On the holographic dual of a topological symmetry operator

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We study the holographic dual of a topological symmetry operator in the context of the AdS/CFT correspondence. Symmetry operators arise from topological field theories localized on a subspace of the boundary conformal field theory spacetime. We use bottom up considerations to construct the topological sector associated with their bulk counterparts. In particular, by exploiting the structure of entanglement wedge reconstruction we argue that the bulk counterpart has a nontopological world volume action, i.e., it describes a dynamical object. As a consequence, we find that there are no global p-form symmetries for $p \geq 0$ in asymptotically anti–de Sitter spacetimes, which includes the case of noninvertible symmetries. Provided one has a suitable notion of subregion-subregion duality, our argument for the absence of bulk global symmetries applies to more general spacetimes. These considerations also motivate us to consider for general QFTs (holographic or not) the notion of lower-form symmetries, namely, $(-m)$-form symmetries for $m \geq 2$.

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I. INTRODUCTION

Symmetries provide important constraints on observables in physical systems. An important recent lesson is that global symmetries also encode a rich topological structure in a quantum field theory (QFT) [1]. In a D-dimensional QFT a topological symmetry operator supported on a closed manifold Y of dimension $(D-p-1)$ (codimension $p+1$) implements a p-form symmetry action on objects of dimension at least p.[1] *A priori*, the theory on Y could support a nontrivial topological field theory (TFT). In this broader context, a nongroup-like fusion rule reflects the appearance of a noninvertible symmetry.[2]

Now, in the context of the AdS/CFT correspondence [8–10] relating a gravitational theory on $(D+1)$-dimensional anti–de Sitter (AdS) space and a D-dimensional conformal field theory (CFT), one expects that any operator of the boundary CFT should have a bulk counterpart. Recent progress in the context of stringy holographic and geometric engineering setups indicates that these topological operators "come to life" in the bulk as the topological sector of dynamical branes [11–19].

Our aim in this note will be to give a bottom up explanation for some of these observations without reference to a specific top down construction.

A helpful clue for how to proceed is the associated symmetry topological field theory (SymTFT) which governs the global symmetries of a QFT.[3] For a D-dimensional QFT with a given set of categorical symmetries, there is a $(D+1)$-dimensional SymTFT_{D+1} which governs the global form of the QFT_D; this involves extending the D-dimensional QFT by an interval; at one boundary we have the (relative) QFT, and at the other end we introduce topological/gapped boundary conditions to specify the global form of the QFT.[4] This is of course highly

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[1]One might prefer to restrict to ordinary linking, but it is helpful to work more broadly. For example, in a 3D Chern-Simons theory with charge conjugation symmetry, we have a 0-form symmetry operator of codimension 2 which acts on the line operators.

[2]See, e.g., the reviews [2–7] and references therein.

[3]See, e.g., Refs. [20–35].

[4]The structure of the SymTFT is best understood in the case of finite and discrete symmetries. In the case of continuous symmetries, there are some subtleties with demanding a purely gapped bulk, and to a certain extent it is not necessary (and sometimes undesirable) to enforce this structure since the "global form" for continuous symmetries is less ambiguous; we simply need to enforce suitable boundary conditions for continuous gauge fields near a boundary. For a recent discussion on SymTFTs for continuous symmetries see Ref. [35]. Let us also comment that in all of these circumstances we can still consider a bulk TFT which detects the anomalies of a boundary theory which by (a mild) abuse of terminology we shall refer to as the SymTFT.

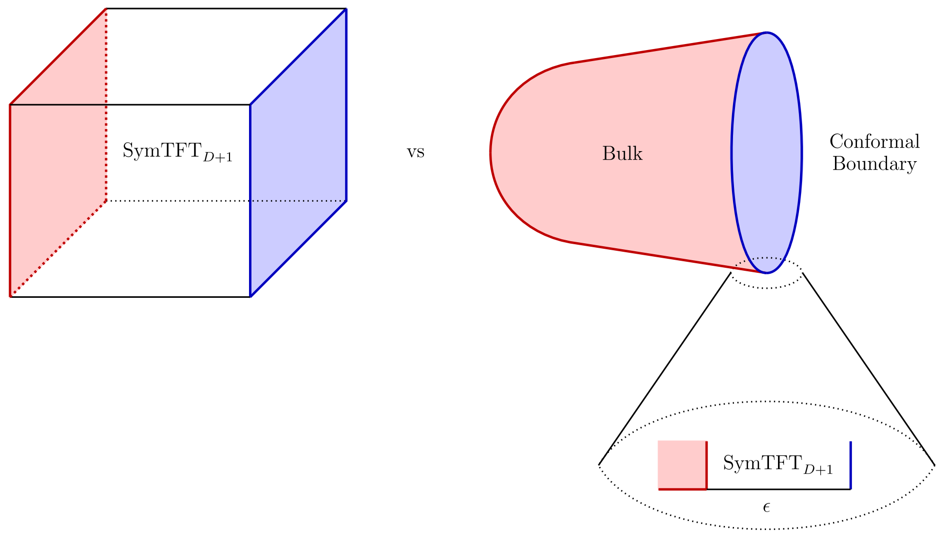

FIG. 1. Left: we depict the standard SymTFT sandwich. The SymTFT$_{D+1}$ is supported on a slab of dimension $D+1$ with boundaries of dimension D at which physical and topological boundary conditions are imposed as indicated by the shaded regions. Right: in holographic setups the full bulk is dual to the physical theory and therefore sets the physical boundary conditions, topological boundary conditions are imposed asymptotically at the conformal boundary. The SymTFT$_{D+1}$ can be thought of as supported between these on a slab / sliver of width $\epsilon \to 0$. The physical boundary is now determined by a gravitational system in dimension $D+1$.

reminiscent of the AdS/CFT correspondence, although in the latter, we really only have the single conformal boundary of AdS; to some extent the physical boundary conditions of the SymTFT have instead been "smeared" over the entire bulk (as it must, since the gravitational theory "knows" about all the local interactions of the CFT). See Fig. 1 for a depiction of the two bulk/boundary correspondences.

Another clue is the recent argument presented in [36,37] that certain global symmetries cannot be present in asymptotically AdS spacetimes.[5] The argument follows via "proof by contradiction," showing that any topological operator of the boundary theory cannot reach "far enough" into the bulk to influence quasilocalized bulk operators.[6] Indeed, the essential point in this line of reasoning is that entanglement wedges have only finite extent in the bulk (see, e.g., [43–45]). A final complication is that the arguments presented in [36,37] appear to require $D \geq p+2$.

While intuitively appealing, this line of reasoning also raises some questions. For one, the topological nature of the symmetry operators suggests that they can be deformed arbitrarily, provided linking between operators is maintained. From this perspective, one reaches a puzzle: how could the gravity dual of a topological operator "know" about the location of something as metric dependent as an entanglement wedge in the first place?

Here we address some of these issues, showing that bottom up considerations concerning bulk reconstruction/holographic renormalization group (RG) flows are enough to argue that bulk symmetry operators are never purely topological, and so there are no global symmetries. Our line of reasoning shares some similarities with that presented in [36,37] but is also complementary in many aspects.

With this aim in mind, we consider for $D \geq 2$, a CFT$_D$ with a semiclassical gravity dual, but we do not assume that global symmetries of the boundary theory are gauged in the bulk.[7] Rather, we argue that bulk reconstruction means that symmetry operators must have a bulk dual which sources stress energy in the sense that these operators are now sensitive to small fluctuations in the metric. As such, they cannot be purely topological. While detailed properties of the resulting object in gravity are model dependent, we can also extract some qualitative properties of the resulting brane action world volume theory. Summarizing, the existence of a symmetry operator in the boundary CFT$_D$ allows us to *predict* the existence of a brane in the gravitational bulk.

For completeness, we also revisit the no global symmetries proof given in [36,37], showing how to also cover the case of noninvertible symmetries which fuse to

[5]The general expectation is that in a theory of quantum gravity there are no global symmetries. See, e.g., Refs. [38–42].

[6]It is important to note that this proof does not directly construct a candidate dual for a topological symmetry operator of the boundary theory. Additionally, there are some technical complications in extending this discussion to the case of noninvertible symmetries, where the fusion rules for symmetry operators fail to obey a grouplike product rule.

[7]We expect our considerations to also apply for $D = 1$, but in this case there is not much to discuss other than (-1)-form symmetries, which involve parameters of the theory. In known AdS$_2$/CFT$_1$ pairs one often has to entertain an ensemble average over parameters right from the start (see, e.g., [46]) so the analysis will be somewhat different. We defer this case to future work.

condensation defects. We also comment on the construction of $(-m)$-form symmetries of a general QFT for $m \geq 2$, as well as their holographic dual description in the context of the AdS/CFT correspondence. We conclude by briefly discussing some potential extensions to more general spacetimes.

