Semiclassical bounce with strong minimal assumptions

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We explore the possibility of avoiding cosmological singularity with a bounce solution in the early Universe. The main finding is that simple and well-known semiclassical correction, which describes the mixing of radiation and gravity in the effective action, may provide an analytic solution with a bounce. The solution requires a positive beta function for the total radiation term and the contraction of the Universe at the initial instant. The numerical estimate shows that the bounce may occur in an acceptable range of energies, but only under strong assumptions about the particle physics beyond the Standard Model.

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I. INTRODUCTION

The initial cosmological singularity is considered an important indication to either modify general relativity or introduce exotic forms of "matter" with an unusual equation of state (see, e.g., [1]). One may also think about taking into account the effects of quantum gravity. The last is a direct consequence of the fact that the Planck density of matter is achieved in the vicinity of a singularity. In this sense, singularity may be a kind of window to observe the quantum gravity effects.

The safest way to avoid the singularity is to have a cosmological solution with a bounce, as pioneered by Tolman in 1931 [2]. Starting from the 1970s, there are numerous bouncing models [3,4], partially related to the interest in taking quantum effects into account. Since then, different bouncing cosmological scenarios attracted a lot of attention (see Refs. [5,6] for the reviews of the literature). In most of the existing models, the bounce is achieved by using a scalar field with the specially designed potential, or by using modified gravity actions. A new recent trend is related to the introduction of the nonlocalities into the gravitational action (see, e.g., [7,8]). The same purpose can be achieved by taking into account the nonlocal semiclassical corrections [9,10]. One of the challenges in building bounce models is to avoid pathologies related to quantum instabilities [11].

The conventional assumption is that the consistent theory of quantum gravity would be as an ultimate solution for the problem of singularities. The dimensional arguments indicate that the quantum gravity effects should become relevant at the Planck scale $M_P \approx 10^{19}$ GeV. On the other hand, the effects of quantum matter fields on the classical gravitational background (semiclassical gravity) may produce changes in the action of gravity and matter such that the solution of the effective equations is free of singularity. In this way, the mentioned window may be closed to the observer from the later Universe. The purpose of the present paper is to explore this possibility by constructing the solution with a cosmological bounce, where the contraction of the Universe goes on until a minimum point, after which the expansion starts. This minimal point should correspond to the energy densities far below the Planck density M_P^4, such that the quantum gravity effects and also the possible higher derivative terms in the gravitational action are Planck suppressed and hence irrelevant. Thus, we take into account only the quantum effects of matter fields.

The first necessary condition to meet a bounce is to have a decreasing conformal factor of the metric, $a(t)$, at some initial instant before the bounce. Since the bounce is a form to remove the singularity, in its vicinity we can assume that the typical distances are small, the energy density is high, and the quantum effects of matter fields are relevant. In such a UV regime, the typical energies are such that all fields are approximately massless. This feature has the following two consequences:

(i) One can use a massless approximation for at least most of the matter fields in the UV limit. For the sake of simplicity, let us assume that the nongravitational contents of the Universe are pure electromagnetic radiation. Later on, we discuss how other kinds of matter may change the conditions of the bounce.

(ii) Since the matter content can be described by pure radiation, the relevant semiclassical diagrams are those with two external lines of photons and an arbitrary number of linearized gravity tails, as shown in Fig. 1.

In the far UV, when the masses of all quantum fields are small compared to the energies of the photons (important:

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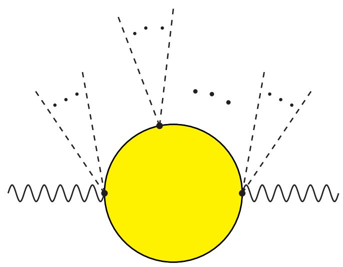

FIG. 1. The loop kernel of matter fields connects to the two photon lines and an unrestricted number of the dashed lines of the linearized metric $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$.

not gravitons), one can ignore the effect of quantum decoupling, i.e., the Appelquist and Carazzone theorem [12], and take into account the quantum effects of matter fields by using the minimal subtraction scheme of renormalization. In this case, the leading quantum effect is the conformal anomaly. While the classical radiation decouples from the dynamical equation for $a(t)$ and affects only the initial condition (we elaborate on this point below), at the quantum level, the matter contents of the Universe enter the equation for $a(t)$.

There is extensive literature on the cosmological models based on conformal anomaly, starting from [13] and [14], where the anomaly-induced effective action served as an extreme case of the first inflationary model. However, our present purpose is different, as we are interested only in the radiation part of the anomaly, which may become relevant at the lower energies. The explanation is that the typical energies of the photons in Fig. 1 are much greater than those of gravitons, which define the energy scale of the vacuum quantum effects. The electromagnetic part of the anomaly has been used previously in [15] to explain the seeds of magnetic fields in the epoch of forming galaxies. In the present work, we apply the same approach to describe the dynamics of the whole Universe and discuss whether it is sufficient to avoid singularity in the contracting Universe.

