma\lambda}g^{\beta\mu} + 4p_1^\lambda p_2^\alpha g^{\sigma\nu}g^{\beta\mu}\\
&- 4g^{\alpha\sigma}g^{\lambda\nu}p_1\cdot p_2 g^{\beta\mu} + 6g^{\alpha\nu}g^{\lambda\sigma}p_1\cdot p_2 g^{\beta\mu} + 6g^{\alpha\nu}g^{\sigma\lambda}p_1\cdot p_2 g^{\beta\mu} - 4g^{\alpha\lambda}g^{\sigma\nu}p_1\cdot p_2 g^{\beta\mu} - 4p_1^\sigma p_2^\lambda g^{\alpha\mu}g^{\beta\nu}\\
&- 4p_1^\lambda p_2^\sigma g^{\alpha\mu}g^{\beta\nu} - 4p_1^\sigma p_2^\nu g^{\alpha\beta}g^{\lambda\mu} + 4p_1^\sigma p_2^\beta g^{\alpha\nu}g^{\lambda\mu} + 4p_1^\beta p_2^\nu g^{\alpha\sigma}g^{\lambda\mu} - 4p_1^\sigma p_2^\nu g^{\beta\alpha}g^{\lambda\mu} + 4p_1^\sigma p_2^\alpha g^{\beta\nu}g^{\lambda\mu}\\
&+ 4p_1^\alpha p_2^\nu g^{\beta\sigma}g^{\lambda\mu} - 4p_1^\sigma p_2^\mu g^{\alpha\beta}g^{\lambda\nu} + 4p_1^\sigma p_2^\beta g^{\alpha\mu}g^{\lambda\nu} + 4p_1^\beta p_2^\mu g^{\alpha\sigma}g^{\lambda\nu} - 4p_1^\sigma p_2^\mu g^{\beta\alpha}g^{\lambda\nu} + 4p_1^\alpha p_2^\mu g^{\beta\sigma}g^{\lambda\nu}\\
&- 2p_1^\beta p_2^\nu g^{\alpha\mu}g^{\lambda\sigma} - 2p_1^\beta p_2^\mu g^{\alpha\nu}g^{\lambda\sigma} - 2p_1^\alpha p_2^\mu g^{\beta\nu}g^{\lambda\sigma} + 2p_1^\sigma p_2^\lambda g^{\alpha\beta}g^{\mu\nu} + 2p_1^\lambda p_2^\sigma g^{\alpha\beta}g^{\mu\nu} - 2p_1^\beta p_2^\sigma g^{\alpha\lambda}g^{\mu\nu}\\
&- 2p_1^\beta p_2^\lambda g^{\alpha\sigma}g^{\mu\nu} + 2p_1^\sigma p_2^\lambda g^{\beta\alpha}g^{\mu\nu} + 2p_1^\lambda p_2^\sigma g^{\beta\alpha}g^{\mu\nu} - 2p_1^\alpha p_2^\sigma g^{\beta\lambda}g^{\mu\nu} - 2p_1^\alpha p_2^\lambda g^{\beta\sigma}g^{\mu\nu} + 2p_1^\beta p_2^\alpha g^{\lambda\sigma}g^{\mu\nu}\\
&+ 2p_1^\alpha p_2^\beta g^{\lambda\sigma}g^{\mu\nu} - 2p_1^\beta p_2^\alpha g^{\lambda\nu}g^{\mu\sigma} - 2p_1^\alpha p_2^\beta g^{\lambda\nu}g^{\mu\sigma} + 2p_1^\sigma p_2^\lambda g^{\alpha\beta}g^{\nu\mu} + 2p_1^\lambda p_2^\sigma g^{\alpha\beta}g^{\nu\mu} - 2p_1^\beta p_2^\sigma g^{\alpha\lambda}g^{\nu\mu}\\
&- 2p_1^\beta p_2^\lambda g^{\alpha\sigma}g^{\nu\mu} + 2p_1^\sigma p_2^\lambda g^{\beta\alpha}g^{\nu\mu} + 2p_1^\lambda p_2^\sigma g^{\beta\alpha}g^{\nu\mu} - 2p_1^\alpha p_2^\sigma g^{\beta\lambda}g^{\nu\mu} - 2p_1^\alpha p_2^\lambda g^{\beta\sigma}g^{\nu\mu} + 2p_1^\beta p_2^\alpha g^{\lambda\sigma}g^{\nu\mu}\\