II. BULK DUAL OF A SYMMETRY OPERATOR

To begin, we consider a CFT on a D-dimensional spacetime and assume that it admits a topological symmetry operator $\mathcal{N}$ supported on a closed q-manifold Y. This is a codimension $D-q$ symmetry operator and so specifies a p-form symmetry with $p+q=D-1$. For $\mathcal{N}$ to be a symmetry operator, we require that it commute with the stress tensor $T_{\mu\nu}$ of the CFT. In general, $\mathcal{N}$ is specified by a nontrivial topological field theory (TFT) supported on Y so we schematically write this as[8]

$$\mathcal{N} \sim \int [da] \exp\left(2\pi i \int_Y \mathcal{L}_{\mathrm{TFT}}\right), \tag{2.1}$$

in the obvious notation. It is topological because local variations in the shape of Y do not alter the action of the topological defect on operators of the QFT. One can view this as being enforced by the requirement that the TFT on Y is independent of the spacetime metric for the CFT.

As is the case for any defect, the bulk fields of the CFT specify background fields/sources for the TFT action. We say that an operator $\mathcal{O}$ supported on a subspace of dimension $k \geq p$ of the CFT is charged under this symmetry operator when it can nontrivially intersect/link with the symmetry operator.[9] The action of the symmetry operator on $\mathcal{O}$ will be denoted as $\mathcal{O} \mapsto \mathcal{O}^{(\mathcal{N})}$. We first consider $p \geq 0$, returning to $p=-1$ later. Recall that a global (-1)-form symmetry is associated with picking parameters of the CFT.

We now make the further assumption that our CFT has a semiclassical gravity dual.[10] In what follows we shall not assume that the global symmetry of the boundary theory is gauged. Rather, we shall ask whether having a global symmetry in the bulk is compatible with bulk reconstruction, reaching a contradiction.

We would like to characterize the bulk object corresponding to $\mathcal{N}$, as well as its possible linking with $\mathcal{O}$. Our general expectation is that to characterize the bulk dual, we ought to convolve a boundary CFT operator with a suitable smearing kernel and its improvement/reinterpretation in terms of a quantum error correcting code [47,48] (see, e.g., [49] for a review); objects close to the conformal boundary will be sharply localized but as we move deeper into the interior of AdS, these bulk dual objects will become more spread out. For example, in the case of $\mathcal{O}=\mathcal{O}(x)$ a local operator, we have a smearing kernel $\mathcal{K}(x',x;z)$ so that the resulting bulk operator is of the form (see, e.g., [47,50–53])

$$\tilde{\mathcal{O}}(x,z) \sim \int dx' \mathcal{O}(x')\mathcal{K}(x',x;z), \tag{2.2}$$

where z denotes the local radial coordinate in the Poincaré patch, i.e., we have the empty AdS metric:

$$ds^2 = \ell_{\mathrm{AdS}}^2 \frac{ds_{\mathrm{CFT}}^2 + dz^2}{z^2}, \tag{2.3}$$

with ds_{CFT}^2 the metric of the boundary CFT. Similar considerations apply for extended operators of the CFT, i.e., they also smear out in the bulk. We write this as a convolution:

$$\tilde{\mathcal{O}} \sim \mathcal{O} * \mathcal{K}. \tag{2.4}$$

Let us now turn to the main focus of this note: the bulk dual of a topological symmetry operator $\mathcal{N}$ of the CFT_D. In keeping with our discussion of smearing for CFT operators, we shall refer to the putative bulk dual as $\tilde{\mathcal{N}}$. There are two issues one can immediately raise. First of all, how do we know that $\tilde{\mathcal{N}}$ even exists, and moreover, should we expect it to be topological in the bulk?[11]

First of all, there are good reasons to expect that some object such as $\tilde{\mathcal{N}}$ does exist in the bulk. For one, note that for a general QFT_D in D spacetime dimensions we can speak of the associated SymTFT_{D+1}. In this picture, the SymTFT_{D+1} is defined on a slab with one boundary carrying gapped boundary conditions and the other supporting physical boundary conditions, i.e., a relative QFT. Defects of the boundary relative theory are extended by one dimension, and symmetry operators simply "pull off" of the boundary, thus maintaining linking in both the boundary and the bulk. So, at least in the SymTFT there is a natural bulk object which we shall refer to as $\mathcal{N}^{\mathrm{stft}}$. An additional comment here is that $\mathcal{N}$ is constructed from fields of the QFT_D so smearing of these fields should in principle produce an object in any putative bulk dual.

Consider next the issue of whether $\tilde{\mathcal{N}}$ is topological. To see why this issue is so subtle, suppose we consider integrating a candidate smearing kernel against some $\mathcal{N}$ supported on a subspace Y. Provided we remain close

[8] We keep implicit the specific symmetry generator, as implemented by the TFT.

[9] Note, for example, that a line operator can thus be charged under both a 1-form symmetry as well as a 0-form symmetry.

[10] A loose form of holography would assert that any QFT should have some gravity dual. In practice this statement has little content/practical value.

[11] We thank J. McNamara and H. Ooguri for helpful questions and comments on this point.

enough to the boundary so that any nontrivial topological features in the bulk are avoided, the only possible dual is some object supported on a manifold $Y(z)$ homotopic to Y. On the other hand, precisely because $\mathcal{N}$ is topological, convolution with a smearing kernel ought to have no effect on $\tilde{\mathcal{N}}$ at all.

With this motivation in mind, our aim will be to establish not only that $\tilde{\mathcal{N}}$ exists but also that it is not topological. There are two conflicting intuitions which we will need to reconcile: on the one hand "smearing" with a topological operator would seem to produce no effect at all. On the other hand, linking between $\mathcal{N}$ and $\mathcal{O}$ will eventually get smeared out, making it difficult to separately reconstruct $\mathcal{N}$ and $\mathcal{O}$.

Once we establish that $\tilde{\mathcal{N}}$ exists, we also know that in the $z \to 0$ limit it must reduce to $\mathcal{N}$. Since $\mathcal{N}$ is topological, we can deduce that near the conformal boundary of AdS, the world volume theory on a radial slice $\tilde{\mathcal{N}}(z)$ will have to be of the form[12]

$$\tilde{\mathcal{N}}(z) \sim \mathcal{Z} \times \int [da] \exp\left(2\pi i \int_{Y(z)} \mathcal{L}_{\mathrm{TFT}} + \text{Nontopological}\right), \tag{2.5}$$

where $Y(z)$ is homotopic to $Y(0) = Y$. This is simply because any conventional smearing kernel cannot alter the TFT world volume terms. On the other hand, it could happen that once we move into the bulk there are additional nontopological terms which depend on local fluctuations of the bulk metric.[13] We will shortly argue that the world volume action used to specify $\tilde{\mathcal{N}}(z)$ is nontopological. As such, a global symmetry in the CFT cannot remain a global symmetry in the bulk. In addition, we have included the possibility of a further dressing term $\mathcal{Z}$ as associated with a theory supported on a chain which stretches from $Y(z)$ to $Y(0)$. This only occurs in situations where the naive SymTFT extrapolation of $\mathcal{L}_{\mathrm{TFT}}$ to the bulk is, on its own, ill defined.

For each radial slice of the local AdS geometry, we expect that $\tilde{\mathcal{O}}(z)$ still links with $\tilde{\mathcal{N}}(z)$. From the perspective of the dual CFT, this is expected: if we introduce a small UV cutoff length scale, then we induce an RG flow and so we can track, as a function of RG time, the action of topological symmetry operators on CFT operators.[14] The RG flow as we proceed into the infrared could involve nontrivial operator mixing/transport into the bulk, so there is an "RG line" which extends out from $\tilde{\mathcal{O}}(z)$ back to the boundary at $z = 0$. Likewise, we can consider the full evolution of $\tilde{\mathcal{N}}(z)$ back to the boundary at $z = 0$. Observe that the linking dimension for $\mathcal{O}$ and $\mathcal{N}$ in the CFT is different from that of their smeared counterparts $\tilde{\mathcal{O}}$ and $\tilde{\mathcal{N}}$ in the bulk (which is higher dimensional). This does not directly imply a contradiction since along each radial slice we still observe a linking.

In the next two subsections we argue that $\tilde{\mathcal{N}}(z)$ exists, and moreover it cannot be purely topological. To illustrate the main idea we shall go through the argument twice, once in Euclidean signature, where all operator linking/RG statements are on the same footing, and again in Lorentzian signature, where the physical interpretation of smearing and bulk reconstruction is more apparent.

This will be enough to also establish the absence of bulk global symmetries. Indeed, given a putative topological symmetry operator $\mathcal{N}^{\mathrm{AdS}}$ for a global symmetry of the bulk, pushing it to the boundary would produce a topological symmetry operator $\mathcal{N}$ for a global symmetry of the boundary CFT. Pulling it back into the bulk via bulk reconstruction would then yield a nontopological $\tilde{\mathcal{N}} = \mathcal{N}^{\mathrm{AdS}}$. This is a contradiction, since on the one hand $\mathcal{N}^{\mathrm{AdS}}$ as a global symmetry operator should not depend on local metric fluctuations, but on the other hand its counterpart $\tilde{\mathcal{N}}$ does depend on such fluctuations. The contradiction implies the absence of bulk global symmetries.