The paper is organized as follows. Section II contains a short survey of the anomaly-induced action in the radiation sector. In Sec. III, we derive the analytic bounce solution in a theory formed by the Einstein-Hilbert action with the anomalous contribution mixing the radiation term with gravity. This solution is supported by the plots obtained using the numerical solution, which also includes the nonzero cosmological constant case. Section IV reports on the numerical estimates for the bounce and discusses the possibility of overcoming the dramatic physical inconsistency which we met in the simplest electromagnetic radiation case. Finally, in Sec. V, we draw our conclusions.

We adopt the natural units such that $c = \hbar = 1$ and use the signature $(+,-,-,-)$ for the Minkowski metric $\eta_{\mu\nu}$.

II. ANOMALY-INDUCED ACTION WITH RADIATION

It proves useful to present the one-loop beta function for the square of the gauge coupling g in the unconventional form βg^4, where[1]

$$\beta = -\frac{2}{(4\pi)^2}\left(\frac{11}{3}C_1 - \frac{1}{6}N_{cs} - \frac{4}{3}N_f\right). \tag{1}$$

Here N_{cs} and N_f are the numbers of complex scalars and fermions coupled to the given vector field. C_1 is the Casimir operator of the corresponding gauge group, which is zero in the Abelian case. The g^4 factor was separated from the beta function for the sake of further convenience. In the non-Abelian theory, C_1 is positive, and this opens the possibility of the asymptotic freedom in the theory [17,18]. At the relatively low energies, for the electromagnetic field, obviously $C_1 = 0$. However, above the scale of electroweak phase transition, the electromagnetic fields mix with other vector bosons and become part of the asymptotic freedom scheme. Thus, depending on the energy scale both signs are, in principle, possible. We assume that the one-loop effects are dominating and ignore the higher loop effects, except for the discussion in the last sections.

The trace anomaly in the radiation sector has the form (see, e.g., [16] for the details)

$$\begin{aligned}\langle T^\mu_\mu\rangle &= -\frac{2}{\sqrt{-g}}g_{\alpha\beta}\frac{\delta\Gamma_r}{\delta g_{\alpha\beta}} \\ &= -\frac{1}{\sqrt{-\bar{g}}}\frac{\delta\Gamma_r[\bar{g}_{\alpha\beta}e^{2\sigma}]}{\delta\sigma}\bigg|_{\bar{g}_{\alpha\beta}\to g_{\alpha\beta},\sigma\to 0} = \frac{\beta}{4}g^2F^2,\end{aligned} \tag{2}$$

where $F^2 = F_{\mu\nu}F^{\mu\nu}$ is the square of the gauge field strength tensor and Γ_r is the one-loop renormalized effective action in the radiation sector. Also, we introduced the parametrization of the metric

$$g_{\alpha\beta} = \bar{g}_{\alpha\beta}e^{2\sigma} = \bar{g}_{\alpha\beta}a^2, \tag{3}$$

which will prove useful below. In the homogeneous and isotropic Universe, the unique spacetime coordinate is the conformal time η, related to the physical time t by the formula $dt = a(\eta)d\eta$. We shall write the next few formulas in a covariant way and then switch to the flat-space cosmological metric.

Equation (2) can be used to find a solution to the effective action, and its covariant nonlocal form [19,20] (see also further developments in [21] and [22]) is

[1]The detailed derivation of this expression can be found in many Quantum Field Theory (QFT) textbooks, e.g., in [16].

$$\Gamma_r = -\frac{\beta g^2}{16}\iint_{x,y}\left(E_4 - \frac{2}{3}\Box R\right)_x G(x,y)F^2(y), \quad (4)$$

where $E_4 = R^2_{\mu\nu\alpha\beta} - 4R^2_{\alpha\beta} + R^2$ is the Gauss-Bonnet invariant, $G(x,y)$ is the conformally covariant Green function of the operator $\Delta_4 = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^\mu R)\nabla_\mu$ and $\int_x = \int d^4x\sqrt{-g(x)}$. It is possible to formulate the induced action in the covariant local form [19], including with two auxiliary scalar fields [23]. The last is the most useful formulation for many applications, such as the classification of vacuum states [24] or the reaction of gravitational waves to the presence of higher derivatives [25]. A qualitatively similar representation, with certain simplifications [21,22], should be most useful for the analysis of cosmological perturbations. We leave this part for future work and, in the rest of this paper, will restrict the consideration by the basic elements of the cosmological model, i.e., the dynamics of the homogeneous and isotropic Universe. In this case, one can use a much simpler form of induced action, which is equivalent to (4) for this special metric.

In covariant form, the anomaly-induced term mixes radiation and curvature-dependent terms. In the cosmological framework, we assume that the fiducial metric is flat and then (4), with an additional Einstein-Hilbert term and cosmological constant, boils down to the noncovariant local form

$$\Gamma = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}(R+2\Lambda) - \frac{\beta g^2}{4}\int d^4x\sqrt{-\bar{g}}\bar{F}^2\sigma, \quad (5)$$

where the bars denote quantities defined using the fiducial metric, $\bar{g} = \det(\bar{g}_{\mu\nu})$ and $\bar{F}^2 = \bar{g}^{\mu\alpha}\bar{g}^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$. It is fairly easy to check that the last term in (5) is a solution of (2).

