Two-Higgs-doublet model effective field theory

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We construct the general two-Higgs doublet model effective field theory where the effects of additional new physics are parametrized by operators up to mass dimension-six. We further transform this effective theory to the Higgs basis and provide matching of the Wilson coefficients between the two descriptions. We illustrate the advantages of the Higgs basis which include the separation of operators that modify standard model couplings and masses from operators that contribute to scattering processes only, transparent correlations between scattering processes resulting from the same operator, and derivation of correlations between different operators in specific UV completions. For completeness, we also construct specific versions corresponding to four types of two-Higgs-doublet models: type-I, -II, -X, and -Y, distinguished by Z_2 symmetries which restrict the couplings of the Higgs doublets to standard model fermions. Furthermore, we derive general vacuum and stability conditions of the scalar potential in the presence of higher-dimensional terms.

I. INTRODUCTION

The effective field theories are very useful for general explorations of effects of new physics without relying on specific complete models. Among these, the standard model effective field theory (SMEFT) describes possible effects of new physics, provided that the low energy degrees of freedom are the known particles in the standard model (SM) and the new physics enters at a new scale, Λ, significantly above the electroweak (EW) scale. Below Λ, heavy degrees of freedom are integrated out and their effects are parametrized by nonrenormalizable, higher-dimensional operators with Wilson coefficients $C_i \propto \Lambda^{4-d}$, where $d > 4$ is the mass dimension of the operator [1]. Constructing all operators up to mass dimension-six [2] and up to mass dimension-eight [3] has led to a broad theoretical and experimental effort to constrain possible effects of new physics.

The two-Higgs-doublet model (2HDM) is one of the simplest extensions of the SM [4]. While the new particles (two neutral and a pair of charged Higgs bosons) may be very heavy and thus their effects would be well described by the SMEFT resulting from such a model, they could also be light, within the reach of the Large Hadron Collider (LHC) or future colliders, in which case the new particles

should be kept in the low energy theory. However, the 2HDM could by further extended by new particles, for example new quarks and leptons, an even richer scalar sector, or new gauge bosons. Just like for the SMEFT, it is possible to parametrize the effects of these extra particles by operators of higher dimensions, provided that the extra particles are above the scale of additional Higgs bosons.

In this paper, we construct the general 2HDM effective field theory (2HDM EFT) where the low-energy theory is that of the 2HDM and the effects of additional new physics, possibly entering at a scale far above the masses of new Higgs bosons, are parametrized by operators of mass dimension-five and -six. We assume the $SU(2)_L \times U(1)_Y$ theory is linear and all parameters in the scalar potential are complex. We also construct specific versions of 2HDM EFTs, corresponding to four types of 2HDMs: type-I, -II, -X, and -Y, distinguished by Z_2 symmetries that restrict the couplings of Higgs doublets to SM fermions. We will show that each model allows a unique set of operators as a consequence of specific Z_2 charge assignments for fermions. Furthermore, we transform the general 2HDM EFT to the Higgs basis and provide matching of the Wilson coefficients between the two descriptions. In the Higgs basis the SM degrees of freedom are contained in one doublet, H_1, and all additional Higgses are in another doublet, H_2 [5–8]. We illustrate the advantages of working with the 2HDM EFT in the Higgs basis that include the separation of operators that modify SM couplings and masses from operators that contribute to scattering processes only, transparent correlations between scattering processes resulting from the same operator, and derivation of correlations between different operators in specific UV

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completions. For completeness, we also derive general vacuum and stability conditions of the scalar potential in the presence of higher-dimensional terms.

Previously, the 2HDM EFT has been studied in the case of a CP-conserving Higgs potential [9]. In this paper, we find agreement with the results therein, with the exception that in all four types of CP-conserving 2HDMs, we find twice as many operators which modify quark and lepton masses, as well as operators involving covariant derivatives acting on either doublet contracted with right-handed quark currents.

Other groups have extended this study toward a general 2HDM EFT [10,11], where Ref. [11] has also counted the independent operators through the Hilbert series method. We note however several disagreements between these references, Ref. [9], and this paper, notably on constructing independent derivative operators involving the Higgs doublets. The 2HDM EFT in the Higgs basis has not been previously discussed.

Other formalisms include a nonlinear type of EFT, called Higgs effective field theory, which assumes the Higgs is a SM gauge singlet and all additional SM scalars such as G and $G^{\pm}$, longitudinal Goldstone modes of the Z and $W^{\pm}$, are placed in an $SU(2)$ triplet [12,13]. Comparisons between this and the SM effective field theory have been collected in Ref. [14]. More recently, a 2HDM equivalent of this has been studied [15]. Furthermore, SMEFT resulting from theories with extended Higgs sectors has been presented in Refs. [16–18]. Integrating out the additional Higgses is accomplished by working in the Higgs basis where the mass scale of heavy Higgses is the scale of new physics, Λ. In this paper, we make no assumptions about the masses of new Higgses that can be anywhere between the EW scale and the UV cutoff Λ.

This paper is organized as follows. In Sec. II, we define all parameters and conventions, an construct the entire set of effective operators in the general 2HDM up to mass dimension-six. In Sec. III, we discuss the four specific kinds of 2HDM EFTs resulting from specific Z_2 charge

assignments and discuss what operators are common to all 2HDMs and what are unique for a specified model. We transform the general 2HDM EFT into the Higgs basis, and discuss advantages of working in the Higgs basis in Sec. IV. We conclude in Sec. V. We also provide two Appendixes: in Appendix A we derive general vacuum and stability conditions of the scalar potential in the presence of higher-dimensional terms; and in Appendix B we list the complete expressions for all dimension-four, -five, and -six terms translated to the Higgs basis.

II. CONSTRUCTING THE EFFECTIVE FIELD THEORY

A. The standard basis definitions and parameters

To start, we adopt the conventions of [2] and construct the basis of linearly independent operators in a general 2HDM similar to that of the SMEFT Warsaw basis. For a given UV completion, where heavy degrees of freedom are decoupled at some high scale Λ, the effects of new physics are parametrized by higher-dimensional terms appearing alongside the fully renormalizable two-Higgs doublet model. Up to mass dimension-six, the Lagrangian is defined as

$$\mathcal{L} = \mathcal{L}_{\text{2HDM}}^{(4)} + \sum_i C_i^{(5)} \mathcal{O}_i^{(5)} + \sum_i C_i^{(6)} \mathcal{O}_i^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right), \quad (1)$$

where the superscript denotes the mass dimension of the interactions. $\mathcal{L}_{\text{2HDM}}^{(4)}$ is the renormalizable 2HDM Lagrangian, which contains gauge and Yukawa interactions of SM fermions, as well as the scalar potential. $C_i^{(5)}$ and $C_i^{(6)}$ are the Wilson coefficients of the mass dimension-five and -six operators, respectively, and for compactness, the mass scale is contained in the Wilson coefficients.

The renormalizable 2HDM Lagrangian (summarized in [19] except for the mixed kinetic term) is given by

$$\begin{aligned}
\mathcal{L}_{\text{2HDM}}^{(4)} = &-\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} + (D_\mu\Phi_1)^\dagger D^\mu\Phi_1 + (D_\mu\Phi_2)^\dagger D^\mu\Phi_2 + (\eta(D_\mu\Phi_1)^\dagger D^\mu\Phi_2 + \text{H.c.}) \\
&- V(\Phi_1,\Phi_2) + i\bar{l}_L\slashed{D}l_L + i\bar{e}_R\slashed{D}e_R + i\bar{q}_L\slashed{D}q_L + i\bar{d}_R\slashed{D}d_R + i\bar{u}_R\slashed{D}u_R \\
&- \left(y_e^{(1)}\bar{l}_L e_R\Phi_1 + y_e^{(2)}\bar{l}_L e_R\Phi_2 + y_d^{(1)}\bar{q}_L d_R\Phi_1 + y_d^{(2)}\bar{q}_L d_R\Phi_2 + y_u^{(1)}\bar{q}_L u_R\cdot\Phi_1^\dagger + y_u^{(2)}\bar{q}_L u_R\cdot\Phi_2^\dagger + \text{H.c.}\right),
\end{aligned} \quad (2)$$

where the lepton and quark doublets are defined as $l_L = (\nu_L, e_L)^T$ and $q_L = (u_L, d_L)^T$. The gauge covariant derivative acting on an object charged under $SU(3) \times SU(2)_L \times U(1)_Y$ is defined as

$$(D_\mu q)_{\alpha i} = \left[\delta_{\alpha\beta}\delta_{ij}\partial_\mu + \frac{ig}{2}\delta_{\alpha\beta}(\tau^a)_{ij}W_\mu^a + \frac{ig_s}{2}(\lambda^a)_{\alpha\beta}\delta_{ij}G_\mu^a + ig'Y_q B_\mu\delta_{\alpha\beta}\delta_{ij}\right]q_{\beta j}, \quad (3)$$

TABLE I. $SU(3) \times SU(2)_L \times U(1)_Y$ quantum numbers of standard model leptons, quarks, and Higgs doublets. The electric charge generated after electroweak symmetry breaking is $Q = T^3 + Y$. The bold font is to denote the representation of the object charged under the respective group.

	l_L	e_R	q_L	u_R	d_R	Φ_1	Φ_2
$SU(3)$	**1**	**1**	**3**	**3**	**3**	**1**	**1**
$SU(2)_L$	**2**	**1**	**2**	**1**	**1**	**2**	**2**
$U(1)_Y$	$-\frac{1}{2}$	-1	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$

where $\tau^a/2$ and $\lambda^a/2$ are the generators of $SU(2)$ and $SU(3)$, respectively, with τ^a and λ^a being the Pauli matrices and Gell-Mann matrices. $SU(2)$ and $SU(3)$ fundamental indices are written with (ij) and $(\alpha\beta)$ where needed, respectively. The hypercharges of the particle content are provided in Table I. Field strength tensors of the gauge fields are defined as

$$
\begin{aligned}
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\
W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\epsilon^{abc}W^b_\mu W^c_\nu, \\
G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc}G^b_\mu G^c_\nu,
\end{aligned}
\tag{4}
$$

and for later purposes, the dual field strength tensor is $\tilde{X}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}X^{\rho\sigma}/2$ for each gauge field $X = B$, W, G and $\epsilon_{0123} = +1$.

The two Higgs fields $\Phi_{1,2}$ are complex $SU(2)$ doublets, each with hypercharge $Y_{1,2} = +1/2$, defined as [8]

$$
\Phi_1 = \begin{pmatrix} \Phi_1^+ \\ \Phi_1^0 \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} \Phi_2^+ \\ \Phi_2^0 \end{pmatrix}.
\tag{5}
$$

For $SU(2)$ invariant combinations of doublets involving $\mathbf{2} \times \mathbf{2}$ or $\bar{\mathbf{2}} \times \bar{\mathbf{2}}$, contraction of $SU(2)$ indices with antisymmetric ϵ_{ij} are noted with an explicit "$\cdot$", e.g. $\bar{q}_L u_R \cdot \Phi^\dagger_{1,2} = (\bar{q}_L)_1 u_R \epsilon_{12}(\Phi^\dagger_{1,2})_2 + (\bar{q}_L)_2 u_R \epsilon_{21}(\Phi^\dagger_{1,2})_1 = \bar{u}_L u_R \Phi^{0*}_{1,2} - \bar{d}_L u_R \Phi^-_{1,2}$, where $\epsilon_{12} = -\epsilon_{21} = +1$.

The most general scalar potential in a 2HDM is

$$
\begin{aligned}
V(\Phi_1, \Phi_2) = {}& m_1^2(\Phi_1^\dagger\Phi_1) + m_2^2(\Phi_2^\dagger\Phi_2) + (m_{12}^2\Phi_1^\dagger\Phi_2 + \text{H.c.}) \\
& + \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
& + \left(\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \lambda_6(\Phi_1^\dagger\Phi_1)\Phi_1^\dagger\Phi_2 + \lambda_7(\Phi_2^\dagger\Phi_2)\Phi_1^\dagger\Phi_2 + \text{H.c.}\right),
\end{aligned}
\tag{6}
$$

where the parameters $m_1^2, m_2^2, \lambda_1, \lambda_2, \lambda_3$, and λ_4 are real by Hermiticity, while $m_{12}^2, \lambda_5, \lambda_6$, and λ_7 are in general complex and can introduce CP-violating interactions in the scalar sector. Moreover, specifying a Z_2 symmetry on Φ_1 and Φ_2 restricts the potential to containing only λ_5 (and m_{12}^2 if soft-breaking terms are allowed).[1] Enforcing a Z_2 symmetry will be discussed in detail in the next section. This scalar potential will be modified in the presence of higher-dimensional terms, affecting the vacuum conditions of the extrema, concavity, and stability of the potential for asymptotically large values in field space. These conditions are collected in Appendix A and should be compared to the corresponding ones resulting from Eq. (6) only [21].

The mixed kinetic term containing η can be rotated away via the nonunitary transformation

$$
(\Phi_1, \Phi_2) \rightarrow \left(\frac{\sqrt{\eta^*}\Phi_1 + \sqrt{\eta}\Phi_2}{2\sqrt{|\eta|(1+|\eta|)}} \pm \frac{\sqrt{\eta^*}\Phi_1 - \sqrt{\eta}\Phi_2}{2\sqrt{|\eta|(1-|\eta|)}}\right);
\tag{7}
$$

however, in the most general sense (e.g. in the CP-violating 2HDM where $m_{12}^2, \lambda_{5,6,7} \in \mathbb{C}$), η will be complex and generated at the loop level even in this diagonal basis. The renormalization of the 2HDM with a softly broken Z_2 symmetry was discussed in great detail in [22]. A general 2HDM can include this kinetic term or not by choosing different renormalization schemes, where these are related by scale-dependent field redefinitions [23]. This detail about considering the mixed kinetic term was not mentioned previously in [10,11] and we do so here to make the reader aware as we choose to include it. Including the mixed kinetic term in the Lagrangian is relevant for the equations of motion of Φ_1 and Φ_2, and after electroweak symmetry breaking (EWSB), the proper canonical definitions of scalar and vector boson fields.

After EWSB, the neutral components of the scalar doublets develop vacuum expectation values (VEVs) $\langle\Phi_1^0\rangle = v_1$ and $\langle\Phi_2^0\rangle = v_2$, whereby $v = \sqrt{v_1^2 + v_2^2} = 174$ GeV and their ratio is parametrized by $v_2/v_1 = \tan\beta$. The angle $\hat{\alpha}$ rotates the CP-even scalars h and H to the physical basis, while $\hat{\beta}$ diagonalizes the CP-odd scalars G and A, and $\hat{\beta}^\pm$ diagonalizes the

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[1]The softly broken term is vital as it allows for a large mass spectrum for additional Higgses while respecting unitarity and perturbativity. When $m_{12}^2 \gg 8\pi v^2$ for sufficiently heavy masses, $\tan\beta$ becomes largely unconstrained by direct searches [20].

TABLE II.   Operators with left- and right-handed fermions with either scalar doublets or vector bosons in the standard basis for a general 2HDM. Each operator has a distinct Hermitian conjugate, giving a total of 80 operators. Dipole operators are defined with $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$.

$\psi^2\phi^3$	$\mathcal{O}_{l\Phi_1}^{(11)} = \bar{l}_L e_R \Phi_1(\Phi_1^\dagger\Phi_1)$	$\mathcal{O}_{d\Phi_1}^{(11)} = \bar{q}_L d_R \Phi_1(\Phi_1^\dagger\Phi_1)$	$\mathcal{O}_{u\Phi_1}^{(11)} = \bar{q}_L u_R \cdot \Phi_1^\dagger(\Phi_1^\dagger\Phi_1)$
	$\mathcal{O}_{l\Phi_1}^{(22)} = \bar{l}_L e_R \Phi_1(\Phi_2^\dagger\Phi_2)$	$\mathcal{O}_{d\Phi_1}^{(22)} = \bar{q}_L d_R \Phi_1(\Phi_2^\dagger\Phi_2)$	$\mathcal{O}_{u\Phi_1}^{(22)} = \bar{q}_L u_R \cdot \Phi_1^\dagger(\Phi_2^\dagger\Phi_2)$
	$\mathcal{O}_{l\Phi_1}^{(21)} = \bar{l}_L e_R \Phi_1(\Phi_2^\dagger\Phi_1)$	$\mathcal{O}_{d\Phi_1}^{(21)} = \bar{q}_L d_R \Phi_1(\Phi_2^\dagger\Phi_1)$	$\mathcal{O}_{u\Phi_1}^{(21)} = \bar{q}_L u_R \cdot \Phi_1^\dagger(\Phi_2^\dagger\Phi_1)$
	$\mathcal{O}_{l\Phi_1}^{(12)} = \bar{l}_L e_R \Phi_1(\Phi_1^\dagger\Phi_2)$	$\mathcal{O}_{d\Phi_1}^{(12)} = \bar{q}_L d_R \Phi_1(\Phi_1^\dagger\Phi_2)$	$\mathcal{O}_{u\Phi_1}^{(12)} = \bar{q}_L u_R \cdot \Phi_1^\dagger(\Phi_1^\dagger\Phi_2)$
	$\mathcal{O}_{l\Phi_2}^{(22)} = \bar{l}_L e_R \Phi_2(\Phi_2^\dagger\Phi_2)$	$\mathcal{O}_{d\Phi_2}^{(22)} = \bar{q}_L d_R \Phi_2(\Phi_2^\dagger\Phi_2)$	$\mathcal{O}_{u\Phi_2}^{(22)} = \bar{q}_L u_R \cdot \Phi_2^\dagger(\Phi_2^\dagger\Phi_2)$
	$\mathcal{O}_{l\Phi_2}^{(11)} = \bar{l}_L e_R \Phi_2(\Phi_1^\dagger\Phi_1)$	$\mathcal{O}_{d\Phi_2}^{(11)} = \bar{q}_L d_R \Phi_2(\Phi_1^\dagger\Phi_1)$	$\mathcal{O}_{u\Phi_2}^{(11)} = \bar{q}_L u_R \cdot \Phi_2^\dagger(\Phi_1^\dagger\Phi_1)$
	$\mathcal{O}_{l\Phi_2}^{(21)} = \bar{l}_L e_R \Phi_2(\Phi_2^\dagger\Phi_1)$	$\mathcal{O}_{d\Phi_2}^{(21)} = \bar{q}_L d_R \Phi_2(\Phi_2^\dagger\Phi_1)$	$\mathcal{O}_{u\Phi_2}^{(21)} = \bar{q}_L u_R \cdot \Phi_2^\dagger(\Phi_2^\dagger\Phi_1)$
	$\mathcal{O}_{l\Phi_2}^{(12)} = \bar{l}_L e_R \Phi_2(\Phi_1^\dagger\Phi_2)$	$\mathcal{O}_{d\Phi_2}^{(12)} = \bar{q}_L d_R \Phi_2(\Phi_1^\dagger\Phi_2)$	$\mathcal{O}_{u\Phi_2}^{(12)} = \bar{q}_L u_R \cdot \Phi_2^\dagger(\Phi_1^\dagger\Phi_2)$
$\psi^2 X\phi$	$\mathcal{O}_{lB\Phi_1} = \bar{l}_L \sigma^{\mu\nu} e_R \Phi_1 B_{\mu\nu}$	$\mathcal{O}_{dB\Phi_1} = \bar{q}_L \sigma^{\mu\nu} d_R \Phi_1 B_{\mu\nu}$	$\mathcal{O}_{uB\Phi_1} = \bar{q}_L \sigma^{\mu\nu} u_R \cdot \Phi_1^\dagger B_{\mu\nu}$
	$\mathcal{O}_{lW\Phi_1} = \bar{l}_L \sigma^{\mu\nu} e_R \tau^a \Phi_1 W_{\mu\nu}^a$	$\mathcal{O}_{dW\Phi_1} = \bar{q}_L \sigma^{\mu\nu} d_R \tau^a \Phi_1 W_{\mu\nu}^a$	$\mathcal{O}_{uW\Phi_1} = \bar{q}_L \sigma^{\mu\nu} u_R \tau^a \cdot \Phi_1^\dagger W_{\mu\nu}^a$
	$\mathcal{O}_{lB\Phi_2} = \bar{l}_L \sigma^{\mu\nu} e_R \Phi_2 B_{\mu\nu}$	$\mathcal{O}_{dG\Phi_1} = \bar{q}_L \sigma^{\mu\nu} \lambda^a d_R \Phi_1 G_{\mu\nu}^a$	$\mathcal{O}_{uG\Phi_1} = \bar{q}_L \sigma^{\mu\nu} \lambda^a u_R \cdot \Phi_1^\dagger G_{\mu\nu}^a$
	$\mathcal{O}_{lW\Phi_2} = \bar{l}_L \sigma^{\mu\nu} e_R \tau^a \Phi_2 W_{\mu\nu}^a$	$\mathcal{O}_{dB\Phi_2} = \bar{q}_L \sigma^{\mu\nu} d_R \Phi_2 B_{\mu\nu}$	$\mathcal{O}_{uB\Phi_2} = \bar{q}_L \sigma^{\mu\nu} u_R \cdot \Phi_2^\dagger B_{\mu\nu}$
		$\mathcal{O}_{dW\Phi_2} = \bar{q}_L \sigma^{\mu\nu} d_R \tau^a \Phi_2 W_{\mu\nu}^a$	$\mathcal{O}_{uW\Phi_2} = \bar{q}_L \sigma^{\mu\nu} u_R \tau^a \cdot \Phi_2^\dagger W_{\mu\nu}^a$
		$\mathcal{O}_{dG\Phi_2} = \bar{q}_L \sigma^{\mu\nu} \lambda^a d_R \Phi_2 G_{\mu\nu}^a$	$\mathcal{O}_{uG\Phi_2} = \bar{q}_L \sigma^{\mu\nu} \lambda^a u_R \cdot \Phi_2^\dagger G_{\mu\nu}^a$

charged fields $G^\pm$ and $H^\pm$, yielding the following doublet components[2]:

$$\Phi_1^0 = v_1 + \frac{1}{\sqrt{2}}(-h\sin\hat\alpha + H\cos\hat\alpha)$$
$$+ \frac{i}{\sqrt{2}}(G\cos\hat\beta - A\sin\hat\beta),$$
$$\Phi_2^0 = v_2 + \frac{1}{\sqrt{2}}(h\cos\hat\alpha + H\sin\hat\alpha)$$
$$+ \frac{i}{\sqrt{2}}(G\sin\hat\beta + A\cos\hat\beta),$$
$$\Phi_1^\pm = G^\pm\cos\hat\beta^\pm - H^\pm\sin\hat\beta^\pm,$$
$$\Phi_2^\pm = G^\pm\sin\hat\beta^\pm + H^\pm\cos\hat\beta^\pm, \tag{8}$$

where h is the SM Higgs field, G and $G^\pm$ are the longitudinal Goldstone modes for the Z and $W^\pm$ bosons, respectively, and H, A, and $H^\pm$ are the additional Higgs degrees of freedom.

In the mass sector, note that the presence of two doublets does not guarantee a simultaneous diagonalization of the lepton and quark mass matrices $y_{(e,d,u)}^{(1)}$ and $y_{(e,d,u)}^{(2)}$. This introduces so-called flavor changing neutral currents of SM

fermions, where tree-level vertices involving scalars are nondiagonal in flavor space. These can be large due to $\tan\beta$ enhancements, but are highly constrained by many experimental measurements.

B. The complete set of operators up to dimension-six

Given the quantum numbers of SM fields in addition to two new doublets defined in Table I, we now proceed to construct the entire set of independent operators. The procedure is a straightforward extension of [2], notably, one can imagine replacing the SM Higgs doublet with either Φ_1 or Φ_2 since they all have identical quantum numbers. However, this generalization introduces non-Hermitian terms which mix Φ_1 and Φ_2 currents, generating new singlet combinations.

In the SMEFT, there is only one dimension-five operator, the so-called Weinberg operator [24]. However, in the general 2HDM, there are three dimension-five operators present [25] which may affect the mass generation of the three neutrinos. They are defined as

$$\mathcal{O}_{\nu\nu\Phi}^{(11)} = (\Phi_1 \cdot l_L)^T\mathbf{C}(\Phi_1 \cdot l_L),$$
$$\mathcal{O}_{\nu\nu\Phi}^{(22)} = (\Phi_2 \cdot l_L)^T\mathbf{C}(\Phi_2 \cdot l_L),$$
$$\mathcal{O}_{\nu\nu\Phi}^{(12)} = (\Phi_1 \cdot l_L)^T\mathbf{C}(\Phi_2 \cdot l_L), \tag{9}$$

where $\mathbf{C}$ is the charge conjugation matrix in the Dirac representation, $\mathbf{C} = i\gamma^2\gamma^0$. The combination of $(\Phi_2 \cdot l_L)^T\mathbf{C}(\Phi_1 \cdot l_L)$ is redundant with the last operator.

Now, we proceed with constructing operators at mass dimension-six. Operators involving fermions and either scalar doublets or vector bosons are provided in Table II,

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[2]The presence of dimension-six terms affects the physical Higgs spectrum; canonical field redefinitions and corrections to their masses must be considered. The vacuum angle β thus no longer diagonalizes the CP-odd scalars and charged fields in Eq. (8) as in the dimension-four theory. The difference between the vacuum and each diagonalization angle is $\beta - \hat\beta^{(\pm)} \sim \mathcal{O}(v^4/\Lambda^2 M^2)$, where M^2 is either m_A^2 or $m_{H^\pm}^2$ depending on the angle.

TABLE III. Operators with left- or right-handed fermion currents involving covariant derivatives in the standard basis for a general 2HDM. There are a total of 34 operators.

