

Dark matter relic density in strongly interacting dark sectors with light vector mesons

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Stable dark matter particles may arise as pseudo-Goldstone bosons from the confinement of dark quarks interacting via a non-Abelian gauge force. Their relic abundance is determined not by annihilations into visible particles but by dark pion number-changing processes within the dark sector, such as $3\pi_D \rightarrow 2\pi_D$. However, if the dark vector mesons ρ_D are light enough for $3\pi_D \rightarrow \pi_D\rho_D$ annihilations to be kinematically allowed, this process dominates and significantly delays freeze-out. As a result, the preferred dark matter mass scale increases and bounds from the Bullet Cluster can be evaded.

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I. INTRODUCTION

An attractive alternative to the paradigm of weakly interacting massive particles is the idea that dark matter (DM) is part of a strongly interacting dark sector [1]. At high energies, such a dark sector can be described in terms of dark quarks interacting via the dark gluons of a non-Abelian extension of the Standard Model (SM) gauge group. At low energies, on the other hand, the dark sector confines, and the dark quarks and gluons are bound in dark mesons and dark baryons. In analogy to SM quantum chromodynamics, the pseudoscalar mesons, called dark pions, are expected to be the lightest dark sector state, because they are the pseudo-Goldstone bosons of chiral symmetry breaking. Indeed, if the dark quark masses are sufficiently small, the dark pions can be significantly lighter than the confinement scale. In contrast to the SM, however, the dark pions may be stable, due to either a $U(1)$ or a parity symmetry [2–4] (although the latter may be broken through gravitational effects [5]), making them attractive DM candidates [6–8].

Strongly interacting dark sectors are an attractive target for collider searches due to their striking signature: If a pair

of dark quarks is produced in a hard process, it will generate a shower of dark hadrons, some of which may decay into SM particles, while others evade detection [9–18]. The results are one or more semivisible [19,20] or emerging [21] jets. These signatures have been explored in a bottom-up way by varying phenomenological parameters such as the masses of dark mesons and the fraction of invisible particles [22–25], and corresponding searches have been carried out by ATLAS [26] and CMS [27]. In order to make a connection to the DM puzzle, however, it becomes necessary to explore the cosmological history of strongly interacting dark sectors and to understand whether the dark pions can be produced in the right amount to explain observations of the cosmic microwave background [8].

A central result in this context is the so-called strongly interacting massive particle (SIMP) mechanism [6,28]: dark pions may participate in number-changing processes, such as $3\pi_D \rightarrow 2\pi_D$ via the Wess-Zumino-Witten anomaly. At low temperatures, these processes lead to the conversion of rest mass into kinetic energy and hence a depletion of the dark sector. The freeze-out temperature, when number-changing processes become inefficient, then determines the relic abundance of dark pions. Comparison to observations leads to a strict upper bound of $m_{\pi_D} \lesssim 100$ MeV in order to avoid an overabundance of DM [6,29–31]. This bound is in tension with a lower bound on the dark pion mass obtained from observations of the Bullet Cluster, which constrains the DM self-interaction cross section [32–34].

In this paper, we point out an important modification of the SIMP mechanism: In addition to number-changing processes involving only dark pions, there may also be processes involving heavier dark sector states, in particular,

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the vector mesons, called dark ρ mesons [35–37]. Indeed, if the dark quarks have masses comparable to the confinement scale, it is possible to have $m_{\rho_D} < 2m_{\pi_D}$. This scenario is particularly interesting for collider experiments, since the dark ρ meson in this case cannot decay into dark pions and must instead decay into SM particles (e.g., via kinetic mixing with the SM photon). The lifetime of the dark ρ mesons can be quite long, such that their decays lead to displaced vertices in the detector [17,38].

We show that, if the process $3\pi_D \rightarrow \pi_D \rho_D$ is kinematically allowed at low temperatures, it will typically dominate the rate of number-changing processes. This is due to a favorable velocity dependence (the process proceeds via s wave, whereas the process involving only pions proceeds via d wave) and a resonant enhancement if the internal dark pions can be nearly on shell. We calculate the relevant cross sections, discuss how thermal effects determine the width of the resonance, and solve the resulting Boltzmann equation. Our central result is that the presence of dark ρ mesons relaxes the cosmological bound on the dark pion mass. It thus becomes possible to realize the SIMP mechanism for heavier dark pions and smaller couplings, thereby evading the Bullet Cluster constraint.