III. THE BOUNCE SOLUTION

Let us consider an analytical cosmological solution in the theory (5) and use it for making general conclusions that go beyond QED and even beyond the Standard Model.

Taking the variational derivative with respect to $\sigma(\eta)$ and changing the variables to the physical time t and $a(t) = \exp\{\sigma(t)\}$, we arrive at the equation (note the change of notations compared to [22])

$$\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = \frac{\mathcal{M}}{2a^4} + \frac{16\pi}{3M_P^2}\rho_\Lambda. \quad (6)$$

In this expression and below, we use the notations

$$M_P^2 = \frac{1}{G}, \qquad \mathcal{M} = \frac{2\pi\beta g^2}{3M_P^2}\bar{F}^2, \qquad \rho_\Lambda = \frac{\Lambda}{8\pi G}. \quad (7)$$

Previously, the bounce with a cosmological constant had been considered, e.g., in [26]. Our expression (7) includes the cosmological constant term and the anomalous part described above. Usually, one can assume that the cosmological constant is irrelevant at extremely high energies where the cosmological singularity or a bounce should take place. On the other hand, at the energies above the scale of the electroweak phase transition, the cosmological constant is supposed to change its magnitude by many orders [27]. Regardless of the cosmological term ρ_Λ is subleading [28] compared to the radiation energy density, we take it into account. That is especially important because classical radiation does not enter directly [Eq. (6)] and, as we shall see in a moment, shows up only after the first integration.

The first integration, or order reduction, can be done by taking the Hubble parameter as a function of the conformal factor $H(a) = \dot{a}/a$. This approach brings the relation

$$H^2 = \frac{C}{a^4} + \frac{\mathcal{M}}{a^4}\log\frac{a}{a_0} + \frac{\Lambda}{3}. \quad (8)$$

It is worth noting that, in the classical approach, there are both a dynamical equation for the scale factor and a constraint equation. In case of the terms generated by trace anomaly in both gravitational and radiation sectors, the constraint equation also takes place in the form $\langle\nabla_\mu T^{\mu\nu}\rangle = 0$, reflecting the general covariance of the anomaly-induced action, including the nonlocal term (2). This part was elaborated in detail in Ref. [22], so we can skip the details and just give the equation for the pressure of radiation in the presence of the anomaly, which supplements the energy density formula equivalent to (8),

$$p_r = \frac{1}{3}\left(\rho_r - \frac{1}{4}\frac{|\beta|g^2\bar{F}^2}{a^4}\right). \quad (9)$$

The last term on the rhs represents a quantum correction to the equation of state for the radiation. In the present work, we will not use this expression, but it may be useful for the analysis of cosmic perturbations. On the other hand, a more solid approach should be based on the local version of the covariant expression (4).

Coming back to our consideration, the second term in the rhs of (8) vanishes in the classical limit, and this enables us to identify the integration constant C with the product $\rho_{r0}a_0^4M_P^{-2}$, where ρ_{r0} is a radiation energy density at $a = a_0$. The comparison with our previous parametrization of the metric (3) makes us assume that $\bar{g}_{\mu\nu}$ corresponds to the value of a_0. Consequently, we replace $\bar{F}^2 \to F_0^2a_0^4$ in the formula for $\mathcal{M}$ in (7). After that, the previous relation (8) is cast into the form

$$H^2 = \frac{a_0^4}{a^4}\left(\frac{\rho_{r0}}{M_P^2} + \mathcal{M}\log\frac{a}{a_0}\right) + \frac{\Lambda}{3}. \tag{10}$$

In all these relations, the value a_0 corresponds to the size of the Universe where our approximations apply. That means, a_0 should provide sufficiently high energies to have either (i) a radiation-dominated regime, when the role of massive particles (in the form of dust or larger objects) is irrelevant compared to radiation; or (ii) all matter particles at such high temperatures that their masses are negligible.

Thus, the questions to address are as follows: (a) whether Eq. (10) admits an analytic solution corresponding to a bounce and (b) if the required difference in size between a_0 and the value a_m corresponding to a bounce, takes us to the trans-Planckian energies. The successful bounce model should answer negatively to the last question, as otherwise, we cannot justify ignoring the quantum gravity effects. Here we consider part (a) and leave the more complicated question (b) to the next section.

A. Analytic solution for a bounce

As we know [28], for a sufficiently small $a_m \ll a_0$, the cosmological term is small compared to other terms on the rhs of (10). Thus, we can explore the bounce solution for $\Lambda = 0$ and then include a nonzero Λ term, treating it as a small perturbation. In this way, using (7), we arrive at the condition of $H(a_m) = 0$ in the form

$$\rho_{r0} = -\mathcal{M}M_P^2 \log\frac{a_m}{a_0} = \frac{2\pi}{3}\beta g^2 F_0^2 \log\frac{a_0}{a_m}. \tag{11}$$

Since we suppose that the Universe is initially contracting, $a_0 > a_m$ and hence the necessary condition of the bounce is that $\beta F_0^2 > 0$.