$\psi^2\phi^2 D$		
	$\mathcal{O}_{\Phi e}^{(11)} = (\Phi_1^\dagger i\overleftrightarrow{D}_\mu \Phi_1)(\bar{e}_R \gamma^\mu e_R)$	$\mathcal{O}_{\Phi d}^{(11)} = (\Phi_1^\dagger i\overleftrightarrow{D}_\mu \Phi_1)(\bar{d}_R \gamma^\mu d_R)$
	$\mathcal{O}_{\Phi e}^{(22)} = (\Phi_2^\dagger i\overleftrightarrow{D}_\mu \Phi_2)(\bar{e}_R \gamma^\mu e_R)$	$\mathcal{O}_{\Phi d}^{(22)} = (\Phi_2^\dagger i\overleftrightarrow{D}_\mu \Phi_2)(\bar{d}_R \gamma^\mu d_R)$
	$\mathcal{O}_{\Phi e}^{(12)} = (\Phi_1^\dagger i\overleftrightarrow{D}_\mu \Phi_2)(\bar{e}_R \gamma^\mu e_R) + \text{H.c.}$	$\mathcal{O}_{\Phi d}^{(12)} = (\Phi_1^\dagger i\overleftrightarrow{D}_\mu \Phi_2)(\bar{d}_R \gamma^\mu d_R) + \text{H.c.}$
	$\mathcal{O}_{\Phi l}^{(11)[1]} = (\Phi_1^\dagger i\overleftrightarrow{D}_\mu \Phi_1)(\bar{l}_L \gamma^\mu l_L)$	$\mathcal{O}_{\Phi u}^{(11)} = (\Phi_1^\dagger i\overleftrightarrow{D}_\mu \Phi_1)(\bar{u}_R \gamma^\mu u_R)$
	$\mathcal{O}_{\Phi l}^{(22)[1]} = (\Phi_2^\dagger i\overleftrightarrow{D}_\mu \Phi_2)(\bar{l}_L \gamma^\mu l_L)$	$\mathcal{O}_{\Phi u}^{(22)} = (\Phi_2^\dagger i\overleftrightarrow{D}_\mu \Phi_2)(\bar{u}_R \gamma^\mu u_R)$
	$\mathcal{O}_{\Phi l}^{(12)[1]} = (\Phi_1^\dagger i\overleftrightarrow{D}_\mu \Phi_2)(\bar{l}_L \gamma^\mu l_L) + \text{H.c.}$	$\mathcal{O}_{\Phi u}^{(12)} = (\Phi_1^\dagger i\overleftrightarrow{D}_\mu \Phi_2)(\bar{u}_R \gamma^\mu u_R) + \text{H.c.}$
	$\mathcal{O}_{\Phi l}^{(11)[3]} = (\Phi_1^\dagger i\overleftrightarrow{D}_\mu^a \Phi_1)(\bar{l}_L \tau^a \gamma^\mu l_L)$	$\mathcal{O}_{\Phi q}^{(11)[1]} = (\Phi_1^\dagger i\overleftrightarrow{D}_\mu \Phi_1)(\bar{q}_L \gamma^\mu q_L)$
	$\mathcal{O}_{\Phi l}^{(22)[3]} = (\Phi_2^\dagger i\overleftrightarrow{D}_\mu^a \Phi_2)(\bar{l}_L \tau^a \gamma^\mu l_L)$	$\mathcal{O}_{\Phi q}^{(22)[1]} = (\Phi_2^\dagger i\overleftrightarrow{D}_\mu \Phi_2)(\bar{q}_L \gamma^\mu q_L)$
	$\mathcal{O}_{\Phi l}^{(12)[3]} = (\Phi_1^\dagger i\overleftrightarrow{D}_\mu^a \Phi_2)(\bar{l}_L \tau^a \gamma^\mu l_L) + \text{H.c.}$	$\mathcal{O}_{\Phi q}^{(12)[1]} = (\Phi_1^\dagger i\overleftrightarrow{D}_\mu \Phi_2)(\bar{q}_L \gamma^\mu q_L) + \text{H.c.}$
		$\mathcal{O}_{\Phi q}^{(11)[3]} = (\Phi_1^\dagger i\overleftrightarrow{D}_\mu^a \Phi_1)(\bar{q}_L \tau^a \gamma^\mu q_L)$
		$\mathcal{O}_{\Phi q}^{(22)[3]} = (\Phi_2^\dagger i\overleftrightarrow{D}_\mu^a \Phi_2)(\bar{q}_L \tau^a \gamma^\mu q_L)$
		$\mathcal{O}_{\Phi q}^{(12)[3]} = (\Phi_1^\dagger i\overleftrightarrow{D}_\mu^a \Phi_2)(\bar{q}_L \tau^a \gamma^\mu q_L) + \text{H.c.}$
		$\mathcal{O}_{\Phi ud}^{(11)} = (\Phi_1 \cdot iD_\mu \Phi_1)(\bar{u}_R \gamma^\mu d_R) + \text{H.c.}$
		$\mathcal{O}_{\Phi ud}^{(22)} = (\Phi_2 \cdot iD_\mu \Phi_2)(\bar{u}_R \gamma^\mu d_R) + \text{H.c.}$
		$\mathcal{O}_{\Phi ud}^{(21)} = (\Phi_2 i \cdot \overleftrightarrow{D}_\mu \Phi_1)(\bar{u}_R \gamma^\mu d_R) + \text{H.c.}$

whereas purely bosonic operators are displayed in Tables III and IV, and four-fermion currents in Table V. There are a total of $80 + 34 + 76 + 38 = 228$ operators (including Hermitian conjugation when needed); 144 more than the operators present in the SMEFT Warsaw basis. This agrees with the independent counting introduced via the 2HDM Hilbert series in Ref. [11]. The 38 four-fermion operators [including baryon-number (B)-violating terms] and 4 pure gauge operators are identical to the SMEFT Warsaw basis and are unaffected by the additional Higgs doublet. However, we will see in the next section that the four-fermion operators become restricted once couplings to fermions are specified by a Z_2 symmetry in different types of 2HDMs.

While keeping the notation as close as possible to SMEFT operators in the Warsaw basis, we choose a simple labeling of operators where the superscript $(\cdot\cdot)$ represents the contraction of Higgs doublets, where the first entry corresponds to the doublet under Hermitian conjugation (except for operators containing ϵ in charged quark currents, e.g. $\mathcal{O}_{\Phi ud}^{(11)}$ in Table III). Labels containing square brackets denote operators contracting $SU(2)$ doublets with δ_{ij}, [1], or with $(\tau^a)_{ij}$, [3]. Subscripts labeled first with the lepton doublet, l_L, or quark doublet, $q_L = (u_L, d_L)^T$ followed by a scalar doublet are contracted together in operators such as $\mathcal{O}_{(l,u,d)\Phi_1}^{(11)}$ in Table II. Furthermore, for simplicity, note that we suppress flavor indices in Tables II, III, and V. Note that the counting of the 228 operators (as well as any subset of operators) does not include flavor indices.

Operators containing covariant derivatives acting on the Higgs doublets $\Phi_{1,2}$ are constructed in an antisymmetric way, i.e.

$$\Phi_{1,2}^\dagger i\overleftrightarrow{D}_\mu \Phi_{1,2} \equiv i\left(\Phi_{1,2}^\dagger(D_\mu \Phi_{1,2}) - (D_\mu \Phi_{1,2})^\dagger \Phi_{1,2}\right),$$

$$\Phi_{1,2}^\dagger i\overleftrightarrow{D}_\mu^a \Phi_{1,2} \equiv i\left(\Phi_{1,2}^\dagger \tau^a(D_\mu \Phi_{1,2}) - (D_\mu \Phi_{1,2})^\dagger \tau^a \Phi_{1,2}\right).$$

(10)

If the doublets in the operator are identical, the above equations are Hermitian. However, because Φ_1 and Φ_2 have the same quantum numbers, non-Hermitian combinations such as $\Phi_1^\dagger i\overleftrightarrow{D}_\mu \Phi_2$ are also permitted. Other combinations involving antisymmetric ϵ and covariant derivatives appear as $\Phi_{1,2} \cdot i\overleftrightarrow{D}_\mu \Phi_{1,2} = 2\Phi_{1,2} \cdot iD_\mu \Phi_{1,2}$. The symmetric combination, $(\Phi_{1,2}^\dagger(D_\mu \Phi_{1,2}) + (D_\mu \Phi_{1,2})^\dagger \Phi_{1,2}) = \partial_\mu(\Phi_{1,2}^\dagger \Phi_{1,2})$, is used to construct the other independent combination of operators. All other conventions and definitions on objects other than the new Higgs doublets are defined in [2].

In general, the Wilson coefficients of operators in the class of $\psi^2\phi^3$ and $\psi^2 X\phi$ are complex, and any operator labeled $\mathcal{O}_\Phi^{(21)}$ contained in $\psi^2\phi^2 D$ is non-Hermitian. Derivative operators involving the same fermion object and same $\Phi_{1,2}$ currents are Hermitian. For purely bosonic operators X^3 and $X^2\phi^2$, all operators are Hermitian with the exception of those labeled with $\mathcal{O}_\Phi^{(21)}$. ϕ^6 operators labeled with an odd number of Φ_1 or Φ_2 in the label are complex.

TABLE IV. All purely bosonic operators in the standard basis for a general 2HDM. There are a total of 76 operators.

X^3	$\mathcal{O}_W = \epsilon^{abc} W_\mu^{a\nu} W_\nu^{b\sigma} W_\sigma^{c\mu}$	$\mathcal{O}_{\tilde{W}} = \epsilon^{abc} \tilde{W}_\mu^{a\nu} W_\nu^{b\sigma} W_\sigma^{c\mu}$
	$\mathcal{O}_G = f^{abc} G_\mu^{a\nu} G_\nu^{b\sigma} G_\sigma^{c\mu}$	$\mathcal{O}_{\tilde{G}} = f^{abc} \tilde{G}_\mu^{a\nu} G_\nu^{b\sigma} G_\sigma^{c\mu}$
$X^2\phi^2$	$\mathcal{O}_{\Phi G}^{(11)} = (\Phi_1^\dagger \Phi_1) G_{\mu\nu}^a G^{a\mu\nu}$	$\mathcal{O}_{\Phi W}^{(11)} = (\Phi_1^\dagger \Phi_1) W_{\mu\nu}^a W^{a\mu\nu}$
	$\mathcal{O}_{\Phi G}^{(22)} = (\Phi_2^\dagger \Phi_2) G_{\mu\nu}^a G^{a\mu\nu}$	$\mathcal{O}_{\Phi W}^{(22)} = (\Phi_2^\dagger \Phi_2) W_{\mu\nu}^a W^{a\mu\nu}$
	$\mathcal{O}_{\Phi G}^{(21)} = (\Phi_2^\dagger \Phi_1) G_{\mu\nu}^a G^{a\mu\nu} + \text{H.c.}$	$\mathcal{O}_{\Phi W}^{(21)} = (\Phi_2^\dagger \Phi_1) W_{\mu\nu}^a W^{a\mu\nu} + \text{H.c.}$
	$\mathcal{O}_{\Phi\tilde{G}}^{(11)} = (\Phi_1^\dagger \Phi_1) \tilde{G}_{\mu\nu}^a G^{a\mu\nu}$	$\mathcal{O}_{\Phi\tilde{W}}^{(11)} = (\Phi_1^\dagger \Phi_1) \tilde{W}_{\mu\nu}^a W^{a\mu\nu}$
	$\mathcal{O}_{\Phi\tilde{G}}^{(22)} = (\Phi_2^\dagger \Phi_2) \tilde{G}_{\mu\nu}^a G^{a\mu\nu}$	$\mathcal{O}_{\Phi\tilde{W}}^{(22)} = (\Phi_2^\dagger \Phi_2) \tilde{W}_{\mu\nu}^a W^{a\mu\nu}$
	$\mathcal{O}_{\Phi\tilde{G}}^{(21)} = (\Phi_2^\dagger \Phi_1) \tilde{G}_{\mu\nu}^a G^{a\mu\nu} + \text{H.c.}$	$\mathcal{O}_{\Phi\tilde{W}}^{(21)} = (\Phi_2^\dagger \Phi_1) \tilde{W}_{\mu\nu}^a W^{a\mu\nu} + \text{H.c.}$
	$\mathcal{O}_{\Phi B}^{(11)} = (\Phi_1^\dagger \Phi_1) B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{\Phi WB}^{(11)} = (\Phi_1^\dagger \tau^a \Phi_1) W_{\mu\nu}^a B^{\mu\nu}$
	$\mathcal{O}_{\Phi B}^{(22)} = (\Phi_2^\dagger \Phi_2) B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{\Phi WB}^{(22)} = (\Phi_2^\dagger \tau^a \Phi_2) W_{\mu\nu}^a B^{\mu\nu}$
	$\mathcal{O}_{\Phi B}^{(21)} = (\Phi_2^\dagger \Phi_1) B_{\mu\nu} B^{\mu\nu} + \text{H.c.}$	$\mathcal{O}_{\Phi WB}^{(21)} = (\Phi_2^\dagger \tau^a \Phi_1) W_{\mu\nu}^a B^{\mu\nu} + \text{H.c.}$
	$\mathcal{O}_{\Phi\tilde{B}}^{(11)} = (\Phi_1^\dagger \Phi_1) \tilde{B}_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{\Phi\tilde{W}B}^{(11)} = (\Phi_1^\dagger \tau^a \Phi_1) \tilde{W}_{\mu\nu}^a B^{\mu\nu}$
	$\mathcal{O}_{\Phi\tilde{B}}^{(22)} = (\Phi_2^\dagger \Phi_2) \tilde{B}_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{\Phi\tilde{W}B}^{(22)} = (\Phi_2^\dagger \tau^a \Phi_2) \tilde{W}_{\mu\nu}^a B^{\mu\nu}$
	$\mathcal{O}_{\Phi\tilde{B}}^{(21)} = (\Phi_2^\dagger \Phi_1) \tilde{B}_{\mu\nu} B^{\mu\nu} + \text{H.c.}$	$\mathcal{O}_{\Phi\tilde{W}B}^{(21)} = (\Phi_2^\dagger \tau^a \Phi_1) \tilde{W}_{\mu\nu}^a B^{\mu\nu} + \text{H.c.}$
$\phi^4 D^2$	$\mathcal{O}_{\Phi\partial^2}^{(11)(11)} = \partial_\mu(\Phi_1^\dagger \Phi_1) \partial^\mu(\Phi_1^\dagger \Phi_1)$	$\mathcal{O}_{\Phi D}^{(11)(11)} = (\Phi_1^\dagger \overset{\leftrightarrow}{D}_\mu \Phi_1)(\Phi_1^\dagger \overset{\leftrightarrow}{D}{}^\mu \Phi_1)$
	$\mathcal{O}_{\Phi\partial^2}^{(22)(22)} = \partial_\mu(\Phi_2^\dagger \Phi_2) \partial^\mu(\Phi_2^\dagger \Phi_2)$	$\mathcal{O}_{\Phi D}^{(22)(22)} = (\Phi_2^\dagger \overset{\leftrightarrow}{D}_\mu \Phi_2)(\Phi_2^\dagger \overset{\leftrightarrow}{D}{}^\mu \Phi_2)$
	$\mathcal{O}_{\Phi\partial^2}^{(11)(22)} = \partial_\mu(\Phi_1^\dagger \Phi_1) \partial^\mu(\Phi_2^\dagger \Phi_2)$	$\mathcal{O}_{\Phi D}^{(11)(22)} = (\Phi_1^\dagger \overset{\leftrightarrow}{D}_\mu \Phi_1)(\Phi_2^\dagger \overset{\leftrightarrow}{D}{}^\mu \Phi_2)$
	$\mathcal{O}_{\Phi\partial^2}^{(21)(21)} = \partial_\mu(\Phi_2^\dagger \Phi_1) \partial^\mu(\Phi_2^\dagger \Phi_1) + \text{H.c.}$	$\mathcal{O}_{\Phi D}^{(21)(21)} = (\Phi_2^\dagger \overset{\leftrightarrow}{D}_\mu \Phi_1)(\Phi_2^\dagger \overset{\leftrightarrow}{D}{}^\mu \Phi_1) + \text{H.c.}$
	$\mathcal{O}_{\Phi\partial^2}^{(21)(12)} = \partial_\mu(\Phi_2^\dagger \Phi_1) \partial^\mu(\Phi_1^\dagger \Phi_2)$	$\mathcal{O}_{\Phi D}^{(21)(12)} = (\Phi_2^\dagger \overset{\leftrightarrow}{D}_\mu \Phi_1)(\Phi_1^\dagger \overset{\leftrightarrow}{D}{}^\mu \Phi_2)$
	$\mathcal{O}_{\Phi\partial^2}^{(21)(11)} = \partial_\mu(\Phi_2^\dagger \Phi_1) \partial^\mu(\Phi_1^\dagger \Phi_1) + \text{H.c.}$	$\mathcal{O}_{\Phi D}^{(21)(11)} = (\Phi_2^\dagger \overset{\leftrightarrow}{D}_\mu \Phi_1)(\Phi_1^\dagger \overset{\leftrightarrow}{D}{}^\mu \Phi_1) + \text{H.c.}$
	$\mathcal{O}_{\Phi\partial^2}^{(21)(22)} = \partial_\mu(\Phi_2^\dagger \Phi_1) \partial^\mu(\Phi_2^\dagger \Phi_2) + \text{H.c.}$	$\mathcal{O}_{\Phi D}^{(21)(22)} = (\Phi_2^\dagger \overset{\leftrightarrow}{D}_\mu \Phi_1)(\Phi_2^\dagger \overset{\leftrightarrow}{D}{}^\mu \Phi_2) + \text{H.c.}$
ϕ^6	$\mathcal{O}_\Phi^{(11)(11)(11)} = (\Phi_1^\dagger \Phi_1)^3$	$\mathcal{O}_\Phi^{(11)(21)(21)} = (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_1)^2 + \text{H.c.}$
	$\mathcal{O}_\Phi^{(11)(11)(22)} = (\Phi_1^\dagger \Phi_1)^2(\Phi_2^\dagger \Phi_2)$	$\mathcal{O}_\Phi^{(11)(21)(12)} = (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2)$
	$\mathcal{O}_\Phi^{(11)(22)(22)} = (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2)^2$	$\mathcal{O}_\Phi^{(22)(21)(21)} = (\Phi_2^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)^2 + \text{H.c.}$
	$\mathcal{O}_\Phi^{(11)(11)(21)} = (\Phi_1^\dagger \Phi_1)^2(\Phi_2^\dagger \Phi_1) + \text{H.c.}$	$\mathcal{O}_\Phi^{(22)(21)(12)} = (\Phi_2^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2)$
	$\mathcal{O}_\Phi^{(22)(22)(21)} = (\Phi_2^\dagger \Phi_2)^2(\Phi_2^\dagger \Phi_1) + \text{H.c.}$	$\mathcal{O}_\Phi^{(21)(21)(21)} = (\Phi_2^\dagger \Phi_1)^3 + \text{H.c.}$
	$\mathcal{O}_\Phi^{(22)(22)(22)} = (\Phi_2^\dagger \Phi_2)^3$	$\mathcal{O}_\Phi^{(21)(21)(12)} = (\Phi_2^\dagger \Phi_1)^2(\Phi_1^\dagger \Phi_2) + \text{H.c.}$
		$\mathcal{O}_\Phi^{(11)(22)(21)} = (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \text{H.c.}$

Lastly, ψ^4 operators with identical pairings of fermion fields in the first row of Table V are Hermitian.

Before we move forward, some special care is needed to construct the $\phi^4 D^2$ operators. Notably, there seems to be disagreements between Refs. [10,11] for the general 2HDM EFT, while in the limit the 2HDM is CP-conserving, there is an additional disagreement with [9] on those operators. Disagreements between Refs. [9–11] were discussed in the last reference. However, we find disagreement with all three references, and here we focus on the disagreements with Ref. [11].

Through the counting of independent operators in the Hilbert series of the 2HDM, [11] has correctly counted the $14 + 6$ (Hermitian conjugation) $= 20$ operators predicted. However, operators containing the $\Box = \partial^2$ operator called

$\mathcal{O}_\Box^{11(22)} = (\Phi_1^\dagger \Phi_1)\Box(\Phi_2^\dagger \Phi_2)$ and $\mathcal{O}_\Box^{22(11)} = (\Phi_2^\dagger \Phi_2)\Box(\Phi_1^\dagger \Phi_1)$ therein are not independent from each other due to integration by parts, yielding a total derivative and the latter combination. Furthermore, their operator labeled $\mathcal{O}_{\phi D}^{12(1)(2)} = (\Phi_1^\dagger \Phi_2)[(D^\mu \Phi_1)^\dagger (D_\mu \Phi_2)]$ can be rewritten via equations of motions in terms of box operator $(\Phi_1^\dagger \Phi_2)\Box(\Phi_1^\dagger \Phi_2)$, terms involving $\psi^2\phi^3, \phi^6$, and ϕ^4, in which the first is the Hermitian conjugate of their $\mathcal{O}_\Box^{21(21)} = (\Phi_2^\dagger \Phi_1)\Box(\Phi_2^\dagger \Phi_1)$ operator. A proper way to construct linearly independent combinations is by separating $SU(2)$ invariant combinations through the derivative operator [Eq. (10)] and its symmetric counterpart. We can begin with constructing all 16 of the following triplet combinations:

TABLE V. All four-fermion operators for a general 2HDM, identical to the SMEFT Warsaw basis. There are 38 operators, where entries in the second column of the second row are B-violating terms.

ψ^4		
	$\mathcal{O}_{ll} = (\bar{l}_L\gamma^\mu l_L)(\bar{l}_L\gamma_\mu l_L)$	$\mathcal{O}_{ee} = (\bar{e}_R\gamma^\mu e_R)(\bar{e}_R\gamma_\mu e_R)$
	$\mathcal{O}_{qq}^{(1)} = (\bar{q}_L\gamma^\mu q_L)(\bar{q}_L\gamma_\mu q_L)$	$\mathcal{O}_{dd} = (\bar{d}_R\gamma^\mu d_R)(\bar{d}_R\gamma_\mu d_R)$
	$\mathcal{O}_{qq}^{(3)} = (\bar{q}_L\tau^a\gamma^\mu q_L)(\bar{q}_L\tau^a\gamma_\mu q_L)$	$\mathcal{O}_{uu} = (\bar{u}_R\gamma^\mu u_R)(\bar{u}_R\gamma_\mu u_R)$
	$\mathcal{O}_{lq}^{(1)} = (\bar{l}_L\gamma^\mu l_L)(\bar{q}_L\gamma_\mu q_L)$	$\mathcal{O}_{ed} = (\bar{e}_R\gamma^\mu e_R)(\bar{d}_R\gamma_\mu d_R)$
	$\mathcal{O}_{le} = (\bar{l}_L\gamma^\mu l_L)(\bar{e}_R\gamma_\mu e_R)$	$\mathcal{O}_{eu} = (\bar{e}_R\gamma^\mu e_R)(\bar{u}_R\gamma_\mu u_R)$
	$\mathcal{O}_{ld} = (\bar{l}_L\gamma^\mu l_L)(\bar{d}_R\gamma_\mu d_R)$	$\mathcal{O}_{ud}^{(1)} = (\bar{u}_R\gamma^\mu u_R)(\bar{d}_R\gamma_\mu d_R)$
	$\mathcal{O}_{lu} = (\bar{l}_L\gamma^\mu l_L)(\bar{u}_R\gamma_\mu u_R)$	$\mathcal{O}_{ud}^{(8)} = (\bar{u}_R\lambda^a\gamma^\mu u_R)(\bar{d}_R\lambda^a\gamma_\mu d_R)$
	$\mathcal{O}_{qe} = (\bar{q}_L\gamma^\mu q_L)(\bar{e}_R\gamma_\mu e_R)$	$\mathcal{O}_{qd}^{(1)} = (\bar{q}_L\gamma^\mu q_L)(\bar{d}_R\gamma_\mu d_R)$
	$\mathcal{O}_{lq}^{(3)} = (\bar{l}_L\tau^a\gamma^\mu l_L)(\bar{q}_L\tau^a\gamma_\mu q_L)$	$\mathcal{O}_{qd}^{(8)} = (\bar{q}_L\lambda^a\gamma^\mu q_L)(\bar{d}_R\lambda^a\gamma_\mu d_R)$
		$\mathcal{O}_{qu}^{(1)} = (\bar{q}_L\gamma^\mu q_L)(\bar{u}_R\gamma_\mu u_R)$
		$\mathcal{O}_{qu}^{(8)} = (\bar{q}_L\lambda^a\gamma^\mu q_L)(\bar{u}_R\lambda^a\gamma_\mu u_R)$
	$\mathcal{O}_{ledq} = (\bar{l}_L e_R)(\bar{d}_R q_L) + \text{H.c.}$	$\mathcal{O}_{duq} = \epsilon^{\alpha\beta\gamma}\epsilon_{jk}((d_R^\alpha)^T\mathbf{C}u_R^\beta)((q_{Lj}^\gamma)^T\mathbf{C}l_{Lk}) + \text{H.c.}$
	$\mathcal{O}_{quqd}^{(1)} = (\bar{q}_L u_R)\cdot(\bar{q}_L d_R) + \text{H.c.}$	$\mathcal{O}_{qqu} = \epsilon^{\alpha\beta\gamma}\epsilon_{jk}((q_{Lj}^\alpha)^T\mathbf{C}q_{Lk}^\beta)((u_R^\gamma)^T\mathbf{C}e_R) + \text{H.c.}$
	$\mathcal{O}_{quqd}^{(8)} = (\bar{q}_L\lambda^a u_R)\cdot(\bar{q}_L\lambda^a d_R) + \text{H.c.}$	$\mathcal{O}_{qqq} = \epsilon^{\alpha\beta\gamma}\epsilon_{jn}\epsilon_{km}((q_{Lj}^\alpha)^T\mathbf{C}q_{Lk}^\beta)((q_{Lm}^\gamma)^T\mathbf{C}l_{Ln}) + \text{H.c.}$
	$\mathcal{O}_{lequ}^{(1)} = (\bar{l}_L e_R)\cdot(\bar{q}_L u_R) + \text{H.c.}$	$\mathcal{O}_{duu} = \epsilon^{\alpha\beta\gamma}((d_R^\alpha)^T\mathbf{C}u_R^\beta)((u_R^\gamma)^T\mathbf{C}e_R) + \text{H.c.}$
	$\mathcal{O}_{lequ}^{(3)} = (\bar{l}_L\sigma^{\mu\nu} e_R)\cdot(\bar{q}_L\sigma_{\mu\nu} u_R) + \text{H.c.}$	

$$(\Phi_1^\dagger\tau^a\Phi_1)((D_\mu\Phi_1)^\dagger\tau^a(D^\mu\Phi_1)), \qquad (\Phi_2^\dagger\tau^a\Phi_2)((D_\mu\Phi_2)^\dagger\tau^a(D^\mu\Phi_2)),$$

$$(\Phi_1^\dagger\tau^a\Phi_1)((D_\mu\Phi_2)^\dagger\tau^a(D^\mu\Phi_2)), \qquad (\Phi_2^\dagger\tau^a\Phi_2)((D_\mu\Phi_1)^\dagger\tau^a(D^\mu\Phi_1)),$$

$$(\Phi_2^\dagger\tau^a\Phi_1)((D_\mu\Phi_2)^\dagger\tau^a(D^\mu\Phi_1)) + \text{H.c.}, \qquad (\Phi_2^\dagger\tau^a\Phi_1)((D_\mu\Phi_1)^\dagger\tau^a(D^\mu\Phi_2)) + \text{H.c.},$$

$$(\Phi_1^\dagger\tau^a\Phi_1)((D_\mu\Phi_2)^\dagger\tau^a(D^\mu\Phi_1)) + \text{H.c.}, \qquad (\Phi_2^\dagger\tau^a\Phi_2)((D_\mu\Phi_2)^\dagger\tau^a(D^\mu\Phi_1)) + \text{H.c.},$$

$$(\Phi_2^\dagger\tau^a\Phi_1)((D_\mu\Phi_1)^\dagger\tau^a(D^\mu\Phi_1)) + \text{H.c.}, \qquad (\Phi_2^\dagger\tau^a\Phi_1)((D_\mu\Phi_2)^\dagger\tau^a(D^\mu\Phi_2)) + \text{H.c.} \tag{11}$$

By exploiting the $SU(2)$ relation $(\tau^a)_{ij}(\tau^a)_{kl} = 2\delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl}$, we obtain singlet terms in which by separating derivatives in symmetric and antisymmetric combinations, we then find the results listed in Table IV. These are independent from one another and free of ambiguity from where the partial derivatives act on. Note that operators $\mathcal{O}_{\Phi\partial^2}^{(mn)(pq)}$ in the class of $\phi^4 D^2$, $\partial_\mu(\Phi_m^\dagger\Phi_n)\partial^\mu(\Phi_p^\dagger\Phi_q)$, can be equivalently written with box operator on $either$ singlet pair via a total derivative: $(\Phi_m^\dagger\Phi_n)\Box(\Phi_p^\dagger\Phi_q)$ or $(\Phi_p^\dagger\Phi_q)\Box(\Phi_m^\dagger\Phi_n)$. However, we favor our notation not only because it is the symmetric combination of covariant derivative operators which is by definition orthogonal to the antisymmetric combination, $\mathcal{O}_{\Phi D}^{(mn)(pq)}$, but also writing

those kinds of operators in that form helps collect dimension-six terms contributing to field redefinitions of the physical scalar spectrum after electroweak symmetry breaking.