The remainder of this paper is structured as follows. We first introduce the strongly interacting dark sector model and discuss the spectrum of dark mesons expected from nonperturbative calculations. We then present the relevant Boltzmann equations and the required reaction rates and finally calculate the dark pion relic density and the Bullet Cluster constraint.

II. MODEL DETAILS

We start from an $SU(N_{c_D})$ gauge group, with N_{f_D} mass-degenerate Dirac fermions q_D in its fundamental representation. The high-energy Lagrangian for such a theory is given by

$$\mathcal{L}_{\text{UV}} = -\frac{1}{4} G_{D\mu\nu}^a G_D^{\mu\nu,a} + \bar{q}_D (i\gamma^\mu D_\mu - M_{q_D}) q_D, \quad (1)$$

where M_{q_D} is the mass of the dark quarks, $G_D^{\mu\nu}$ denotes the dark gluon field strength tensor, and D_μ is the gauge covariant derivative. While we leave N_{c_D}, N_{f_D} as free parameters, it is important to note that chiral symmetry breaking only takes place for $N_{f_D} < 3N_{c_D}$.

At low energies, chiral symmetry breaking leads to confinement. The dynamics of the resulting pseudo-Nambu-Goldstone bosons, called dark pions π_D , are described by a chiral Lagrangian. The kinetic term and the mass term are contained in

$$\mathcal{L}_{\text{Ch}} = \frac{f_{\pi_D}^2}{4} \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \left[\frac{\mu_D^3}{2} \text{Tr}(M_{q_D} U^\dagger) + \text{H.c.} \right] \quad (2)$$

with the $SU(N_{f_D})$ matrix $U \equiv \exp(2i\pi_D/f_{\pi_D})$, the dark pion decay constant f_{π_D} , and the quark condensate μ_D^3 . In addition, the terms in Eq. (2) give rise to interactions between even numbers of dark pions. In the chirally broken phase, the dark pions are the lightest mesons in the spectrum, and they can be stabilized by a suitable discrete or continuous symmetry.

In addition, there will also be heavier dark mesons in the spectrum, in particular, vector mesons analogous to the SM ρ mesons. Interactions between dark pions and dark vector mesons are introduced by using the massive Yang-Mills approach. The corresponding covariant derivative is given by [39]

$$D_\mu U = \partial_\mu U + ig[U, \rho_{D\mu}], \quad (3)$$

with the $\pi_D \pi_D \rho_D$ coupling $g \approx m_{\rho_D}/(\sqrt{2}f_{\pi_D})$ obtained from the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation [40,41]. Hence, the chiral Lagrangian of dark pions and dark vector mesons, expanded up to terms with at most four dark pion fields, reads

$$\begin{aligned} \mathcal{L}_{\text{Ch}} \supset & \text{Tr}(D_\mu \pi_D D^\mu \pi_D) + m_{\pi_D}^2 \text{Tr}(\pi_D^2) + \frac{m_{\pi_D}^2}{3f_{\pi_D}^2} \text{Tr}(\pi_D^4) \\ & - \frac{2}{3f_{\pi_D}^2} \text{Tr}(\pi_D^2 D_\mu \pi_D D^\mu \pi_D - \pi_D D_\mu \pi_D \pi_D D^\mu \pi_D) \end{aligned} \quad (4)$$

in agreement with the literature. For $N_{f_D} \geq 3$, the Wess-Zumino-Witten (WZW) term induces an anomalous five-point interaction given by

$$\mathcal{L}_{\text{WZW}} = \frac{2N_{c_D}}{15\pi^2 f_{\pi_D}^5} \epsilon^{\mu\nu\rho\sigma} \text{Tr}(\pi_D \partial_\mu \pi_D \partial_\nu \pi_D \partial_\rho \pi_D \partial_\sigma \pi_D). \quad (5)$$

The free parameters in the above chiral Lagrangian ($m_{\pi_D}, m_{\rho_D}, f_{\pi_D}$) need to be calculated using nonperturbative methods. Along with N_{c_D}, N_{f_D} , a QCD-like strongly interacting theory with mass-degenerate quarks has two additional free parameters, one mass ratio and one mass scale [18]. Once these inputs are fixed, the dark meson spectrum can be computed using nonperturbative methods, e.g., lattice simulations. In the UV the two free parameters could be considered $M_{q_D}, M_{q_D}/\Lambda_D$, which are traded in for, e.g., $m_{\pi_D}, m_{\pi_D}/f_{\pi_D}$ in the chirally broken phase.

