Simplest model of a scalarized black hole in the Einstein-Klein-Gordon theory

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We investigate scalarized black holes in the Einstein minimally coupled scalar theory with a negative potential $V(\phi) = -\alpha^2\phi^6$. The tachyonic instability is absent from analyzing the linearized scalar equation, which could not allow for spontaneous scalarization. However, we obtain the black hole solutions with scalar hair by solving three full equations because this scalar potential violates the weak energy condition. This shows clearly that a single branch of scalarized black holes can be obtained without introducing a nonminimal scalar coupling term. We perform the stability analysis for scalarized black holes by adopting radial perturbations, implying that all scalarized black holes belonging to a single branch are unstable.

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I. INTRODUCTION

The no-hair theorem in general relativity (GR) prevents the existence of asymptotically flat black hole solutions with scalar hair except the mass M, angular momentum J, and charge Q of the black hole [1–3]. This is based on the Einstein minimally coupled scalar theory [4].

If one introduces a conformally coupled scalar (Einstein conformally coupled scalar theory), it gave us the Bocharova-Bronnikov-Melnikov-Bekenstein (BBMB) black holes with a conformal scalar hair, indicating an evasion of no-hair theorem [5,6]. In this case, however, the conformal scalar hair blows up at the horizon and these black holes are unstable under linear perturbations.

Recently, the spontaneous scalarization has implied that infinite branches of scalarized (charged) black holes were obtained numerically from the Einstein-Gauss-Bonnet-scalar theory [7–10] (Einstein-Maxwell-scalar theory [11]) through the nonminimal coupling function $f(\phi)$ with $f(0) = 0, f'(0) = 0$, and $f''(0) \neq 0$ to the Gauss-Bonnet term (Maxwell term). In these linearized theories ($\bar{\nabla}^2\delta\phi - m^2_{\rm eff}\delta\phi = 0$), the tachyonic instability of $s(l = 0)$-scalar mode propagating around the GR black holes indicates the onset of spontaneous scalarization. This arose from either the negative Gauss-Bonnet coupling term ($m^2_{\rm eff} = -48\lambda^2M^2/r^6$) or the negative Maxwell coupling term ($m^2_{\rm eff} = -2\alpha Q^2/r^4$), where λ^2 and α are positive scalar coupling parameters. These negative coupling terms actually induce potential wells near the horizon and as coupling parameters increase, leading to tachyonic instabilities. Furthermore, the linearized static scalar equation has played an important role in obtaining scalar clouds (bound states). Requiring an asymptotically vanishing scalar, the condition for obtaining a smooth scalar selects a discrete set of the bifurcation points for scalarized solutions: $M/\lambda = \{0.587, 0.226, 0.140, \ldots\}$ $[\alpha(q = 0.7) = \{8.019, 40.84, 99.89, \ldots\}]$ for the Gauss-Bonnet term [12] (as well as the Maxwell term [13]). They have admitted scalar hairy black holes in the full theory. Actually, it has induced infinite branches of scalarized [charged] black holes: $n = 0(0 < M/\lambda < 0.587), 1(0 < M/\lambda < 0.226), 2(0 < M/\lambda < 0.140), \cdots [n = 0(\alpha > 8.019), n = 1(\alpha > 40.84), n = 2(\alpha > 99.89), \cdots]$.

However, it turned out that the $n = 0$ branch of scalarized (charged) black holes is stable against radial perturbations, whereas all other branches of $n \neq 0$ are unstable [14]. This implies that the $n = 0$ branch of scalarized (charged) black holes could survive for further implications of scalarized black holes. In this direction, the dynamics of scalarized black holes and binary mergers were addressed in the Einstein-Gauss-Bonnet-scalar theory [15–21]. Furthermore, the photon spheres and observational appearance of scalarized black holes are investigated in the Einstein-Maxwell-scalar theory [22–29].