Mass operators, $\psi^2\phi^3$, found in the first row of Table II now modify the masses of the SM fields, in addition to the dimension-four Yukawa terms in Eq. (2). The mass matrices for each type of fermion with explicit flavor indices are given by

$$\mathcal{L}_M = -\bar{e}_{L,a}(M_e)_{ab}e_{R,b} - \bar{d}_{L,a}(M_d)_{ab}d_{R,b}$$
$$- \bar{u}_{L,a}(M_u)_{ab}u_{R,b} + \text{H.c.}, \tag{12}$$

where

$$(M_e)_{ab} = v(y_{e,a}^{(1)}\cos\beta + y_{e,a}^{(2)}\sin\beta)\delta_{ab}$$
$$- v^3\Big(\cos^3\beta(C_{l\Phi_1}^{(11)})_{ab} + \sin^2\beta\cos\beta(C_{l\Phi_1}^{(22)})_{ab} + \sin\beta\cos^2\beta(C_{l\Phi_1}^{(12)})_{ab} + \sin\beta\cos^2\beta(C_{l\Phi_1}^{(21)})_{ab}$$
$$+ \sin^3\beta(C_{l\Phi_2}^{(22)})_{ab} + \sin\beta\cos^2\beta(C_{l\Phi_2}^{(11)})_{ab} + \sin^2\beta\cos\beta(C_{l\Phi_2}^{(12)})_{ab} + \sin^2\beta\cos\beta(C_{l\Phi_2}^{(21)})_{ab}\Big), \tag{13}$$

$$
\begin{aligned}
(M_d)_{ab} = {}& v(y_{d,a}^{(1)}\cos\beta + y_{d,a}^{(2)}\sin\beta)\delta_{ab} \\
& - v^3\Big(\cos^3\beta(C_{d\Phi_1}^{(11)})_{ab} + \sin^2\beta\cos\beta(C_{d\Phi_1}^{(22)})_{ab} + \sin\beta\cos^2\beta(C_{d\Phi_1}^{(12)})_{ab} + \sin\beta\cos^2\beta(C_{d\Phi_1}^{(21)})_{ab} \\
& + \sin^3\beta(C_{d\Phi_2}^{(22)})_{ab} + \sin\beta\cos^2\beta(C_{d\Phi_2}^{(11)})_{ab} + \sin^2\beta\cos\beta(C_{d\Phi_2}^{(12)})_{ab} + \sin^2\beta\cos\beta(C_{d\Phi_2}^{(21)})_{ab}\Big),
\end{aligned}
\tag{14}
$$

and

$$
\begin{aligned}
(M_u)_{ab} = {}& v(y_{u,a}^{(1)}\cos\beta + y_{u,a}^{(2)}\sin\beta)\delta_{ab} \\
& - v^3\Big(\cos^3\beta(C_{u\Phi_1}^{(11)})_{ab} + \sin^2\beta\cos\beta(C_{u\Phi_1}^{(22)})_{ab} + \sin\beta\cos^2\beta(C_{u\Phi_1}^{(12)})_{ab} + \sin\beta\cos^2\beta(C_{u\Phi_1}^{(21)})_{ab} \\
& + \sin^3\beta(C_{u\Phi_2}^{(22)})_{ab} + \sin\beta\cos^2\beta(C_{u\Phi_2}^{(11)})_{ab} + \sin^2\beta\cos\beta(C_{u\Phi_2}^{(12)})_{ab} + \sin^2\beta\cos\beta(C_{u\Phi_2}^{(21)})_{ab}\Big).
\end{aligned}
\tag{15}
$$

Rotating the fields to the mass eigenstate basis can be done via bi-unitary transformations $\psi_L \rightarrow U_L\hat{\psi}_L$ and $\psi_R \rightarrow U_R\hat{\psi}_R$, whereby the unitary matrices $U_{L,R}$ are such that the diagonal entries of

$$
\begin{aligned}
\hat{M}_e &= U_L^{e\dagger}M_eU_R^e = \mathrm{diag}(m_e, m_\mu, m_\tau), \\
\hat{M}_d &= U_L^{d\dagger}M_dU_R^d = \mathrm{diag}(m_d, m_s, m_b), \\
\hat{M}_u &= U_L^{u\dagger}M_uU_R^u = \mathrm{diag}(m_u, m_c, m_t),
\end{aligned}
\tag{16}
$$

are real and positive. To obtain the operators in Tables II and III after field rotations, one can redefine the Yukawa couplings and Wilson coefficients to ones with carrots via

$$
\begin{aligned}
(\hat{y}_{(e,d,u)}^{(1,2)})_{ab} &= \Big(U_L^{(e,d,u)\dagger}y_{(e,d,u)}^{(1,2)}U_R^{(e,d,u)}\Big)_{ab}, \\
(\hat{C}_{\nu\nu\Phi}^{(11,22,12)})_{ab} &= \Big(U_L^{\nu T}C_{\nu\nu\Phi}^{(11,22,12)}U_L^{\nu}\Big)_{ab}, \\
(\hat{C}_{(l,d,u)\Phi_{(1,2)}}^{(11,22,12,21)})_{ab} &= \Big(U_L^{(e,d,u)\dagger}C_{(l,d,u)\Phi_{(1,2)}}^{(11,22,12,21)}U_R^{(e,d,u)}\Big)_{ab}, \\
(\hat{C}_{l(B,W)\Phi_{(1,2)}})_{ab} &= \Big(U_L^{e\dagger}C_{l(B,W)\Phi_{(1,2)}}U_R^{e}\Big)_{ab}, \\
(\hat{C}_{d(B,W,G)\Phi_{(1,2)}})_{ab} &= \Big(U_L^{d\dagger}C_{d(B,W,G)\Phi_{(1,2)}}U_R^{d}\Big)_{ab}, \\
(\hat{C}_{u(B,W,G)\Phi_{(1,2)}})_{ab} &= \Big(U_L^{u\dagger}C_{u(B,W,G)\Phi_{(1,2)}}U_R^{u}\Big)_{ab}, \\
(\hat{C}_{\Phi(e,d,u)}^{(11,22,12)})_{ab} &= \Big(U_R^{(e,d,u)\dagger}C_{\Phi(e,d,u)}^{(11,22,12)}U_R^{(e,d,u)}\Big)_{ab}, \\
(\hat{C}_{\Phi(l,q)}^{(11,22,12)[1]})_{ab} &= \Big(U_L^{(e,d)\dagger}C_{\Phi(l,q)}^{(11,22,12)[1]}U_L^{(e,d)}\Big)_{ab}, \\
(\hat{C}_{\Phi(l,q)}^{(11,22,12)[3]})_{ab} &= \Big(U_L^{(e,d)\dagger}C_{\Phi(l,q)}^{(11,22,12)[3]}U_L^{(e,d)}\Big)_{ab}, \\
(\hat{C}_{\Phi ud}^{(11,22,21)})_{ab} &= \Big(U_R^{u\dagger}C_{\Phi ud}^{(11,22,21)}U_R^{d}\Big)_{ab}.
\end{aligned}
\tag{17}
$$

Four-fermion operators in V are rotated just as in SMEFT and can be found in Ref. [26].

Similarly, the Majorana neutrino mass matrix generated by the dimension-five operators is given by

$$
\mathcal{L}_M = -\frac{1}{2}\nu_{L,a}^T(M_\nu)_{ab}\mathbf{C}\nu_{L,b} + \mathrm{H.c.},
\tag{18}
$$

where

$$
\begin{aligned}
(M_\nu)_{ab} = -2v^2\Big(&\cos^2\beta(C_{\nu\nu\Phi}^{(11)})_{ab} + \sin^2\beta(C_{\nu\nu\Phi}^{(22)})_{ab} \\
& + \sin\beta\cos\beta(C_{\nu\nu\Phi}^{(12)})_{ab}\Big).
\end{aligned}
\tag{19}
$$

The diagonal mass matrix is given by

$$
\hat{M}_\nu = U_L^{\nu T}M_\nu U_L^{\nu} = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}).
\tag{20}
$$

We can also introduce $\bar{\nu}_L^c = \nu_L^T\mathbf{C}$, in which case $\mathcal{L}_M$ can be written as $-\bar{\nu}_L^cM_\nu\nu_L/2 + \mathrm{H.c.}$ (note that the Majorana condition is $\nu = \nu_L + \nu_L^c = \nu^c$).

III. SPECIFIC TYPES OF 2HDMS

Different types of 2HDMs are motivated by a variety of reasons. The type-I 2HDM is unique in the sense that all fermionic masses are generated by a single Higgs doublet Φ_2, while the remaining Φ_1 doublet affects the generation of Z and $W^\pm$ masses [27]. The type-II 2HDM is among the most simple and well studied; phenomenologically popular [28] and well motivated in supersymmetric models [29] where masses of the down quarks and leptons are generated from Φ_1 while up-type quarks acquire masses from Φ_2. The lepton-specific type of 2HDM (known also as type-X) has been attractive in explaining neutrino mass generation and dark matter [30,31]. In this case, the doublet Φ_1 is responsible for generating the lepton masses while the remaining Φ_2 generates all quark masses. For example, both type-II and -X models have been known to generate enhanced lepton couplings by factors of $\tan\beta$, useful for explaining anomalous magnetic dipole moments of the electron and/or the muon [20,32–35] (note that type-X models require a light scalar spectrum to accomplish this while simultaneously, couplings of additional Higgses to

quarks are suppressed, allowing for weaker constraints than in type-II models). The last type of model is known as a flipped 2HDM (or type-Y), where Φ_2 is responsible for generating the masses of the up and lepton sectors, while Φ_1 couples only to down-type quarks [36,37]. In this model, lepton and up-type couplings can be suppressed while down-type couplings can be simultaneously enhanced. For a comprehensive review of all types of 2HDMs and their phenomenology, see [19].

In all types of 2HDMs, we assume the standard Z_2 symmetry on the doublets $\Phi_1 \to -\Phi_1$ and $\Phi_2 \to +\Phi_2$ [8]. This requirement restricts terms involving λ_6 and λ_7 in the scalar potential [Eq. (6)] (and possible softly broken effects if enforced exactly). Note that there is a difference between Refs. [9,10] with respect to resulting operators in CP-conserving 2HDMs, where the former mandates no mixed contracted pairs $(\Phi_{1,2}^\dagger \Phi_{2,1})$ and the latter uses the convention $\Phi_1 \to +\Phi_1$ and $\Phi_2 \to -\Phi_2$. Specifying how each of the doublets couple to SM fermions is provided in Table VI, where we use the convention that Φ_2 always couples to the up-type sector. In any of the four models, the Z_2 symmetry prevents flavor changing neutral currents among fermionic species at the tree level. The Yukawa interactions for each type of model are

Type-I: $\mathcal{L} \supset -y_e \bar{l}_L e_R \Phi_2 - y_d \bar{q}_L d_R \Phi_2 - y_u \bar{q}_L u_R \cdot \Phi_2^\dagger + \text{H.c.}$,

Type-II: $\mathcal{L} \supset -y_e \bar{l}_L e_R \Phi_1 - y_d \bar{q}_L d_R \Phi_1 - y_u \bar{q}_L u_R \cdot \Phi_2^\dagger + \text{H.c.}$,

Type-X: $\mathcal{L} \supset -y_e \bar{l}_L e_R \Phi_1 - y_d \bar{q}_L d_R \Phi_2 - y_u \bar{q}_L u_R \cdot \Phi_2^\dagger + \text{H.c.}$,

Type-Y: $\mathcal{L} \supset -y_e \bar{l}_L e_R \Phi_2 - y_d \bar{q}_L d_R \Phi_1 - y_u \bar{q}_L u_R \cdot \Phi_2^\dagger + \text{H.c.}$,

(21)

where the superscripts (1,2) on the Yukawa couplings are omitted since there is only one type of Yukawa matrix for each fermion. Under the Z_2 symmetry of Φ_1 and Φ_2, there are a total of 76 operators common in all 4 types of 2HDMs, collected in Table VII, and includes operators involving left- or right-handed fermion currents. Note that operators of those kinds in the general 2HDM which involved odd pairs of $\Phi_1 \Phi_2^{(\dagger)}$ or $\Phi_1^{(\dagger)} \Phi_2$ are now forbidden. Model-specific operators are then collected in Table VIII

TABLE VI. Z_2 charge assignments of leptons, quarks, and Higgs doublets in each type of 2HDM. The middle column indicates which scalar doublet couples to each sector, where by convention, Φ_2 always couples to the up-type sector, while $\Phi_1 \to -\Phi_1$ and $\Phi_2 \to +\Phi_2$.

Model	u	d	e	l_L	e_R	q_L	u_R	d_R	Φ_1	Φ_2
Type-I	Φ_2	Φ_2	Φ_2	+	+	+	+	+	−	+
Type-II	Φ_2	Φ_1	Φ_1	+	−	+	+	−	−	+
Type-X (Lepton-specific)	Φ_2	Φ_2	Φ_1	+	−	+	+	+	−	+
Type-Y (Flipped)	Φ_2	Φ_1	Φ_2	+	+	+	−	−	−	+

for type-I couplings, Table IX for type-II couplings, Table X for type-X couplings, and Table XI for type-Y couplings. A type-II 2HDM EFT is the most restrictive, containing $24 + \text{H.c.} = 48$ specific operators, whereas the type-I model is less restrictive having $31 + \text{H.c.} = 62$ specific operators. Note that all type-specific operators are non-Hermitian.

In the case that m_{12}^2 and λ_5 are real, the Higgs potential is CP-conserving, which was first studied in the context of a 2HDM EFT in [9]. The authors comment on the couplings of doublets to SM fermions, restricting the couplings to only right-handed fields. After translating to their notation, we find agreement with the operators in Ref. [9], with the exception of several: $\mathcal{O}_{\Phi ud}^{(21)}$, which should be present in type-II and -Y models and moreover, they report only the mass operators $\mathcal{O}_{(l,d,u)\Phi_2}^{(22)}$ and $\mathcal{O}_{(l,d,u)\Phi_2}^{(11)}$ are present in a type-I model, $\mathcal{O}_{(l,d)\Phi_1}^{(11)}$, $\mathcal{O}_{(l,d)\Phi_1}^{(22)}$, $\mathcal{O}_{u\Phi_2}^{(22)}$, and $\mathcal{O}_{u\Phi_2}^{(11)}$ in a type-II, $\mathcal{O}_{(d,u)\Phi_2}^{(22)}$, $\mathcal{O}_{(d,u)\Phi_2}^{(11)}$, $\mathcal{O}_{l\Phi_1}^{(11)}$, and $\mathcal{O}_{l\Phi_1}^{(22)}$ in a type-X and finally in a type-Y model, $\mathcal{O}_{(l,u)\Phi_2}^{(22)}$, $\mathcal{O}_{(l,u)\Phi_2}^{(11)}$, $\mathcal{O}_{d\Phi_1}^{(11)}$, and $\mathcal{O}_{d\Phi_1}^{(22)}$. The additional $\psi^2 \phi^3$ operators we find were perhaps overlooked by mandating no mixed contracted pairs $(\Phi_{1,2}^\dagger \Phi_{2,1})$ in the operators. We find twice as many mass operators in every type of model.

A. Comments on H_d and H_u notation for the type-II 2HDM

It is well known that the Higgs potential of the minimal supersymmetric SM (MSSM) is that of a type-II 2HDM (see [38] for definitions). In the MSSM, the doublets are defined in a more suggestive way by

$$H_d = \begin{pmatrix} H_d^+ \\ H_d^0 \end{pmatrix} \equiv \Phi_1 = \begin{pmatrix} \Phi_1^+ \\ \Phi_1^0 \end{pmatrix},$$

$$H_u = \begin{pmatrix} H_u^0 \\ H_u^- \end{pmatrix} \equiv i\sigma^2 \Phi_2^\dagger = \begin{pmatrix} \Phi_2^{0*} \\ -\Phi_2^- \end{pmatrix}, \quad (22)$$

where the subscripts denote which doublet explicitly couples to the down (H_d) and up (H_u) sectors, and $i\sigma^2 = \epsilon$. The hypercharges are then $Y_{H_d} = +1/2$ and $Y_{H_u} = -1/2$. For a type-II 2HDM, one can make the appropriate replacements in Table IX to this notation. These notations and definitions in the context of a type-II 2HDM as a low energy effective field theory were previously used in [20,39], the former in the context of generating electric and magnetic dipole moments of the muon after integrating out heavy vectorlike leptons and the latter for signals of a modified Yukawa coupling. The notation of the mass operators therein are slightly different than presented here, utilizing $H_{d,u}$ explicitly: $\mathcal{O}_{l\Phi_1}^{(11)} \to \mathcal{O}_{lH_d} = \bar{l}_L e_R H_d (H_d^\dagger H_d)$, $\mathcal{O}_{l\Phi_1}^{(22)} \to \mathcal{O}_{lH_u}^{(1)} = \bar{l}_L e_R H_d (H_u^\dagger H_u)$,

TABLE VII. Common operators in all types of 2HDMs. Note that all X^3 (4) and the first row of ψ^4 operators (20) are present, giving a total of 76 common operators.

$\phi^4 D^2$	$\mathcal{O}_{\Phi\partial^2}^{(11)(11)} = \partial_\mu(\Phi_1^\dagger\Phi_1)\partial^\mu(\Phi_1^\dagger\Phi_1)$	$\mathcal{O}_{\Phi\partial^2}^{(21)(12)} = \partial_\mu(\Phi_2^\dagger\Phi_1)\partial^\mu(\Phi_1^\dagger\Phi_2)$
	$\mathcal{O}_{\Phi\partial^2}^{(22)(22)} = \partial_\mu(\Phi_2^\dagger\Phi_2)\partial^\mu(\Phi_2^\dagger\Phi_2)$	$\mathcal{O}_{\Phi\partial^2}^{(21)(21)} = \partial_\mu(\Phi_2^\dagger\Phi_1)\partial^\mu(\Phi_2^\dagger\Phi_1) + \text{H.c.}$
	$\mathcal{O}_{\Phi\partial^2}^{(11)(22)} = \partial_\mu(\Phi_1^\dagger\Phi_1)\partial^\mu(\Phi_2^\dagger\Phi_2)$	
	$\mathcal{O}_{\Phi D}^{(11)(11)} = (\Phi_1^\dagger \overset{\leftrightarrow}{D}_\mu \Phi_1)(\Phi_1^\dagger \overset{\leftrightarrow}{D}^\mu \Phi_1)$	$\mathcal{O}_{\Phi D}^{(21)(12)} = (\Phi_2^\dagger \overset{\leftrightarrow}{D}_\mu \Phi_1)(\Phi_1^\dagger \overset{\leftrightarrow}{D}^\mu \Phi_2)$
	$\mathcal{O}_{\Phi D}^{(22)(22)} = (\Phi_2^\dagger \overset{\leftrightarrow}{D}_\mu \Phi_2)(\Phi_2^\dagger \overset{\leftrightarrow}{D}^\mu \Phi_2)$	$\mathcal{O}_{\Phi D}^{(21)(21)} = (\Phi_2^\dagger \overset{\leftrightarrow}{D}_\mu \Phi_1)(\Phi_2^\dagger \overset{\leftrightarrow}{D}^\mu \Phi_1) + \text{H.c.}$
	$\mathcal{O}_{\Phi D}^{(11)(22)} = (\Phi_1^\dagger \overset{\leftrightarrow}{D}_\mu \Phi_1)(\Phi_2^\dagger \overset{\leftrightarrow}{D}^\mu \Phi_2)$	
ϕ^6	$\mathcal{O}_\Phi^{(11)(11)(11)} = (\Phi_1^\dagger\Phi_1)^3$	$\mathcal{O}_\Phi^{(11)(21)(21)} = (\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1)^2 + \text{H.c.}$
	$\mathcal{O}_\Phi^{(22)(22)(22)} = (\Phi_2^\dagger\Phi_2)^3$	$\mathcal{O}_\Phi^{(22)(21)(21)} = (\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)^2 + \text{H.c.}$
	$\mathcal{O}_\Phi^{(11)(11)(22)} = (\Phi_1^\dagger\Phi_1)^2(\Phi_2^\dagger\Phi_2)$	$\mathcal{O}_\Phi^{(11)(21)(12)} = (\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2)$
	$\mathcal{O}_\Phi^{(11)(22)(22)} = (\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)^2$	$\mathcal{O}_\Phi^{(22)(21)(12)} = (\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2)$
$X^2\phi^2$	$\mathcal{O}_{\Phi G}^{(11)} = (\Phi_1^\dagger\Phi_1)G_{\mu\nu}^a G^{a\mu\nu}$	$\mathcal{O}_{\Phi\tilde{G}}^{(11)} = (\Phi_1^\dagger\Phi_1)\tilde{G}_{\mu\nu}^a G^{a\mu\nu}$
	$\mathcal{O}_{\Phi G}^{(22)} = (\Phi_2^\dagger\Phi_2)G_{\mu\nu}^a G^{a\mu\nu}$	$\mathcal{O}_{\Phi\tilde{G}}^{(22)} = (\Phi_2^\dagger\Phi_2)\tilde{G}_{\mu\nu}^a G^{a\mu\nu}$
	$\mathcal{O}_{\Phi W}^{(11)} = (\Phi_1^\dagger\Phi_1)W_{\mu\nu}^a W^{a\mu\nu}$	$\mathcal{O}_{\Phi\tilde{W}}^{(11)} = (\Phi_1^\dagger\Phi_1)\tilde{W}_{\mu\nu}^a W^{a\mu\nu}$
	$\mathcal{O}_{\Phi W}^{(22)} = (\Phi_2^\dagger\Phi_2)W_{\mu\nu}^a W^{a\mu\nu}$	$\mathcal{O}_{\Phi\tilde{W}}^{(22)} = (\Phi_2^\dagger\Phi_2)\tilde{W}_{\mu\nu}^a W^{a\mu\nu}$
	$\mathcal{O}_{\Phi B}^{(11)} = (\Phi_1^\dagger\Phi_1)B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{\Phi\tilde{B}}^{(11)} = (\Phi_1^\dagger\Phi_1)\tilde{B}_{\mu\nu} B^{\mu\nu}$
	$\mathcal{O}_{\Phi B}^{(22)} = (\Phi_2^\dagger\Phi_2)B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{\Phi\tilde{B}}^{(22)} = (\Phi_2^\dagger\Phi_2)\tilde{B}_{\mu\nu} B^{\mu\nu}$
	$\mathcal{O}_{\Phi WB}^{(11)} = (\Phi_1^\dagger\tau^a\Phi_1)W_{\mu\nu}^a B^{\mu\nu}$	$\mathcal{O}_{\Phi\tilde{W}B}^{(11)} = (\Phi_1^\dagger\tau^a\Phi_1)\tilde{W}_{\mu\nu}^a B^{\mu\nu}$
	$\mathcal{O}_{\Phi WB}^{(22)} = (\Phi_2^\dagger\tau^a\Phi_2)W_{\mu\nu}^a B^{\mu\nu}$	$\mathcal{O}_{\Phi\tilde{W}B}^{(22)} = (\Phi_2^\dagger\tau^a\Phi_2)\tilde{W}_{\mu\nu}^a B^{\mu\nu}$
$\psi^2\phi^2 D$	$\mathcal{O}_{\Phi e}^{(11)} = (\Phi_1^\dagger i\overset{\leftrightarrow}{D}_\mu \Phi_1)(\bar{e}_R\gamma^\mu e_R)$	$\mathcal{O}_{\Phi l}^{(11)[1]} = (\Phi_1^\dagger i\overset{\leftrightarrow}{D}_\mu \Phi_1)(\bar{l}_L\gamma^\mu l_L)$
	$\mathcal{O}_{\Phi e}^{(22)} = (\Phi_2^\dagger i\overset{\leftrightarrow}{D}_\mu \Phi_2)(\bar{e}_R\gamma^\mu e_R)$	$\mathcal{O}_{\Phi l}^{(22)[1]} = (\Phi_2^\dagger i\overset{\leftrightarrow}{D}_\mu \Phi_2)(\bar{l}_L\gamma^\mu l_L)$
	$\mathcal{O}_{\Phi d}^{(11)} = (\Phi_1^\dagger i\overset{\leftrightarrow}{D}_\mu \Phi_1)(\bar{d}_R\gamma^\mu d_R)$	$\mathcal{O}_{\Phi q}^{(11)[1]} = (\Phi_1^\dagger i\overset{\leftrightarrow}{D}_\mu \Phi_1)(\bar{q}_L\gamma^\mu q_L)$
	$\mathcal{O}_{\Phi d}^{(22)} = (\Phi_2^\dagger i\overset{\leftrightarrow}{D}_\mu \Phi_2)(\bar{d}_R\gamma^\mu d_R)$	$\mathcal{O}_{\Phi q}^{(22)[1]} = (\Phi_2^\dagger i\overset{\leftrightarrow}{D}_\mu \Phi_2)(\bar{q}_L\gamma^\mu q_L)$
	$\mathcal{O}_{\Phi u}^{(11)} = (\Phi_1^\dagger i\overset{\leftrightarrow}{D}_\mu \Phi_1)(\bar{u}_R\gamma^\mu u_R)$	$\mathcal{O}_{\Phi l}^{(11)[3]} = (\Phi_1^\dagger i\overset{\leftrightarrow}{D}{}_\mu^a \Phi_1)(\bar{l}_L\tau^a\gamma^\mu l_L)$
	$\mathcal{O}_{\Phi u}^{(22)} = (\Phi_2^\dagger i\overset{\leftrightarrow}{D}_\mu \Phi_2)(\bar{u}_R\gamma^\mu u_R)$	$\mathcal{O}_{\Phi l}^{(22)[3]} = (\Phi_2^\dagger i\overset{\leftrightarrow}{D}{}_\mu^a \Phi_2)(\bar{l}_L\tau^a\gamma^\mu l_L)$
		$\mathcal{O}_{\Phi q}^{(11)[3]} = (\Phi_1^\dagger i\overset{\leftrightarrow}{D}{}_\mu^a \Phi_1)(\bar{q}_L\tau^a\gamma^\mu q_L)$
		$\mathcal{O}_{\Phi q}^{(22)[3]} = (\Phi_2^\dagger i\overset{\leftrightarrow}{D}{}_\mu^a \Phi_2)(\bar{q}_L\tau^a\gamma^\mu q_L)$

$\mathcal{O}_{l\Phi_2}^{(21)} \rightarrow \mathcal{O}_{lH_u}^{(2)} = \bar{l}_L e_R \cdot H_u^\dagger(H_d \cdot H_u)$, and $\mathcal{O}_{l\Phi_2}^{(12)} \rightarrow \mathcal{O}_{lH_u}^{(3)} = \bar{l}_L e_R \cdot H_u^\dagger(H_d^\dagger \cdot H_u^\dagger)$. By exploiting the $SU(2)$ algebra $\epsilon_{ij}\epsilon_{kl} = \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}$ on the second and third operators, one can perform a basis change and define a new operator as a linear combination of the two:

$$\mathcal{O}_{lH_u}' = \mathcal{O}_{lH_u}^{(1)} - \mathcal{O}_{lH_u}^{(2)} = \bar{l}_L e_R H_u(H_u^\dagger H_d), \qquad (23)$$

where indices are contracted purely with δ_{ij}. After EWSB, this operator does not contribute to the lepton's mass. Note that in the standard notation, this operator corresponds to

$$\mathcal{O}_{l\Phi}' = \mathcal{O}_{l\Phi_1}^{(22)} - \mathcal{O}_{l\Phi_2}^{(21)} = \bar{l}_L e_R \cdot \Phi_2^\dagger(\Phi_1 \cdot \Phi_2). \quad (24)$$

One can also perform this basis change through the $SU(2)$ algebra on other operators in favor of new combinations. Considering different bases may be useful when considering specific UV completions, for example, different representations of vectorlike leptons restrict which kind of (mass) operator can be generated at tree level. In a model where *only* the operator in Eq. (23) is generated, one can immediately see that working in the basis that contains this operator shows that there is no contribution to the lepton's mass. Otherwise, in the standard basis, one would find a precise cancellation after writing all contributions. Note that in all kinds of models, it is expected that each mass operator can be generated via loop corrections [20,39].