Finally, let us comment that while the style of our argument is geared towards a "proof by contradiction" namely we assume at the outset that the bulk has a global symmetry and then derive a contradiction, the argument carries through in the same fashion even if we assume that symmetry in the bulk is gauged. This is because we can compute (via the use of a connection/Wilson line) the difference of $\tilde{\mathcal{O}}$ and its smeared counterpart. There is still a contribution to the stress energy tensor in this case, and this again establishes that $\tilde{\mathcal{N}}$ is not topological.

A. Euclidean signature analysis

To begin, suppose our CFT is formulated on a Euclidean signature manifold of dimension D. We can view this as preparing a specific state of the Lorentzian signature theory. In this case, the empty AdS geometry is topologically a $(D+1)$ ball; the radial direction of the ball can be viewed as the RG time of the Euclidean CFT (after introducing a short distance cutoff).

We consider $\mathcal{N}$ a topological operator with support on Y. Partitioning up the Euclidean spacetime into a collection of D-dimensional "pixels" P_i, we can track the effects of RG flow (i.e., smearing in the bulk) by increasing the size of the P_i as we move to the infrared. Note that in the linking between $\mathcal{O}$ and $\mathcal{N}$, a far away observer will only see the action of $\mathcal{N}$ on $\mathcal{O}$, i.e., the combination $\mathcal{O}^{(\mathcal{N})}$. In other

[12]The TFT here is specified by the boundary theory. There can often be difficulties in constructing this TFT purely in terms of bulk objects; we discuss this issue when we turn to examples with continuous non-Abelian symmetries.

[13]One can of course entertain topological couplings such as the Pontryagin density, but these do not produce a source of stress energy.

[14]Recall that the practical implementation of holographic RG requires us to move the CFT a "small amount" into the interior to initiate a flow. See, e.g., [54,55] as well as the review [56].

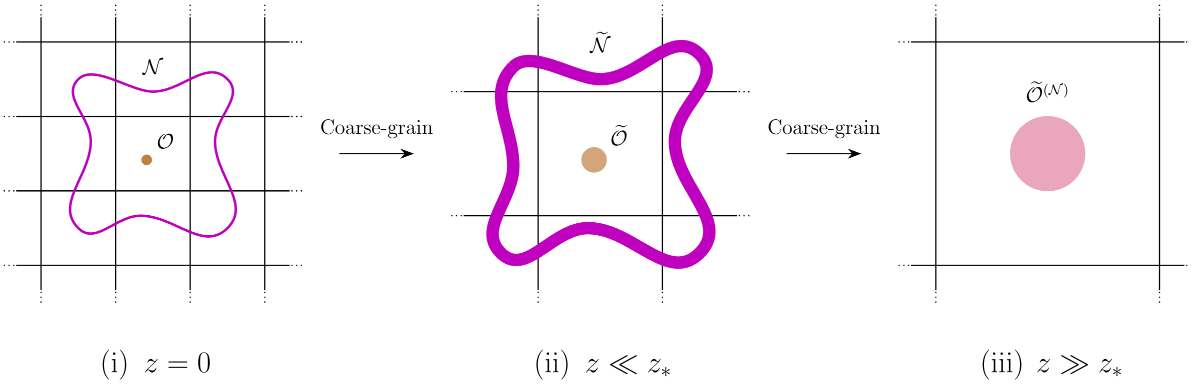

FIG. 2. (i) At zero RG time $z = 0$ the operators $\mathcal{N}$ and $\mathcal{O}$ are not smeared. (ii) At small RG times $z \ll z_*$, some smearing has occurred but an observer can still probe the symmetry operator $\tilde{\mathcal{N}}$ and the operator $\tilde{\mathcal{O}}$ individually. We denote this as $\tilde{\mathcal{O}} \oplus \tilde{\mathcal{N}}$. (iii) At large RG times $z \gg z_*$ an observer can no longer distinguish the individual objects, and we instead have the single merged object $\tilde{\mathcal{O}}^{(\mathcal{N})}$.

words, a far away observer cannot microscopically probe $\mathcal{O}$ and $\mathcal{N}$ separately. This crossover behavior happens at some scale $z = z_*$.[15] Said differently, for $z \ll z_*$, the bulk configuration consists of $\tilde{\mathcal{O}}$, and—provided it exists—will be accompanied by $\tilde{\mathcal{N}}$, so we denote this by $\tilde{\mathcal{O}} \oplus \tilde{\mathcal{N}}$. For $z \gg z_*$, we can only detect $\tilde{\mathcal{O}}^{(\mathcal{N})}$ due to smearing effects. See Fig. 2 for a depiction.

Formally speaking, we can also estimate where this crossover takes place. To see how, consider in the CFT a spatial D-dimensional ball B which includes the $\mathcal{O} \oplus \mathcal{N}$ configuration. We can build a minimal area "surface" in the bulk Euclidean AdS which is homologous to B and which has the same boundary ∂B. This region extends out a finite amount into the bulk, and this is the demarcation region $z = z_*$. So, there is a maximum depth z_* to which it can penetrate. This is the crossover scale from the individual objects $\tilde{\mathcal{O}} \oplus \tilde{\mathcal{N}}$ to $\tilde{\mathcal{O}}^{(\mathcal{N})}$. By general scaling arguments, we also know that $z_* \sim r(B)$, the radius of B.

Consider now a field configuration with profile which transitions from $\tilde{\mathcal{O}}$ to $\tilde{\mathcal{O}}^{(\mathcal{N})}$ and compare this with a field configuration which is simply $\tilde{\mathcal{O}}$ throughout (no $\mathcal{N}$ insertion in the CFT dual):

$$(\tilde{\mathcal{O}} \to \tilde{\mathcal{O}}^{(\mathcal{N})}) \quad \text{vs} \quad (\tilde{\mathcal{O}} \to \tilde{\mathcal{O}}). \tag{2.6}$$

The two configurations have different bulk stress energy simply because there is a nonzero jump in the z profile near $z = z_*$; derivatives D_z along the RG line are different.[16] Here, D_z refers to a formal covariant derivative with any possible background connections switched on.

[15]The precise value of z_* is scheme dependent. Indeed, a change of RG scheme in the boundary CFT will be reflected as a choice of diffeomorphism in the bulk. Even so, there is clearly a characteristic length scale associated with this "threshold correction" to the RG evolution.

[16]Note that even a jump by a phase rotation is enough to signal a discontinuity in the z direction.

Let us elaborate on this point. Since we expect in an actual AdS/CFT pair that the bulk symmetry will be gauged anyway, one might ask whether in such a situation the resultant $\tilde{\mathcal{O}}^{(\mathcal{N})}$ is actually "gauge equivalent" to $\tilde{\mathcal{O}}$.[17] To properly calculate differences in the values of the $\tilde{\mathcal{O}}$ operator, we introduce a formal Wilson defect operator $\tilde{\mathcal{W}}(z', z)$ which serves to properly compute differences between charged operators located at different radial slices:

$$\tilde{\mathcal{O}}(z')\tilde{\mathcal{W}}(z', z) - \tilde{\mathcal{O}}(z). \tag{2.7}$$

This can be done for any candidate categorical symmetry which has a corresponding symmetry TFT/symmetry theory. Now, the defect $\tilde{\mathcal{W}}$ has support on a subspace which has one higher dimension than $\tilde{\mathcal{O}}$.[18] Returning to line (2.6), we can now properly compare the differences:

$$\tilde{\mathcal{O}}^{(\mathcal{N})}(z')\tilde{\mathcal{W}}^{(\mathcal{N})}(z', z) - \tilde{\mathcal{O}}(z) \quad \text{vs} \quad \tilde{\mathcal{O}}(z')\tilde{\mathcal{W}}(z', z) - \tilde{\mathcal{O}}(z). \tag{2.8}$$

For $z' = z + \delta z$, these finite differences can be viewed as approximating a covariant derivative $\delta z D_z \tilde{\mathcal{O}}(z)$ associated with the corresponding symmetry. In particular, these differences are what will directly enter into any stress energy tensor.[19] Our smearing argument has established that $\tilde{\mathcal{O}}^{(\mathcal{N})}\tilde{\mathcal{W}}^{(\mathcal{N})} \neq \tilde{\mathcal{O}}\tilde{\mathcal{W}}$. This establishes the expected jump in the stress energy.

[17]We thank the anonymous referee for a helpful question of clarification.

[18]In particular, in the symmetry TFT/symmetry theory this defect would topologically link with $\tilde{\mathcal{N}}$.

[19]Consider for example the case of a complex scalar charged under a $U(1)$ symmetry. Then, the stress energy tensor has contributions from covariant derivatives of the form $T_{ij} \supset D_i\phi D_j\phi^\dagger$.