As the first example, consider the simplest case when the Universe is very hot and its contents can be described by the energy density of radiation ρ_r. On the other hand, the space is conducting owing to the presence of a hot gas of charged particles. For the sake of simplicity, we assume that, in the initial point of the relevant phase of the contracting Universe, most of ρ_{r0} consists of the electromagnetic radiation [15]. Then we have

$$\rho_r \approx \frac{\vec{H}^2 + \vec{E}^2}{2} \quad \text{and} \quad F^2 \approx \frac{\vec{H}^2 - \vec{E}^2}{2}. \tag{12}$$

Owing to the conducting media, $\vec{E}^2 \approx 0$ we arrive at the estimate $\rho_{r0} \approx F_0^2$. Thus, we arrive at the solution for a_m in the form

$$a_m = a_m(\Lambda = 0) = a_0 \exp\left\{-\frac{3}{2\pi\beta g^2}\right\}. \tag{13}$$

Another possibility is to use relation (10) with $\Lambda = 0$ and get the general solution

$$t - t_0 = \pm\sqrt{\frac{\pi}{2\mathcal{M}}}e^{-2C/\mathcal{M}}\left[\mathrm{erfi}\left(\sqrt{2\log a + 2C/\mathcal{M}}\right) - \mathrm{erfi}\left(\sqrt{2C/\mathcal{M}}\right)\right], \tag{14}$$

where $\mathrm{erfi}(x) = -i\,\mathrm{erf}(ix)$ is the imaginary error function. Treating $\mathcal{M}$ as a small perturbation, we can use the asymptotic expansion

$$\mathrm{erfi}(x)|_{x\to\infty} \simeq -i + \frac{e^{x^2}}{\sqrt{\pi}x} + O(x^{-1}), \tag{15}$$

and derive the following approximate solution:

$$t \simeq \pm\frac{1}{2\sqrt{C}}\left[a^2\left(1 - \frac{\mathcal{M}}{2C}\log a\right) - 1\right], \qquad t_0 = 0. \tag{16}$$

In the limit $t \to 0$, we verify that $a(t) \to 1$, which is consistent with the numerical solutions, as we will see in the next subsection. Additionally, taking the limits $t \to \pm\infty$, we find $a(t) \to +\infty$. Let us note that this scheme is opposite to what is required for the bounce since, in the last case, $\mathcal{M}$ cannot be regarded as small.

Taking the cosmological constant term as a small perturbation in Eq. (10) is a technically simple exercise, and we give only the final result:

$$a_m(\Lambda) = a_m\left(1 - \frac{4\pi\rho_\Lambda}{\beta g^2 F_0^2}\frac{a_m^4}{a_0^4}\right). \tag{17}$$

Typically, this formula describes a small correction to the basic solution (13).

B. Plots corresponding to the bounce

Let us first illustrate the analytic solution presented above by a few plots obtained by the numerical solution of Eq. (6) with $\rho_\Lambda = 0$ using *Mathematica* [29]. Imposing the initial conditions corresponding to contraction, one arrives at the bounce type plots of $a(t)$, with a smooth transition between the contracting and expanding phases. These plots are shown in Fig. 2. The last curve clearly shows that the Hubble parameter H evolves smoothly through the bounce region.

The last point concerning the solutions without the cosmological constant is that the general shape of the bouncing solutions does not depend on the values of parameters and on the details of initial conditions. Thus, the analytic results from the previous subsection are perfectly well confirmed, and there are no issues with the stability in this model of the bounce. When the cosmological constant term is positive, the numerical analysis shows that the bounce-type solutions remain. However, with the growth of the magnitude of Λ, the plots become narrow. The plots obtained with different values of