In the following, we are particularly interested in the process $3\pi_D \rightarrow \pi_D \rho_D$ (see Fig. 1 for the corresponding Feynman diagrams) and its effect on the relic density of dark pions. For this process to be allowed in the nonrelativistic limit (i.e., for negligible kinetic energy of the initial state particles), we require $m_{\rho_D}/m_{\pi_D} < 2$. Incidentally, this condition also implies that dark ρ mesons cannot decay into pairs of dark pions. Nevertheless, dark ρ mesons do, in general, decay into SM particles (for example, through kinetic mixing with the SM photon [42,43], see Refs. [7,12,17,44,45]

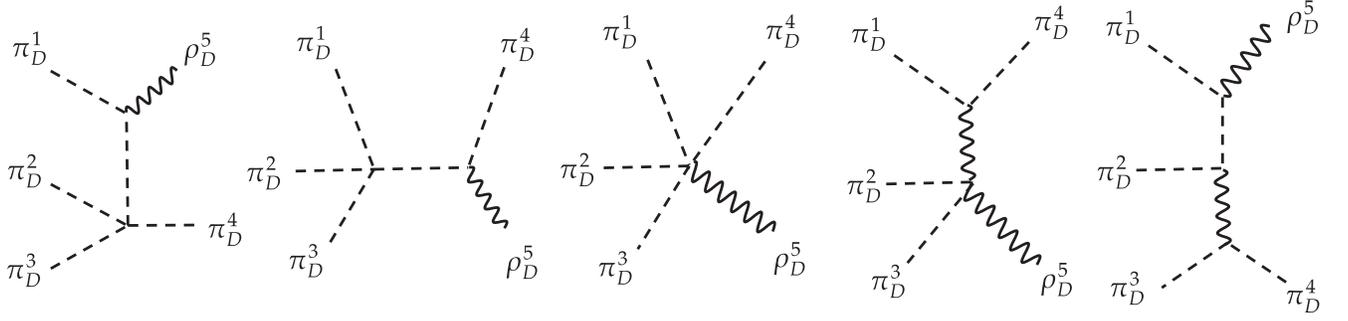


FIG. 1. Representative Feynman diagrams for the processes that contribute to the $3\pi_D \rightarrow \pi_D \rho_D$ cross section. Additional diagrams are obtained from the first, third, fourth, and fifth diagram through permutations of the pion states.

for explicit constructions). These (inverse) decays can efficiently transfer energy and entropy between the dark sector and the SM and keep the two sectors in thermal equilibrium.

It will therefore be convenient to use m_{π_D} and the mass ratio m_{ρ_D}/m_{π_D} as free parameters for our analysis. Using the results from Ref. [46] and neglecting the effect of varying N_{c_D} and N_{f_D} , we obtain an approximate relation between f_{π_D} , m_{π_D} , and m_{ρ_D} ,

$$\xi \equiv \frac{m_{\pi_D}}{f_{\pi_D}} = 7.79 \frac{m_{\pi_D}}{m_{\rho_D}} + 0.57 \left(\frac{m_{\pi_D}}{m_{\rho_D}} \right)^2, \quad (6)$$

which is valid for $1 < m_{\rho_D}/m_{\pi_D} \lesssim 20$, where the lower limit on m_{ρ_D}/m_{π_D} corresponds to $\xi \sim 8$.

In principle, one could consider the entire range of above fit. For large ξ , however, the validity of chiral perturbation theory becomes increasingly dubious and higher-order corrections become relevant [29]. In the present work, we therefore only consider dark ρ meson masses larger than $1.45m_{\pi_D}$ (equivalently $\xi \lesssim 5.7$), corresponding to the range where the Gell-Mann–Oakes–Renner relation [47] is expected to be satisfied [48–51]. This bound implies, in particular, that the forbidden annihilation process $\pi_D \pi_D \rightarrow \rho_D \rho_D$ studied in Ref. [8] is not relevant for the calculation of the dark pion relic density. The case $m_{\rho_D} > 2m_{\pi_D}$, on the other hand, has previously been studied in Ref. [35]. In this case, the process $3\pi_D \rightarrow 2\pi_D$ may receive a strong resonant enhancement from on-shell intermediate dark vector mesons.