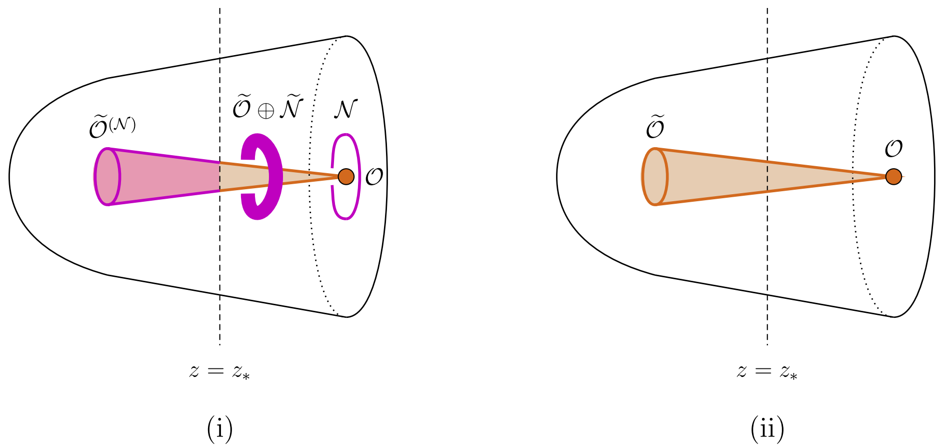

FIG. 3. (i) Depiction of a field configuration, as specified by $\tilde{\mathcal{N}}$, in which $\tilde{\mathcal{O}}$ transitions to $\tilde{\mathcal{O}}^{(\mathcal{N})}$. We indicate the symmetry operator $\mathcal{N}$ and charged operator $\mathcal{O}$ of the CFT. When $z \ll z_*$, they smear to $\tilde{\mathcal{O}} \oplus \tilde{\mathcal{N}}$. When $z \gg z_*$, they smear to the symmetry transformed operator $\tilde{\mathcal{O}}^{(\mathcal{N})}$. Near $z = z_*$ we have a discontinuity in the z direction as we jump from $\tilde{\mathcal{O}}$ to $\tilde{\mathcal{O}}^{(\mathcal{N})}$. (ii) Depiction of the same setup as in (i) with constant field profile.

Said differently, this jump in the profile of the bulk $\tilde{\mathcal{O}}$ before and after $z = z_*$ allows us to establish two things in one shot. First of all, there must be *something else* present besides just $\tilde{\mathcal{O}}$ (for $z \ll z_*$) and $\tilde{\mathcal{O}}^{(\mathcal{N})}$ (for $z \gg z_*$). This establishes the existence of $\tilde{\mathcal{N}}(z)$.

Second, if $\tilde{\mathcal{N}}$ contributes no stress energy, then the two configurations ($\tilde{\mathcal{O}} \to \tilde{\mathcal{O}}^{(\mathcal{N})}$) and ($\tilde{\mathcal{O}} \to \tilde{\mathcal{O}}$) would have had the same stress energy, a contradiction. See Fig. 3 for a depiction of these two situations.

The resolution of the contradiction is that the insertion of $\tilde{\mathcal{N}}$ itself contributes some stress energy to the configuration. Said differently, returning to line (2.5), we conclude that there is a nontrivial coupling of the bulk metric and its local fluctuations to the theory specified by $\tilde{\mathcal{N}}(z)$.

Summarizing, we have argued that coarse graining in the Euclidean CFT leads to a maximal depth of penetration in the Euclidean AdS, as captured by an associated D-dimensional ball of the Euclidean CFT which "dips" into the bulk AdS. One might view this as a Euclidean signature generalization of an RT surface, but it is clear that this differs from the standard notion which would have made reference to a $(D-1)$-dimensional ball. Here, we simply view this as a formal device to figure out the crossover region $z = z_*$. In any event, the existence of a crossover scale is clear, and this also establishes that $\tilde{\mathcal{N}}$ exists and is not topological in the bulk AdS space.

B. Lorentzian signature analysis

We now repeat our coarse graining analysis but in Lorentzian signature. Here, there is an important distinction in the dimension of support for our p-form symmetry operator. Working at a fixed time slice, observe that for $p = 0$, the corresponding symmetry operator would fill all of space, while for $p \geq 1$ we can put the topological operator on a smaller spatial region. This is unpleasant, and one workaround is to assume that we can decompose the domain of the TFT into smaller regions, and analyze the entanglement wedge of each of these objects separately. This is the approach of Refs. [36,37], and we return to this treatment in Sec. IV.

Here, we shall instead take a different tack: to keep our treatment of all p-form symmetries on an equal footing, we consider operators $\mathcal{O}$ supported on a spatial subspace, and so our $\tilde{\mathcal{N}}$ will necessarily have some finite extent in both space and time. For example, in the case of a 2D Lorentzian signature CFT we can speak of a local operator $\mathcal{O}(x)$ which links with $\mathcal{N}$, supported on a closed one-dimensional curve which has both spacelike and timelike pieces. See Fig. 4 for a depiction.

In this case, we can draw a large causal diamond R around the $\mathcal{O} \oplus \mathcal{N}$ configuration. On short distance scales, we can again resolve the two individual constituents, but at long distance scales we instead only detect $\mathcal{O}^{(\mathcal{N})}$. This is apparent in the holographic dual by constructing the Hubeny-Rangamani-Takayanagi (HRT) surface $EW(R)$ associated to our region R. Inside $EW(R)$, we can resolve $\tilde{\mathcal{O}} \oplus \tilde{\mathcal{N}}$, but outside, the two constituents have merged to $\tilde{\mathcal{O}}^{(\mathcal{N})}$. Now everything proceeds as before; we get a maximum depth of resolution set by a characteristic scale z_*, and again, the "jump" in comparing the stress energy of the ($\tilde{\mathcal{O}} \to \tilde{\mathcal{O}}$) line and the ($\tilde{\mathcal{O}} \to \tilde{\mathcal{O}}^{(\mathcal{N})}$) line tells us that $\tilde{\mathcal{N}}$ exists and cannot be purely topological in the bulk. See Fig. 4 for a depiction of the causal diamond in the boundary CFT, as well as the smearing in the bulk.

As a final comment, note that if we had restricted to $p \geq 1$ with a topological operator supported on a purely spatial region then we could have run precisely the same coarse graining argument used in our Euclidean signature analysis, but now for just spatial subregions of the CFT.

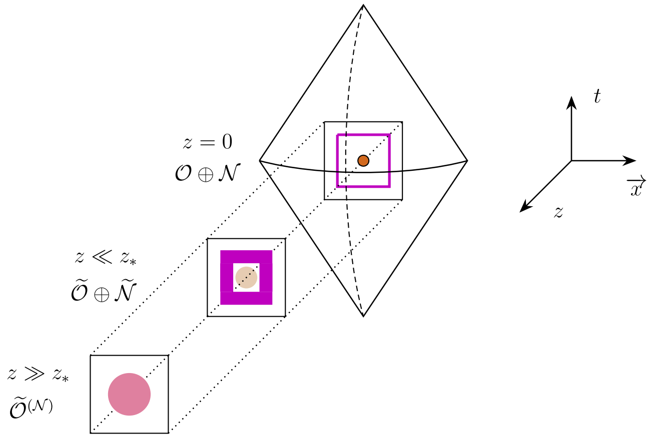

FIG. 4. Depiction of smearing for the $\mathcal{O} \oplus \mathcal{N}$ configuration of the boundary CFT. In Lorentzian signature the subspace filled by $\mathcal{N}$ sweeps out both a spatial and temporal directions. Surrounding the configuration with a causal diamond in the CFT, we can track the associated entanglement wedge bounded by the HRT surface. This leads to a maximal depth for disambiguating the two constituents; beyond this depth we instead have a single $\tilde{\mathcal{O}}^{(\mathcal{N})}$ in the bulk. This again leads to a contradiction unless the bulk $\tilde{\mathcal{N}}$ is nontopological.

C. (−1)-form symmetries

We now return to the case of (−1)-form symmetries of the boundary CFT. Our aim here will be to give a bottom up proposal for how to make sense of this case in a way compatible with holographic considerations.

Recall that (−1)-form symmetries are associated with specifying the parameters of the CFT, which we label as $\{\lambda\}$. The associated topological operator $\mathcal{N}$ fills the spacetime so the notion of "smearing" itself would appear to be somewhat ill defined. Nevertheless, we can construct a space Λ from the $\{\lambda\}$ and equip it with the Zamolodchikov metric [57] for the continuous parameters and the discrete topology for any discrete parameters. From this perspective, we get a family of CFTs fibered over Λ, and so we can speak of a spacetime filling topological operator $\mathcal{N}(\lambda)$ which sits at a particular point $\lambda \in \Lambda$ and fills the directions of the CFT. Inserting $\mathcal{N}(\lambda)$ moves us from the point λ to the point $\lambda^{(\mathcal{N}(\lambda))}$.