On the other hand, the nonlinear mechanism was introduced to obtain a single branch of scalarized black holes in Einstein-scalar-Gauss-Bonnet gravity, which is surely beyond the previous spontaneous scalarization. Introducing a different coupling function $f(\phi)$ satisfying $f(0) = 0, f'(0) = 0, f''(0) = 0$, one found from its linearized equation ($\bar{\nabla}^2\delta\phi = 0$) that the Schwarzschild black hole is linearly stable against scalar perturbation, whereas it is unstable against nonlinear scalar perturbation when the

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amplitude of a perturbed scalar is large enough [21,30–32]. This provides another mechanism to obtain a single branch of scalarized black holes via nonlinear scalarization [33,34].

In this work, we wish to introduce another model to find a single branch of scalarized black holes. Its linearized theory is stable against scalar perturbation, whereas its full theory indicates a violation of the weak energy condition (WEC) because the scalar potential includes a negative region. We are interested in a negative potential term $V(\phi) = -\alpha^2\phi^6 < 0$ violating the WEC in the Einstein minimally coupled scalar theory without introducing any coupling function $f(\phi)$. Hence, the tachyonic instability is absent. In this case, we may obtain a single branch of scalarized black holes in Einstein minimally coupled scalar theory. Carrying out stability analysis for scalarized black holes by adopting radial perturbations, we find that all scalarized black holes belonging to a single branch are unstable. This implies that, even though one obtains easily a scalarized black hole from the simplest scalar-tensor action, they are hard to survive for further implications.

The present work is motivated partly from the hairy black hole solutions in Einstein-Weyl massive conformally coupled scalar theory where the tachyonic scalar mass has played an important role in obtaining scalar hairy black holes [35]. In this case, in the absence of a tachyonic mass term, we have obtained the non-BBMB black hole solution in the new massive conformal gravity [36]. Also, this work is motivated partly from finding scalarized black holes in the Einstein minimally coupled scalar theory with an asymmetric scalar potential that contains a negative region, indicating a violation of the WEC [37–40]. Additionally, the asymmetric potential has also been employed to construct the fermionic stars [41,42]. A symmetric potential violating the WEC was also introduced to obtain the scalarized black holes [43], which can be smoothly connected with the counterpart gravitating scalaron in the small horizon limit [44]. Note that this gravitating scalaron can possess the positive Arnowitt-Deser-Misner (ADM) mass, but previously another type of gravitating scalaron with negative ADM mass was constructed by employing the Higgs-like potential with a phantom field [45]; hence this demonstrates that the use of a phantom field can be avoided for the construction of a gravitating scalaron. The no-hair theorem suggests that asymptotically flat black holes with scalar hair do not exist if a scalar matter satisfies the WEC [46]. Hence, if the WEC for a scalar matter is violated, scalarized black holes could be found from the Einstein minimally coupled scalar theory. As far as we know, our model corresponds to a simplest model that violates the WEC because of a negative potential and, thus, it could induce scalarized black holes without introducing a non-minimal scalar coupling term. Other types of scalarized black holes in the Einstein minimally coupled scalar theory can be found in Refs. [47–58]. Additionally, recently this similar concept has been adopted to construct a traversable wormhole in the Einstein 3-form theory with the Higgs-like potential, which is sufficient to violate the null energy condition where the phantom field is no longer needed and the kinetic term still can remain the correct sign [59].

II. NO TACHYONIC INSTABILITY OF SCHWARZSCHILD BLACK HOLE

We start with the Einstein minimally coupled scalar theory defined by

$$S_{\mathrm{Es}} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right], \quad (1)$$

where $V(\phi)$ is a negative scalar potential

$$V(\phi) = -\alpha^2\phi^6 \quad (2)$$

with $\alpha > 0$.

The Einstein equation is derived from (1) as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G\left[\nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\nabla_\alpha\phi\nabla^\alpha\phi + V(\phi)\right)\right]. \quad (3)$$

On the other hand, the scalar equation is given by

$$\nabla^2\phi = \frac{dV(\phi)}{d\phi}. \quad (4)$$

Considering

$$\bar{R}_{\mu\rho\nu\sigma} \neq 0, \qquad \bar{R}_{\mu\nu} = 0, \qquad \bar{R} = 0, \qquad \bar{\phi} = 0, \quad (5)$$

Equations (3) and (4) imply the GR (Schwarzschild) black hole solution

$$ds^2_{\mathrm{Sch}} = \bar{g}_{\mu\nu}dx^\mu dx^\nu = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2 \quad (6)$$

with the metric function

$$f(r) = 1 - \frac{r_0}{r}. \quad (7)$$

Here the event horizon appears at $r_H = r_0$. We note that $\bar{\phi} = \mathrm{const}$ is not allowed for getting a GR black hole.