&+ 2p_1^\alpha p_2^\beta g^{\lambda\sigma}g^{\nu\mu} - 2p_1^\beta p_2^\alpha g^{\lambda\mu}g^{\nu\sigma} - 2p_1^\alpha p_2^\beta g^{\lambda\mu}g^{\nu\sigma} - 2p_1^\beta p_2^\nu g^{\alpha\mu}g^{\sigma\lambda} - 2p_1^\beta p_2^\mu g^{\alpha\nu}g^{\sigma\lambda} - 2p_1^\alpha p_2^\mu g^{\beta\nu}g^{\sigma\lambda}\\
&+ 2p_1^\beta p_2^\alpha g^{\mu\nu}g^{\sigma\lambda} + 2p_1^\alpha p_2^\beta g^{\mu\nu}g^{\sigma\lambda} + 2p_1^\beta p_2^\alpha g^{\nu\mu}g^{\sigma\lambda} + 2p_1^\alpha p_2^\beta g^{\nu\mu}g^{\sigma\lambda} - 4p_1^\lambda p_2^\nu g^{\alpha\beta}g^{\sigma\mu} + 4p_1^\beta p_2^\nu g^{\alpha\lambda}g^{\sigma\mu}\\
&+ 4p_1^\lambda p_2^\beta g^{\alpha\nu}g^{\sigma\mu} - 4p_1^\lambda p_2^\nu g^{\beta\alpha}g^{\sigma\mu} + 4p_1^\alpha p_2^\nu g^{\beta\lambda}g^{\sigma\mu} + 4p_1^\lambda p_2^\alpha g^{\beta\nu}g^{\sigma\mu} - 2p_1^\beta p_2^\alpha g^{\nu\lambda}g^{\sigma\mu} - 2p_1^\alpha p_2^\beta g^{\nu\lambda}g^{\sigma\mu}\\
&- 2p_1^\nu(-2p_2^\lambda g^{\alpha\sigma}g^{\beta\mu} + 2p_2^\alpha g^{\lambda\sigma}g^{\beta\mu} + 2p_2^\alpha g^{\sigma\lambda}g^{\beta\mu} - 2p_2^\lambda g^{\alpha\mu}g^{\beta\sigma} + p_2^\sigma(-2g^{\alpha\mu}g^{\beta\lambda} - 2g^{\alpha\lambda}g^{\beta\mu} + (g^{\alpha\beta}+g^{\beta\alpha})g^{\lambda\mu})\\
&+ 2p_2^\beta g^{\alpha\mu}g^{\lambda\sigma} + 2p_2^\beta g^{\alpha\mu}g^{\sigma\lambda} + 2p_2^\mu(g^{\alpha\sigma}g^{\beta\lambda} + g^{\alpha\lambda}g^{\beta\sigma} - (g^{\alpha\beta}+g^{\beta\alpha})(g^{\lambda\sigma}+g^{\sigma\lambda})) + p_2^\lambda g^{\alpha\beta}g^{\sigma\mu} + p_2^\lambda g^{\beta\alpha}g^{\sigma\mu})\\
&- 4p_1^\lambda p_2^\mu g^{\alpha\beta}g^{\sigma\nu} + 4p_1^\beta p_2^\mu g^{\alpha\lambda}g^{\sigma\nu} + 4p_1^\lambda p_2^\beta g^{\alpha\mu}g^{\sigma\nu} - 4p_1^\lambda p_2^\mu g^{\beta\alpha}g^{\sigma\nu} + 4p_1^\alpha p_2^\mu g^{\beta\lambda}g^{\sigma\nu} - 2p_1^\beta p_2^\alpha g^{\mu\lambda}g^{\sigma\nu} - 2p_1^\alpha p_2^\beta g^{\mu\lambda}g^{\sigma\nu}\\
&- 2p_1^\mu(-2p_2^\lambda g^{\alpha\sigma}g^{\beta\nu} + 2p_2^\alpha g^{\lambda\sigma}g^{\beta\nu} + 2p_2^\alpha g^{\sigma\lambda}g^{\beta\nu} - 2p_2^\lambda g^{\alpha\nu}g^{\beta\sigma} + p_2^\sigma(-2g^{\alpha\nu}g^{\beta\lambda} - 2g^{\alpha\lambda}g^{\beta\nu} + (g^{\alpha\beta}+g^{\beta\alpha})g^{\lambda\nu})\\