Furthermore, in the MSSM, the couplings to SM fermions may appear as a type-II; however, they are not

TABLE VIII.   Type-I 2HDM-specific operators, each with a distinct Hermitian conjugate.

Type-I (31 + H.c. = 62)		
$\psi^2\phi^3$		
$\mathcal{O}^{(21)}_{l\Phi_1} = \bar{l}_L e_R \Phi_1(\Phi_2^\dagger\Phi_1)$	$\mathcal{O}^{(21)}_{d\Phi_1} = \bar{q}_L d_R \Phi_1(\Phi_2^\dagger\Phi_1)$	$\mathcal{O}^{(21)}_{u\Phi_1} = \bar{q}_L u_R \cdot \Phi_1^\dagger(\Phi_2^\dagger\Phi_1)$
$\mathcal{O}^{(12)}_{l\Phi_1} = \bar{l}_L e_R \Phi_1(\Phi_1^\dagger\Phi_2)$	$\mathcal{O}^{(12)}_{d\Phi_1} = \bar{q}_L d_R \Phi_1(\Phi_1^\dagger\Phi_2)$	$\mathcal{O}^{(12)}_{u\Phi_1} = \bar{q}_L u_R \cdot \Phi_1^\dagger(\Phi_1^\dagger\Phi_2)$
$\mathcal{O}^{(22)}_{l\Phi_2} = \bar{l}_L e_R \Phi_2(\Phi_2^\dagger\Phi_2)$	$\mathcal{O}^{(22)}_{d\Phi_2} = \bar{q}_L d_R \Phi_2(\Phi_2^\dagger\Phi_2)$	$\mathcal{O}^{(22)}_{u\Phi_2} = \bar{q}_L u_R \cdot \Phi_2^\dagger(\Phi_2^\dagger\Phi_2)$
$\mathcal{O}^{(11)}_{l\Phi_2} = \bar{l}_L e_R \Phi_2(\Phi_1^\dagger\Phi_1)$	$\mathcal{O}^{(11)}_{d\Phi_2} = \bar{q}_L d_R \Phi_2(\Phi_1^\dagger\Phi_1)$	$\mathcal{O}^{(11)}_{u\Phi_2} = \bar{q}_L u_R \cdot \Phi_2^\dagger(\Phi_1^\dagger\Phi_1)$
$\psi^2 X\phi$		
$\mathcal{O}_{lB\Phi_2} = \bar{l}_L \sigma^{\mu\nu} e_R \Phi_2 B_{\mu\nu}$	$\mathcal{O}_{dB\Phi_2} = \bar{q}_L \sigma^{\mu\nu} d_R \Phi_2 B_{\mu\nu}$	$\mathcal{O}_{uB\Phi_2} = \bar{q}_L \sigma^{\mu\nu} u_R \cdot \Phi_2^\dagger B_{\mu\nu}$
$\mathcal{O}_{lW\Phi_2} = \bar{l}_L \sigma^{\mu\nu} e_R \tau^a \Phi_2 W^a_{\mu\nu}$	$\mathcal{O}_{dW\Phi_2} = \bar{q}_L \sigma^{\mu\nu} d_R \tau^a \Phi_2 W^a_{\mu\nu}$	$\mathcal{O}_{uW\Phi_2} = \bar{q}_L \sigma^{\mu\nu} u_R \tau^a \cdot \Phi_2^\dagger W^a_{\mu\nu}$
	$\mathcal{O}_{dG\Phi_2} = \bar{q}_L \sigma^{\mu\nu} \lambda^a d_R \Phi_2 G^a_{\mu\nu}$	$\mathcal{O}_{uG\Phi_2} = \bar{q}_L \sigma^{\mu\nu} \lambda^a u_R \cdot \Phi_2^\dagger G^a_{\mu\nu}$

$\psi^2\phi^2 D$		
$\mathcal{O}^{(11)}_{\Phi ud} = (\Phi_1 \cdot iD_\mu\Phi_1)(\bar{u}_R\gamma^\mu d_R)$		$\mathcal{O}^{(22)}_{\Phi ud} = (\Phi_2 \cdot iD_\mu\Phi_2)(\bar{u}_R\gamma^\mu d_R)$
ψ^4		
$\mathcal{O}_{ledq} = (\bar{l}_L e_R)(\bar{d}_R q_L)$		$\mathcal{O}_{duq} = \epsilon^{\alpha\beta\gamma}\epsilon_{jk}((d_R^\alpha)^T \mathbf{C} u_R^\beta)((q_{Lj}^\gamma)^T \mathbf{C} l_{Lk})$
$\mathcal{O}^{(1)}_{quqd} = (\bar{q}_L u_R) \cdot (\bar{q}_L d_R)$		$\mathcal{O}_{qqu} = \epsilon^{\alpha\beta\gamma}\epsilon_{jk}((q_{Lj}^\alpha)^T \mathbf{C} q_{Lk}^\beta)((u_R^\gamma)^T \mathbf{C} e_R)$
$\mathcal{O}^{(8)}_{quqd} = (\bar{q}_L \lambda^a u_R) \cdot (\bar{q}_L \lambda^a d_R)$		$\mathcal{O}_{qqq} = \epsilon^{\alpha\beta\gamma}\epsilon_{jn}((q_{Lj}^\alpha)^T \mathbf{C} q_L^\beta) \cdot ((q_L^\gamma)^T \mathbf{C} l_{Ln})$
$\mathcal{O}^{(1)}_{lequ} = (\bar{l}_L e_R) \cdot (\bar{q}_L u_R)$		$\mathcal{O}_{duu} = \epsilon^{\alpha\beta\gamma}((d_R^\alpha)^T \mathbf{C} u_R^\beta)((u_R^\gamma)^T \mathbf{C} e_R)$
$\mathcal{O}^{(3)}_{lequ} = (\bar{l}_L \sigma^{\mu\nu} e_R) \cdot (\bar{q}_L \sigma_{\mu\nu} u_R)$		

TABLE IX.   Type-II 2HDM-specific operators, each with a distinct Hermitian conjugate.

Type-II (24 + H.c. = 48)		
$\psi^2\phi^3$		
$\mathcal{O}^{(11)}_{l\Phi_1} = \bar{l}_L e_R \Phi_1(\Phi_1^\dagger\Phi_1)$	$\mathcal{O}^{(11)}_{d\Phi_1} = \bar{q}_L d_R \Phi_1(\Phi_1^\dagger\Phi_1)$	$\mathcal{O}^{(21)}_{u\Phi_1} = \bar{q}_L u_R \cdot \Phi_1^\dagger(\Phi_2^\dagger\Phi_1)$
$\mathcal{O}^{(22)}_{l\Phi_1} = \bar{l}_L e_R \Phi_1(\Phi_2^\dagger\Phi_2)$	$\mathcal{O}^{(22)}_{d\Phi_1} = \bar{q}_L d_R \Phi_1(\Phi_2^\dagger\Phi_2)$	$\mathcal{O}^{(12)}_{u\Phi_1} = \bar{q}_L u_R \cdot \Phi_1^\dagger(\Phi_1^\dagger\Phi_2)$
$\mathcal{O}^{(21)}_{l\Phi_2} = \bar{l}_L e_R \Phi_2(\Phi_2^\dagger\Phi_1)$	$\mathcal{O}^{(21)}_{d\Phi_2} = \bar{q}_L d_R \Phi_2(\Phi_2^\dagger\Phi_1)$	$\mathcal{O}^{(22)}_{u\Phi_2} = \bar{q}_L u_R \cdot \Phi_2^\dagger(\Phi_2^\dagger\Phi_2)$
$\mathcal{O}^{(12)}_{l\Phi_2} = \bar{l}_L e_R \Phi_2(\Phi_1^\dagger\Phi_2)$	$\mathcal{O}^{(12)}_{d\Phi_2} = \bar{q}_L d_R \Phi_2(\Phi_1^\dagger\Phi_2)$	$\mathcal{O}^{(11)}_{u\Phi_2} = \bar{q}_L u_R \cdot \Phi_2^\dagger(\Phi_1^\dagger\Phi_1)$
$\psi^2 X\phi$		
$\mathcal{O}_{lB\Phi_1} = \bar{l}_L \sigma^{\mu\nu} e_R \Phi_1 B_{\mu\nu}$	$\mathcal{O}_{dB\Phi_1} = \bar{q}_L \sigma^{\mu\nu} d_R \Phi_1 B_{\mu\nu}$	$\mathcal{O}_{uB\Phi_2} = \bar{q}_L \sigma^{\mu\nu} u_R \cdot \Phi_2^\dagger B_{\mu\nu}$
$\mathcal{O}_{lW\Phi_1} = \bar{l}_L \sigma^{\mu\nu} e_R \tau^a \Phi_1 W^a_{\mu\nu}$	$\mathcal{O}_{dW\Phi_1} = \bar{q}_L \sigma^{\mu\nu} d_R \tau^a \Phi_1 W^a_{\mu\nu}$	$\mathcal{O}_{uW\Phi_2} = \bar{q}_L \sigma^{\mu\nu} u_R \tau^a \cdot \Phi_2^\dagger W^a_{\mu\nu}$
	$\mathcal{O}_{dG\Phi_1} = \bar{q}_L \sigma^{\mu\nu} \lambda^a d_R \Phi_1 G^a_{\mu\nu}$	$\mathcal{O}_{uG\Phi_2} = \bar{q}_L \sigma^{\mu\nu} \lambda^a u_R \cdot \Phi_2^\dagger G^a_{\mu\nu}$

$\psi^2\phi^2 D$		
$\mathcal{O}^{(21)}_{\Phi ud} = (\Phi_2 i \overset{\leftrightarrow}{D}_\mu \Phi_1)(\bar{u}_R\gamma^\mu d_R)$		
ψ^4		
$\mathcal{O}_{ledq} = (\bar{l}_L e_R)(\bar{d}_R q_L)$		$\mathcal{O}_{qqq} = \epsilon^{\alpha\beta\gamma}\epsilon_{jn}((q_{Lj}^\alpha)^T \mathbf{C} q_L^\beta) \cdot ((q_L^\gamma)^T \mathbf{C} l_{Ln})$
		$\mathcal{O}_{duu} = \epsilon^{\alpha\beta\gamma}((d_R^\alpha)^T \mathbf{C} u_R^\beta)((u_R^\gamma)^T \mathbf{C} e_R)$

enforced by a Z_2 symmetry but rather the condition that the superpotential is holomorphic with respect to doublets H_d and H_u in Eq. (22). Holomorphicity in the operators will drastically reduce the general list of 2HDM EFT operators to 31 operators containing only $\Phi_1 \to H_d$ and $\Phi_2^\dagger \to H_u$.

IV. THE 2HDM EFT IN THE HIGGS BASIS

From Eq. (8), one can see that the SM degrees of freedom $h, G, G^\pm$ are mixed with new scalars $H, A, H^\pm$, which in general can have masses anywhere from the EW scale up to the scale of new physics Λ. It is convenient to work in the Higgs basis [5–8] where, in the alignment limit, the SM fields and additional scalars are separated into two doublets H_1 and H_2. We can rotate the doublets (Φ_1, Φ_2) via angle β by

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}, \qquad (25)$$

TABLE X.   Type-X 2HDM-specific operators, each with a distinct Hermitian conjugate.

Type-X (26 + H.c. = 52)		
$\psi^2\phi^3$		
$\mathcal{O}_{l\Phi_1}^{(11)} = \bar{l}_L e_R \Phi_1(\Phi_1^\dagger\Phi_1)$	$\mathcal{O}_{d\Phi_1}^{(21)} = \bar{q}_L d_R \Phi_1(\Phi_2^\dagger\Phi_1)$	$\mathcal{O}_{u\Phi_1}^{(21)} = \bar{q}_L u_R \cdot \Phi_1^\dagger(\Phi_2^\dagger\Phi_1)$
$\mathcal{O}_{l\Phi_1}^{(22)} = \bar{l}_L e_R \Phi_1(\Phi_2^\dagger\Phi_2)$	$\mathcal{O}_{d\Phi_1}^{(12)} = \bar{q}_L d_R \Phi_1(\Phi_1^\dagger\Phi_2)$	$\mathcal{O}_{u\Phi_1}^{(12)} = \bar{q}_L u_R \cdot \Phi_1^\dagger(\Phi_1^\dagger\Phi_2)$
$\mathcal{O}_{l\Phi_2}^{(21)} = \bar{l}_L e_R \Phi_2(\Phi_2^\dagger\Phi_1)$	$\mathcal{O}_{d\Phi_2}^{(22)} = \bar{q}_L d_R \Phi_2(\Phi_2^\dagger\Phi_2)$	$\mathcal{O}_{u\Phi_2}^{(22)} = \bar{q}_L u_R \cdot \Phi_2^\dagger(\Phi_2^\dagger\Phi_2)$
$\mathcal{O}_{l\Phi_2}^{(12)} = \bar{l}_L e_R \Phi_2(\Phi_1^\dagger\Phi_2)$	$\mathcal{O}_{d\Phi_2}^{(11)} = \bar{q}_L d_R \Phi_2(\Phi_1^\dagger\Phi_1)$	$\mathcal{O}_{u\Phi_2}^{(11)} = \bar{q}_L u_R \cdot \Phi_2^\dagger(\Phi_1^\dagger\Phi_1)$
$\psi^2 X\phi$		
$\mathcal{O}_{lB\Phi_1} = \bar{l}_L \sigma^{\mu\nu} e_R \Phi_1 B_{\mu\nu}$	$\mathcal{O}_{dB\Phi_2} = \bar{q}_L \sigma^{\mu\nu} d_R \Phi_2 B_{\mu\nu}$	$\mathcal{O}_{uB\Phi_2} = \bar{q}_L \sigma^{\mu\nu} u_R \cdot \Phi_2^\dagger B_{\mu\nu}$
$\mathcal{O}_{lW\Phi_1} = \bar{l}_L \sigma^{\mu\nu} e_R \tau^a \Phi_1 W_{\mu\nu}^a$	$\mathcal{O}_{dW\Phi_2} = \bar{q}_L \sigma^{\mu\nu} d_R \tau^a \Phi_2 W_{\mu\nu}^a$	$\mathcal{O}_{uW\Phi_2} = \bar{q}_L \sigma^{\mu\nu} u_R \tau^a \cdot \Phi_2^\dagger W_{\mu\nu}^a$
	$\mathcal{O}_{dG\Phi_2} = \bar{q}_L \sigma^{\mu\nu} \lambda^a d_R \Phi_2 G_{\mu\nu}^a$	$\mathcal{O}_{uG\Phi_2} = \bar{q}_L \sigma^{\mu\nu} \lambda^a u_R \cdot \Phi_2^\dagger G_{\mu\nu}^a$
$\psi^2\phi^2 D$	$\mathcal{O}_{\Phi ud}^{(11)} = (\Phi_1 \cdot iD_\mu\Phi_1)(\bar{u}_R\gamma^\mu d_R)$	$\mathcal{O}_{\Phi ud}^{(22)} = (\Phi_2 \cdot iD_\mu\Phi_2)(\bar{u}_R\gamma^\mu d_R)$
ψ^4	$\mathcal{O}_{quqd}^{(1)} = (\bar{q}_L u_R)\cdot(\bar{q}_L d_R)$	$\mathcal{O}_{duq} = \epsilon^{\alpha\beta\gamma}\epsilon_{jk}((d_R^\alpha)^T C u_R^\beta)((q_{Lj}^\gamma)^T C l_{Lk})$
	$\mathcal{O}_{quqd}^{(8)} = (\bar{q}_L \lambda^a u_R)\cdot(\bar{q}_L \lambda^a d_R)$	$\mathcal{O}_{qqq} = \epsilon^{\alpha\beta\gamma}\epsilon_{jn}((q_{Lj}^\alpha)^T C q_L^\beta)\cdot((q_L^\gamma)^T C l_{Ln})$

TABLE XI.   Type-Y 2HDM-specific operators, each with a distinct Hermitian conjugate.

Type-Y (25 + H.c. = 50)		
$\psi^2\phi^3$		
$\mathcal{O}_{l\Phi_1}^{(21)} = \bar{l}_L e_R \Phi_1(\Phi_2^\dagger\Phi_1)$	$\mathcal{O}_{d\Phi_1}^{(11)} = \bar{q}_L d_R \Phi_1(\Phi_1^\dagger\Phi_1)$	$\mathcal{O}_{u\Phi_1}^{(21)} = \bar{q}_L u_R \cdot \Phi_1^\dagger(\Phi_2^\dagger\Phi_1)$
$\mathcal{O}_{l\Phi_1}^{(12)} = \bar{l}_L e_R \Phi_1(\Phi_1^\dagger\Phi_2)$	$\mathcal{O}_{d\Phi_1}^{(22)} = \bar{q}_L d_R \Phi_1(\Phi_2^\dagger\Phi_2)$	$\mathcal{O}_{u\Phi_1}^{(12)} = \bar{q}_L u_R \cdot \Phi_1^\dagger(\Phi_1^\dagger\Phi_2)$
$\mathcal{O}_{l\Phi_2}^{(22)} = \bar{l}_L e_R \Phi_2(\Phi_2^\dagger\Phi_2)$	$\mathcal{O}_{d\Phi_2}^{(21)} = \bar{q}_L d_R \Phi_2(\Phi_2^\dagger\Phi_1)$	$\mathcal{O}_{u\Phi_2}^{(22)} = \bar{q}_L u_R \cdot \Phi_2^\dagger(\Phi_2^\dagger\Phi_2)$
$\mathcal{O}_{l\Phi_2}^{(11)} = \bar{l}_L e_R \Phi_2(\Phi_1^\dagger\Phi_1)$	$\mathcal{O}_{d\Phi_2}^{(12)} = \bar{q}_L d_R \Phi_2(\Phi_1^\dagger\Phi_2)$	$\mathcal{O}_{u\Phi_2}^{(11)} = \bar{q}_L u_R \cdot \Phi_2^\dagger(\Phi_1^\dagger\Phi_1)$
$\psi^2 X\phi$		
$\mathcal{O}_{lB\Phi_2} = \bar{l}_L \sigma^{\mu\nu} e_R \Phi_2 B_{\mu\nu}$	$\mathcal{O}_{dB\Phi_1} = \bar{q}_L \sigma^{\mu\nu} d_R \Phi_1 B_{\mu\nu}$	$\mathcal{O}_{uB\Phi_2} = \bar{q}_L \sigma^{\mu\nu} u_R \cdot \Phi_2^\dagger B_{\mu\nu}$
$\mathcal{O}_{lW\Phi_2} = \bar{l}_L \sigma^{\mu\nu} e_R \tau^a \Phi_2 W_{\mu\nu}^a$	$\mathcal{O}_{dW\Phi_1} = \bar{q}_L \sigma^{\mu\nu} d_R \tau^a \Phi_1 W_{\mu\nu}^a$	$\mathcal{O}_{uW\Phi_2} = \bar{q}_L \sigma^{\mu\nu} u_R \tau^a \cdot \Phi_2^\dagger W_{\mu\nu}^a$
	$\mathcal{O}_{dG\Phi_1} = \bar{q}_L \sigma^{\mu\nu} \lambda^a d_R \Phi_1 G_{\mu\nu}^a$	$\mathcal{O}_{uG\Phi_2} = \bar{q}_L \sigma^{\mu\nu} \lambda^a u_R \cdot \Phi_2^\dagger G_{\mu\nu}^a$
$\psi^2\phi^2 D$	$\mathcal{O}_{\Phi ud}^{(21)} = (\Phi_2 i \cdot \overleftrightarrow{D}_\mu\Phi_1)(\bar{u}_R\gamma^\mu d_R)$	
ψ^4	$\mathcal{O}_{lequ}^{(1)} = (\bar{l}_L e_R)\cdot(\bar{q}_L u_R)$	$\mathcal{O}_{qqu} = \epsilon^{\alpha\beta\gamma}\epsilon_{jk}((q_{Lj}^\alpha)^T C q_{Lk}^\beta)((u_R^\gamma)^T C e_R)$
	$\mathcal{O}_{lequ}^{(3)} = (\bar{l}_L \sigma^{\mu\nu} e_R)\cdot(\bar{q}_L \sigma_{\mu\nu} u_R)$	$\mathcal{O}_{qqq} = \epsilon^{\alpha\beta\gamma}\epsilon_{jn}((q_{Lj}^\alpha)^T C q_L^\beta)\cdot((q_L^\gamma)^T C l_{Ln})$

which leads to the following new doublets H_1 and H_2:

$$H_1 = \begin{pmatrix} G^+ \cos(\beta - \hat{\beta}^\pm) + H^+ \sin(\beta - \hat{\beta}^\pm) \\ v + \frac{1}{\sqrt{2}}(h \sin(\beta - \hat{\alpha}) + H \cos(\beta - \hat{\alpha}) + iG \cos(\beta - \hat{\beta}) + iA \sin(\beta - \hat{\beta})) \end{pmatrix} \rightarrow \begin{pmatrix} G^+ \\ v + \frac{1}{\sqrt{2}}(h + iG) \end{pmatrix} + \mathcal{O}\left(\frac{v^4}{\Lambda^2 M^2}\right),$$

$$H_2 = \begin{pmatrix} -G^+ \sin(\beta - \hat{\beta}^\pm) + H^+ \cos(\beta - \hat{\beta}^\pm) \\ \frac{1}{\sqrt{2}}(h \cos(\beta - \hat{\alpha}) - H \sin(\beta - \hat{\alpha}) - iG \sin(\beta - \hat{\beta}) + iA \cos(\beta - \hat{\beta})) \end{pmatrix} \rightarrow \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(-H + iA) \end{pmatrix} + \mathcal{O}\left(\frac{v^4}{\Lambda^2 M^2}\right), \quad (26)$$

where the arrow denotes evaluating $\beta - \hat{\alpha} \rightarrow \pi/2$ in the alignment limit [21]. The alignment limit also enforces that couplings to the light eigenstate h are SM-like, and the masses of H, A, and $H^\pm$ are comparable. In this new basis $\langle H_1^0 \rangle = v$ and $\langle H_2^0 \rangle = 0$, where H_1 now only contains the SM degrees of freedom and H_2 the remaining Higgs

fields, up to terms of order $v^4/\Lambda^2 M^2$. Misalignment of the angles $\beta \neq \hat{\beta}^{(\pm)}$ is expected when additional higher-dimensional operators are present. Contributions from the dimension-six operators are suppressed by v^2/Λ^2 in addition to v^2/M^2, where M^2 is m_A^2 or $m_{H^\pm}^2$ depending on which rotation angle is present in the approximation of

$\cos(\beta - \hat{\beta}^{(\pm)}) \simeq 1 + \mathcal{O}((v^4/\Lambda^2 M^2)^2)$ and $\sin(\beta - \hat{\beta}^{(\pm)}) \simeq \mathcal{O}(v^4/\Lambda^2 M^2)$. Inverting the transformations in Eq. (25), we have

$$\Phi_1 = \cos\beta H_1 - \sin\beta H_2, \qquad (27)$$

$$\Phi_2 = \sin\beta H_1 + \cos\beta H_2. \qquad (28)$$

The complete lists of operators in the Higgs basis are provided in Tables XII–XVII, and their Wilson coefficients written in terms of the coefficients in the standard basis are provided in Appendixes B 3–B 8, for each table, respectively. Since dimension-five and -six operators scale as $1/\Lambda$ and $1/\Lambda^2$, respectively, the effects of the mismatch between diagonalization angles are of higher order. There are

TABLE XII. Mass operators in the Higgs basis, all of which have a distinct Hermitian conjugate.

$\mathcal{O}_{lH_1}^{(11)} = \bar{l}_L e_R H_1 (H_1^\dagger H_1)$	$\mathcal{O}_{dH_1}^{(11)} = \bar{q}_L d_R H_1 (H_1^\dagger H_1)$	$\mathcal{O}_{uH_1}^{(11)} = \bar{q}_L u_R \cdot H_1^\dagger (H_1^\dagger H_1)$
$\mathcal{O}_{lH_1}^{(22)} = \bar{l}_L e_R H_1 (H_2^\dagger H_2)$	$\mathcal{O}_{dH_1}^{(22)} = \bar{q}_L d_R H_1 (H_2^\dagger H_2)$	$\mathcal{O}_{uH_1}^{(22)} = \bar{q}_L u_R \cdot H_1^\dagger (H_2^\dagger H_2)$
$\mathcal{O}_{lH_1}^{(21)} = \bar{l}_L e_R H_1 (H_2^\dagger H_1)$	$\mathcal{O}_{dH_1}^{(21)} = \bar{q}_L d_R H_1 (H_2^\dagger H_1)$	$\mathcal{O}_{uH_1}^{(21)} = \bar{q}_L u_R \cdot H_1^\dagger (H_2^\dagger H_1)$
$\mathcal{O}_{lH_1}^{(12)} = \bar{l}_L e_R H_1 (H_1^\dagger H_2)$	$\mathcal{O}_{dH_1}^{(12)} = \bar{q}_L d_R H_1 (H_1^\dagger H_2)$	$\mathcal{O}_{uH_1}^{(12)} = \bar{q}_L u_R \cdot H_1^\dagger (H_1^\dagger H_2)$
$\mathcal{O}_{lH_2}^{(22)} = \bar{l}_L e_R H_2 (H_2^\dagger H_2)$	$\mathcal{O}_{dH_2}^{(22)} = \bar{q}_L d_R H_2 (H_2^\dagger H_2)$	$\mathcal{O}_{uH_2}^{(22)} = \bar{q}_L u_R \cdot H_2^\dagger (H_2^\dagger H_2)$
$\mathcal{O}_{lH_2}^{(11)} = \bar{l}_L e_R H_2 (H_1^\dagger H_1)$	$\mathcal{O}_{dH_2}^{(11)} = \bar{q}_L d_R H_2 (H_1^\dagger H_1)$	$\mathcal{O}_{uH_2}^{(11)} = \bar{q}_L u_R \cdot H_2^\dagger (H_1^\dagger H_1)$
$\mathcal{O}_{lH_2}^{(21)} = \bar{l}_L e_R H_2 (H_2^\dagger H_1)$	$\mathcal{O}_{dH_2}^{(21)} = \bar{q}_L d_R H_2 (H_2^\dagger H_1)$	$\mathcal{O}_{uH_2}^{(21)} = \bar{q}_L u_R \cdot H_2^\dagger (H_2^\dagger H_1)$
$\mathcal{O}_{lH_2}^{(12)} = \bar{l}_L e_R H_2 (H_1^\dagger H_2)$	$\mathcal{O}_{dH_2}^{(12)} = \bar{q}_L d_R H_2 (H_1^\dagger H_2)$	$\mathcal{O}_{uH_2}^{(12)} = \bar{q}_L u_R \cdot H_2^\dagger (H_1^\dagger H_2)$

TABLE XIII. Dipole operators in the Higgs basis, all of which have a distinct Hermitian conjugate.