What is the holographic dual $\tilde{\mathcal{N}}(\lambda, z)$? To begin, recall that via the standard holographic dictionary, the parameter λ is to be viewed as the asymptotic profile of the non-normalizable component of a bulk field ϕ with $\phi|_{\partial \mathrm{Ads}} \sim \lambda$ [9,10]. Suppose that we now pull $\mathcal{N}(\lambda)$ off the boundary CFT located at $z = 0$. We now have a bulk insertion of a codimension one object (i.e., a "wall"). Crossing from one side to the other modifies the value of ϕ; i.e., it induces a jump in the bulk modulus field.

As far as establishing that $\tilde{\mathcal{N}}(\lambda_1 \to \lambda_2)$ is not topological in the bulk, the argument proceeds much as in the previous cases; i.e., we proceed via proof by contradiction. We observe that the bulk modulus field "charged" under $\tilde{\mathcal{N}}(\lambda_1 \to \lambda_2)$ jumps, and so the presence of a kink profile would induce a nonzero stress energy. There is no contradiction provided $\tilde{\mathcal{N}}(\lambda_1 \to \lambda_2)$ supports a nontopological world volume action in the bulk.

It is also of interest to consider possible wormhole configurations which connect different values of λ, as in Refs. [58,59]. For two values of parameters λ_1 and λ_2 in the CFT, suppose we have a (−1)-form symmetry operator $\mathcal{N}(\lambda_1 \to \lambda_2)$ which connects the two. In the gravity dual, we have an interpolating bulk geometry with a $\tilde{\mathcal{N}}(\lambda_1 \to \lambda_2)$ wall inserted. Observe that this insertion is orientable, and so $\tilde{\mathcal{N}}(\lambda_1 \to \lambda_2)^\dagger$ implements the opposite transformation.[20]

See Fig. 5 for a depiction of these bulk configurations with a wall.

D. SymTFT considerations

Summarizing, we have shown that in the context of the AdS/CFT correspondence, any putative topological global symmetry operator of the boundary CFT becomes nontopological in the bulk. Conversely, we have also shown that, in the bulk, there are no candidate topological symmetry operators for a p-form symmetry for $p \geq 0$, thus excluding possible global symmetries in the gravity dual.

This is in accord with what we expect to happen in any D-dimensional QFT with global categorical symmetries. From this broader perspective, one can consider a $(D+1)$-dimensional topological field theory SymTFT_{D+1} which captures these global symmetries. Heavy defects of the QFT_D are extended by one dimension in the bulk, and the topological operators remain of the same dimension so that linking is maintained. This naturally suggests that the SymTFT is simply a topological subsector of the AdS/CFT correspondence, and this is indeed how it arises in all known stringy realizations of SymTFTs [31,34,60,61]. This extension by one further dimension indicates that the p-form symmetries extends to a bulk gauge field A_{p+1} which produces an extended defect operator, with the boundary operator attached to the end (see Fig. 6).

The SymTFT_{D+1} should be viewed as the low energy limit of a gravitational system. In terms of embedding it in the AdS/CFT correspondence, one can view the physical boundary condition as being smeared out over much of the $D+1$ dimensions, with a small purely topological sliver, as in Fig. 6. The minimal way to incorporate gravity is to give each of the fields of the TFT some nonzero kinetic term. Note that this makes sense even for torsional fields since we can instead work in terms of continuous valued forms,

[20] If $\mathcal{N}(\lambda_1 \to \lambda_2)$ is noninvertible, then this can lead to some hysteresis.

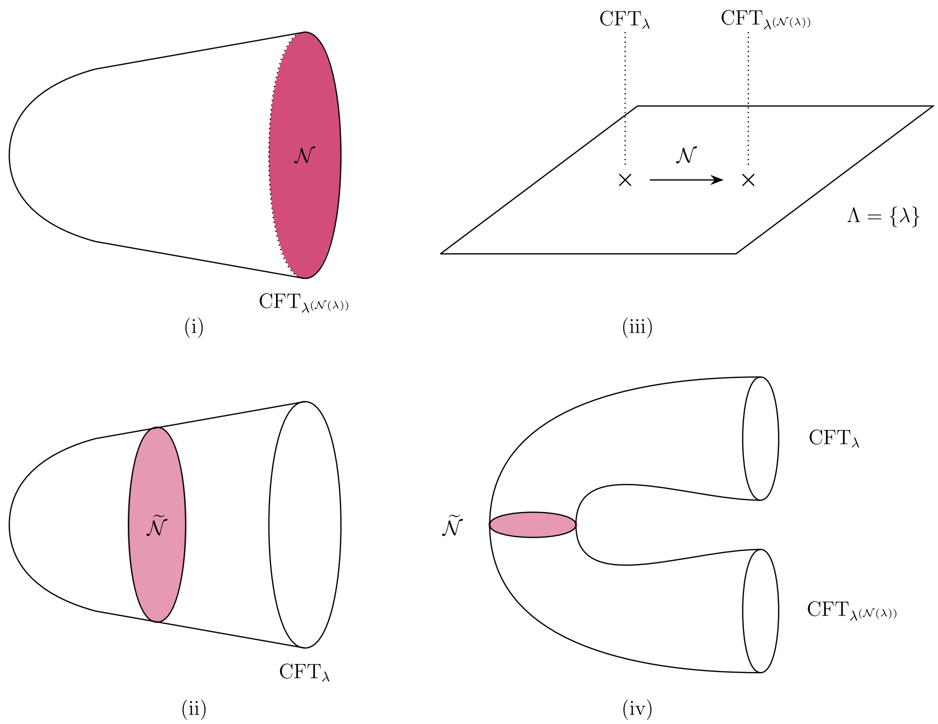

FIG. 5. (i) Sketch of a CFT with parameter $\lambda^{(\mathcal{N}(\lambda))}$ as a CFT with parameter λ stacked with the spacetime filling operator $\mathcal{N}$. (ii) Deformation of $\mathcal{N}$ into the bulk giving $\tilde{\mathcal{N}}$. (iii) Stackings of $\mathcal{N}$ are interpreted as transformations in the parameter space Λ. (iv) Wormholes between asymptotically AdS spaces with asymptotic CFTs located at different points in Λ contain a codimension one wall set by $\tilde{\mathcal{N}}$.

reaching the formulation in terms of torsional fields by performing a suitable rescaling.[21]

E. Comments on topological vs nontopological

The crux of the argument presented above is that the world volume action used to define $\tilde{\mathcal{N}}$ must have nontrivial metric dependence because otherwise we would observe a jump in the stress energy between $\tilde{\mathcal{O}}$ and $\tilde{\mathcal{O}}^{(\mathcal{N})}$. On the other hand, one might ask what role gravity played in these considerations. Said differently, can there be such jumps induced in a QFT decoupled from gravity and if so, is the line of reasoning just presented truly valid?

To illustrate, consider a 0-form symmetry operator $\mathcal{N}$ of a CFT_D which admits nontrivial line operators which are charged under this symmetry. This can happen, especially in theories with a 2-group symmetry where a line can be charged under both a 1-form and 0-form symmetry. Now, suppose we take a line operator $\mathcal{L}$ which pierces through the codimension one wall defined by the 0-form symmetry. On one side we have $\mathcal{L}$, while on the other side we have $\mathcal{L}^{(\mathcal{N})}$. There is a jump in the operator in this case as well. This looks similar to the case just considered, so it is natural to ask whether we are right in arguing for the nontopological nature of $\tilde{\mathcal{N}}$.

The phenomenon of line-changing operators is rather commonplace, and can occur in both CFTs as well as TFTs. For some discussion of the relation between such effects and 2-groups, see for example [62,63]. In all of these cases, the jump in the line is encapsulated in terms of a defect operator $\mathcal{D}$ localized at the transition point. The defect $\mathcal{D}$ is typically also topological, and this can also be verified in terms of the stringy "branes at infinity" picture [11–13]. The point here is that in the QFT case, we clearly see some additional degree of freedom has been explicitly inserted near the jumping point.[22]

[21] As an example, consider the 4D BF theory with topological action $\frac{k}{2\pi}\int B \wedge F$. In this formulation B and F are $U(1)$ valued. We can rescale each field by $2\pi/k$ to instead work in terms of torsional valued fields with action $\frac{2\pi}{k}\int b \wedge f$, in the obvious notation.

[22] For example, consider a stack of D3-branes filling $\mathbb{R}^{3,1}$ and sitting at a point of a transverse $\mathbb{C}^3$. We get duality defects by wrapping constant axio-dilaton 7-branes on the boundary S^5 "at infinity," as in [14]. We get a line defect via an F1/D1 string (the choice depends on the polarization) which stretches from the origin out to the boundary S^5. Consider a line operator which crosses from one side of the duality wall to the other. At the wall crossing, there is a light degree of freedom stretching from the string to the 7-brane, but this is localized at the boundary S^5, far from the D3-brane stack. This is the stringy implementation of $\mathcal{D}$.