&+ 2p_2^\beta g^{\alpha\nu}g^{\lambda\sigma} + 2p_2^\beta g^{\alpha\nu}g^{\sigma\lambda} + 2p_2^\nu(g^{\alpha\sigma}g^{\beta\lambda} + g^{\alpha\lambda}g^{\beta\sigma} - (g^{\alpha\beta}+g^{\beta\alpha})(g^{\lambda\sigma}+g^{\sigma\lambda})) + p_2^\lambda g^{\alpha\beta}g^{\sigma\nu} + p_2^\lambda g^{\beta\alpha}g^{\sigma\nu})\\
&- 4g^{\alpha\sigma}g^{\beta\nu}g^{\lambda\mu}p_1\cdot p_2 - 4g^{\alpha\nu}g^{\beta\sigma}g^{\lambda\mu}p_1\cdot p_2 - 4g^{\alpha\mu}g^{\beta\sigma}g^{\lambda\nu}p_1\cdot p_2 + 6g^{\alpha\mu}g^{\beta\nu}g^{\lambda\sigma}p_1\cdot p_2 + 2g^{\alpha\sigma}g^{\beta\lambda}g^{\mu\nu}p_1\cdot p_2\\
&+ 2g^{\alpha\lambda}g^{\beta\sigma}g^{\mu\nu}p_1\cdot p_2 - 3g^{\alpha\beta}g^{\lambda\sigma}g^{\mu\nu}p_1\cdot p_2 - 3g^{\beta\alpha}g^{\lambda\sigma}g^{\mu\nu}p_1\cdot p_2 + 3g^{\alpha\beta}g^{\lambda\nu}g^{\mu\sigma}p_1\cdot p_2 + 3g^{\beta\alpha}g^{\lambda\nu}g^{\mu\sigma}p_1\cdot p_2\\
&+ 2g^{\alpha\sigma}g^{\beta\lambda}g^{\nu\mu}p_1\cdot p_2 + 2g^{\alpha\lambda}g^{\beta\sigma}g^{\nu\mu}p_1\cdot p_2 - 3g^{\alpha\beta}g^{\lambda\sigma}g^{\nu\mu}p_1\cdot p_2 - 3g^{\beta\alpha}g^{\lambda\sigma}g^{\nu\mu}p_1\cdot p_2 + 3g^{\alpha\beta}g^{\lambda\mu}g^{\nu\sigma}p_1\cdot p_2\\
&+ 3g^{\beta\alpha}g^{\lambda\mu}g^{\nu\sigma}p_1\cdot p_2 + 6g^{\alpha\mu}g^{\beta\nu}g^{\sigma\lambda}p_1\cdot p_2 - 3g^{\alpha\beta}g^{\mu\nu}g^{\sigma\lambda}p_1\cdot p_2 - 3g^{\beta\alpha}g^{\mu\nu}g^{\sigma\lambda}p_1\cdot p_2 - 3g^{\alpha\beta}g^{\nu\mu}g^{\sigma\lambda}p_1\cdot p_2\\
&- 3g^{\beta\alpha}g^{\nu\mu}g^{\sigma\lambda}p_1\cdot p_2 - 4g^{\alpha\nu}g^{\beta\lambda}g^{\sigma\mu}p_1\cdot p_2 - 4g^{\alpha\lambda}g^{\beta\nu}g^{\sigma\mu}p_1\cdot p_2 + 3g^{\alpha\beta}g^{\nu\lambda}g^{\sigma\mu}p_1\cdot p_2 + 3g^{\beta\alpha}g^{\nu\lambda}g^{\sigma\mu}p_1\cdot p_2\\
&- 4g^{\alpha\mu}g^{\beta\lambda}g^{\sigma\nu}p_1\cdot p_2 + 3g^{\alpha\beta}g^{\mu\lambda}g^{\sigma\nu}p_1\cdot p_2 + 3g^{\beta\alpha}g^{\mu\lambda}g^{\sigma\nu}p_1\cdot p_2).
\end{aligned}
\tag{A20}
$$

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