$\mathcal{O}_{lBH_1} = \bar{l}_L \sigma^{\mu\nu} e_R H_1 B_{\mu\nu}$	$\mathcal{O}_{dBH_1} = \bar{q}_L \sigma^{\mu\nu} d_R H_1 B_{\mu\nu}$	$\mathcal{O}_{uBH_1} = \bar{q}_L \sigma^{\mu\nu} u_R \cdot H_1^\dagger B_{\mu\nu}$
$\mathcal{O}_{lWH_1} = \bar{l}_L \sigma^{\mu\nu} e_R \tau^a H_1 W_{\mu\nu}^a$	$\mathcal{O}_{dWH_1} = \bar{q}_L \sigma^{\mu\nu} d_R \tau^a H_1 W_{\mu\nu}^a$	$\mathcal{O}_{uWH_1} = \bar{q}_L \sigma^{\mu\nu} u_R \tau^a \cdot H_1^\dagger W_{\mu\nu}^a$
$\mathcal{O}_{lBH_2} = \bar{l}_L \sigma^{\mu\nu} e_R H_2 B_{\mu\nu}$	$\mathcal{O}_{dGH_1} = \bar{q}_L \sigma^{\mu\nu} \lambda^a d_R H_1 G_{\mu\nu}^a$	$\mathcal{O}_{uGH_1} = \bar{q}_L \sigma^{\mu\nu} \lambda^a u_R \cdot H_1^\dagger G_{\mu\nu}^a$
$\mathcal{O}_{lWH_2} = \bar{l}_L \sigma^{\mu\nu} e_R \tau^a H_2 W_{\mu\nu}^a$	$\mathcal{O}_{dBH_2} = \bar{q}_L \sigma^{\mu\nu} d_R H_2 B_{\mu\nu}$	$\mathcal{O}_{uBH_2} = \bar{q}_L \sigma^{\mu\nu} u_R \cdot H_2^\dagger B_{\mu\nu}$
	$\mathcal{O}_{dWH_2} = \bar{q}_L \sigma^{\mu\nu} d_R \tau^a H_2 W_{\mu\nu}^a$	$\mathcal{O}_{uWH_2} = \bar{q}_L \sigma^{\mu\nu} u_R \tau^a \cdot H_2^\dagger W_{\mu\nu}^a$
	$\mathcal{O}_{dGH_2} = \bar{q}_L \sigma^{\mu\nu} \lambda^a d_R H_2 G_{\mu\nu}^a$	$\mathcal{O}_{uGH_2} = \bar{q}_L \sigma^{\mu\nu} \lambda^a u_R \cdot H_2^\dagger G_{\mu\nu}^a$

TABLE XIV. Derivative operators with fermionic currents in the Higgs basis.

$\mathcal{O}_{He}^{(11)} = (H_1^\dagger i \overleftrightarrow{D}_\mu H_1)(\bar{e}_R \gamma^\mu e_R)$	$\mathcal{O}_{Hd}^{(11)} = (H_1^\dagger i \overleftrightarrow{D}_\mu H_1)(\bar{d}_R \gamma^\mu d_R)$
$\mathcal{O}_{He}^{(22)} = (H_2^\dagger i \overleftrightarrow{D}_\mu H_2)(\bar{e}_R \gamma^\mu e_R)$	$\mathcal{O}_{Hd}^{(22)} = (H_2^\dagger i \overleftrightarrow{D}_\mu H_2)(\bar{d}_R \gamma^\mu d_R)$
$\mathcal{O}_{He}^{(12)} = (H_1^\dagger i \overleftrightarrow{D}_\mu H_2)(\bar{e}_R \gamma^\mu e_R) + \text{H.c.}$	$\mathcal{O}_{Hd}^{(12)} = (H_1^\dagger i \overleftrightarrow{D}_\mu H_2)(\bar{d}_R \gamma^\mu d_R) + \text{H.c.}$
$\mathcal{O}_{Hl}^{(11)[1]} = (H_1^\dagger i \overleftrightarrow{D}_\mu H_1)(\bar{l}_L \gamma^\mu l_L)$	$\mathcal{O}_{Hu}^{(11)} = (H_1^\dagger i \overleftrightarrow{D}_\mu H_1)(\bar{u}_R \gamma^\mu u_R)$
$\mathcal{O}_{Hl}^{(22)[1]} = (H_2^\dagger i \overleftrightarrow{D}_\mu H_2)(\bar{l}_L \gamma^\mu l_L)$	$\mathcal{O}_{Hu}^{(22)} = (H_2^\dagger i \overleftrightarrow{D}_\mu H_2)(\bar{u}_R \gamma^\mu u_R)$
$\mathcal{O}_{Hl}^{(12)[1]} = (H_1^\dagger i \overleftrightarrow{D}_\mu H_2)(\bar{l}_L \gamma^\mu l_L) + \text{H.c.}$	$\mathcal{O}_{Hu}^{(12)} = (H_1^\dagger i \overleftrightarrow{D}_\mu H_2)(\bar{u}_R \gamma^\mu u_R) + \text{H.c.}$
$\mathcal{O}_{Hl}^{(11)[3]} = (H_1^\dagger i \overleftrightarrow{D}_\mu^a H_1)(\bar{l}_L \tau^a \gamma^\mu l_L)$	$\mathcal{O}_{Hq}^{(11)[1]} = (H_1^\dagger i \overleftrightarrow{D}_\mu H_1)(\bar{q}_L \gamma^\mu q_L)$
$\mathcal{O}_{Hl}^{(22)[3]} = (H_2^\dagger i \overleftrightarrow{D}_\mu^a H_2)(\bar{l}_L \tau^a \gamma^\mu l_L)$	$\mathcal{O}_{Hq}^{(22)[1]} = (H_2^\dagger i \overleftrightarrow{D}_\mu H_2)(\bar{q}_L \gamma^\mu q_L)$
$\mathcal{O}_{Hl}^{(12)[3]} = (H_1^\dagger i \overleftrightarrow{D}_\mu^a H_2)(\bar{l}_L \tau^a \gamma^\mu l_L) + \text{H.c.}$	$\mathcal{O}_{Hq}^{(12)[1]} = (H_1^\dagger i \overleftrightarrow{D}_\mu H_2)(\bar{q}_L \gamma^\mu q_L) + \text{H.c.}$
	$\mathcal{O}_{Hq}^{(11)[3]} = (H_1^\dagger i \overleftrightarrow{D}_\mu^a H_1)(\bar{q}_L \tau^a \gamma^\mu q_L)$
	$\mathcal{O}_{Hq}^{(22)[3]} = (H_2^\dagger i \overleftrightarrow{D}_\mu^a H_2)(\bar{q}_L \tau^a \gamma^\mu q_L)$
	$\mathcal{O}_{Hq}^{(12)[3]} = (H_1^\dagger i \overleftrightarrow{D}_\mu^a H_2)(\bar{q}_L \tau^a \gamma^\mu q_L) + \text{H.c.}$
	$\mathcal{O}_{Hud}^{(11)} = (H_1 \cdot i D_\mu H_1)(\bar{u}_R \gamma^\mu d_R) + \text{H.c.}$
	$\mathcal{O}_{Hud}^{(22)} = (H_2 \cdot i D_\mu H_2)(\bar{u}_R \gamma^\mu d_R) + \text{H.c.}$
	$\mathcal{O}_{Hud}^{(21)} = (H_2 i \cdot \overleftrightarrow{D}_\mu H_1)(\bar{u}_R \gamma^\mu d_R) + \text{H.c.}$

TABLE XV. Derivative operators with scalar doublets in the Higgs basis.

$\mathcal{O}_{H\partial^2}^{(11)(11)} = \partial_\mu(H_1^\dagger H_1)\partial^\mu(H_1^\dagger H_1)$	$\mathcal{O}_{HD}^{(11)} = (H_1^\dagger \overleftrightarrow{D}_\mu H_1)(H_1^\dagger \overleftrightarrow{D}^\mu H_1)$
$\mathcal{O}_{H\partial^2}^{(22)(22)} = \partial_\mu(H_2^\dagger H_2)\partial^\mu(H_2^\dagger H_2)$	$\mathcal{O}_{HD}^{(22)(22)} = (H_2^\dagger \overleftrightarrow{D}_\mu H_2)(H_2^\dagger \overleftrightarrow{D}^\mu H_2)$
$\mathcal{O}_{H\partial^2}^{(11)(22)} = \partial_\mu(H_1^\dagger H_1)\partial^\mu(H_2^\dagger H_2)$	$\mathcal{O}_{HD}^{(11)(22)} = (H_1^\dagger \overleftrightarrow{D}_\mu H_1)(H_2^\dagger \overleftrightarrow{D}^\mu H_2)$
$\mathcal{O}_{H\partial^2}^{(21)(21)} = \partial_\mu(H_2^\dagger H_1)\partial^\mu(H_2^\dagger H_1) + \text{H.c.}$	$\mathcal{O}_{HD}^{(21)(21)} = (H_2^\dagger \overleftrightarrow{D}_\mu H_1)(H_2^\dagger \overleftrightarrow{D}^\mu H_1) + \text{H.c.}$
$\mathcal{O}_{H\partial^2}^{(21)(12)} = \partial_\mu(H_2^\dagger H_1)\partial^\mu(H_1^\dagger H_2)$	$\mathcal{O}_{HD}^{(21)(12)} = (H_2^\dagger \overleftrightarrow{D}_\mu H_1)(H_1^\dagger \overleftrightarrow{D}^\mu H_2)$
$\mathcal{O}_{H\partial^2}^{(21)(11)} = \partial_\mu(H_2^\dagger H_1)\partial^\mu(H_1^\dagger H_1) + \text{H.c.}$	$\mathcal{O}_{HD}^{(21)(11)} = (H_2^\dagger \overleftrightarrow{D}_\mu H_1)(H_1^\dagger \overleftrightarrow{D}^\mu H_1) + \text{H.c.}$
$\mathcal{O}_{H\partial^2}^{(21)(22)} = \partial_\mu(H_2^\dagger H_1)\partial^\mu(H_2^\dagger H_2) + \text{H.c.}$	$\mathcal{O}_{HD}^{(21)(22)} = (H_2^\dagger \overleftrightarrow{D}_\mu H_1)(H_2^\dagger \overleftrightarrow{D}^\mu H_2) + \text{H.c.}$

TABLE XVI. Scalar operators in the Higgs basis.

$\mathcal{O}_H^{(11)(11)(11)} = (H_1^\dagger H_1)^3$	$\mathcal{O}_H^{(11)(21)(21)} = (H_1^\dagger H_1)(H_2^\dagger H_1)^2 + \text{H.c.}$
$\mathcal{O}_H^{(11)(11)(22)} = (H_1^\dagger H_1)^2(H_2^\dagger H_2)$	$\mathcal{O}_H^{(11)(21)(12)} = (H_1^\dagger H_1)(H_2^\dagger H_1)(H_1^\dagger H_2)$
$\mathcal{O}_H^{(11)(22)(22)} = (H_1^\dagger H_1)(H_2^\dagger H_2)^2$	$\mathcal{O}_H^{(22)(21)(21)} = (H_2^\dagger H_2)(H_2^\dagger H_1)^2 + \text{H.c.}$
$\mathcal{O}_H^{(11)(11)(21)} = (H_1^\dagger H_1)^2(H_2^\dagger H_1) + \text{H.c.}$	$\mathcal{O}_H^{(22)(21)(12)} = (H_2^\dagger H_2)(H_2^\dagger H_1)(H_1^\dagger H_2)$
$\mathcal{O}_H^{(22)(22)(21)} = (H_2^\dagger H_2)^2(H_2^\dagger H_1) + \text{H.c.}$	$\mathcal{O}_H^{(21)(21)(21)} = (H_2^\dagger H_1)^3 + \text{H.c.}$
$\mathcal{O}_H^{(22)(22)(22)} = (H_2^\dagger H_2)^3$	$\mathcal{O}_H^{(21)(21)(12)} = (H_2^\dagger H_1)^2(H_1^\dagger H_2) + \text{H.c.}$
	$\mathcal{O}_H^{(11)(22)(21)} = (H_1^\dagger H_1)(H_2^\dagger H_2)(H_2^\dagger H_1) + \text{H.c.}$

TABLE XVII. Mixed bosonic operators in the Higgs basis.

$\mathcal{O}_{HG}^{(11)} = (H_1^\dagger H_1)G_{\mu\nu}^a G^{a\mu\nu}$	$\mathcal{O}_{HW}^{(11)} = (H_1^\dagger H_1)W_{\mu\nu}^a W^{a\mu\nu}$
$\mathcal{O}_{HG}^{(22)} = (H_2^\dagger H_2)G_{\mu\nu}^a G^{a\mu\nu}$	$\mathcal{O}_{HW}^{(22)} = (H_2^\dagger H_2)W_{\mu\nu}^a W^{a\mu\nu}$
$\mathcal{O}_{HG}^{(21)} = (H_2^\dagger H_1)G_{\mu\nu}^a G^{a\mu\nu} + \text{H.c.}$	$\mathcal{O}_{HW}^{(21)} = (H_2^\dagger H_1)W_{\mu\nu}^a W^{a\mu\nu} + \text{H.c.}$
$\mathcal{O}_{H\tilde{G}}^{(11)} = (H_1^\dagger H_1)\tilde{G}_{\mu\nu}^a G^{a\mu\nu}$	$\mathcal{O}_{H\tilde{W}}^{(11)} = (H_1^\dagger H_1)\tilde{W}_{\mu\nu}^a W^{a\mu\nu}$
$\mathcal{O}_{H\tilde{G}}^{(22)} = (H_2^\dagger H_2)\tilde{G}_{\mu\nu}^a G^{a\mu\nu}$	$\mathcal{O}_{H\tilde{W}}^{(22)} = (H_2^\dagger H_2)\tilde{W}_{\mu\nu}^a W^{a\mu\nu}$
$\mathcal{O}_{H\tilde{G}}^{(21)} = (H_2^\dagger H_1)\tilde{G}_{\mu\nu}^a G^{a\mu\nu} + \text{H.c.}$	$\mathcal{O}_{H\tilde{W}}^{(21)} = (H_2^\dagger H_1)\tilde{W}_{\mu\nu}^a W^{a\mu\nu} + \text{H.c.}$
$\mathcal{O}_{HB}^{(11)} = (H_1^\dagger H_1)B_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{HWB}^{(11)} = (H_1^\dagger \tau^a H_1)W_{\mu\nu}^a B^{\mu\nu}$
$\mathcal{O}_{HB}^{(22)} = (H_2^\dagger H_2)B_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{HWB}^{(22)} = (H_2^\dagger \tau^a H_2)W_{\mu\nu}^a B^{\mu\nu}$
$\mathcal{O}_{HB}^{(21)} = (H_2^\dagger H_1)B_{\mu\nu}B^{\mu\nu} + \text{H.c.}$	$\mathcal{O}_{HWB}^{(21)} = (H_2^\dagger \tau^a H_1)W_{\mu\nu}^a B^{\mu\nu} + \text{H.c.}$
$\mathcal{O}_{H\tilde{B}}^{(11)} = (H_1^\dagger H_1)\tilde{B}_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{H\tilde{W}B}^{(11)} = (H_1^\dagger \tau^a H_1)\tilde{W}_{\mu\nu}^a B^{\mu\nu}$
$\mathcal{O}_{H\tilde{B}}^{(22)} = (H_2^\dagger H_2)\tilde{B}_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{H\tilde{W}B}^{(22)} = (H_2^\dagger \tau^a H_2)\tilde{W}_{\mu\nu}^a B^{\mu\nu}$
$\mathcal{O}_{H\tilde{B}}^{(21)} = (H_2^\dagger H_1)\tilde{B}_{\mu\nu}B^{\mu\nu} + \text{H.c.}$	$\mathcal{O}_{H\tilde{W}B}^{(21)} = (H_2^\dagger \tau^a H_1)\tilde{W}_{\mu\nu}^a B^{\mu\nu} + \text{H.c.}$

exactly the same number of operators in the Higgs basis as the standard basis at dimension-six, 228 (including Hermitian conjugates). However, in a specified type of 2HDM, all of the operators except four-fermion operators are identical; the type of model only restricts which Wilson coefficients contribute to operators in the Higgs basis. For convenience, the notation is identical to the general 2HDM of the main text with the replacement of $\Phi \to H$ in the label.

A. Advantages of the Higgs basis

The convenience of the Higgs basis originates from the separation of the SM fields (contained in H_1 with the characteristic scale $M_1 \simeq v$) from the additional scalars, H, A, and $H^\pm$ contained in H_2 with masses at a different scale M_2. In addition to numerous calculations in the 2HDM, the Higgs basis has also been used to obtain SMEFT from the 2HDM in the alignment limit [16–18]

by integrating out the H_2 doublet at the scale $M_2 \simeq \Lambda \gg v$. Here we will point out further advantages of the Higgs basis when working within the 2HDM EFT. These include the separation of operators that modify SM couplings and masses from operators that contribute to scattering processes only, transparent correlations between scattering processes resulting from the same operator, and derivation of correlations between different operators in specific UV completions.

An example of the separation of operators that modifies the SM couplings and masses is given in Fig. 1. In the standard basis, there are 8 mass operators, $\psi^2 \phi^3$, for each fermion type (see Table II). Correspondingly, there are 8 mass operators in the Higgs basis. However, although all 8 operators in the standard basis modify the fermion mass and the Yukawa coupling, only one operator in the Higgs basis [Fig. 1(a)] does this. All other operators contribute only to scattering processes involving at least one additional Higgs boson.

Furthermore, the correlations between different scattering processes resulting from the same operator are obvious in the Higgs basis. The coefficients under the diagrams in Fig. 1 are proportionality factors assuming the operators result from the $\mathcal{O}_{\psi\Phi_1}^{(11)}$ operator in the standard basis. Proportionality factors for other cases can be easily read out from Appendix B 3. Thus, if the $\mathcal{O}_{\psi\Phi_1}^{(11)}$ is the dominant operator (as already mentioned, specific UV completions often generate only one mass operator at tree level [20,39]), we can immediately see a number of interesting correlations. For example, $\sigma(\psi\bar\psi \to HHH)/\sigma(\psi\bar\psi \to hhh) \propto \tan^6\beta$, and similar $\tan^6\beta$ enhancement is expected for other heavy tri-Higgs final states: AAA, HAA, AHH, HH^+H^-, or AH^+H^-. Correlations with many other processes featuring mixed h, H, A, and $H^\pm$ final states can be readily obtained. Enhancements of this type were recently studied in the connection with the sensitivity to the muon Yukawa coupling at a muon collider [39], but are clearly not limited to muons.

Working in the Higgs basis is also crucial when deriving correlations between different operators in specific UV completions. To illustrate this, consider a specific UV completion that generates $\mathcal{O}_{\psi\Phi_1}^{(11)}$ via tree-level mixing of SM fermions with new vectorlike fermion fields, depicted
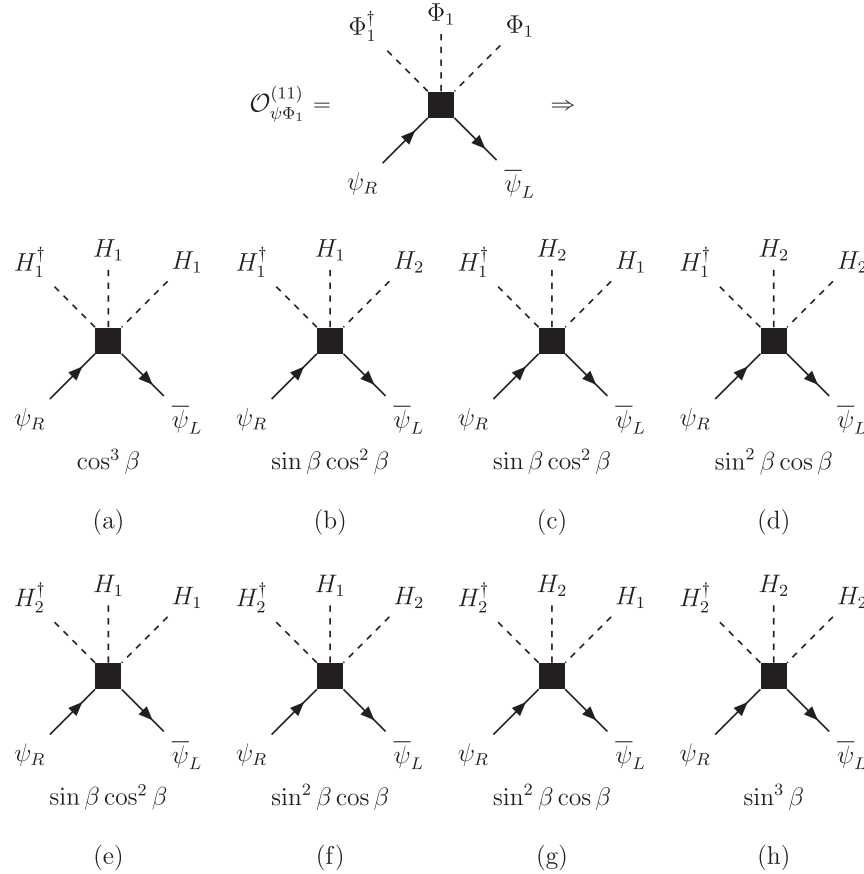

FIG. 1. Top: diagram of the $\mathcal{O}_{\psi\Phi_1}^{(11)}$ dimension-six mass operator. Bottom: the same operator decomposed in the Higgs basis. There are a total of eight diagrams (a)–(h), and their corresponding Wilson coefficients are proportional to factors of $\cos\beta$ for each H_1 and $\sin\beta$ for every H_2 (indicated under the diagrams).
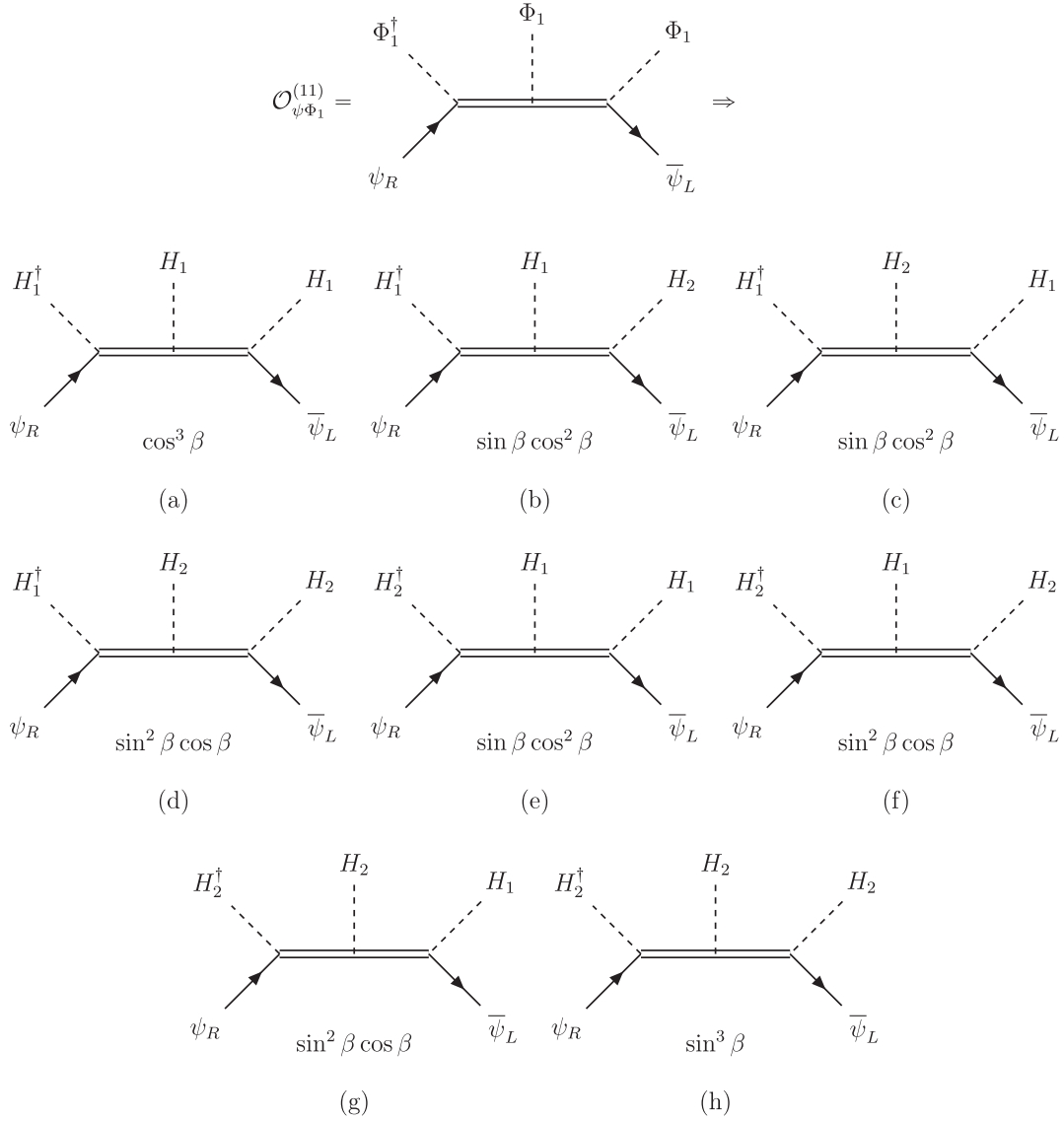
$$\mathcal{O}^{(11)}_{\psi\Phi_1} =$$

FIG. 2. Top: tree-level diagram generating $\mathcal{O}^{(11)}_{\psi\Phi_1}$ mass operator in a UV completion with vectorlike fermions. Bottom: The same operator decomposed in the Higgs basis. There are a total of eight diagrams (a)–(h), and their corresponding Wilson coefficients are proportional to factors of $\cos\beta$ for each H_1 and $\sin\beta$ for every H_2 (indicated under the diagrams).

in Fig. 2 [40]. Such a UV completion will also generate dipole operators leading to dipole moments of the fermion that mixes with new fermions. To calculate the dipole operators, consider a subset of the diagrams generating mass operators with H_1 and close the loop on pairs of remaining $H_1^\dagger H_1$ and $H_2^\dagger H_2$ in an $SU(2)_L \times U(1)_Y$-invariant way when possible. Then, dress the loop of each diagram with the B and W^a fields. The resulting diagrams are shown in Fig. 3 where the labels indicate the diagrams from which they were obtained. These diagrams generate the dipole operators $\mathcal{O}_{\psi(B,W)H_1}$ in Table XIII. Moreover, since dipole operators are obtained from the mass

operators, their Wilson coefficients are correlated.[3] This connection between mass and dipole operators, and the resulting correlations between the modification of the Yukawa coupling and electric and magnetic dipole

---

[3]The loop functions can be evaluated thanks to the separation of SM fields from heavy Higgses. Note also, that it is straightforward to see that the loops involving H_2 are $\tan^2\beta$ enhanced compared to loops involving H_1. The same results can be obtained by the complete calculation in the mass eigenstate basis in a given UV completion [20,34,35] which is however much more complicated and does not provide a simple understanding of the results.
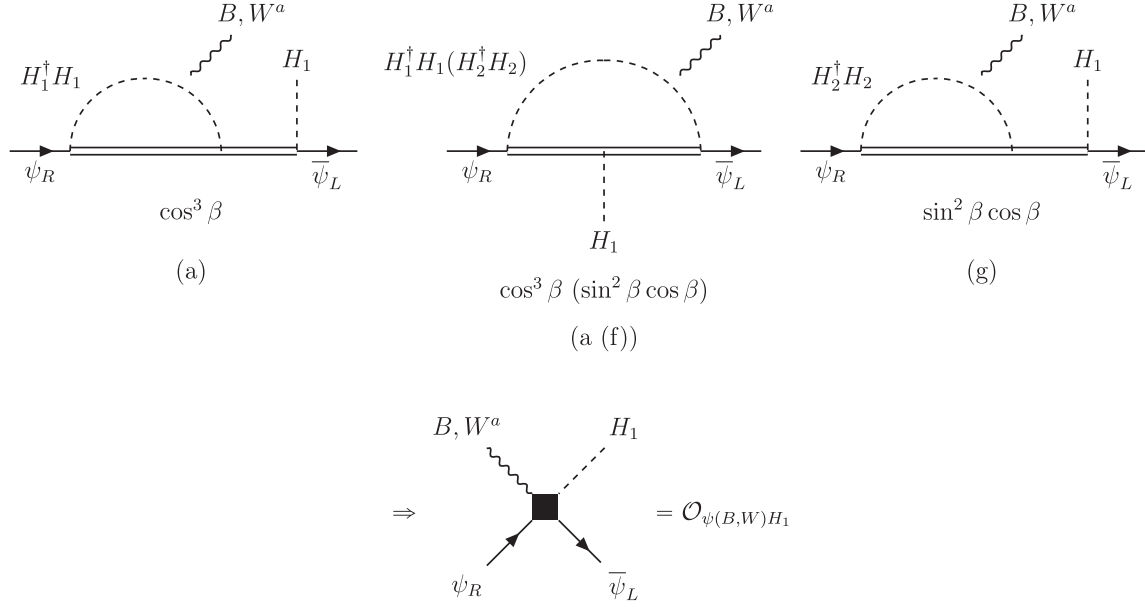
FIG. 3.   A subset of diagrams in Fig. 2, where $H_1^\dagger H_1$ or $H_2^\dagger H_2$ pairs are closed in $SU(2)$-invariant ways. The loops are dressed with a B or W^a fields, generating the dipole operators $\mathcal{O}_{\psi(B,W)H_1}$.

moments was extensively studied for the muon in Refs. [20,40,41]. Similarly, we may also consider the subset of mass diagrams containing H_2 and dress the closed loops of the remaining $H_1^\dagger H_1$ and $H_2^\dagger H_2$ pairs with B and W^a fields, generating the $\mathcal{O}_{\psi(B,W)H_2}$ operators in Table XIII. However, these operators will only affect scattering processes since H_2 does not acquire a VEV in this basis. This is yet another example of the separation of operators that illustrates the usefulness of the Higgs basis.