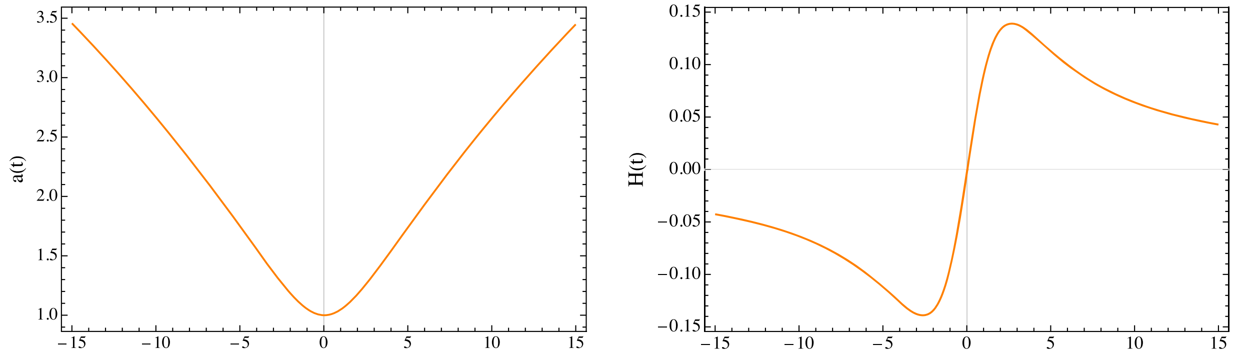

FIG. 2. Numerical solution for the scale factor $a(t)$ in the presence of the anomalous radiation term. We assumed the value $\beta g^2 F_0^2 = 0.1$ in the Planck units and the initial conditions $a(0) = 1$ and $\dot{a}(0) = -10^{-3} H_0$. The left plot shows $a(t)$ in the range $-15 \leq t \leq 15$. The right plot shows the Hubble parameter $H(t)$.

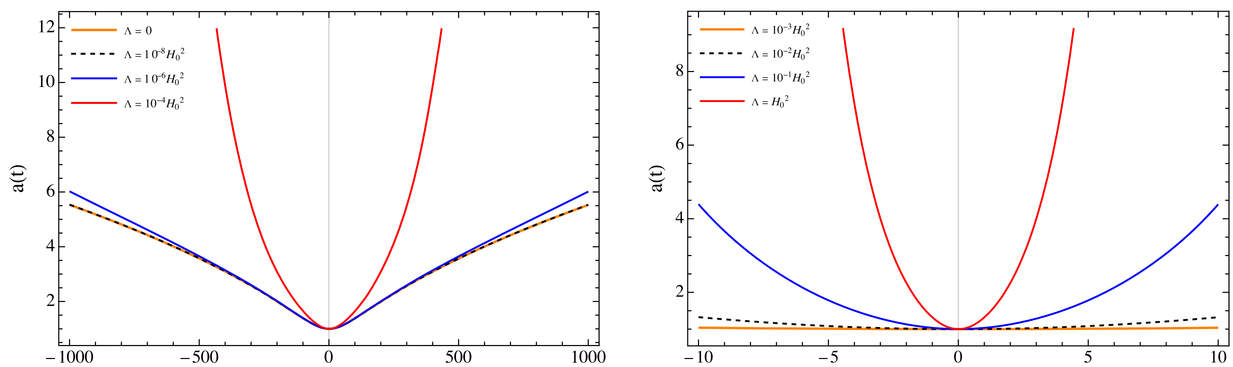

FIG. 3. Numerical solution for the scale factor $a(t)$ in the presence of the anomalous radiation term and positive cosmological constant. The values used for getting the numerical solutions are indicated on the plots.

Λ and ranges of t, are presented in Fig. 3. We adopt Planck units for time t in all the plots.

All the mentioned features concern only the positive cosmological constant. Let me mention that, in the case with the negative cosmological constant, there is a nonsingular cyclic behavior of $a(t)$. The difference with the known cyclic models (see, e.g., [30], also [31,32], and references therein) is that, in the present case the frequency of the oscillations is very high. Since we do not have physical interpretation of this type of solution, it will not be discussed in detail here.

IV. QUANTITATIVE ESTIMATES AND ANALYSIS

The consideration of the physical significance of the bounce solution starts with the note that, in the expanding or contracting Universe, the typical energy of a photon, or the temperature of the background radiation, is inverse to the scale factor, i.e.,

$$\frac{a}{a_0} = \frac{T_0}{T}. \tag{18}$$

Thus, the solution (13) implies the following estimate for the energy in the bounce point:

$$T_m = T_0 \exp\left\{\frac{3}{2\pi\beta g^2}\right\}. \tag{19}$$

Taking the values corresponding to QED, the coupling satisfies $\alpha = g^2/(4\pi) \approx 1/137$, which provides a very pessimistic estimate of $T_m \gg 10^{100}$ GeV. This means, the bounce solution occurs at the energies in the very deep trans-Planckian regime, i.e., far beyond the framework of the approximation we use. The conclusion is that the consideration based on QED does not form a sound basis for the semiclassical bounce model without additional assumptions. Is it possible to get a better estimate in more general theories?

The expression (19) is quite sensible to the magnitude of the product βg^2, owing to the exponential dependence. It is clear that the numerical estimate for the bounce may be improved in two ways, namely by increasing the value of g and increasing the beta function, according to the general formula (1) and beyond this formula. According to interpretation (ii) in Sec. III, we can assume that the temperature T_0 is of the order of grand unification scale M_X or slightly lower, such that all matter particles have high kinetic energies and their masses are negligible. Then the definition (7) should be modified. Indeed, it is not sufficient to replace $\bar{F}^2 \to \bar{\mathcal{L}}$, where the last symbol indicates the covariant Lagrangian of the whole theory, including fermions, scalars, and vectors, at the point a_0. The reason is that the term ρ_{r0} in the main equation (10), should be interpreted as the energy density of the whole contents of the Universe at the corresponding high energy scale. The product $\beta g^2 F^2$ should be replaced by the sum of the terms corresponding to different parts of the Lagrangian. Then the expression (19) should be replaced by

$$T_m = T_0 \exp\left\{\frac{3\sum_k \bar{\rho}_k}{2\pi \sum_k \beta_k g_k^2 \bar{\mathcal{L}}_k}\right\}, \tag{20}$$

where index k runs over all fields in the Lagrangian. This expression is model dependent, and its evaluation is beyond the scope of the present work. Let us, anyway, list the requirements for the acceptable bounce in this framework.