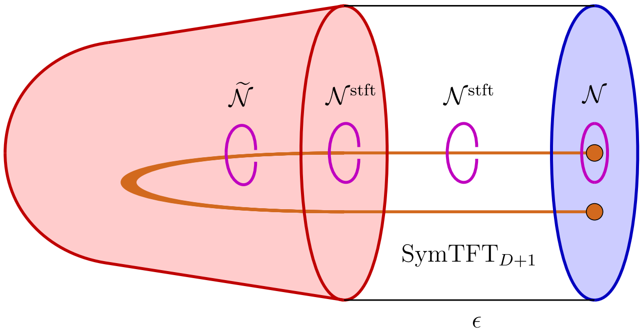

FIG. 6. Sketch of symmetry operators (purple) in the holographic setup linking with defect operators (brown). The topological symmetry operator $\mathcal{N}$ is localized at the boundary CFT. This operator can be pushed into the bulk of the SymTFT_{D+1} which is now viewed as a subsector of the bulk gravitational dual.

The most important difference in comparing our AdS analysis with this QFT example is that in the latter we clearly see an extra defect has been included at the transition point; in the AdS case, however, there is a transition from a specific field profile from $\tilde{\mathcal{O}}$ to $\tilde{\mathcal{O}}^{(\mathcal{N})}$ with seemingly no such accompanying defect. The absence of a new degree of freedom (topological or otherwise) produces a more dramatic discontinuity, and as we argued previously, this can be resolved by assuming that $\tilde{\mathcal{N}}(z)$ for $z \neq 0$ also contributes to the bulk stress energy. Indeed, along each z slicing we have no such collision between symmetry defects and CFT operators in the first place; the $\tilde{\mathcal{N}}$ has simply dissolved into $\tilde{\mathcal{O}}$ for $z > z_*$. This is qualitatively different from the way line-changing operators arise in the QFT with gravity decoupled.

F. Properties of $\tilde{\mathcal{N}}$

With these observations in place, we now argue that we can indeed interpret the world volume action for $\tilde{\mathcal{N}}$ as that of a dynamical brane. To frame the discussion to follow, it is helpful to note that in stringy constructions of generalized symmetry operators, there is a natural expectation that many examples will result from suitably wrapped branes in backgrounds of the form $AdS \times X$. In this context, the world volume theory in the AdS directions is given by a suitable dimensional reduction of the Dirac-Born-Infeld (DBI) and Wess–Zumino (WZ) terms supported on the D-brane. Pushing the brane to the conformal boundary of the AdS freezes out the metric dependent terms, resulting in a purely topological action in the CFT [11] (see also [12,13]). Of course, one can also contemplate more general solitonic objects, but this provides a natural first class of examples.

In all of these examples, the brane in question will source some amount of stress energy. For a gravitational brane wrapped on a q-dimensional subspace Y, we can parametrize our ignorance of the brane dynamics by introducing an unknown world volume tensor $M_{ij}(g_{AB}, \ldots)$ which depends on the pullback of the bulk metric g_{AB}, and possibly other bulk and world volume degrees of freedom on the brane:

$$S_{\text{brane}} \sim \tau_q \int_Y d^q\xi \sqrt{\det M} + \text{topological terms}, \qquad (2.9)$$

with τ_q an overall dimensionful constant which we refer to as the tension. For example, in the special case of a D-brane, we would write $M_{ij} = h_{ij} + b_{ij} + 2\pi\alpha' F_{ij} + \ldots$, with h_{ij} the pullback of the bulk metric to the brane, b_{ij} the pullback of the NS 2-form, and F_{ij} the field strength tensor of a $U(1)$ gauge field. In this case, the "topological terms" are just the WZ terms.

What can we say about the value of τ_q in our case? Without providing further details on the precise AdS/CFT pair, we should likely only expect to obtain crude estimates. That being said, general bottom up considerations provide some insight.

On dimensional analysis grounds, we know that τ_q ought to scale as

$$\tau_q \sim \frac{1}{\ell_*^q} \qquad (2.10)$$

for some characteristic length ℓ_*. We view this as specifying the Compton wavelength for our object $\tilde{\mathcal{N}}$. Now, if $\tilde{\mathcal{N}}$ were of size the AdS radius there would be no sense in which we could localize it in a small region near the boundary of the CFT; it would immediately be spread over the entirety of our spacetime. Since the coarse graining takes some RG time to proceed, we conclude that the radius of curvature $\ell_{\text{AdS}} \gg \ell_*$. On the other hand, we also know that this characteristic scale must be greater than the $(D+1)$-dimensional Planck length ℓ_{pl}; otherwise there would be no regime of validity to consider this object in the effective field theory in the first place. In principle, however, it could be separated from the Planck scale, and this possibility does occur in many situations.

Putting these observations together, we learn

$$\ell_{\text{AdS}} \gg \ell_* \gtrsim \ell_{\text{pl}}, \qquad (2.11)$$

which is not altogether surprising. We comment here that in the special case of a D-brane action we would of course interpret ℓ_* as the minimal resolving power of a D-brane [64].

III. EXAMPLES

In this section we briefly discuss some examples illustrating the general structure found above.[23] In most cases, we can directly deduce the topological sector of $\tilde{\mathcal{N}}$

[23]We thank M. Montero and H. Ooguri for several questions in this direction.

by appealing to its construction in the SymTFT_{D+1}. Many examples with holographic duals have also been constructed from a "top down" point of view, see, e.g., Refs. [11–19,65–68]. In most of these cases one considers a brane wrapped "at infinity" (prior to taking the near horizon limit) which nonetheless still links with the defect in question. In these cases the world volume theory for the $\tilde{\mathcal{N}}$ is simply that of the corresponding brane. This provides a systematic way to construct symmetry operators (and their categorical generalizations) for various discrete and continuous symmetries.

On the other hand, it is natural to ask whether we need the full machinery of string theory to identify more detailed properties of $\tilde{\mathcal{N}}$. Here we consider a few examples of this sort, focusing on the symmetry operators for some continuous 0-form symmetries. We first discuss in detail the case of a $U(1)$ symmetry and then turn to G a continuous Lie group with a single connected component.

Consider, then, a CFT_D with a $U(1)$ 0-form symmetry which is dual to a $U(1)$ gauge symmetry in the bulk. Since we have a conserved current in the boundary theory, there is a current $(D-1)$-form j_{D-1} which we can integrate over a $(D-1)$-dimensional closed subspace Y_{D-1} to form a symmetry operator:

$$\mathcal{N}_\eta = \exp\left(2\pi i\eta \int_{Y_{D-1}} j_{D-1}\right). \tag{3.1}$$

This links with local operators which are charged under the $U(1)$ global symmetry. In the CFT_D we can introduce a background gauge field a_1 associated with this global symmetry.

Now, in the bulk AdS_{D+1} we have a $U(1)$ gauge field A_1 which approaches the background value a_1 at the boundary. Denote by $\mathcal{W}$ an electric line operator built from A_1. Let us consider the idealized situation where gravity is switched off (as in the SymTFT_{D+1}). Then, we can construct a 1-form symmetry operator which links with $\mathcal{W}$:

$$\mathcal{N}_\eta^{\mathrm{stft}}(z) = \exp\left(2\pi i\eta \int_{Y_{D-1}(z)} F^{\mathrm{dual}}\right), \tag{3.2}$$

where in the above we have assumed that the operator is close enough to the conformal boundary that we can take $Y_{D-1}(z)$ homotopic to Y_{D-1}.

What happens as we take $z \to 0$? Since it is topological, we can push $\mathcal{N}_\eta^{\mathrm{stft}}(z)$ all the way to the boundary. Doing so, we get

$$\mathcal{N}_\eta^{\mathrm{stft}}(z=0) = \exp\left(2\pi i\eta \int_{Y_{D-1}} F^{\mathrm{dual}}\right). \tag{3.3}$$

Next, introduce a D-chain C_D which terminates in AdS_{D+1} and has boundary $\partial C_D = Y_{D-1}$. Then, we can equivalently write

$$\mathcal{N}_\eta^{\mathrm{stft}}(z=0) = \exp\left(2\pi i\eta \int_{C_D} dF^{\mathrm{dual}}\right). \tag{3.4}$$

On the other hand, we also know, via the equations of motion, that

$$dF^{\mathrm{dual}} = J_D. \tag{3.5}$$

As such, we also have

$$\begin{aligned} \mathcal{N}_\eta^{\mathrm{stft}}(z=0) &= \exp\left(2\pi i\eta \int_{C_D} J_D\right) \\ &= \exp\left(2\pi i\eta \int_{Y_{D-1}} j_{D-1}\right), \end{aligned} \tag{3.6}$$

where in the last equality we used the fact that the topological linking in the CFT_D can equivalently be computed in terms of an intersection number in the bulk AdS_{D+1}.[24] Summarizing, we have found that in the bulk, the topological part of $\tilde{\mathcal{N}}(z)$ is obtained via line (3.2).