Now, let us introduce the scalar and metric perturbations around the Schwarzschild black hole

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \qquad \phi = 0 + \delta\phi. \quad (8)$$

First, we consider the scalar perturbation. Considering (4), its linearized scalar equation is given by

$$\bar{\nabla}^2\delta\phi = 0. \tag{9}$$

Reminding the reader of the spherically symmetric background (6), it is convenient to separate the scalar perturbation into modes

$$\delta\phi(t, r, \theta, \varphi) = e^{-i\omega t} Y_{lm}(\theta, \varphi)\frac{u(r)}{r}, \tag{10}$$

where $Y_{lm}(\theta, \varphi)$ is spherical harmonics with $-m \le l \le m$. Introducing a tortoise coordinate $r_* = r + r_0 \ln(r/r_0 - 1)$ defined by $dr_*(r) = dr/f(r)$, a radial part of the wave equation leads to the Schrödinger-type equation as

$$\frac{d^2u}{dr_*^2} + [\omega^2 - V_s(r)]u(r) = 0, \tag{11}$$

where the scalar potential $V_s(r)$ takes the form

$$V_s(r) = f(r)\left[\frac{r_0}{r^3} + \frac{l(l+1)}{r^2}\right], \tag{12}$$

which is always positive definite for any l outside the horizon. Therefore, there is no tachyonic instability and the Schwarzschild black hole is stable against scalar perturbation.

Now, we briefly mention the tensor perturbation. The linearized Einstein equation around the Schwarzschild black hole is simply given by

$$\delta G_{\mu\nu} = 0, \tag{13}$$

where the linearized Einstein tensor is expressed in terms of the linearized Ricci tensor and Ricci scalar as

$$\delta G_{\mu\nu} = \delta R_{\mu\nu} - \frac{1}{2}\delta R \bar{g}_{\mu\nu}, \tag{14}$$

$$\delta R_{\mu\nu} = \frac{1}{2}\left(\bar{\nabla}^\rho \bar{\nabla}_\mu h_{\nu\rho} + \bar{\nabla}^\rho \bar{\nabla}_\nu h_{\mu\rho} - \bar{\nabla}^2 h_{\mu\nu} - \bar{\nabla}_\mu \bar{\nabla}_\nu h\right), \tag{15}$$

$$\delta R = \bar{g}^{\mu\nu}\delta R_{\mu\nu} = \bar{\nabla}^\mu \bar{\nabla}^\nu h_{\mu\nu} - \bar{\nabla}^2 h \tag{16}$$

with $h = h^\rho{}_\rho$. Taking the trace of the linearized Einstein equation (13), one has

$$\delta R = 0. \tag{17}$$

Plugging $\delta R = 0$ into Eq. (13) leads to the linearized GR equation for the linearized Ricci tensor

$$\delta R_{\mu\nu} = 0. \tag{18}$$

It is well known that the Schwarzschild black hole is stable against metric perturbation $h_{\mu\nu}$ in GR [60–63]. This implies that the linearized stability analysis has no prediction on exploring scalarized black holes.

III. SCALARIZED BLACK HOLES

Even though the Schwarzschild black hole appears stable against scalar perturbation, we may obtain scalarized black holes because the potential in (2) violates the WEC. First of all, we introduce a spherically symmetric metric to construct the scalarized black hole solutions

$$ds^2_{\mathrm{Sb}} = -N(r)e^{-2\sigma(r)}dt^2 + \frac{dr^2}{N(r)} + r^2 d\Omega_2^2 \tag{19}$$

with $N(r) = 1–2m(r)/r$, where $m(r)$ denotes the Misner-Sharp mass function. We note that $m(\infty) = M$, the total mass of the configuration.