V. CONCLUSIONS

We have constructed the general 2HDM EFT in a Warsaw-like basis and found 228 linearly independent operators of mass dimension-six. We have also constructed specific versions of 2HDM EFTs, corresponding to four types of 2HDMs: type-I, -II, -X, and -Y, distinguished by Z_2 symmetries that restrict the couplings of Higgs doublets to SM fermions. We demonstrated that in all specific types of 2HDM EFTs, there are 76 common operators due to imposing a Z_2 symmetry on the Higgs doublets, $\Phi_1 \to -\Phi_1$ and $\Phi_2 \to +\Phi_2$, while the Z_2 charges of fermions lead to a different number of model specific operators: the type-I model predicts 62 operators, type-II predicts 48 operators, type-X comes with 52 operators, and type-Y has 50 model specific operators. We have also discussed disagreements with previous works on 2HDM EFT [9–11]. These include most notably twice as many operators which modify quark and lepton masses, as well as operators involving covariant derivatives acting on either doublet contracted with right-handed quark currents in the case of a CP-conserving

Higgs potential, compared to Ref. [9], and disagreements with Ref. [11] in constructing independent derivative operators involving only the Higgs doublets.

Furthermore, we have transformed the general 2HDM EFT to the Higgs basis and provided matching of the Wilson coefficients between the two descriptions. In the Higgs basis, in the alignment limit favored in all types of 2HDMs [42], the SM degrees of freedom are contained in one doublet, H_1, and all additional Higgses are in another doublet, H_2. There is exactly the same number of independent dimension-six operators, 228, as in the general 2HDM in the standard basis. For specific types of 2HDM EFTs, all of the operators except four-fermion operators are now identical; the type of model only restricts which Wilson coefficients contribute to operators in the Higgs basis. On specific examples, we have illustrated the advantages of working with the 2HDM EFT in the Higgs basis that included the separation of operators that modify SM couplings and masses from operators that contribute to scattering processes only, transparent correlations between scattering processes resulting from the same operator, and derivation of correlations between different operators in specific UV completions.

Generally, ϕ^6 terms affect the scalar potential, and thus the vacuum and stability conditions are modified compared to those in the renormalizable 2HDM [21]. For completeness, we have derived general vacuum and stability conditions of the scalar potential in the presence of dimension-six terms.

The SMEFT, parametrizing possible effects of new physics with characteristic scale far above the EW scale has led to a broad theoretical and experimental

effort to constrain possible effects of new physics. New contact interactions resulting from nonrenormalizable operators [26] could lead to numerous signals at the LHC and future colliders [43]. By power counting, processes mediated by these contact interactions grow with energy, and can supersede the SM background, until the theory breaks down by perturbativity and unitarity, constraining the involved Wilson coefficients [44–46]. Traditional techniques of partial wave analysis and analyticity of the amplitude [47–49] have been successful in limiting the parameter space of these energy-growing processes in the SMEFT and could be tailored to 2HDM EFT that predicts a significantly larger number of new contact interactions. Furthermore, studying the effects of renormalization group equations and one-loop mixing of the SMEFT operators [50–55] could be also extended to the 2HDM EFT.

APPENDIX A: CORRECTIONS TO THE SCALAR POTENTIAL

In this appendix, we provide details and conditions about how the presence of higher-dimensional terms affect the two vacua and stability of the scalar potential. The inclusion of higher-dimensional operators from the class ϕ^6 in the theory will introduce corrections to the potential, which now reads as $V'(\Phi_1, \Phi_2) = V(\Phi_1, \Phi_2) - \sum_{i=\phi^6} C_i \mathcal{O}_i$. Extemizing the potential of Eq. (6) while including all ϕ^6 operators in Table III and defining $\sin\beta \equiv s_\beta, \cos\beta \equiv c_\beta$ and $\tan\beta \equiv t_\beta$, we find at the vacuum v

$$
\begin{aligned}
\frac{1}{v_1}\frac{\partial V'}{\partial \Phi_1}\bigg|_v =\ & m_1^2 + t_\beta (m_{12}^2)^* + v^2(c_\beta^2\lambda_1 + s_\beta^2(\lambda_3 + \lambda_4 + \lambda_5^*)) + s_\beta c_\beta(\lambda_6 + 2\lambda_6^*) + s_\beta^2 t_\beta \lambda_7^* \\
& - v^4\Big(3c_\beta^4 C_\Phi^{(11)(11)(11)} + 3s_\beta^3 c_\beta C_\Phi^{(21)(21)} + 2s_\beta^3 c_\beta C_\Phi^{(21)(21)(12)} + s_\beta^3 c_\beta C_\Phi^{(21)(21)(12)*} \\
& + 2s_\beta^3 c_\beta C_\Phi^{(11)(22)(21)} + s_\beta^3 c_\beta C_\Phi^{(11)(22)(21)*} + 2s_\beta^2 c_\beta^2 C_\Phi^{(11)(11)(22)} + 3s_\beta^2 c_\beta^2 C_\Phi^{(11)(21)(21)} \\
& + s_\beta^2 c_\beta^2 C_\Phi^{(11)(21)(21)*} + 2s_\beta^2 c_\beta^2 C_\Phi^{(11)(21)(12)} + s_\beta^4 C_\Phi^{(11)(22)(22)} + 2s_\beta^4 C_\Phi^{(22)(21)(21)} \\
& + s_\beta^4 C_\Phi^{(22)(21)(12)} + 3s_\beta c_\beta^3 C_\Phi^{(11)(11)(21)} + 2s_\beta c_\beta^3 C_\Phi^{(11)(11)(21)*} + s_\beta^4 t_\beta C_\Phi^{(22)(22)(21)}\Big) = 0,
\end{aligned}
\tag{A1}
$$

$$
\begin{aligned}
\frac{1}{v_2}\frac{\partial V'}{\partial \Phi_2}\bigg|_v =\ & m_2^2 + \frac{m_{12}^2}{t_\beta} + v^2\Big(s_\beta^2\lambda_2 + c_\beta^2(\lambda_3 + \lambda_4 + \lambda_5) + \frac{c_\beta^2}{t_\beta}\lambda_6 + s_\beta c_\beta(2\lambda_7 + \lambda_7^*)\Big) \\
& - v^4\Big(3s_\beta^4 C_\Phi^{(22)(22)(22)} + 3s_\beta c_\beta^3 C_\Phi^{(21)(21)(21)*} + s_\beta c_\beta^3 C_\Phi^{(21)(21)(12)} + 2s_\beta c_\beta^3 C_\Phi^{(21)(21)(12)*} \\
& + s_\beta c_\beta^3 C_\Phi^{(11)(22)(21)} + 2s_\beta c_\beta^3 C_\Phi^{(11)(22)(21)*} + c_\beta^4 C_\Phi^{(11)(11)(22)} + 2c_\beta^4 C_\Phi^{(11)(21)(21)*} \\
& + c_\beta^4 C_\Phi^{(11)(21)(12)} + 2s_\beta^2 c_\beta^2 C_\Phi^{(11)(22)(22)} + s_\beta^2 c_\beta^2 C_\Phi^{(22)(21)(21)} + 3s_\beta^2 c_\beta^2 C_\Phi^{(22)(21)(21)*} \\
& + 2s_\beta^2 c_\beta^2 C_\Phi^{(22)(21)(12)} + \frac{c_\beta^4}{t_\beta} C_\Phi^{(11)(11)(21)*} + 2s_\beta^3 c_\beta C_\Phi^{(22)(22)(21)} + 3s_\beta^3 c_\beta C_\Phi^{(22)(22)(21)*}\Big) = 0.
\end{aligned}
\tag{A2}
$$

Since the parameters $m_1^2, m_2^2, \lambda_1, \lambda_2, \lambda_3,$ and λ_4 and Wilson coefficients $C_\Phi^{(11)(11)(11)}, C_\Phi^{(22)(22)(22)}, C_\Phi^{(11)(11)(22)}, C_\Phi^{(11)(21)(12)},$ $C_\Phi^{(11)(22)(22)},$ and $C_\Phi^{(22)(21)(12)}$ are Hermitian, the phases of the remaining contributions can be constrained from either extrema condition:

$$
\begin{aligned}
-\mathrm{Im}[m_{12}^2] =\ & v^2(s_\beta c_\beta \mathrm{Im}[\lambda_5] + c_\beta^2 \mathrm{Im}[\lambda_6] + s_\beta^2 \mathrm{Im}[\lambda_7]) \\
& + v^4\Big(3s_\beta^2 c_\beta^2 \mathrm{Im}[C_\Phi^{(21)(21)(21)}] + s_\beta^2 c_\beta^2 \mathrm{Im}[C_\Phi^{(21)(21)(12)}] + s_\beta^2 c_\beta^2 \mathrm{Im}[C_\Phi^{(11)(22)(21)}] \\
& + 2s_\beta c_\beta^3 \mathrm{Im}[C_\Phi^{(11)(21)(21)}] + 2s_\beta^3 c_\beta \mathrm{Im}[C_\Phi^{(22)(21)(21)}] + c_\beta^4 \mathrm{Im}[C_\Phi^{(11)(11)(21)}] + s_\beta^4 \mathrm{Im}[C_\Phi^{(22)(22)(21)}]\Big).
\end{aligned}
\tag{A3}
$$

For a complex scalar function of two variables, the conditions for positive concavity needed to guarantee these extrema are truly minima are

$$\left|\begin{matrix} \frac{\partial^2 V'}{\partial \Phi_1^\dagger \partial \Phi_1} & \frac{\partial^2 V'}{\partial \Phi_1^\dagger \partial \Phi_2} \\ \frac{\partial^2 V'}{\partial \Phi_2^\dagger \partial \Phi_1} & \frac{\partial^2 V'}{\partial \Phi_2^\dagger \partial \Phi_2} \end{matrix}\right|_v = \left(\frac{\partial^2 V'}{\partial \Phi_1^\dagger \partial \Phi_1}\right)\left(\frac{\partial^2 V'}{\partial \Phi_2^\dagger \partial \Phi_2}\right) - \left|\frac{\partial^2 V'}{\partial \Phi_1^\dagger \partial \Phi_2}\right|^2\Bigg|_v > 0,$$

$$\frac{\partial^2 V'}{\partial \Phi_1^\dagger \partial \Phi_2}\bigg|_v = \left(\frac{\partial^2 V'}{\partial \Phi_2^\dagger \partial \Phi_1}\right)^*\bigg|_v,$$

$$\frac{\partial^2 V'}{\partial \Phi_{1,2}^\dagger \partial \Phi_{1,2}}\bigg|_v > 0. \tag{A4}$$

The partial second derivatives at the vacuum are

$$\frac{\partial^2 V'}{\partial \Phi_1^\dagger \partial \Phi_1}\bigg|_v = m_1^2 + v^2(2c_\beta^2\lambda_1 + s_\beta^2(\lambda_3 + \lambda_4) + 4s_\beta c_\beta \mathrm{Re}[\lambda_6])$$

$$- v^4\Big[9c_\beta^4 C_\Phi^{(11)(11)(11)} + 4s_\beta^3 c_\beta \mathrm{Re}[C_\Phi^{(21)(21)(12)}] + 4s_\beta^3 c_\beta \mathrm{Re}[C_\Phi^{(11)(22)(21)}] + 4s_\beta^2 c_\beta^2 C_\Phi^{(11)(11)(22)} + s_\beta^4 C_\Phi^{(11)(22)(22)}$$

$$+ 6s_\beta^2 c_\beta^2 \mathrm{Re}[C_\Phi^{(11)(21)(21)}] + 4s_\beta^2 c_\beta^2 C_\Phi^{(11)(21)(12)} + s_\beta^4 C_\Phi^{(22)(21)(12)} + 12s_\beta c_\beta^3 \mathrm{Re}[C_\Phi^{(11)(11)(21)}]\Big], \tag{A5}$$

$$\frac{\partial^2 V'}{\partial \Phi_2^\dagger \partial \Phi_2}\bigg|_v = m_2^2 + v^2(2s_\beta^2\lambda_2 + c_\beta^2(\lambda_3 + \lambda_4) + 4s_\beta c_\beta \mathrm{Re}[\lambda_7])$$

$$- v^4\Big[9s_\beta^4 C_\Phi^{(22)(22)(22)} + 4s_\beta c_\beta^3 \mathrm{Re}[C_\Phi^{(21)(21)(12)}] + 4s_\beta c_\beta^3 \mathrm{Re}[C_\Phi^{(11)(22)(21)}] + c_\beta^4 C_\Phi^{(11)(11)(22)} + 4s_\beta^2 c_\beta^2 C_\Phi^{(11)(22)(22)}$$

$$+ 6s_\beta^2 c_\beta^2 \mathrm{Re}[C_\Phi^{(22)(21)(21)}] + c_\beta^4 C_\Phi^{(11)(21)(12)} + 4s_\beta^2 c_\beta^2 C_\Phi^{(22)(21)(12)} + 12s_\beta^3 c_\beta \mathrm{Re}[C_\Phi^{(22)(22)(21)}]\Big], \tag{A6}$$

and

$$\frac{\partial^2 V'}{\partial \Phi_2^\dagger \partial \Phi_1}\bigg|_v = \left(\frac{\partial^2 V'}{\partial \Phi_1^\dagger \partial \Phi_2}\right)^*\bigg|_v = (m_{12}^2)^* + v^2(s_\beta c_\beta(\lambda_3 + \lambda_4 + 2\lambda_5^*) + 2c_\beta^2\lambda_6^* + 2s_\beta^2\lambda_7^*)$$

$$- v^4\Big[9s_\beta^2 c_\beta^2 C_\Phi^{(21)(21)(21)} + 4s_\beta^2 c_\beta^2 C_\Phi^{(21)(21)(12)} + s_\beta^2 c_\beta^2 C_\Phi^{(21)(21)(12)*} + 4s_\beta^2 c_\beta^2 C_\Phi^{(11)(22)(21)}$$

$$+ s_\beta^2 c_\beta^2 C_\Phi^{(11)(22)(21)*} + 2s_\beta c_\beta^3 C_\Phi^{(11)(11)(22)} + 2s_\beta^3 c_\beta C_\Phi^{(11)(22)(22)} + 6s_\beta c_\beta^3 C_\Phi^{(11)(21)(21)}$$

$$+ 6s_\beta^3 c_\beta C_\Phi^{(22)(21)(21)} + 2s_\beta c_\beta^3 C_\Phi^{(11)(21)(12)} + 2s_\beta^3 c_\beta C_\Phi^{(22)(21)(12)} + 3c_\beta^4 C_\Phi^{(11)(11)(21)} + 3s_\beta^4 c_\beta C_\Phi^{(22)(22)(21)}\Big]. \tag{A7}$$

Finally, the presence of ϕ^6 operators may asymptotically grow over the dimension-four terms in the potential, provided the Wilson coefficients of the operators are sufficiently large. Mandating the modified potential V' is bounded below in all directions by large values of fields, we can marginalize over them in field space. Defining $A \equiv \Phi_1^\dagger \Phi_1$, $B \equiv \Phi_2^\dagger \Phi_2$, $C \equiv \mathrm{Re}[\Phi_1^\dagger \Phi_2]$, and $D \equiv \mathrm{Im}[\Phi_1^\dagger \Phi_2]$ whereby $C = \xi D$ and $AB \geq C^2 + D^2$ and keeping terms which grow asymptotically, we first set $C = D = 0$ and find

$$\frac{1}{2}\lambda_1 A^2 + \frac{1}{2}\lambda_2 B^2 + \lambda_3 AB - \left[A^3 C_\Phi^{(11)(11)(11)} + B^3 C_\Phi^{(22)(22)(22)} + A^2 B C_\Phi^{(11)(11)(22)} + AB^2 C_\Phi^{(11)(22)(22)}\right] > 0. \tag{A8}$$

If we define $A' \equiv \lambda_1^{1/2} A \equiv \sqrt{A'^2 + B'^2}s_\zeta$, $B' \equiv \lambda_2^{1/2} B \equiv \sqrt{A'^2 + B'^2}c_\zeta$ by parametrizing on a right triangle, this condition can be written as

$$(\lambda_1\lambda_2)^{3/2} + 2\lambda_1\lambda_2\lambda_3 s_\zeta c_\zeta - 2\sqrt{A'^2 + B'^2}\Big[s_\zeta^3\lambda_2^{3/2}C_\Phi^{(11)(11)(11)} + c_\zeta^3\lambda_1^{3/2}C_\Phi^{(22)(22)(22)} + s_\zeta^2 c_\zeta\lambda_1^{1/2}\lambda_2 C_\Phi^{(11)(11)(22)}$$

$$+ s_\zeta c_\zeta^2\lambda_1\lambda_2^{1/2}C_\Phi^{(11)(22)(22)}\Big] > 0. \tag{A9}$$

By setting $\zeta = \pi/2$ and 0, respectively, we find

$$\lambda_1 - 2AC_\Phi^{(11)(11)(11)} > 0, \qquad \lambda_2 - 2BC_\Phi^{(22)(22)(22)} > 0, \tag{A10}$$

and by setting $\zeta = \pi/4$, we have $A' = B'$ and the above condition can be written in terms of either $A' = \lambda_1^{1/2}A$ or $B' = \lambda_2^{1/2}B$. For all values of A and B, we find

$$(\lambda_1\lambda_2)^{3/2} + \lambda_1\lambda_2\lambda_3 - A\Big[C_\Phi^{(11)(11)(11)}\lambda_1^{1/2}\lambda_2^{3/2} + C_\Phi^{(22)(22)(22)}\lambda_1^2 + C_\Phi^{(11)(11)(22)}\lambda_1\lambda_2 + C_\Phi^{(11)(22)(22)}\lambda_1^{3/2}\lambda_2^{1/2}\Big] > 0,$$

$$(\lambda_1\lambda_2)^{3/2} + \lambda_1\lambda_2\lambda_3 - B\Big[C_\Phi^{(11)(11)(11)}\lambda_2^2 + C_\Phi^{(22)(22)(22)}\lambda_1^{3/2}\lambda_2^{1/2} + C_\Phi^{(11)(11)(22)}\lambda_1^{1/2}\lambda_2^{3/2} + C_\Phi^{(11)(22)(22)}\lambda_1\lambda_2\Big] > 0. \tag{A11}$$

Marginalizing over other values of field space when $AB = C^2 + D^2$ for $\lambda_1^{1/2}A = \lambda_2^{1/2}B$ leads to a complicated polynomial in ξ that cannot be solved analytically for all values of ξ. Notice that in the limit all Wilson coefficients are zero, we recover the same conditions on the dimension-four scalar potential [21].

APPENDIX B: MATCHING TO THE HIGGS BASIS

In this appendix, we list the translation of all terms from the standard basis to the Higgs basis using Eqs. (27) and (28). For dimension-six terms, once the type of 2HDM is specified in Tables VI and VIII–X, or XI, one can set the irrelevant Wilson coefficients to zero in this basis. For compactness, we suppress flavor indices for higher-dimensional operators, where we define $\bar{\psi}C_i\psi \equiv \bar{\psi}_a(C_i)_{ab}\psi_b$ and $\bar{\psi}C_i^*\psi \equiv \bar{\psi}_a(C_i^*)_{ba}\psi_b$. Note that for flavor-diagonal terms below, the combination of Wilson coefficients reduces to $C_i + C_i^* = 2\text{Re}[C_i]$. For convenience, the notation is identical to the general 2HDM of the main text with the replacement of $\Phi \to H$ in the label.

1. Dimension-four terms

The scalar potential [Eq. (6)] translated to the Higgs basis is

$$V(H_1, H_2) = M_1^2(H_1^\dagger H_1) + M_2^2(H_2^\dagger H_2) + (M_{12}^2 H_1^\dagger H_2 + \text{H.c.}) + \frac{1}{2}\Lambda_1(H_1^\dagger H_1)^2 + \frac{1}{2}\Lambda_2(H_2^\dagger H_2)^2 + \Lambda_3(H_1^\dagger H_1)(H_2^\dagger H_2)$$

$$+ \Lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1) + \Big(\frac{1}{2}\Lambda_5(H_1^\dagger H_2)^2 + \Lambda_6(H_1^\dagger H_1)H_1^\dagger H_2 + \Lambda_7(H_2^\dagger H_2)H_1^\dagger H_2 + \text{H.c.}\Big), \tag{B1}$$

where the new quartic couplings are, if we use the shorthand notation $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \text{Re}[\lambda_5]$,

$$\Lambda_1 = \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + 2\lambda_{345}s_\beta^2 c_\beta^2 + 4s_\beta c_\beta(c_\beta^2\text{Re}[\lambda_6] + s_\beta^2\text{Re}[\lambda_7]),$$

$$\Lambda_2 = \lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + 2\lambda_{345}s_\beta^2 c_\beta^2 - 4s_\beta c_\beta(s_\beta^2\text{Re}[\lambda_6] + c_\beta^2\text{Re}[\lambda_7]),$$

$$\Lambda_3 = (\lambda_1 + \lambda_2 - 2\lambda_{345})s_\beta^2 c_\beta^2 + \lambda_3 - 2s_\beta c_\beta(c_\beta^2 - s_\beta^2)\text{Re}[\lambda_6 - \lambda_7],$$

$$\Lambda_4 = (\lambda_1 + \lambda_2 - 2\lambda_{345})s_\beta^2 c_\beta^2 + \lambda_4 - 2s_\beta c_\beta(c_\beta^2 - s_\beta^2)\text{Re}[\lambda_6 - \lambda_7],$$

$$\Lambda_5 = (\lambda_1 + \lambda_2 - 2\lambda_{345})s_\beta^2 c_\beta^2 + c_\beta^2\lambda_5 + s_\beta^2\lambda_5^* - 2s_\beta c_\beta(c_\beta^2(\lambda_6 - \lambda_7) - s_\beta^2(\lambda_6^* - \lambda_7^*)),$$

$$\Lambda_6 = -(\lambda_1 - \lambda_3 - \lambda_4 - \lambda_5)s_\beta c_\beta^3 + (\lambda_2 - \lambda_3 - \lambda_4 - \lambda_5^*)s_\beta^3 c_\beta + c_\beta^2((c_\beta^2 - s_\beta^2)\lambda_6 - 2s_\beta^2\lambda_6^*) + s_\beta^2((c_\beta^2 - s_\beta^2)\lambda_7^* + 2c_\beta^2\lambda_7),$$

$$\Lambda_7 = -(\lambda_1 - \lambda_3 - \lambda_4 - \lambda_5^*)s_\beta^3 c_\beta + (\lambda_2 - \lambda_3 - \lambda_4 - \lambda_5)s_\beta c_\beta^3 + s_\beta^2((c_\beta^2 - s_\beta^2)\lambda_6^* + 2c_\beta^2\lambda_6) + c_\beta^2((c_\beta^2 - s_\beta^2)\lambda_7 - 2s_\beta^2\lambda_7^*). \tag{B2}$$

Note that in general, each doublet $\Phi_{1,2}$ can contain possible phases and by performing a $U(1)_Y$ rotation on Φ_2, the phase ξ can be placed on the VEV of Φ_2: $v_2 e^{i\xi}$. Additional symmetries of the scalar potential in the Higgs basis include $U(1)$ transformations of Higgs flavors $H_1 \rightarrow e^{-i\chi}$ and $H_2 \rightarrow e^{i\chi}H_2$, introducing another arbitrary phase χ. For our discussion, it suffices to neglect all possible phase factors. Otherwise, these would introduce additional phase factors on m_{12}^2, λ_5, λ_6, λ_7, and M_{12}^2, Λ_5, Λ_6, and Λ_7 (see [8] for a detailed discussion). The dimensionful parameters listed below can be simplified as

$$M_1^2 = m_1^2 c_\beta^2 + m_2^2 s_\beta^2 + 2s_\beta c_\beta \text{Re}[m_{12}^2] = -v^2\Lambda_1,$$
$$M_2^2 = m_1^2 s_\beta^2 + m_2^2 c_\beta^2 - 2s_\beta c_\beta \text{Re}[m_{12}^2]$$
$$M_{12}^2 = -(m_1^2 - m_2^2)s_\beta c_\beta + m_{12}^2 c_\beta^2 - (m_{12}^2)^* s_\beta^2 = -v^2\Lambda_6. \quad \text{(B3)}$$

Kinetic terms involving the doublets H_1 and H_2 are

$$\mathcal{L} \supset (1 + 2s_\beta c_\beta \text{Re}[\eta])(D_\mu H_1)^\dagger D^\mu H_1$$
$$+ (1 - 2s_\beta c_\beta \text{Re}[\eta])(D_\mu H_2)^\dagger D^\mu H_2$$
$$+ (\eta c_\beta^2 - \eta^* s_\beta^2)(D_\mu H_1)^\dagger D^\mu H_2 + \text{H.c.} \quad \text{(B4)}$$