(1) To have a sufficiently small ratio in the exponential in (20), at least some of the coupling constants should be large. That implies that phenomenologically successful bounce can be achieved without modifications of gravity or introducing a dedicated scalar field. However, at least part of the couplings need to be strong in this case. Consequently, one has to account for the nonperturbative effects in the corresponding QFT.

(2) The sign of the denominator in the exponential in (20) should be positive as otherwise equation (10) would not have bounce solutions.

(3) The magnitude of the ratio in the exponential in (20) should be such that T_m belongs to the interval between the masses of at least some of the quantum particles and the Planck scale, where we assume modifications of the action of gravity and, probably, quantum gravitational effects.

(4) To provide a correspondence with the observational data concerning inflations, it is important that the bounce region starts and ends with a very high $|H_0|$, e.g., in the interval 10^{11}–10^{13} GeV. Without modifying the gravitational action, this means that the initial point a_0 corresponds to the temperature (typical energy) $T_0 \sim [H_0^2 M_P^2]^{1/4}$. On the other hand, the simplest description of inflation is the Starobinsky model [14], that corresponds to adding the R^2 term with the coefficient about 5×10^8 [33]. We plan to explore this extension of the model described above in the future work [34], but assuming that this extra term does not have a dramatic effect on the value of T_0, we arrive at the narrow interval of Hubble parameters $-|H_0| < H < |H_0|$ and the temperatures $T_0 < T < M_P$.

The last observation concerns the first of the listed points. In the case of strong coupling, the one-loop approximation which we used here is not appropriate. The required modifications do not reduce to the change of the beta functions and the corresponding modifications in the anomaly. The point is that the first order in σ in Eq. (5) and in the similar extended formulas related to (20) reflect only the violation of local conformal symmetry corresponding to the first logarithms, such as terms proportional to $L = \log(\Box/\mu^2)$ in the UV form factors (see, e.g., [16] for detailed explanation).

Let us use this information as a hint to what may happen at higher loops. At the second loop, there is certainly the L^2-type addition in the form factor of the $F_{\mu\nu}F^{\mu\nu}$ term in the electromagnetic sector; in the third loop there will be the L^3-type addition, etc. Let us stress that these extra logarithms are companions of the leading divergences of the theory before the renormalization is applied. Thus, since the underlying theory is renormalizable, the structure of the terms in the action remains the same, and the changes concern only the form factors. As a result, the leading-log terms in the nonperturbative regime will give the complication in the action (5),

$$\Gamma_{np} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}(R + 2\Lambda) - \frac{1}{4}\int d^4x\sqrt{-\bar{g}}\bar{F}^2\sigma B(g^2\sigma), \tag{21}$$

where $B(x)$ is some unknown function corresponding to the summation of the perturbative series.[2] Let us note that the leading logarithmic terms always enter with the coefficient g^2, and the same is true for the powers of σ, such that the argument of B should be the product $g^2\sigma$. It is clear that this modification may change the solution such as (13), including it may wash out the bounce, or modify the shape of the $a(t)$ dependencies, etc. The only thing we can say at this point is that the bounce of the described type, completely based on particle physics and without additional inputs, is possible. On the other hand, its detailed investigation requires better knowledge of many issues, such as UV completion of the Standard Model and summing up the leading logs in the UV regime.

V. CONCLUSIONS AND DISCUSSIONS

We have found an analytic solution describing the cosmological bounce without modifying the action of gravity, introducing a scalar field, or accounting for the vacuum quantum effects. The bounce occurs owing to the equilibrium between the classical radiation term and the quantum

[2] Assuming that this series is convergent, in some sense.

correction in the radiation-gravitational sector (4). The form of these loop contributions is well known and does not require anything besides the well-established results of quantum field theory. If comparing with the previously known models with bounce, the anomaly-induced correction to radiation plays the role of the phantom scalar [35].

The numerical estimates show that, in the minimalist QED framework, the bounce occurs at absurdly high energies, making the aforementioned analytic solution physically senseless. On the other hand, this estimate is exponentially dependent on the value of the strongest coupling constant of the theory. The physically acceptable bounce is possible, but this imposes strong restrictions on the underlying particle physics model beyond the Standard Model. In particular, there should be a UV regime with a strong coupling, similar to what is required for the fully QFT-based stable version of Starobinsky inflation [36].