Plugging Eq. (19) into (3) and (4), three differential equations for metric functions (m, σ) and scalar (ϕ) are found as

$$m' = 2\pi G r^2(N\phi'^2 + 2V), \qquad \sigma' = -4\pi G r\phi'^2,$$
$$(e^{-\sigma}r^2 N\phi')' = e^{-\sigma}r^2\frac{dV}{d\phi}, \tag{20}$$

where the prime ($'$) denotes the derivative with respect to the radial coordinate r.

At this stage, one needs to know the asymptotic behavior of these functions at the horizon and the infinity to construct globally defined black hole solutions. Near the horizon ($r \simeq r_H$), the leading forms in the series expansion are given by

$$m(r) = \frac{r_H}{2} + m_1(r - r_H) + O\left((r - r_H)^2\right), \tag{21}$$

$$\sigma(r) = \sigma_H + \sigma_1(r - r_H) + O\left((r - r_H)^2\right), \tag{22}$$

$$\phi(r) = \phi_H + \phi_{H,1}(r - r_H) + O\left((r - r_H)^2\right), \tag{23}$$

where the coefficients are given by

$$m_1 = 4\pi G r_H^2 V(\phi_H), \qquad \sigma_1 = -4\pi G r_H \phi_{H,1}^2,$$
$$\phi_{H,1} = \frac{r_H V'(\phi_H)}{1 - 8\pi G r_H^2 V(\phi_H)}. \tag{24}$$

Here σ_H and ϕ_H are the values of σ and ϕ at the horizon. We note that the denominator of $\phi_{H,1}$ should satisfy the condition of $1 - 8\pi G r_H^2 V(\phi_H) \neq 0$ to keep $\sigma(r)$, and $\phi(r)$ should be finite at the horizon.

Asymptotic expansions of the metric and scalar functions at infinity take the forms when imposing asymptotical flatness and a vanishing scalar as

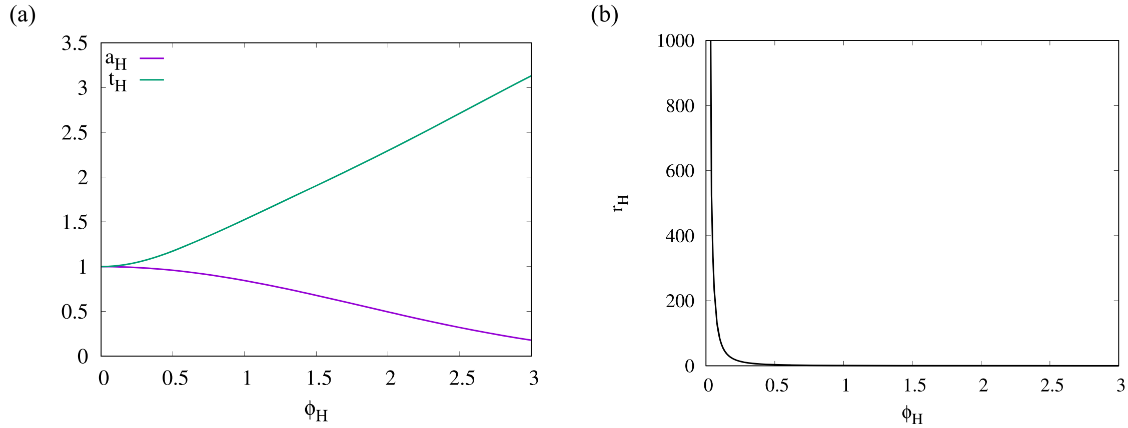

FIG. 1. Two basic quantities of hairy black hole. (a) Reduced area of horizon a_H and reduced Hawking temperature t_H as functions of the scalar field at the horizon ϕ_H. Two are starting with $a_H = t_H = 1$ for the Schwarzschild black hole at $\phi_H = 0$. (b) Horizon radius r_H as a function of ϕ_H.

$$m(r) = M - \frac{D^2}{4r} - \frac{MD^2}{4r^2} - \frac{D^2}{3r^3}(2D^4 + M^2) + \mathcal{O}(r^{-4}), \tag{25}$$

$$\sigma(r) = \frac{D^2}{4r^2} + \frac{2MD^2}{3r^3} + \left(\frac{3D^6}{2} - \frac{D^4}{8} + 3M^2D^2\right)\frac{1}{2r^4} + \mathcal{O}(r^{-5}), \tag{26}$$

$$\phi(r) = -\frac{D}{r} - \frac{DM}{r^2} + \left(-D^4 + \frac{D^2}{12} - \frac{4M^2}{3}\right)\frac{D}{r^3} + \mathcal{O}(r^{-4}), \tag{27}$$

where M and D represent the ADM mass and scalar charge of the hairy black hole, respectively.