Once gravity is switched on there will be additional nontopological contributions to the world volume, as captured by $\tilde{\mathcal{N}}$ rather than $\mathcal{N}^{\mathrm{stft}}$, much as in Fig. 6. What can we say about these nontopological terms? As mentioned earlier, this is a model dependent issue, but we can still deduce the general form to be a DBI-like action in many cases of interest. Suppose, for example, that our $\mathrm{AdS}_{D+1}/\mathrm{CFT}_D$ pair arises from a geometry of the form $\mathrm{AdS}_{D+1} \times X$ with the $U(1)$ an isometry on X. Then, the corresponding symmetry operator is obtained from a solitonic configuration constructed from a diffeomorphism of X. As a soliton, we again expect there to be a DBI-like action. That said, the field content on the brane deserves further study.

Consider next the case of G a non-Abelian Lie group with a single connected component (i.e., all elements can be continuously connected to the identity). In this case the symmetry operators will be labeled by elements of G. These naturally act on objects in $\mathrm{Rep}(G)$, i.e., representations of G. The only change is that now, the symmetry current j_{cd} will transform in a representation of the Lie algebra $\mathfrak{g} = \mathrm{Lie}(G)$, so the generator of interest will also be labeled by a parameter $\mathfrak{t}^{cd} \in \mathfrak{g}$:

$$\mathcal{N}_{\mathfrak{t}} = \exp\left(2\pi i\mathfrak{t}^{cd} \int_{Y_{D-1}} j_{cd}\right), \tag{3.7}$$

in the obvious notation.

[24] In slightly more detail, we can view the electric line operator as an insertion in the action along the Poincaré dual (PD) of J_D, namely $\delta_{\mathrm{PD}(J_D)}$. The integral over J_D thus collapses to the intersection between the supports of C_D and $PD(J_D)$.

We can also attempt to mimic the steps taken in the $U(1)$ case to construct a bulk dual object. The main difference is that now, our equation of motion involves a covariant derivative:

$$d_A F^{\mathrm{dual}} = J_D, \tag{3.8}$$

and as such, we cannot simply use Stokes' theorem to replace the integral over the boundary current by a D-chain which terminates on Y_{D-1}. So, on general grounds, we expect the "naive" $\mathcal{N}_{\mathfrak{t}}$ to also be dressed by $\mathcal{Z}$, as in line (2.5). Said differently, the candidate topological operator in the bulk does not fully detach from the boundary.

This is in accord with the fact that if we do attempt to define an object purely in the bulk, then we should introduce a codimension 2 vortex labeled by the conjugacy class of $\exp(2\pi i \mathfrak{t})$, namely a Gukov-Witten operator [69].[25] One can obtain this by averaging the "naive defect" over gauge orbits, as in [70]. It is also worth noting (see [1]) that in pure gauge theory, the only topological Gukov-Witten operators are associated with elements in the center of the Lie group G. For further discussion, especially in the context of gravity, see Ref. [71].

Of course, on top of all of these complications we also expect there to be additional DBI-like contributions to the world volume action of $\tilde{\mathcal{N}}$. For example, suppose that G arises as a continuous non-Abelian isometry group of some X in a background of the form $\mathrm{AdS}_{D+1} \times X$. Then, the brane world volume action will again descend from a solitonic configuration associated with the diffeomorphisms of X.

IV. PROOFS WITH SPLITTABILITY REVISITED

The line of argument just presented is somewhat different from that in Refs. [36,37], which concentrate on 0-form symmetries as well as the dimensional reduction of p-form symmetries to 0-form symmetries. For ease of exposition we focus on the case of 0-form symmetries, since the other cases follow from similar steps to those presented in [36,37].

Now, one of the central ideas there is to start with a spatial region of the CFT R, and to consider a favorable splitting into smaller regions $R_1, \ldots, R_m$ such that the entanglement wedge for any individual region R_i is sufficiently close to the conformal boundary of AdS. Doing so, one can then consider an operator deep in the interior of AdS, and since no single region penetrates that far, a symmetry operator confined to the region R_i cannot properly act in the bulk. This provides another way to argue for the absence of global symmetries in AdS. Of course, a potential loophole in this argument is that one must somehow argue that a topological operator of the boundary theory "cares" about the metric dependent structure of an entanglement wedge in the first place. Our argument based on coarse graining considerations shows why the gravity dual of operators such as $\mathcal{N}$ are still sensitive to the entanglement wedge.

Another subtlety in this line of reasoning is that in the case of a noninvertible symmetry, we cannot simply take products of operators to get another symmetry generator. Rather, we often get sums of symmetry operators, i.e., there is a nontrivial fusion rule. In what follows we assume that the fusion rule only involves condensation defects, i.e., objects of lower-dimensional support (see, e.g., [72]). The main reason to make this technical assumption is that there is still a notion of a single "big object" which is present in the boundary theory and in the bulk.

Finally, we also face the issue of how to define the $\mathcal{N}[R_j]$ in the first place; when $\partial R_j \neq 0$, we generically expect the TFT on R_j to support edge modes. In principle these could either be gapless or gapped degrees of freedom depending on the choice of boundary conditions for the TFT for the symmetry operator.

The construction just provided in the previous section provides a resolution for these issues. First of all, given a TFT on a region R, we can introduce a "dressed" operator where we explicitly include the edge modes in question. We write this as[26]

$$\bar{\mathcal{N}}[R] = \mathcal{N}[R]\mathcal{E}[\partial R], \tag{4.1}$$

where $\mathcal{N}[R]$ refers to the path integral over the bulk TFT fields on R, and $\mathcal{E}[\partial R]$ refers to the path integral over any edge modes.[27]

Suppose now that we have a region R and we partition it up into a collection of disjoint regions $\{R_i\}_i$ which nevertheless share common boundaries. We would like to understand the relation between $\bar{\mathcal{N}}[R]$ and $\bar{\mathcal{N}}[R_1]\ldots\bar{\mathcal{N}}[R_m]$. Observe that in the product over the split factors we have a collection of edge modes. The bulk perspective indicates that we can fuse these edge modes together, and in so doing integrate them out. At this point it is helpful to recall the coupled wires construction of [74,75], which shows how to explicitly carry this out for a collection of certain 2D CFTs where neighboring left- and right-moving degrees of freedom are pairwise gapped out, leaving behind a 3D bulk Chern-Simons theory with chiral/antichiral modes on the very left and right of the system (see Fig. 7). We expect that something similar holds far more generally. This is indeed in accord with holographic considerations where we view the $\tilde{\mathcal{N}}$ operators as creating a brane in the bulk; edge modes for neighboring branes/antibranes condense, fusing the original configuration to a single large brane.

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[25] We thank H. Ooguri for a comment on this point.

[26] For another perspective on splittability versus concatenation of such topological operators, see Ref. [73].

[27] As an example, consider 3D Chern-Simons coupled to a 2D chiral CFT.

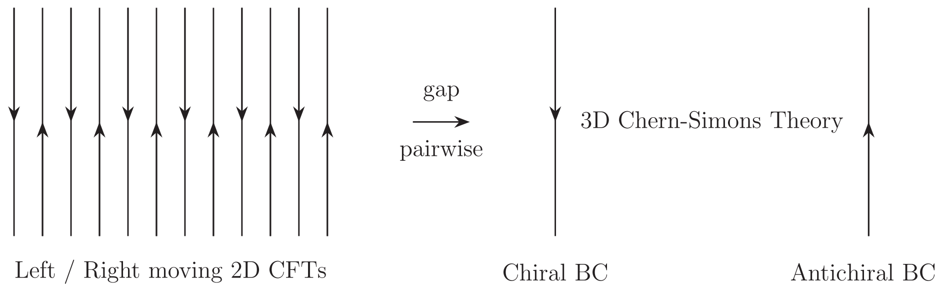

FIG. 7. Left: system of 2D coupled wires. Arrows indicate left/right movers. Right: pairwise gapping out left and right movers results in a 3D Chern-Simons theory confined between the remaining wires which impose chiral/antichiral boundary conditions (BC).

With this, we have a deformation of the original TFT/edge mode system to the single dressed operator:

$$\bar{\mathcal{N}}[R_1]\ldots\bar{\mathcal{N}}[R_m] \rightsquigarrow \bar{\mathcal{N}}[R]. \tag{4.2}$$

See Fig. 8 for a depiction of this merging procedure. One can then proceed with the same style of proof as in [36,37], now extended to the case of noninvertible symmetries.

V. FURTHER COMMENTS

In this section we provide some further, more speculative comments which naturally extend the considerations just presented. We begin by extending the notion of a higher-form symmetry to the case of lower-form symmetry, i.e., a $(-m)$-form global symmetry for $m \geq 2$, and also discuss its holographic interpretation. Next, we discuss how our considerations extend to more general spacetimes with holographic screens.