If the Z_2 symmetry is enforced in the standard basis, $\eta \rightarrow 0$. The Yukawa interactions in this basis are

$$\mathcal{L} \supset -(y_e^{(1)}c_\beta + y_e^{(2)}s_\beta)\bar{l}_L e_R H_1 - (-y_e^{(1)}s_\beta + y_e^{(2)}c_\beta)\bar{l}_L e_R H_2$$
$$- (y_d^{(1)}c_\beta + y_d^{(2)}s_\beta)\bar{q}_L d_R H_1 - (-y_d^{(1)}s_\beta + y_d^{(2)}c_\beta)\bar{q}_L d_R H_2$$
$$- (y_u^{(1)}c_\beta + y_u^{(2)}s_\beta)\bar{q}_L u_R H_1^\dagger - (-y_u^{(1)}s_\beta + y_u^{(2)}c_\beta)\bar{q}_L u_R \cdot H_2^\dagger$$
$$+ \text{H.c.} \quad \text{(B5)}$$

2. Dimension-five terms

The Weinberg-like operators in the Higgs basis become

$$\mathcal{O}_{\nu\nu H}^{(11)} = (H_1 \cdot l_L)^T \mathbf{C}(H_1 \cdot l_L), \quad \text{(B6)}$$

$$\mathcal{O}_{\nu\nu H}^{(22)} = (H_2 \cdot l_L)^T \mathbf{C}(H_2 \cdot l_L), \quad \text{(B7)}$$

$$\mathcal{O}_{\nu\nu H}^{(12)} = (H_1 \cdot l_L)^T \mathbf{C}(H_2 \cdot l_L), \quad \text{(B8)}$$

whose Wilson coefficients are

$$C_{\nu\nu H}^{(11)} = c_\beta^2 C_{\nu\nu\Phi}^{(11)} + s_\beta^2 C_{\nu\nu\Phi}^{(22)} + s_\beta c_\beta C_{\nu\nu\Phi}^{(12)}, \quad \text{(B9)}$$

$$C_{\nu\nu H}^{(22)} = s_\beta^2 C_{\nu\nu\Phi}^{(11)} + c_\beta^2 C_{\nu\nu\Phi}^{(22)} - s_\beta c_\beta C_{\nu\nu\Phi}^{(12)}, \quad \text{(B10)}$$

$$C_{\nu\nu H}^{(12)} = -s_\beta c_\beta C_{\nu\nu\Phi}^{(11)} + s_\beta c_\beta C_{\nu\nu\Phi}^{(22)} + (c_\beta^2 - s_\beta^2)C_{\nu\nu\Phi}^{(12)}. \quad \text{(B11)}$$

3. Dimension-six terms–class $\psi^2\phi^3$ operators

Operators in the class of $\psi^2\phi^3$ translated to the Higgs basis are given in Table XII, whose Wilson coefficients are

$$C_{(l,d,u)H_1}^{(11)} = c_\beta^3 C_{(l,d,u)\Phi_1}^{(11)} + s_\beta^2 c_\beta C_{(l,d,u)\Phi_1}^{(22)} + s_\beta c_\beta^2 C_{(l,d,u)\Phi_1}^{(21)} + s_\beta c_\beta^2 C_{(l,d,u)\Phi_1}^{(12)}$$
$$+ s_\beta^3 C_{(l,d,u)\Phi_2}^{(22)} + s_\beta c_\beta^2 C_{(l,d,u)\Phi_2}^{(11)} + s_\beta^2 c_\beta C_{(l,d,u)\Phi_2}^{(21)} + s_\beta^2 c_\beta C_{(l,d,u)\Phi_2}^{(12)}, \quad \text{(B12)}$$

$$C_{(l,d,u)H_1}^{(22)} = s_\beta^2 c_\beta C_{(l,d,u)\Phi_1}^{(11)} + c_\beta^3 C_{(l,d,u)\Phi_1}^{(22)} - s_\beta c_\beta^2 C_{(l,d,u)\Phi_1}^{(21)} - s_\beta c_\beta^2 C_{(l,d,u)\Phi_1}^{(12)}$$
$$+ s_\beta c_\beta^2 C_{(l,d,u)\Phi_2}^{(22)} + s_\beta^3 C_{(l,d,u)\Phi_2}^{(11)} - s_\beta^2 c_\beta C_{(l,d,u)\Phi_2}^{(21)} - s_\beta^2 c_\beta C_{(l,d,u)\Phi_2}^{(12)}, \quad \text{(B13)}$$

$$C_{(l,d,u)H_1}^{(21)} = -s_\beta c_\beta^2 C_{(l,d,u)\Phi_1}^{(11)} + s_\beta c_\beta^2 C_{(l,d,u)\Phi_1}^{(22)} + c_\beta^3 C_{(l,d,u)\Phi_1}^{(21)} - s_\beta^2 c_\beta C_{(l,d,u)\Phi_1}^{(12)}$$
$$+ s_\beta^2 c_\beta C_{(l,d,u)\Phi_2}^{(22)} - s_\beta^2 c_\beta C_{(l,d,u)\Phi_2}^{(11)} + s_\beta c_\beta^2 C_{(l,d,u)\Phi_2}^{(21)} - s_\beta^3 C_{(l,d,u)\Phi_2}^{(12)}, \quad \text{(B14)}$$

$$C_{(l,d,u)H_1}^{(12)} = -s_\beta c_\beta^2 C_{(l,d,u)\Phi_1}^{(11)} + s_\beta c_\beta^2 C_{(l,d,u)\Phi_1}^{(22)} - s_\beta^2 c_\beta C_{(l,d,u)\Phi_1}^{(21)} + c_\beta^3 C_{(l,d,u)\Phi_1}^{(12)}$$
$$+ s_\beta^2 c_\beta C_{(l,d,u)\Phi_2}^{(22)} - s_\beta^2 c_\beta C_{(l,d,u)\Phi_2}^{(11)} - s_\beta^3 C_{(l,d,u)\Phi_2}^{(21)} + s_\beta c_\beta^2 C_{(l,d,u)\Phi_2}^{(12)}, \quad \text{(B15)}$$

$$C_{(l,d,u)H_2}^{(22)} = -s_\beta^3 C_{(l,d,u)\Phi_1}^{(11)} - s_\beta c_\beta^2 C_{(l,d,u)\Phi_1}^{(22)} + s_\beta^2 c_\beta C_{(l,d,u)\Phi_1}^{(21)} + s_\beta^2 c_\beta C_{(l,d,u)\Phi_1}^{(12)}$$
$$+ c_\beta^3 C_{(l,d,u)\Phi_2}^{(22)} + s_\beta^2 c_\beta C_{(l,d,u)\Phi_2}^{(11)} - s_\beta c_\beta^2 C_{(l,d,u)\Phi_2}^{(21)} - s_\beta c_\beta^2 C_{(l,d,u)\Phi_2}^{(12)}, \quad \text{(B16)}$$

$$C^{(11)}_{(l,d,u)H_2} = -s_\beta c^2_\beta C^{(11)}_{(l,d,u)\Phi_1} - s^3_\beta C^{(22)}_{(l,d,u)\Phi_1} - s^2_\beta c_\beta C^{(21)}_{(l,d,u)\Phi_1} - s^2_\beta c_\beta C^{(12)}_{(l,d,u)\Phi_1}$$
$$+ s^2_\beta c_\beta C^{(22)}_{(l,d,u)\Phi_2} + c^3_\beta C^{(11)}_{(l,d,u)\Phi_2} + s_\beta c^2_\beta C^{(21)}_{(l,d,u)\Phi_2} + s_\beta c^2_\beta C^{(12)}_{(l,d,u)\Phi_2}, \tag{B17}$$

$$C^{(21)}_{(l,d,u)H_2} = s^2_\beta c_\beta C^{(11)}_{(l,d,u)\Phi_1} - s^2_\beta c_\beta C^{(22)}_{(l,d,u)\Phi_1} - s_\beta c^2_\beta C^{(21)}_{(l,d,u)\Phi_1} + s^3_\beta C^{(12)}_{(l,d,u)\Phi_1}$$
$$+ s_\beta c^2_\beta C^{(22)}_{(l,d,u)\Phi_2} - s_\beta c^2_\beta C^{(11)}_{(l,d,u)\Phi_2} + c^3_\beta C^{(21)}_{(l,d,u)\Phi_2} - s^2_\beta c_\beta C^{(12)}_{(l,d,u)\Phi_2}, \tag{B18}$$

and

$$C^{(12)}_{(l,d,u)H_2} = s^2_\beta c_\beta C^{(11)}_{(l,d,u)\Phi_1} - s^2_\beta c_\beta C^{(22)}_{(l,d,u)\Phi_1} + s^3_\beta C^{(21)}_{(l,d,u)\Phi_1} - s_\beta c^2_\beta C^{(12)}_{(l,d,u)\Phi_1}$$
$$+ s_\beta c^2_\beta C^{(22)}_{(l,d,u)\Phi_2} - s_\beta c^2_\beta C^{(11)}_{(l,d,u)\Phi_2} - s^2_\beta c_\beta C^{(21)}_{(l,d,u)\Phi_2} + c^3_\beta C^{(12)}_{(l,d,u)\Phi_2}. \tag{B19}$$

4. Class $\psi^2 X\phi$ operators

Operators in the $\psi^2 X\phi$ class translated to the Higgs basis are given in Table XIII, whose Wilson coefficients are

$$C_{(l,d,u)BH_1} = c_\beta C_{(l,d,u)B\Phi_1} + s_\beta C_{(l,d,u)B\Phi_2}, \tag{B20}$$

$$C_{(l,d,u)BH_2} = -s_\beta C_{(l,d,u)B\Phi_1} + c_\beta C_{(l,d,u)B\Phi_2}, \tag{B21}$$

$$C_{(l,d,u)WH_1} = c_\beta C_{(l,d,u)W\Phi_1} + s_\beta C_{(l,d,u)W\Phi_2}, \tag{B22}$$

$$C_{(l,d,u)WH_2} = -s_\beta C_{(l,d,u)W\Phi_1} + c_\beta C_{(l,d,u)W\Phi_2}, \tag{B23}$$

$$C_{(d,u)GH_1} = c_\beta C_{(d,u)G\Phi_1} + s_\beta C_{(d,u)G\Phi_2}, \tag{B24}$$

and

$$C_{(d,u)GH_2} = -s_\beta C_{(d,u)G\Phi_1} + c_\beta C_{(d,u)G\Phi_2}. \tag{B25}$$

5. Class $\psi^2 \phi^2 D$ operators

Operators in the $\psi^2 \phi^2 D$ class defined in the Higgs basis are provided in Table XIV, with the following Wilson coefficients:

$$C^{(11)}_{H(e,d,u)} = c^2_\beta C^{(11)}_{\Phi(e,d,u)} + s^2_\beta C^{(22)}_{\Phi(e,d,u)} + s_\beta c_\beta (C^{(12)}_{\Phi(e,d,u)} + C^{(12)*}_{\Phi(e,d,u)}), \tag{B26}$$

$$C^{(22)}_{H(e,d,u)} = s^2_\beta C^{(11)}_{\Phi(e,d,u)} + c^2_\beta C^{(22)}_{\Phi(e,d,u)} - s_\beta c_\beta (C^{(12)}_{\Phi(e,d,u)} + C^{(12)*}_{\Phi(e,d,u)}), \tag{B27}$$

$$C^{(12)}_{H(e,d,u)} = -s_\beta c_\beta C^{(11)}_{\Phi(e,d,u)} + s_\beta c_\beta C^{(22)}_{\Phi(e,d,u)} + c^2_\beta C^{(12)}_{\Phi(e,d,u)} - s^2_\beta C^{(12)*}_{\Phi(e,d,u)}, \tag{B28}$$

$$C^{(11)[1,3]}_{H(l,q)} = c^2_\beta C^{(11)[1,3]}_{\Phi(l,q)} + s^2_\beta C^{(22)[1,3]}_{\Phi(l,q)} + s_\beta c_\beta (C^{(12)[1,3]}_{\Phi(l,q)} + C^{(12)[1,3]*}_{\Phi(l,q)}), \tag{B29}$$

$$C^{(22)[1,3]}_{H(l,q)} = s^2_\beta C^{(11)[1,3]}_{\Phi(l,q)} + c^2_\beta C^{(22)[1,3]}_{\Phi(l,q)} - s_\beta c_\beta (C^{(12)[1,3]}_{\Phi(l,q)} + C^{(12)[1,3]*}_{\Phi(l,q)}), \tag{B30}$$

$$C^{(12)[1,3]}_{H(l,q)} = -s_\beta c_\beta C^{(11)[1,3]}_{\Phi(l,q)} + s_\beta c_\beta C^{(22)[1,3]}_{\Phi(l,q)} + c^2_\beta C^{(12)[1,3]}_{\Phi(l,q)} - s^2_\beta C^{(12)[1,3]*}_{\Phi(l,q)}, \tag{B31}$$

$$C^{(11)}_{Hud} = c^2_\beta C^{(11)}_{\Phi ud} + s^2_\beta C^{(22)}_{\Phi ud} + 2s_\beta c_\beta C^{(21)}_{\Phi ud}, \tag{B32}$$

$$C^{(22)}_{Hud} = s^2_\beta C^{(11)}_{\Phi ud} + c^2_\beta C^{(22)}_{\Phi ud} - 2s_\beta c_\beta C^{(21)}_{\Phi ud}, \tag{B33}$$

and

$$C_{Hud}^{(21)} = -s_\beta c_\beta C_{\Phi ud}^{(11)} + s_\beta c_\beta C_{\Phi ud}^{(22)} + (c_\beta^2 - s_\beta^2) C_{\Phi ud}^{(21)}.$$ (B34)

6. Class $\phi^4 D^2$ operators

The $\phi^4 D^2$ class of operators translated to the Higgs basis are listed in Table XV, with Wilson coefficients

$$
\begin{aligned}
C_{H(\partial^2,D)}^{(11)(11)} = {}& c_\beta^4 C_{\Phi(\partial^2,D)}^{(11)(11)} + s_\beta^4 C_{\Phi(\partial^2,D)}^{(22)(22)} + 2s_\beta^2 c_\beta^2 C_{\Phi(\partial^2,D)}^{(11)(22)} + s_\beta^2 c_\beta^2 (C_{\Phi(\partial^2,D)}^{(21)(21)} + C_{\Phi(\partial^2,D)}^{(21)(21)*}) + s_\beta^2 c_\beta^2 (C_{\Phi(\partial^2,D)}^{(21)(12)} + C_{\Phi(\partial^2,D)}^{(21)(12)*}) \\
& + 2s_\beta c_\beta^3 (C_{\Phi(\partial^2,D)}^{(21)(11)} + C_{\Phi(\partial^2,D)}^{(21)(11)*}) + 2s_\beta^3 c_\beta (C_{\Phi(\partial^2,D)}^{(21)(22)} + C_{\Phi(\partial^2,D)}^{(21)(22)*}),
\end{aligned}
$$ (B35)

$$
\begin{aligned}
C_{H(\partial^2,D)}^{(22)(22)} = {}& s_\beta^4 C_{\Phi(\partial^2,D)}^{(11)(11)} + c_\beta^4 C_{\Phi(\partial^2,D)}^{(22)(22)} + 2s_\beta^2 c_\beta^2 C_{\Phi(\partial^2,D)}^{(11)(22)} + s_\beta^2 c_\beta^2 (C_{\Phi(\partial^2,D)}^{(21)(21)} + C_{\Phi(\partial^2,D)}^{(21)(21)*}) + s_\beta^2 c_\beta^2 (C_{\Phi(\partial^2,D)}^{(21)(12)} + C_{\Phi(\partial^2,D)}^{(21)(12)*}) \\
& - 2s_\beta^3 c_\beta (C_{\Phi(\partial^2,D)}^{(21)(11)} + C_{\Phi(\partial^2,D)}^{(21)(11)*}) - 2s_\beta c_\beta^3 (C_{\Phi(\partial^2,D)}^{(21)(22)} + C_{\Phi(\partial^2,D)}^{(21)(22)*}),
\end{aligned}
$$ (B36)

$$
\begin{aligned}
C_{H(\partial^2,D)}^{(11)(22)} = {}& s_\beta^2 c_\beta^2 C_{\Phi(\partial^2,D)}^{(11)(11)} + s_\beta^2 c_\beta^2 C_{\Phi(\partial^2,D)}^{(22)(22)} + (c_\beta^4 + s_\beta^4) C_{\Phi(\partial^2,D)}^{(11)(22)} - s_\beta^2 c_\beta^2 (C_{\Phi(\partial^2,D)}^{(21)(21)} + C_{\Phi(\partial^2,D)}^{(21)(21)*}) - 2s_\beta^2 c_\beta^2 C_{\Phi(\partial^2,D)}^{(21)(12)} \\
& + (s_\beta^3 c_\beta - s_\beta c_\beta^3)(C_{\Phi(\partial^2,D)}^{(21)(11)} + C_{\Phi(\partial^2,D)}^{(21)(11)*}) + (s_\beta c_\beta^3 - s_\beta^3 c_\beta)(C_{\Phi(\partial^2,D)}^{(21)(22)} + C_{\Phi(\partial^2,D)}^{(21)(22)*}),
\end{aligned}
$$ (B37)

$$
\begin{aligned}
C_{H(\partial^2,D)}^{(21)(21)} = {}& s_\beta^2 c_\beta^2 C_{\Phi(\partial^2,D)}^{(11)(11)} + s_\beta^2 c_\beta^2 C_{\Phi(\partial^2,D)}^{(22)(22)} - 2s_\beta^2 c_\beta^2 C_{\Phi(\partial^2,D)}^{(11)(22)} + c_\beta^4 C_{\Phi(\partial^2,D)}^{(21)(21)} + s_\beta^4 C_{\Phi(\partial^2,D)}^{(21)(21)*} - 2s_\beta^2 c_\beta^2 C_{\Phi(\partial^2,D)}^{(21)(12)} \\
& - 2s_\beta c_\beta^3 C_{\Phi(\partial^2,D)}^{(21)(11)} + 2s_\beta^3 c_\beta C_{\Phi(\partial^2,D)}^{(21)(11)*} + 2s_\beta c_\beta^3 C_{\Phi(\partial^2,D)}^{(21)(22)} - 2s_\beta^3 c_\beta C_{\Phi(\partial^2,D)}^{(21)(22)*},
\end{aligned}
$$ (B38)

$$
\begin{aligned}
C_{H(\partial^2,D)}^{(21)(12)} = {}& s_\beta^2 c_\beta^2 C_{\Phi(\partial^2,D)}^{(11)(11)} + s_\beta^2 c_\beta^2 C_{\Phi(\partial^2,D)}^{(22)(22)} - 2s_\beta^2 c_\beta^2 C_{\Phi(\partial^2,D)}^{(11)(22)} - s_\beta^2 c_\beta^2 (C_{\Phi(\partial^2,D)}^{(21)(21)} + C_{\Phi(\partial^2,D)}^{(21)(21)*}) + (c_\beta^4 + s_\beta^4) C_{\Phi(\partial^2,D)}^{(21)(12)} \\
& + (s_\beta^3 c_\beta - s_\beta c_\beta^3)(C_{\Phi(\partial^2,D)}^{(21)(11)} + C_{\Phi(\partial^2,D)}^{(21)(11)*}) + (s_\beta c_\beta^3 - s_\beta^3 c_\beta)(C_{\Phi(\partial^2,D)}^{(21)(22)} + C_{\Phi(\partial^2,D)}^{(21)(22)*}),
\end{aligned}
$$ (B39)

$$
\begin{aligned}
C_{H(\partial^2,D)}^{(21)(11)} = {}& -s_\beta c_\beta^3 C_{\Phi(\partial^2,D)}^{(11)(11)} + s_\beta^3 c_\beta C_{\Phi(\partial^2,D)}^{(22)(22)} + (s_\beta c_\beta^3 - s_\beta^3 c_\beta) C_{\Phi(\partial^2,D)}^{(11)(22)} + s_\beta c_\beta^3 C_{\Phi(\partial^2,D)}^{(21)(21)} - s_\beta^3 c_\beta C_{\Phi(\partial^2,D)}^{(21)(21)*} + (s_\beta c_\beta^3 - s_\beta^3 c_\beta) C_{\Phi(\partial^2,D)}^{(21)(12)} \\
& + (c_\beta^4 - s_\beta^2 c_\beta^2) C_{\Phi(\partial^2,D)}^{(21)(11)} - 2s_\beta^2 c_\beta^2 C_{\Phi(\partial^2,D)}^{(21)(11)*} + 2s_\beta^2 c_\beta^2 C_{\Phi(\partial^2,D)}^{(21)(22)} + (s_\beta^2 c_\beta^2 - s_\beta^4) C_{\Phi(\partial^2,D)}^{(21)(22)*},
\end{aligned}
$$ (B40)

$$
\begin{aligned}
C_{H(\partial^2,D)}^{(21)(22)} = {}& -s_\beta^3 c_\beta C_{\Phi(\partial^2,D)}^{(11)(11)} + s_\beta c_\beta^3 C_{\Phi(\partial^2,D)}^{(22)(22)} + (s_\beta^3 c_\beta - s_\beta c_\beta^3) C_{\Phi(\partial^2,D)}^{(11)(22)} - s_\beta c_\beta^3 C_{\Phi(\partial^2,D)}^{(21)(21)} + s_\beta^3 c_\beta C_{\Phi(\partial^2,D)}^{(21)(21)*} + (s_\beta^3 c_\beta - s_\beta c_\beta^3) C_{\Phi(\partial^2,D)}^{(21)(12)} \\
& + 2s_\beta^2 c_\beta^2 C_{\Phi(\partial^2,D)}^{(21)(11)} + (s_\beta^2 c_\beta^2 - s_\beta^4) C_{\Phi(\partial^2,D)}^{(21)(11)*} + (c_\beta^4 - s_\beta^2 c_\beta^2) C_{\Phi(\partial^2,D)}^{(21)(22)} - 2s_\beta^2 c_\beta^2 C_{\Phi(\partial^2,D)}^{(21)(22)*}.
\end{aligned}
$$ (B41)

7. Class ϕ^6 operators

The ϕ^6 class of operators in the Higgs basis are listed in Table XVI, with Wilson coefficients

$$
\begin{aligned}
C_H^{(11)(11)(11)} = {}& c_\beta^6 C_\Phi^{(11)(11)(11)} + s_\beta^2 c_\beta^4 C_\Phi^{(11)(11)(22)} + s_\beta^4 c_\beta^2 C_\Phi^{(11)(22)(22)} + s_\beta c_\beta^5 (C_\Phi^{(11)(11)(21)} + C_\Phi^{(11)(11)(21)*}) + s_\beta^5 c_\beta (C_\Phi^{(22)(22)(21)} \\
& + C_\Phi^{(22)(22)(21)*}) + s_\beta^6 C_\Phi^{(22)(22)(22)} + s_\beta^2 c_\beta^4 (C_\Phi^{(11)(21)(21)} + C_\Phi^{(11)(21)(21)*}) + s_\beta^2 c_\beta^4 C_\Phi^{(11)(21)(12)} + s_\beta^4 c_\beta^2 (C_\Phi^{(22)(21)(21)} \\
& + C_\Phi^{(22)(21)(21)*}) + s_\beta^4 c_\beta^2 C_\Phi^{(22)(21)(12)} + s_\beta^3 c_\beta^3 (C_\Phi^{(21)(21)(21)} + C_\Phi^{(21)(21)(21)*}) + s_\beta^3 c_\beta^3 (C_\Phi^{(21)(21)(12)} + C_\Phi^{(21)(21)(12)*}) \\
& + s_\beta^3 c_\beta^3 (C_\Phi^{(11)(22)(21)} + C_\Phi^{(11)(22)(21)*}),
\end{aligned}
$$ (B42)

$$C_H^{(11)(11)(22)} = 3s_\beta^2 c_\beta^4 C_\Phi^{(11)(11)(11)} + (c_\beta^6 + 2s_\beta^4 c_\beta^2)C_\Phi^{(11)(11)(22)} + (s_\beta^6 + 2s_\beta^2 c_\beta^4)C_\Phi^{(11)(22)(22)}$$
$$+ (-s_\beta c_\beta^5 + 2s_\beta^3 c_\beta^3)(C_\Phi^{(11)(11)(21)} + C_\Phi^{(11)(11)(21)*}) + (-s_\beta^5 c_\beta + 2s_\beta^3 c_\beta^3)(C_\Phi^{(22)(22)(21)} + C_\Phi^{(22)(22)(21)*})$$
$$+ 3s_\beta^4 c_\beta^2 C_\Phi^{(22)(22)(22)} + (s_\beta^4 c_\beta^2 - 2s_\beta^2 c_\beta^4)(C_\Phi^{(11)(21)(21)} + C_\Phi^{(11)(21)(21)*}) + (s_\beta^4 c_\beta^2 - 2s_\beta^2 c_\beta^4)C_\Phi^{(11)(21)(12)}$$
$$+ (s_\beta^2 c_\beta^4 - 2s_\beta^4 c_\beta^2)(C_\Phi^{(22)(21)(21)} + C_\Phi^{(22)(21)(21)*}) + (s_\beta^2 c_\beta^4 - 2s_\beta^4 c_\beta^2)C_\Phi^{(22)(21)(12)}$$
$$- 3s_\beta^3 c_\beta^3(C_\Phi^{(21)(21)(21)} + C_\Phi^{(21)(21)(21)*}) - 3s_\beta^3 c_\beta^3(C_\Phi^{(21)(21)(12)} + C_\Phi^{(21)(21)(12)*})$$
$$+ (-s_\beta^3 c_\beta^3 + s_\beta^5 c_\beta + s_\beta c_\beta^5)(C_\Phi^{(11)(22)(21)} + C_\Phi^{(11)(22)(21)*}),$$
\hfill (B43)

$$C_H^{(11)(22)(22)} = 3s_\beta^4 c_\beta^2 C_\Phi^{(11)(11)(11)} + (s_\beta^6 + 2s_\beta^2 c_\beta^4)C_\Phi^{(11)(11)(22)} + (c_\beta^6 + 2s_\beta^4 c_\beta^2)C_\Phi^{(11)(22)(22)}$$
$$+ (s_\beta^5 c_\beta - 2s_\beta^3 c_\beta^3)(C_\Phi^{(11)(11)(21)} + C_\Phi^{(11)(11)(21)*}) + (s_\beta c_\beta^5 - 2s_\beta^3 c_\beta^3)(C_\Phi^{(22)(22)(21)} + C_\Phi^{(22)(22)(21)*})$$
$$+ 3s_\beta^2 c_\beta^4 C_\Phi^{(22)(22)(22)} + (s_\beta^2 c_\beta^4 - 2s_\beta^4 c_\beta^2)(C_\Phi^{(11)(21)(21)} + C_\Phi^{(11)(21)(21)*}) + (s_\beta^2 c_\beta^4 - 2s_\beta^4 c_\beta^2)C_\Phi^{(11)(21)(12)}$$
$$+ (s_\beta^4 c_\beta^2 - 2s_\beta^2 c_\beta^4)(C_\Phi^{(22)(21)(21)} + C_\Phi^{(22)(21)(21)*}) + (s_\beta^4 c_\beta^2 - 2s_\beta^2 c_\beta^4)C_\Phi^{(22)(21)(12)}$$
$$+ 3s_\beta^3 c_\beta^3(C_\Phi^{(21)(21)(21)} + C_\Phi^{(21)(21)(21)*}) + 3s_\beta^3 c_\beta^3(C_\Phi^{(21)(21)(12)} + C_\Phi^{(21)(21)(12)*})$$
$$+ (s_\beta^3 c_\beta^3 - s_\beta^5 c_\beta - s_\beta c_\beta^5)(C_\Phi^{(11)(22)(21)} + C_\Phi^{(11)(22)(21)*}),$$
\hfill (B44)