The mentioned conditions do not look completely impossible to satisfy, but, at the present state of the art, it is not feasible to state that this kind of bounce is a realistic scenario to avoid singularity. Anyway, we can conclude that, in principle, the semiclassical effects in the radiation sector at the scale of grand unification may provide the singularity avoidance without additional *ad hoc* assumptions.

Another open question is the stability of the bounce model under discussion under the density and metric perturbations. This issue is typically complicated in all bounce models. The reason is that, in the vicinity of the bounce, the time derivative $\dot{H}$ is necessary positive, and this leads to the violation of the null energy condition (NEC). This feature may lead to instabilities in cosmic perturbations [5,37] (see also [38] for an alternative discussion). The analysis of the cosmological perturbations in a cosmological model is a necessary element of its development, and this is especially true for models with a bounce [6]. Only the analysis of perturbations may show whether the given model is viable or possesses inconsistencies.

In general, additional restrictions on the bounce models come from the feature that the amplitude of the primordial power spectrum is usually proportional to the energy scale of the bounce. The power spectrum is constrained by cosmic microwave background (CMB) data, which can provide more stringent constraints on the bounce energy scale. This part was extensively discussed, e.g., in [6,39,40] and more recently in [41,42]. The general situation is such that the mentioned constraints can be obtained only on the basis of the given cosmological model, as they are sensible to the structure of cosmic perturbations.[3] Thus, we leave the investigation of this issue to the next work with the analysis of perturbations.

In the existing literature, there are strong indications of that the violation of NEC by quantum corrections may not lead to the inconsistencies [43] and that the same is true in the theories with scalar fields [44,45]. Both arguments apply to our case. It is important to note that the perturbations should be analyzed not on the basis of the simplest non-covariant form of induced action (5), but using the covariant form (4), in the local representation. In this case, the theory always includes two auxiliary scalar fields [23–25] and, therefore, there are chances to arrive at the consistent model of bounce, including the perturbations free of pathologies, according to the criterion of [6]. We hope to come back to the detailed consideration of this issue and, as a first step, construct a new simplified formulation of the induced action with auxiliary fields, in a close future.

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[1] P. Coles and F. Lucchin, *Cosmology: The Origin and Evolution of Cosmic Structure*, 2nd ed. (Wiley-VCH, New York, 2002).
[2] R. C. Tolman, On the problem of the entropy of the Universe as a whole, Phys. Rev. **37**, 1639 (1931).
[3] T. Ruzmaikina and A. A. Ruzmaikin, Quadratic corrections to the Lagrangian density of the gravitational field and the singularity, JETP **30**, 372 (1970), http://www.jetp.ras.ru/cgi-bin/e/index/e/30/2/p372?a=list.
[4] A. A. Starobinsky, On a singular isotropic cosmological model, Sov. Astron. Lett. **4**, 82 (1978), https://adsabs.harvard.edu/full/1978SvAL....4...82S.
[5] M. Novello and S. E. P. Bergliaffa, Bouncing cosmologies, Phys. Rep. **463**, 127 (2008).
[6] D. Battefeld and P. Peter, A critical review of classical bouncing cosmologies, Phys. Rep. **571**, 1 (2015).
[7] T. Biswas, A. S. Koshelev, A. Mazumdar, and S. Y. Vernov, Stable bounce and inflation in non-local higher

derivative cosmology, J. Cosmol. Astropart. Phys. 08 (2012) 024.

[8] A. S. Koshelev, Stable analytic bounce in non-local Einstein-Gauss-Bonnet cosmology, Classical Quantum Gravity **30**, 155001 (2013).

[9] J. C. Fabris, A. M. Pelinson, and I. L. Shapiro, Anomaly-induced effective action for gravity and inflation, Gravitation Cosmol. **6**, 59 (2000); J. C. Fabris, A. M. Pelinson, and I. L. ShapiroarXiv:gr-qc/9810032; Nucl. Phys. B, Proc. Suppl. **95**, 78 (2001).

[10] W. C. e Silva and I. L. Shapiro, Bounce and stability in the early cosmology with anomaly-induced corrections, Symmetry **13**, 50 (2021).

[11] J. Karouby and R. Brandenberger, A radiation bounce from the Lee-Wick construction?, Phys. Rev. D **82**, 063532 (2010); J. Karouby, T. Qiu, and R. Brandenberger, On the instability of the Lee-Wick bounce, Phys. Rev. D **84**, 043505 (2011).

[12] T. Appelquist and J. Carazzone, Infrared singularities and massive fields, Phys. Rev. D **11**, 2856 (1975).

[13] M. V. Fischetti, J. B. Hartle, and B. L. Hu, Quantum effects in the early universe. I. Influence of trace anomalies on homogeneous, isotropic, classical geometries, Phys. Rev. D **20**, 1757 (1979).

[14] A. A. Starobinski, A new type of isotropic cosmological models without singularity, Phys. Lett. **91B**, 99 (1980).

[15] A. D. Dolgov, Breaking of conformal invariance and electromagnetic field generation in the Universe, Phys. Rev. D **48**, 2499 (1993).