A. Lower-form symmetries

It is of course tempting to extend our discussion to cover $(-m)$-form symmetries. One reason for doing so is to ask whether the bulk AdS space can support bulk parameters, i.e., can we have a global (-1)-form symmetry in the bulk? Formally speaking this would be a "(-2)-form symmetry" in the boundary CFT. This is of direct interest in the context of a number of questions in quantum gravity, i.e., can quantum gravity support bulk parameters at all? See, e.g., [34,76–79] for some different perspectives. The construction of such objects in the dual CFT_D sounds puzzling since it seems to require a topological operator which fills more than the spacetime dimensions of the system.

We now propose to give a formal definition of a $(-m)$-form symmetry for any QFT, which we can of course apply to the special case of holographic CFTs. As a general comment, we have deferred discussion of this case because it is necessarily somewhat more speculative.

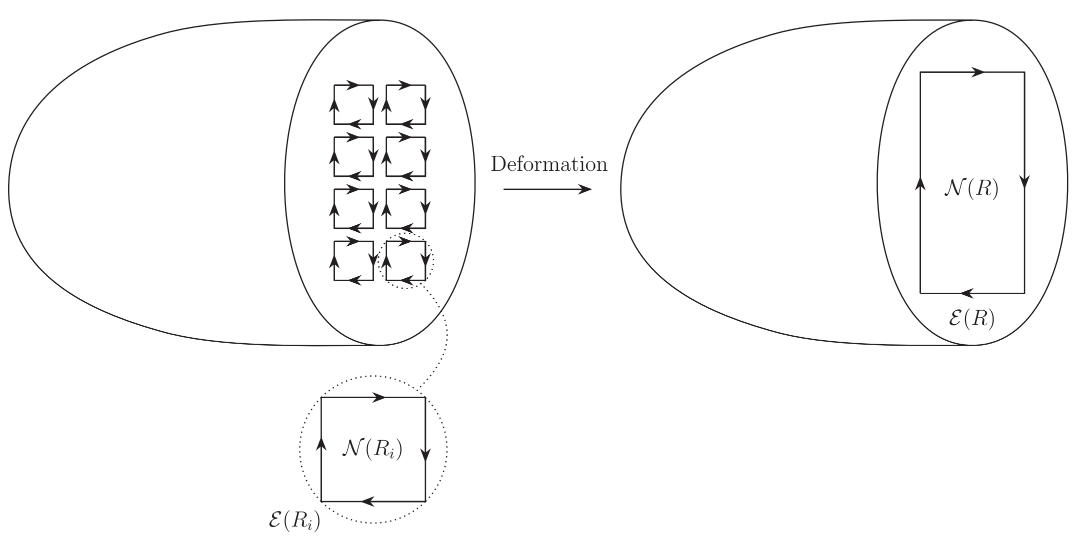

FIG. 8. On the left we depict in the CFT_D a collection of the $\mathcal{N}(R_i)$ supported on compact subspaces R_i with boundary ∂R_i which are dressed by nontopological edge modes $\mathcal{E}(\partial R_i)$. Whenever these constitute a tiling we can turn on deformations to gap out edge modes pairwise, resulting in a larger symmetry operator, with possibly some edge modes remaining.

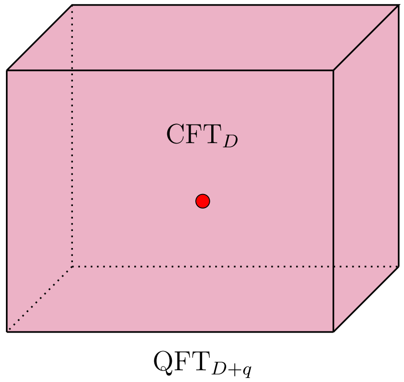

FIG. 9. We can construct $(-m)$-form symmetries for a CFT_D by viewing it as a defect (red dot) inside of an ambient QFT_{D+q} with $q = m - 1$. In the higher-dimensional setting, an ambient (-1)-form symmetry associated with a parameter of QFT_{D+q} formally defines a $(-m)$-form symmetry of the defect system. If the CFT_D has a gravity dual, then this yields a bulk $(-m+1)$-form symmetry.

We propose to view the D-dimensional QFT_D as a defect in a higher-dimensional QFT_{D+q} (see Fig. 9). Setting $q \equiv m - 1$, observe that a spacetime filling (-1)-form symmetry operator of QFT_{D+q} can be interpreted, in the QFT_D theory, as a $(-m)$-form symmetry.[28] It is also natural to consider the entwinement and nested categorical structure of these symmetries. A related comment is that this sort of symmetry inheritance arises quite naturally in many bulk/boundary systems, including some stringy constructions (see, e.g., the recent discussion in Ref. [80]). It would also be natural to ask whether lower-form symmetries can be defined intrinsically or must implicitly always make reference to a bulk QFT.

Before proceeding to the holographic interpretation, let us mention that at least in certain circumstances one might wish to entertain a different notion of lower-form symmetries based on taking a field profile which is sensitive to topological structures in the target space, i.e., topologically nontrivial field excursions in the QFT_D.[29] At least in explicit stringy constructions of QFTs there is certainly overlap with our proposal since we can consider "moving the brane around" in the ambient bulk QFT_{D+q}, but it would clearly be interesting to explore this generalization as well.

What then is the gravity dual of a $(-m)$-form symmetry operator? The most conservative answer is to simply consider a CFT in dimension $D + q = D + m - 1$ with a semiclassical gravity dual on AdS_{D+m}. Then, the

[28]Why not simply demand that we have a p-form symmetry operator of the bulk which fills all the world volume directions of the QFT_D? If we can move the topological operator off of the QFT_D it is unclear whether the QFT_D will continue to see the symmetry operator in the first place. Indeed, otherwise this would likely lead to contradictions with the standard treatment of (-1)-form symmetries.

[29]We thank J. McNamara and M. Montero for helpful comments on this point.

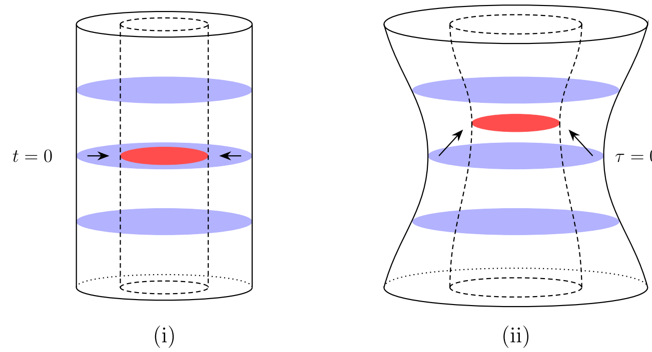

FIG. 10. Depiction of bulk reconstruction in AdS/CFT and more general holographic spacetimes. (i) AdS/CFT setup with leaves of constant time t (blue). RG/bulk flow evolves cylindrical shells radially inwards. For example, leaves shrink in size (red). (ii) More general holographic spacetimes. In this case, the boundary theory is supported on a holographic screen.

$(-m)$-form symmetry of the defect QFT directly lifts to a codimension one wall of the ambient gravity theory, much as in our treatment of (-1)-form symmetries.

On the other hand, if we start with a known $\mathrm{AdS}_{D+1}/\mathrm{CFT}_D$ pair and ask about the fate of a bulk (-1)-form symmetry, we seem to require that our boundary CFT can also be viewed as a defect in a QFT_{D+q} decoupled from gravity. In that setting, we need not (and should not) demand that the QFT_{D+q} has its own semiclassical AdS dual. Rather, we might have a spacetime of the form $\mathrm{AdS}_{D+1} \times X$ with the QFT_{D+q} filling the AdS_{D+1} factor and possibly some subspace of X. It is also tempting to speculate that here, the appropriate notion of "smearing" involves coarse graining in the directions of the X factor of $\mathrm{AdS}_{D+1} \times X$. We leave a full treatment of these intriguing possibilities for future work.

B. More general spacetimes

Similar considerations hold for a general spacetime that admits a bulk-boundary duality with a suitable notion of entanglement wedge reconstruction and coarse-graining. The surface-state correspondence provides this framework for any convex region M in a static spacetime wherein the dual boundary theory lives on ∂M [81,82], see Fig. 10 for a depiction.[30] Coarse-graining is given by the bulk flow procedure [90,91]. Note that there is also a notion of approximate locality in this boundary theory because the entanglement wedges become smaller closer to the boundary theory. We can now repeat our argument establishing that a reconstructed $\tilde{\mathcal{N}}$ sources some stress energy. So, establishing the existence of entanglement wedge reconstruction in more general spacetimes would also exclude global symmetries.

[30]See [83–85] for covariant generalizations wherein the boundary theory lives on the holographic screen [86]. See also [87–89] for other proposals for holography in general spacetimes.

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