$$C_H^{(11)(11)(21)} = -3s_\beta c_\beta^5 C_\Phi^{(11)(11)(11)} + (s_\beta c_\beta^5 - 2s_\beta^3 c_\beta^3)C_\Phi^{(11)(11)(22)} + (-s_\beta^5 c_\beta + 2s_\beta^3 c_\beta^3)C_\Phi^{(11)(22)(22)} + (c_\beta^6 - 2s_\beta^2 c_\beta^4)C_\Phi^{(11)(11)(21)}$$
$$- 3s_\beta^2 c_\beta^4 C_\Phi^{(11)(11)(21)*} + 3s_\beta^4 c_\beta^2 C_\Phi^{(22)(22)(21)} + (-s_\beta^6 + 2s_\beta^4 c_\beta^2)C_\Phi^{(22)(22)(21)*} + 3s_\beta^5 c_\beta C_\Phi^{(22)(22)(22)}$$
$$+ (-s_\beta^3 c_\beta^3 + 2s_\beta c_\beta^5)C_\Phi^{(11)(21)(21)} - 3s_\beta^3 c_\beta^3 C_\Phi^{(11)(21)(21)*} + (-2s_\beta^3 c_\beta^3 + s_\beta c_\beta^5)C_\Phi^{(11)(21)(12)} + 3s_\beta^3 c_\beta^3 C_\Phi^{(22)(21)(21)}$$
$$+ (s_\beta^3 c_\beta^3 - 2s_\beta^5 c_\beta)C_\Phi^{(22)(21)(21)*} + (2s_\beta^3 c_\beta^3 - s_\beta^5 c_\beta)C_\Phi^{(22)(21)(12)} + 3s_\beta^2 c_\beta^4 C_\Phi^{(21)(21)(21)} - 3s_\beta^4 c_\beta^2 C_\Phi^{(21)(21)(21)*}$$
$$+ (-s_\beta^4 c_\beta^2 + 2s_\beta^2 c_\beta^4)C_\Phi^{(21)(21)(12)} + (s_\beta^2 c_\beta^4 - 2s_\beta^4 c_\beta^2)C_\Phi^{(21)(21)(12)*} + (2s_\beta^2 c_\beta^4 - s_\beta^4 c_\beta^2)C_\Phi^{(11)(22)(21)}$$
$$+ (-2s_\beta^4 c_\beta^2 + s_\beta^2 c_\beta^4)C_\Phi^{(11)(22)(21)*},$$
\hfill (B45)

$$C_H^{(22)(22)(21)} = -3s_\beta^5 c_\beta C_\Phi^{(11)(11)(11)} + (s_\beta^5 c_\beta - 2s_\beta^3 c_\beta^3)C_\Phi^{(11)(11)(22)} + (-s_\beta c_\beta^5 + 2s_\beta^3 c_\beta^3)C_\Phi^{(11)(22)(22)} + 3s_\beta^4 c_\beta^2 C_\Phi^{(11)(11)(21)}$$
$$+ (-s_\beta^6 + 2s_\beta^4 c_\beta^2)C_\Phi^{(11)(11)(21)*} + (c_\beta^6 - 2s_\beta^2 c_\beta^4)C_\Phi^{(22)(22)(21)} - 3s_\beta^2 c_\beta^4 C_\Phi^{(22)(22)(21)*} + 3s_\beta c_\beta^5 C_\Phi^{(22)(22)(22)}$$
$$- 3s_\beta^3 c_\beta^3 C_\Phi^{(11)(21)(21)} + (-s_\beta^3 c_\beta^3 + 2s_\beta^5 c_\beta)C_\Phi^{(11)(21)(21)*} + (-2s_\beta^3 c_\beta^3 + s_\beta^5 c_\beta)C_\Phi^{(11)(21)(12)}$$
$$+ (s_\beta^3 c_\beta^3 - 2s_\beta c_\beta^5)C_\Phi^{(22)(21)(21)} + 3s_\beta^3 c_\beta^3 C_\Phi^{(22)(21)(21)*} + (2s_\beta^3 c_\beta^3 - s_\beta c_\beta^5)C_\Phi^{(22)(21)(12)} + 3s_\beta^2 c_\beta^4 C_\Phi^{(21)(21)(21)}$$
$$- 3s_\beta^4 c_\beta^2 C_\Phi^{(21)(21)(21)*} + (-s_\beta^4 c_\beta^2 + 2s_\beta^2 c_\beta^4)C_\Phi^{(21)(21)(12)} + (s_\beta^2 c_\beta^4 - 2s_\beta^4 c_\beta^2)C_\Phi^{(21)(21)(12)*}$$
$$+ (2s_\beta^2 c_\beta^4 - s_\beta^4 c_\beta^2)C_\Phi^{(11)(22)(21)} + (-2s_\beta^4 c_\beta^2 + s_\beta^2 c_\beta^4)C_\Phi^{(11)(22)(21)*},$$
\hfill (B46)

$$C_H^{(22)(22)(22)} = s_\beta^6 C_\Phi^{(11)(11)(11)} + s_\beta^4 c_\beta^2 C_\Phi^{(11)(11)(22)} + s_\beta^2 c_\beta^4 C_\Phi^{(11)(22)(22)} - s_\beta^5 c_\beta(C_\Phi^{(11)(11)(21)} + C_\Phi^{(11)(11)(21)*})$$
$$- s_\beta c_\beta^5(C_\Phi^{(22)(22)(21)} + C_\Phi^{(11)(11)(21)*}) + c_\beta^6 C_\Phi^{(22)(22)(22)} + s_\beta^4 c_\beta^2(C_\Phi^{(11)(21)(21)} + C_\Phi^{(11)(21)(21)*})$$
$$+ s_\beta^4 c_\beta^2 C_\Phi^{(11)(21)(12)} + s_\beta^2 c_\beta^4(C_\Phi^{(22)(21)(21)} + C_\Phi^{(22)(21)(21)*}) + s_\beta^2 c_\beta^4 C_\Phi^{(22)(21)(12)}$$
$$- s_\beta^3 c_\beta^3(C_\Phi^{(21)(21)(21)} + C_\Phi^{(21)(21)(21)*}) - s_\beta^3 c_\beta^3(C_\Phi^{(21)(21)(12)} + C_\Phi^{(21)(21)(12)*})$$
$$- s_\beta^3 c_\beta^3(C_\Phi^{(11)(22)(21)} + C_\Phi^{(11)(22)(21)*}),$$
\hfill (B47)

$$C_H^{(11)(21)(21)} = 3s_\beta^2 c_\beta^4 C_\Phi^{(11)(11)(11)} + (-2s_\beta^2 c_\beta^4 + s_\beta^4 c_\beta^2)C_\Phi^{(11)(11)(22)} + (-2s_\beta^4 c_\beta^2 + s_\beta^2 c_\beta^4)C_\Phi^{(11)(22)(22)} + (s_\beta^3 c_\beta^3 - 2s_\beta c_\beta^5)C_\Phi^{(11)(11)(21)}$$
$$+ 3s_\beta^3 c_\beta^3 C_\Phi^{(11)(11)(21)*} + 3s_\beta^3 c_\beta^3 C_\Phi^{(22)(22)(21)} + (s_\beta^3 c_\beta^3 - 2s_\beta^5 c_\beta)C_\Phi^{(22)(22)(21)*} + 3s_\beta^4 c_\beta^2 C_\Phi^{(22)(22)(22)}$$
$$+ (c_\beta^6 - 2s_\beta^2 c_\beta^4)C_\Phi^{(11)(21)(21)} + 3s_\beta^4 c_\beta^2 C_\Phi^{(11)(21)(21)*} + (-2s_\beta^2 c_\beta^4 + s_\beta^4 c_\beta^2)C_\Phi^{(11)(21)(12)} + 3s_\beta^2 c_\beta^4 C_\Phi^{(22)(21)(21)}$$
$$+ (s_\beta^6 - 2s_\beta^4 c_\beta^2)C_\Phi^{(22)(21)(21)*} + (s_\beta^2 c_\beta^4 - 2s_\beta^4 c_\beta^2)C_\Phi^{(22)(21)(12)} + 3s_\beta c_\beta^5 C_\Phi^{(21)(21)(21)} + 3s_\beta^5 c_\beta C_\Phi^{(21)(21)(21)*}$$
$$+ (s_\beta c_\beta^5 - 2s_\beta^3 c_\beta^3)C_\Phi^{(21)(21)(12)} + (s_\beta^5 c_\beta - 2s_\beta^3 c_\beta^3)C_\Phi^{(21)(21)(12)*} + (-2s_\beta^3 c_\beta^3 + s_\beta c_\beta^5)C_\Phi^{(11)(22)(21)}$$
$$+ (-2s_\beta^3 c_\beta^3 + s_\beta^5 c_\beta)C_\Phi^{(11)(22)(21)*}, \tag{B48}$$

$$C_H^{(11)(21)(12)} = 6s_\beta^2 c_\beta^4 C_\Phi^{(11)(11)(11)} + (-4s_\beta^2 c_\beta^4 + 2s_\beta^4 c_\beta^2)C_\Phi^{(11)(11)(22)} + (-4s_\beta^4 c_\beta^2 + 2s_\beta^2 c_\beta^4)C_\Phi^{(11)(22)(22)}$$
$$+ (4s_\beta^3 c_\beta^3 - 2s_\beta c_\beta^5)(C_\Phi^{(11)(11)(21)} + C_\Phi^{(11)(11)(21)*}) + (4s_\beta^3 c_\beta^3 - 2s_\beta^5 c_\beta)(C_\Phi^{(22)(22)(21)} + C_\Phi^{(22)(22)(21)*})$$
$$+ 6s_\beta^4 c_\beta^2 C_\Phi^{(22)(22)(22)} + (2s_\beta^4 c_\beta^2 - 4s_\beta^2 c_\beta^4)(C_\Phi^{(11)(21)(21)} + C_\Phi^{(11)(21)(21)*}) + (c_\beta^6 + 3s_\beta^4 c_\beta^2 - 2s_\beta^2 c_\beta^4)C_\Phi^{(11)(21)(12)}$$
$$+ (-4s_\beta^4 c_\beta^2 + 2s_\beta^2 c_\beta^4)(C_\Phi^{(22)(21)(21)} + C_\Phi^{(22)(21)(21)*}) + (s_\beta^6 + 3s_\beta^2 c_\beta^4 - 2s_\beta^4 c_\beta^2)C_\Phi^{(22)(21)(12)} - 6s_\beta^3 c_\beta^3(C_\Phi^{(21)(21)(21)}$$
$$+ C_\Phi^{(21)(21)(21)*}) + (2s_\beta^5 c_\beta + 2s_\beta c_\beta^5 - 2s_\beta^3 c_\beta^3)(C_\Phi^{(21)(21)(12)} + C_\Phi^{(21)(21)(12)*})$$
$$+ (-4s_\beta^3 c_\beta^3 + s_\beta c_\beta^5 + s_\beta^5 c_\beta)(C_\Phi^{(11)(22)(21)} + C_\Phi^{(11)(22)(21)*}), \tag{B49}$$

$$C_H^{(22)(21)(21)} = 3s_\beta^4 c_\beta^2 C_\Phi^{(11)(11)(11)} + (-2s_\beta^4 c_\beta^2 + s_\beta^2 c_\beta^4)C_\Phi^{(11)(11)(22)} + (-2s_\beta^2 c_\beta^4 + s_\beta^4 c_\beta^2)C_\Phi^{(11)(22)(22)} - 3s_\beta^3 c_\beta^3 C_\Phi^{(11)(11)(21)}$$
$$+ (-s_\beta^3 c_\beta^3 + 2s_\beta^5 c_\beta)C_\Phi^{(11)(11)(21)*} + (-s_\beta^3 c_\beta^3 + 2s_\beta c_\beta^5)C_\Phi^{(22)(22)(21)} - 3s_\beta^3 c_\beta^3 C_\Phi^{(22)(22)(21)*} + 3s_\beta^2 c_\beta^4 C_\Phi^{(22)(22)(22)}$$
$$+ 3s_\beta^2 c_\beta^4 C_\Phi^{(11)(21)(21)} + (s_\beta^6 - 2s_\beta^4 c_\beta^2)C_\Phi^{(11)(21)(21)*} + (-2s_\beta^4 c_\beta^2 + s_\beta^2 c_\beta^4)C_\Phi^{(11)(21)(12)} + (c_\beta^6 - 2s_\beta^2 c_\beta^4)C_\Phi^{(22)(21)(21)}$$
$$+ 3s_\beta^4 c_\beta^2 C_\Phi^{(22)(21)(21)*} + (-2s_\beta^2 c_\beta^4 + s_\beta^4 c_\beta^2)C_\Phi^{(22)(21)(12)} - 3s_\beta c_\beta^5 C_\Phi^{(21)(21)(21)} - 3s_\beta^5 c_\beta C_\Phi^{(21)(21)(21)*}$$
$$+ (-s_\beta c_\beta^5 + 2s_\beta^3 c_\beta^3)C_\Phi^{(21)(21)(12)} + (-s_\beta^5 c_\beta + 2s_\beta^3 c_\beta^3)C_\Phi^{(21)(21)(12)*} + (2s_\beta^3 c_\beta^3 - s_\beta c_\beta^5)C_\Phi^{(11)(22)(21)}$$
$$+ (2s_\beta^3 c_\beta^3 - s_\beta^5 c_\beta)C_\Phi^{(11)(22)(21)*}, \tag{B50}$$

$$C_H^{(22)(21)(12)} = 6s_\beta^4 c_\beta^2 C_\Phi^{(11)(11)(11)} + (-4s_\beta^4 c_\beta^2 + 2s_\beta^2 c_\beta^4)C_\Phi^{(11)(11)(22)} + (-4s_\beta^2 c_\beta^4 + 2s_\beta^4 c_\beta^2)C_\Phi^{(11)(22)(22)}$$
$$+ (-4s_\beta^3 c_\beta^3 + 2s_\beta^5 c_\beta)(C_\Phi^{(11)(11)(21)} + C_\Phi^{(11)(11)(21)*}) + (-4s_\beta^3 c_\beta^3 + 2s_\beta c_\beta^5)(C_\Phi^{(22)(22)(21)} + C_\Phi^{(22)(22)(21)*})$$
$$+ 6s_\beta^2 c_\beta^4 C_\Phi^{(22)(22)(22)} + (2s_\beta^2 c_\beta^4 - 4s_\beta^4 c_\beta^2)(C_\Phi^{(11)(21)(21)} + C_\Phi^{(11)(21)(21)*}) + (s_\beta^6 + 3s_\beta^2 c_\beta^4 - 2s_\beta^4 c_\beta^2)C_\Phi^{(11)(21)(12)}$$
$$+ (-4s_\beta^2 c_\beta^4 + 2s_\beta^4 c_\beta^2)(C_\Phi^{(22)(21)(21)} + C_\Phi^{(22)(21)(21)*}) + (c_\beta^6 + 3s_\beta^4 c_\beta^2 - 2s_\beta^2 c_\beta^4)C_\Phi^{(22)(21)(12)} + 6s_\beta^3 c_\beta^3(C_\Phi^{(21)(21)(21)}$$
$$+ C_\Phi^{(21)(21)(21)*}) + (-2s_\beta^5 c_\beta - 2s_\beta c_\beta^5 + 2s_\beta^3 c_\beta^3)(C_\Phi^{(21)(21)(12)} + C_\Phi^{(21)(21)(12)*})$$
$$+ (4s_\beta^3 c_\beta^3 - s_\beta c_\beta^5 - s_\beta^5 c_\beta)(C_\Phi^{(11)(22)(21)} + C_\Phi^{(11)(22)(21)*}), \tag{B51}$$

$$C_H^{(21)(21)(21)} = -s_\beta^3 c_\beta^3 C_\Phi^{(11)(11)(11)} + s_\beta^3 c_\beta^3 C_\Phi^{(11)(11)(22)} - s_\beta^3 c_\beta^3 C_\Phi^{(11)(22)(22)} + s_\beta^2 c_\beta^4 C_\Phi^{(11)(11)(21)} - s_\beta^4 c_\beta^2 C_\Phi^{(11)(11)(21)*}$$
$$+ s_\beta^2 c_\beta^4 C_\Phi^{(22)(22)(21)} - s_\beta^4 c_\beta^2 C_\Phi^{(22)(22)(21)*} + s_\beta^3 c_\beta^3 C_\Phi^{(22)(22)(22)} - s_\beta c_\beta^5 C_\Phi^{(11)(21)(21)} - s_\beta^5 c_\beta C_\Phi^{(11)(21)(21)*}$$
$$+ s_\beta^3 c_\beta^3 C_\Phi^{(11)(21)(12)} + s_\beta c_\beta^5 C_\Phi^{(22)(21)(21)} + s_\beta^5 c_\beta C_\Phi^{(22)(21)(21)*} - s_\beta^3 c_\beta^3 C_\Phi^{(22)(21)(12)} + c_\beta^6 C_\Phi^{(21)(21)(21)}$$
$$- s_\beta^6 C_\Phi^{(21)(21)(21)*} - s_\beta^2 c_\beta^4 C_\Phi^{(21)(21)(12)} + s_\beta^4 c_\beta^2 C_\Phi^{(21)(21)(12)*} - s_\beta^2 c_\beta^4 C_\Phi^{(11)(22)(21)} + s_\beta^4 c_\beta^2 C_\Phi^{(11)(22)(21)*}, \tag{B52}$$

$$
\begin{aligned}
C_H^{(21)(21)(12)} =& -s_\beta^3 c_\beta^3 C_\Phi^{(11)(11)(11)} + 3s_\beta^3 c_\beta^3 C_\Phi^{(11)(11)(22)} - 3s_\beta^3 c_\beta^3 C_\Phi^{(11)(22)(22)} + (-s_\beta^4 c_\beta^2 + 2s_\beta^2 c_\beta^4) C_\Phi^{(11)(11)(21)} \\
&+ (s_\beta^2 c_\beta^4 - 2s_\beta^4 c_\beta^2) C_\Phi^{(11)(11)(21)*} + (-s_\beta^4 c_\beta^2 + 2s_\beta^2 c_\beta^4) C_\Phi^{(22)(22)(21)} + (s_\beta^2 c_\beta^4 - 2s_\beta^4 c_\beta^2) C_\Phi^{(22)(22)(21)*} \\
&+ 3s_\beta^3 c_\beta^3 C_\Phi^{(22)(22)(22)} + (-s_\beta c_\beta^5 + 2s_\beta^3 c_\beta^3) C_\Phi^{(11)(21)(21)} + (-s_\beta^5 c_\beta + 2s_\beta^3 c_\beta^3) C_\Phi^{(11)(21)(21)*} \\
&+ (s_\beta^3 c_\beta^3 - s_\beta^5 c_\beta - s_\beta c_\beta^5) C_\Phi^{(11)(21)(12)} + (s_\beta c_\beta^5 - 2s_\beta^3 c_\beta^3) C_\Phi^{(22)(21)(21)} + (s_\beta^5 c_\beta - 2s_\beta^3 c_\beta^3) C_\Phi^{(11)(22)(22)*} \\
&+ (-s_\beta^3 c_\beta^3 + s_\beta^5 c_\beta + s_\beta c_\beta^5) C_\Phi^{(22)(21)(12)} - 3s_\beta^2 c_\beta^4 C_\Phi^{(21)(21)(21)} + 3s_\beta^4 c_\beta^2 C_\Phi^{(21)(21)(21)*} + (c_\beta^6 + 2s_\beta^4 c_\beta^2) C_\Phi^{(21)(21)(12)} \\
&+ (-s_\beta^6 - 2s_\beta^2 c_\beta^4) C_\Phi^{(21)(21)(12)*} + (s_\beta^4 c_\beta^2 - 2s_\beta^2 c_\beta^4) C_\Phi^{(11)(22)(21)} + (-s_\beta^2 c_\beta^4 + 2s_\beta^4 c_\beta^2) C_\Phi^{(11)(22)(21)*},
\end{aligned}
\tag{B53}
$$

and

$$
\begin{aligned}
C_H^{(11)(22)(21)} =& -6s_\beta^3 c_\beta^3 C_\Phi^{(11)(11)(11)} + (-2s_\beta c_\beta^5 - 2s_\beta^5 c_\beta + 2s_\beta^3 c_\beta^3) C_\Phi^{(11)(11)(22)} + (2s_\beta^5 c_\beta + s_\beta c_\beta^5 - 2s_\beta^3 c_\beta^3) C_\Phi^{(11)(22)(22)} \\
&+ (4s_\beta^2 c_\beta^4 - 2s_\beta^4 c_\beta^2) C_\Phi^{(11)(11)(21)} + (2s_\beta^2 c_\beta^4 - 4s_\beta^4 c_\beta^2) C_\Phi^{(11)(11)(21)*} + (4s_\beta^2 c_\beta^4 - 2s_\beta^4 c_\beta^2) C_\Phi^{(22)(22)(21)} \\
&+ (-4s_\beta^4 c_\beta^2 + 2s_\beta^2 c_\beta^4) C_\Phi^{(22)(22)(21)*} + 6s_\beta^3 c_\beta^3 C_\Phi^{(22)(22)(22)} + (4s_\beta^3 c_\beta^3 - 2s_\beta c_\beta^5) C_\Phi^{(11)(21)(21)} \\
&+ (4s_\beta^3 c_\beta^3 - 2s_\beta^5 c_\beta) C_\Phi^{(11)(21)(21)*} + (-s_\beta^5 c_\beta + 4s_\beta^3 c_\beta^3 - s_\beta c_\beta^5) C_\Phi^{(11)(21)(12)} + (-4s_\beta^3 c_\beta^3 + 2s_\beta c_\beta^5) C_\Phi^{(22)(21)(21)} \\
&+ (-4s_\beta^3 c_\beta^3 + 2s_\beta^5 c_\beta) C_\Phi^{(22)(21)(21)*} + (s_\beta c_\beta^5 - 4s_\beta^3 c_\beta^3 + s_\beta^5 c_\beta) C_\Phi^{(22)(21)(12)} - 6s_\beta^2 c_\beta^4 C_\Phi^{(21)(21)(21)} \\
&+ 6s_\beta^4 c_\beta^2 C_\Phi^{(21)(21)(21)*} + (2s_\beta^4 c_\beta^2 - 4s_\beta^2 c_\beta^4) C_\Phi^{(21)(21)(12)} + (-2s_\beta^2 c_\beta^4 + 4s_\beta^4 c_\beta^2) C_\Phi^{(21)(21)(12)*} \\
&+ (c_\beta^6 + 3s_\beta^4 c_\beta^2 - 2s_\beta^2 c_\beta^4) C_\Phi^{(11)(22)(21)} + (-s_\beta^6 - 3s_\beta^2 c_\beta^4 + 2s_\beta^4 c_\beta^2) C_\Phi^{(11)(22)(21)*}.
\end{aligned}
\tag{B54}
$$

8. Class $X^2\phi^2$ operators

Finally, for $X^2\phi^2$ operators listed in Table XVII, the Wilson coefficients are

$$
C_{H(G,W,B)}^{(11)} = c_\beta^2 C_{\Phi(G,W,B)}^{(11)} + s_\beta^2 C_{\Phi(G,W,B)}^{(22)} + s_\beta c_\beta (C_{\Phi(G,W,B)}^{(21)} + C_{\Phi(G,W,B)}^{(21)*}),
\tag{B55}
$$

$$
C_{H(G,W,B)}^{(22)} = s_\beta^2 C_{\Phi(G,W,B)}^{(11)} + c_\beta^2 C_{\Phi(G,W,B)}^{(22)} - s_\beta c_\beta (C_{\Phi(G,W,B)}^{(21)} + C_{\Phi(G,W,B)}^{(21)*}),
\tag{B56}
$$

$$
C_{H(G,W,B)}^{(21)} = -s_\beta c_\beta C_{\Phi(G,W,B)}^{(11)} + s_\beta c_\beta C_{\Phi(G,W,B)}^{(22)} + c_\beta^2 C_{\Phi(G,W,B)}^{(21)} - s_\beta^2 C_{\Phi(G,W,B)}^{(21)*},
\tag{B57}
$$

$$
C_{H(\tilde{G},\tilde{W},\tilde{B})}^{(11)} = c_\beta^2 C_{\Phi(\tilde{G},\tilde{W},\tilde{B})}^{(11)} + s_\beta^2 C_{\Phi(\tilde{G},\tilde{W},\tilde{B})}^{(22)} + s_\beta c_\beta (C_{\Phi(\tilde{G},\tilde{W},\tilde{B})}^{(21)} + C_{\Phi(\tilde{G},\tilde{W},\tilde{B})}^{(21)*}),
\tag{B58}
$$

$$
C_{H(\tilde{G},\tilde{W},\tilde{B})}^{(22)} = s_\beta^2 C_{\Phi(\tilde{G},\tilde{W},\tilde{B})}^{(11)} + c_\beta^2 C_{\Phi(\tilde{G},\tilde{W},\tilde{B})}^{(22)} - s_\beta c_\beta (C_{\Phi(\tilde{G},\tilde{W},\tilde{B})}^{(21)} + C_{\Phi(\tilde{G},\tilde{W},\tilde{B})}^{(21)*}),
\tag{B59}
$$

$$
C_{H(\tilde{G},\tilde{W},\tilde{B})}^{(21)} = -s_\beta c_\beta C_{\Phi(\tilde{G},\tilde{W},\tilde{B})}^{(11)} + s_\beta c_\beta C_{\Phi(\tilde{G},\tilde{W},\tilde{B})}^{(22)} + c_\beta^2 C_{\Phi(\tilde{G},\tilde{W},\tilde{B})}^{(21)} - s_\beta^2 C_{\Phi(\tilde{G},\tilde{W},\tilde{B})}^{(21)*},
\tag{B60}
$$

$$
C_{H(WB,\tilde{W}B)}^{(11)} = c_\beta^2 C_{\Phi(WB,\tilde{W}B)}^{(11)} + s_\beta^2 C_{\Phi(WB,\tilde{W}B)}^{(22)} + s_\beta c_\beta (C_{\Phi(WB,\tilde{W}B)}^{(21)} + C_{\Phi(WB,\tilde{W}B)}^{(21)*}),
\tag{B61}
$$

$$
C_{H(WB,\tilde{W}B)}^{(22)} = s_\beta^2 C_{\Phi(WB,\tilde{W}B)}^{(11)} + c_\beta^2 C_{\Phi(WB,\tilde{W}B)}^{(22)} - s_\beta c_\beta (C_{\Phi(WB,\tilde{W}B)}^{(21)} + C_{\Phi(WB,\tilde{W}B)}^{(21)*}),
\tag{B62}
$$

and

$$
C_{H(WB,\tilde{W}B)}^{(21)} = -s_\beta c_\beta C_{\Phi(WB,\tilde{W}B)}^{(11)} + s_\beta c_\beta C_{\Phi(WB,\tilde{W}B)}^{(22)} + c_\beta^2 C_{\Phi(WB,\tilde{W}B)}^{(21)} - s_\beta^2 C_{\Phi(WB,\tilde{W}B)}^{(21)*}.
\tag{B63}
$$

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