[16] I. L. Buchbinder and I. L. Shapiro, *Introduction to Quantum Field Theory with Applications to Quantum Gravity*, (Oxford University Press, New York, 2021).

[17] D. J. Gross and F. Wilczek, Ultraviolet behavior of non-Abelian gauge theories, Phys. Rev. Lett. **30**, 1343 (1973).

[18] H. D. Politzer, Reliable perturbative results for strong interactions, Phys. Rev. Lett. **30**, 1346 (1973).

[19] R. J. Riegert, A non-local action for the trace anomaly, Phys. Lett. **134B**, 56 (1984).

[20] E. S. Fradkin and A. A. Tseytlin, Conformal anomaly in Weyl theory and anomaly free superconformal theories, Phys. Lett. **134B**, 187 (1984).

[21] M. Giannotti and E. Mottola, The trace anomaly and massless scalar degrees of freedom in gravity, Phys. Rev. D **79**, 045014 (2009).

[22] A. M. Pelinson and I. L. Shapiro, On the scaling rules for the anomaly-induced effective action of metric and electromagnetic field, Phys. Lett. B **694**, 467 (2011).

[23] I. L. Shapiro and A. G. Jacksenaev, Gauge dependence in higher derivative quantum gravity and the conformal anomaly problem, Phys. Lett. B **324**, 286 (1994).

[24] R. Balbinot, A. Fabbri, and I. L. Shapiro, Anomaly induced effective actions and Hawking radiation, Phys. Rev. Lett. **83**, 1494 (1999); Vacuum polarization in Schwarzschild spacetime by anomaly induced effective actions, Nucl. Phys. **B559**, 301 (1999).

[25] J. C. Fabris, A. M. Pelinson, and I. L. Shapiro, On the gravitational waves on the background of anomaly-induced inflation, Nucl. Phys. **B597**, 539 (2001).

[26] R. Maier, S. Pereira, N. Pinto-Neto, and B. B. Siffert, Bouncing models with a cosmological constant, Phys. Rev. D **85**, 023508 (2012).

[27] S. Weinberg, The cosmological constant problem, Rev. Mod. Phys. **61**, 1 (1989).

[28] S. A. Bludman and M. A. Ruderman, Induced cosmological constant expected above the phase transition restoring the broken symmetry, Phys. Rev. Lett. **38**, 255 (1977).

[29] Wolfram Research, Inc., Mathematica (Version 12.0, Champaign, IL, 2019).

[30] P. J. Steinhardt and N. Turok, Cosmic evolution in a cyclic universe, Phys. Rev. D **65**, 126003 (2002).

[31] A. Ijjas and P. J. Steinhardt, A new kind of cyclic universe, Phys. Lett. B **795**, 666 (2019).

[32] I. Torres, J. C. Fabris, and O. F. Piattella, Bouncing and cyclic quantum primordial universes and the ordering problem, Classical Quantum Gravity **37**, 105005 (2020).

[33] A. A. Starobinsky, The perturbation spectrum evolving from a nonsingular initially de-Sitter cosmology and the microwave background anisotropy, Sov. Astron. Lett. **9**, 302 (1983), https://adsabs.harvard.edu/full/1983SvAL....9..302S.

[34] S. W. P. Oliveira, W. C. e Silva, and I. L. Shapiro (to be published).

[35] P. Peter and N. Pinto-Neto, Primordial perturbations in a nonsingular bouncing universe model, Phys. Rev. D **66**, 063509 (2002).

[36] T. d. P. Netto, A. M. Pelinson, I. L. Shapiro, and A. A. Starobinsky, From stable to unstable anomaly-induced inflation, Eur. Phys. J. C **76**, 544 (2016).

[37] C. Molina-Paris and M. Visser, Minimal conditions for the creation of a Friedman-Robertson-Walker universe from a "bounce," Phys. Lett. B **455**, 90 (1999).

[38] P. Peter and N. Pinto-Neto, Has the universe always expanded?, Phys. Rev. D **65**, 023513 (2001).

[39] R. Brandenberger, The matter bounce alternative to inflationary cosmology, arXiv:1206.4196.

[40] R. Brandenberger and P. Peter, Bouncing cosmologies: Progress and problems, Found. Phys. **47**, 797 (2017).

[41] Y.-F. Cai, F. Duplessis, D. A. Easson, and D.-G. Wang, Searching for a matter bounce cosmology with low redshift observations, Phys. Rev. D **93**, 043546 (2016).

[42] P. C. M. Delgado, R. Durrer, and N. Pinto-Neto, The CMB bispectrum from bouncing cosmologies, J. Cosmol. Astropart. Phys. 11 (2021) 024.

[43] L. H. Ford, The classical singularity theorems and their quantum loopholes, Int. J. Theor. Phys. **42**, 1219 (2003).

[44] V. A. Rubakov, The null energy condition and its violation, Phys. Usp. **57**, 128 (2014).

[45] A. Ijjas and P. J. Steinhardt, Classically stable nonsingular cosmological bounces, Phys. Rev. Lett. **117**, 121304 (2016).