Finally, introducing three dimensionless parameters

$$r \to \frac{r}{\alpha\sqrt{8\pi G}}, \qquad m \to \frac{m}{\alpha\sqrt{8\pi G}}, \tag{28}$$

three equations in Eq. (20) can be solved by an ordinary differential equation solver package COLSYS when adapting the Newton-Raphson method to solve the boundary value problem for three coupled nonlinear differential equations [64]. We employ a compactified coordinate $x = 1 - r_H/r (x \in [0, 1])$ for constructing hairy black holes in the numerics. Here, we are left with the three parameters (ϕ_H, σ_H, r_H) where σ_H is determined when imposing $\sigma(\infty) = 0$, while r_H could be determined when all solutions satisfy the boundary conditions. In this case, the state of a scalarized black hole depends only on ϕ_H, which means that different scalarized black holes are encoded in different ϕ_H. When the scalar field is zero on the horizon, the corresponding configuration is solely given by the Schwarzschild black hole. However, when the scalar field on the horizon ϕ_H is nonzero and then one increases it, a branch of hairy black holes bifurcates and behaves quite differently from the Schwarzschild black hole. To represent this behavior, we introduce reduced area a_H and reduced temperature t_H by making use of area of horizon A_H and Hawking temperature T_H of hairy black holes as

$$a_H = \frac{A_H}{16\pi M^2}, \qquad t_H = 8\pi T_M M \quad \text{with} \quad A_H = 4\pi r_H^2,$$
$$T_H = \frac{1}{4\pi} N'(r_H) e^{-\sigma_H}. \tag{29}$$

Figure 1(a) shows the plots of reduced area of horizon a_H and reduced Hawking temperature t_H. We recall that two are $a_H = t_H = 1$ for the Schwarzschild black hole with $\phi_H = 0$. For increasing ϕ_H, a_H decreases monotonically from unity to very close to zero, whereas t_H increases monotonically from unity. Although some hairy black holes were constructed by different $V(\phi)$ [39], their a_H and t_H behave qualitatively similar to Fig. 1(a). Figure 1(b) indicates that the radius of horizon r_H of the hairy black hole is inversely proportional to ϕ_H, which means that the hairy black holes bifurcate from the Schwarzschild black hole with a very large value of r_H and, finally, r_H could shrink to zero as ϕ_H increases. Here, both r_H and ϕ_H could take any arbitrary positive real values.

Now, we are in a position to present the numerical solutions and discuss their properties. Figure 2 exhibits six solutions of hairy black hole with a choice of six different ϕ_H as functions of the compactified coordinate x, where they are regular everywhere outside and on the horizon. The functions $\sigma(x)$ and $\phi(x)$ decrease monotonically from its maximum value at the horizon to zero at infinity. However, the mass function $m(x)$ possesses a local minimum that moves away from the horizon to infinity as ϕ_H increases.

We might understand the presence of scalarized black holes by observing the WEC. We examine the energy condition of the hairy black hole as shown in Fig. 3. The WEC is described by the energy density

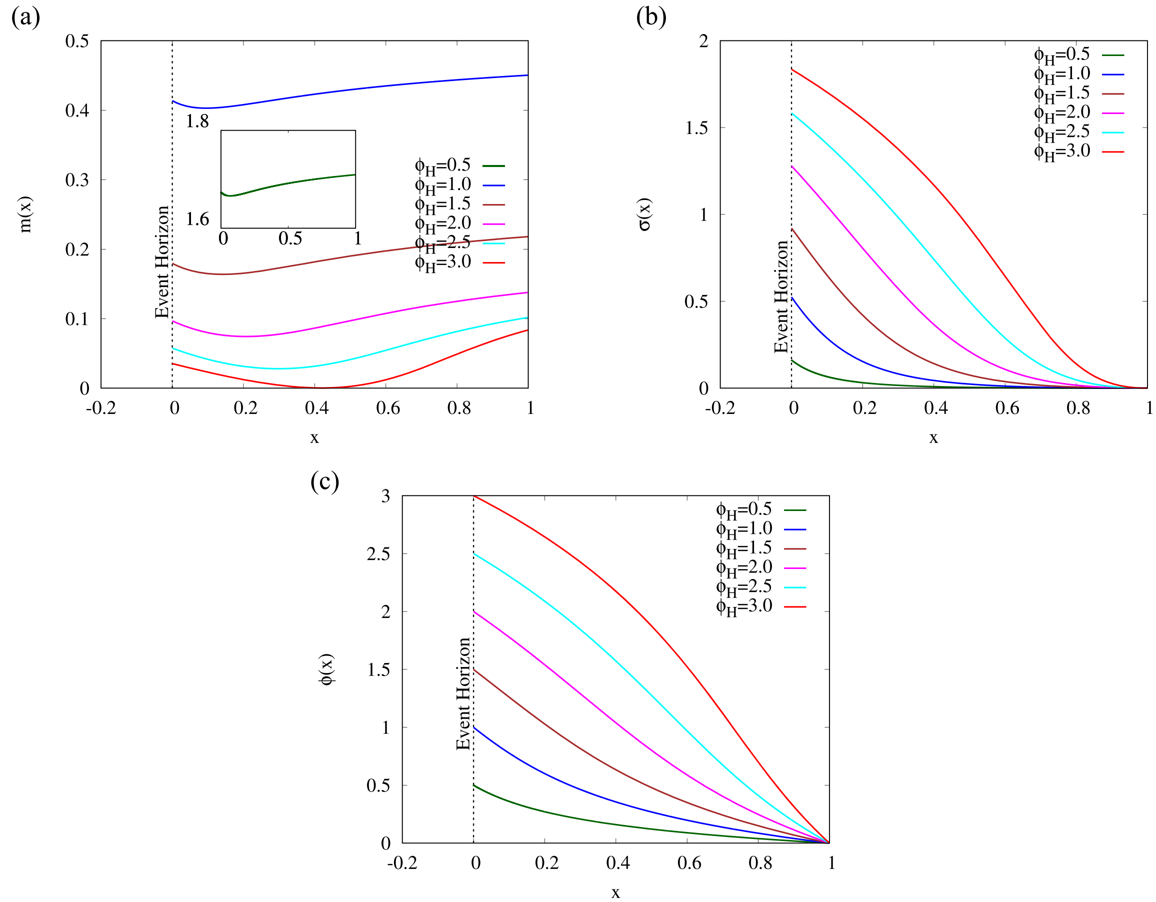

FIG. 2. Six solutions with different ϕ_H of hairy black hole in the compactified coordinate x: (a) mass function $m(x)$ contains an inset for $\phi_H = 0.5$, (b) metric function $\sigma(x)$, and (c) scalar hair $\phi(x)$.

$\rho = -T^t{}_t = N\phi'^2/2 + V(\phi)$, which is negative near the horizon. This shows violation of the WEC clearly. One may evade the no-hair theorem if the scalar matter does not satisfy the WEC [46]. This implies the presence of scalarized black holes. ρ at the horizon decreases very sharply with the increase of ϕ_H. Additionally, ρ possesses a local maximum that is located exactly at the local minimum of $m(x)$, and it moves further away from the horizon to infinity as ϕ_H increases.

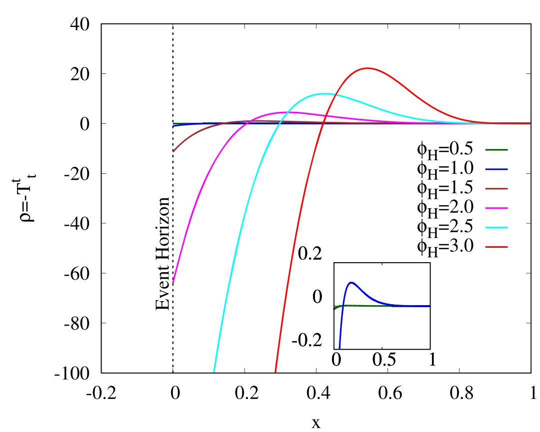

FIG. 3. The energy density ρ for the hairy black holes with six ϕ_H in the compactified coordinate $x \in [0, 1]$. The inset shows ρ for $\phi_H = 0.5,\ 1.0$.

IV. RADIAL PERTURBATIONS AROUND SCALARIZED BLACK HOLES

For further implications of scalarized black holes, we need to perform a stability test for them. For this purpose, we introduce radial perturbations defined by

$$ds^2 = -N(r)e^{-2\sigma(r)}[1 + \epsilon e^{-i\omega t}F_t(r)]dt^2 + \frac{1}{N(r)}[1 + \epsilon e^{-i\omega t}F_r(r)]dr^2 + r^2 d\Omega_2^2, \quad (30)$$

$$\Phi = \phi(r) + \epsilon\Phi_1(r)e^{-i\omega t}, \quad (31)$$

where $F_t(r)$, $F_r(r)$, and $\Phi_1(r)$ are three perturbed fields.

Substituting Eqs. (30) and (31) into (3) and (4), we obtain three linearized equations,

$$F_r = 8\pi G r\phi'\Phi_1, \quad (32)$$

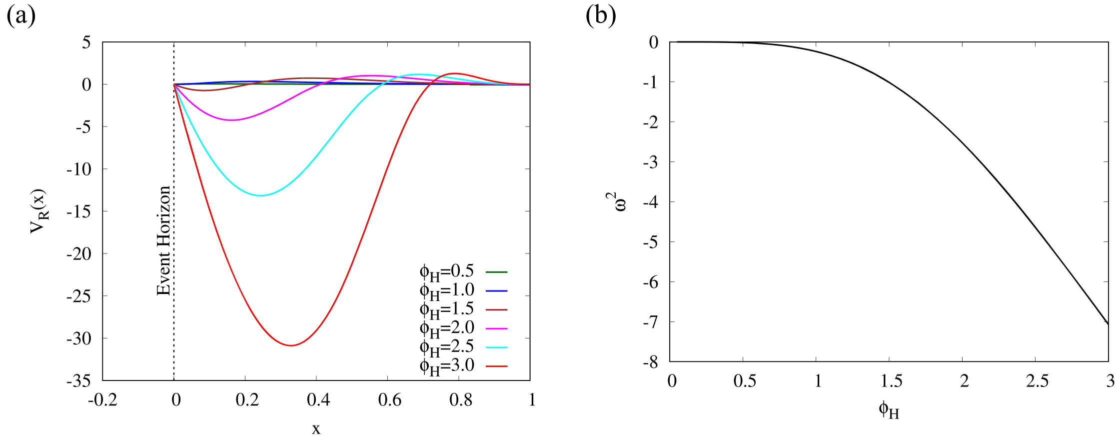

FIG. 4. (a) The effective potential $V_R(x)$ with six different ϕ_H in the compactified coordinate $x \in [0, 1]$ for the Schrödinger-like equation. (b) The eigenvalue ω^2 for unstable modes as a function of ϕ_H. It is obvious that $\omega^2 < 0$.

$$F_t' = -F_r' + 16\pi G r \phi' \Phi_1', \tag{33}$$

$$\Phi_1'' = \left(\sigma' - \frac{N'}{N} - \frac{2}{r}\right)\Phi_1' + \left(\frac{1}{N} V''(\phi) - \omega^2 \frac{e^{2\sigma}}{N^2}\right)\Phi_1 + V'(\phi)\frac{F_r}{N} + \phi' \frac{F_r' - F_t'}{2}. \tag{34}$$

We eliminate the last two terms in Eq. (34) by making use of the first two equations to obtain an independent scalar equation. Then, we can transform Φ_1'' to a Schrödinger-like equation by introducing $Z(r) = r\Phi_1(r)$ and a tortoise coordinate r_* defined by $r_* = \int dr[\frac{e^\sigma}{N}]$ as

$$\frac{d^2 Z}{dr_*^2} + \left[\omega^2 - V_R(r)\right] Z = 0 \tag{35}$$

with the effective potential

$$V_{\mathrm{R}}(r) = N e^{-2\sigma}\left[\frac{N}{r}\left(\frac{N'}{N} - \sigma'\right) - 8\pi G r N \phi'^2 \left(\frac{N'}{N} + \frac{1}{r} - \sigma'\right) + 16\pi G r \phi' V'(\phi) + V''(\phi)\right]. \tag{36}$$

We wish to perform the linear stability of the hairy black hole. Before we proceed, it is worth mentioning that, as Fig. 4(a) has shown, the effective potentials $V_R(x)$ with six different ϕ_H are always negative in some regions of x, implying the possible presence of unstable modes. We note that solving Eq. (35) corresponds to handling an eigenvalue problem. Hence, we obtain the radial mode numerically by using COLSYS to solve it with ω^2 as an eigenvalue. For black holes, we impose that the perturbation fields vanish at two boundaries, $Z(r_H) = Z(\infty) = 0$. In the numerics, we introduce an auxiliary equation of $\frac{d}{dr}[\omega^2] = 0$. This allows us to impose an additional condition of $Z(r_p) = 1$ at some point r_p, which is typically located at the middle of the horizon and infinity. This allows us to obtain a nontrivial and normalizable solution for Z, since Eq. (35) is homogeneous. The eigenvalue ω^2 is determined automatically when Z satisfies all asymptotic boundary conditions. Accordingly, Fig. 4(b) indicates that the $\omega^2 < 0$ (unstable modes) decreases with the increase of ϕ_H where the scalar perturbation increases exponentially with time. The perturbation Z is unstable because $\omega^2 = -\Omega^2 < 0$ where the time-dependent perturbation ($e^{\Omega t}$) grows exponentially with time. This implies that all scalarized black holes belonging to a single branch are unstable.

V. DISCUSSIONS

It is clear that the tachyonic instability as the onset of spontaneous scalarization indicates infinite branches of scalarized black holes. We have explored a single branch of scalarized black holes in the Einstein minimally coupled scalar theory with a negative potential $V(\phi) = -\alpha^2\phi^6$. Here, the tachyonic instability is absent and, thus, it plays no role in predicting infinite branches of scalarized black holes. In this case, one could not meet a condition for spontaneous scalarization, but one meets a condition for nonlinear scalarization to obtain a single branch of scalarized black holes in the Einstein-Gauss-Bonnet-scalar theory with a coupling function $f(\phi)$. This suggests that tachyonic instability is a necessary condition to obtain infinite branches of scalarized black holes.

To generate a single branch of scalarized black holes, there are some sources of nonlinear instability [21,30–32], conformal scalar coupling [5,6,14], superradiant instability [65], and violation of the WEC [37–40]. In our work, it is important to note that the negative scalar potential with $\alpha = 1$ violates the weak energy condition. It is well known that, if the WEC for a scalar matter is violated, scalarized black holes could be found from the Einstein minimally coupled scalar theory without introducing any scalar coupling function $f(\phi)$ to matter. Thus, we have obtained the black hole solutions with scalar hair by solving three nonlinear equations. It includes a single branch of scalarized black holes only because tachyonic instability is absent. Different scalarized black holes are encoded in different ϕ_H because we have chosen $\alpha = 1$. Furthermore, we have studied their thermodynamic properties by introducing reduced horizon area and Hawking temperature.

Then, we have performed the stability analysis for scalarized black holes by adopting radial perturbations. It turned out that six scalarized black holes with six different ϕ_H belonging to a single branch are unstable. Therefore, it is unlikely that this scalarized black hole can be considered as the astrophysical black holes such as M87 and SgrA*, since the detection of their existence from the astrophysical signatures could be very challenging.

Finally, it would be interesting to obtain the other solutions of scalarized black holes by choosing a more simple form of the potential, for instance, $V(\phi) = -m_\phi^2\phi^2, -\Lambda\phi^4$ with m_ϕ^2 and Λ as positive constants, since the previous analysis of Eqs. (8) and (9) in [43] does not rule out the possible existence of scalarized black hole solutions. Therefore, it is meaningful to investigate such possibilities and report them in the future.

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