III. DARK PION RELIC DENSITY

Let us now turn to the calculation of the dark pion relic density. The Boltzmann equation describes the evolution of the density of a particle species as it falls out of equilibrium. Integrating the Boltzmann equation yields the relic abundance of a stable particle given the contributing freeze-out processes. If ρ_D is in equilibrium with the SM bath throughout and $3\pi_D \rightarrow \pi_D \rho_D$ annihilations dominate the depletion of dark pions, we can use the principle of detailed balance to write the Boltzmann equation as

$$\dot{n}_{\pi_D} + 3Hn_{\pi_D} = \langle \sigma v^2 \rangle_{3\pi_D \rightarrow \pi_D \rho_D} n_{\pi_D} \left((n_{\pi_D}^{\text{eq}})^2 - n_{\pi_D}^2 \right), \quad (7)$$

where n_{π_D} is the dark pion number density and H denotes the Hubble rate.

In the nonrelativistic limit, the thermally averaged cross section is given by

$$\langle \sigma v^2 \rangle_{3\pi_D \rightarrow \pi_D \rho_D} = \frac{|\mathcal{M}|_{3\pi_D \rightarrow \pi_D \rho_D}^2}{144\pi S_\alpha S_\beta m_{\pi_D}^3} \sqrt{4 - 5y + y^2}, \quad (8)$$

with $S_\alpha = 3!$ and $S_\beta = 1$ denoting the number of permutations of identical particles in the initial and final state, respectively, and $y = m_{\rho_D}^2/(4m_{\pi_D}^2)$.

Representative diagrams for the process are shown in Fig. 1. Diagrams with an s -channel ρ_D are velocity suppressed and thus negligible in the final amplitude. The sum of all diagrams yields the total squared amplitude

$$|\mathcal{M}|_{3\pi_D \rightarrow \pi_D \rho_D}^2 = \frac{8m_{\pi_D}^4 (1-y)(4-y) \left(\frac{\Gamma_{\text{th}}^2}{m_{\pi_D}^2} + 4y^2 \right) \left(5N_{f_D}^4 \frac{\Gamma_{\text{th}}^2}{m_{\pi_D}^2} (13y+2)^2 + 32(2Ay^2 + 2By + C) \right)}{3f_{\pi_D}^6 \left(\frac{\Gamma_{\text{th}}^2}{m_{\pi_D}^2} + 64 \right) (2y+1)^2 \left(9 \frac{\Gamma_{\text{th}}^2}{m_{\pi_D}^2} + 64(1-y)^2 \right)}, \quad (9)$$

with

$$\begin{aligned}
A &= \frac{(821N_{f_D}^4 - 168N_{f_D}^2 + 36)}{N_{f_D}(N_{f_D}^2 - 1)^2}, & \langle \sigma v^2 \rangle_{3\pi_D \rightarrow 2\pi_D} &= \frac{5\sqrt{5}N_{c_D}^2 \kappa_{3\pi_D \rightarrow 2\pi_D} \xi^{10}}{1536\pi^5 m_{\pi_D}^5 x^2}, \quad (14) \\
B &= \frac{(245N_{f_D}^4 - 114N_{f_D}^2 + 36)}{N_{f_D}(N_{f_D}^2 - 1)^2}, \\
C &= \frac{(37N_{f_D}^4 - 30N_{f_D}^2 + 18)}{N_{f_D}(N_{f_D}^2 - 1)^2}, \quad (10)
\end{aligned}$$

and Γ_{th} being the thermal width of dark pions discussed below. Details of this calculation can be found in Supplemental Material [52].

As m_{ρ_D} approaches $2m_{\pi_D}$ from below, the internal dark pion can go on shell and the cross section grows rapidly. At first sight, this should lead to a divergence, since the dark pion is stable and therefore does not have a decay width. In a thermal plasma, however, dark pions do not have an infinite lifetime, due to interactions with other particles in the plasma. This leads to a thermal self-energy, the imaginary part of which can be interpreted as an effective decay width [56].

We present the derivation of the dark pion thermal width Γ_{th} in Supplemental Material [52]. The main contribution is found to arise from the scattering of two dark pions. For $m_{\rho_D} \approx 2m_{\pi_D}$ and low temperatures, i.e., $x \equiv m_{\pi_D}/T \gg 1$, we find

$$\Gamma_{\text{th}} = \frac{8\pi(N_{f_D}^2 - 1)}{x^2} e^{-x} m_{\pi_D}^3 \sigma_c, \quad (11)$$

with

$$\begin{aligned}
\sigma_c &\approx \frac{1}{64\pi} \frac{3N_{f_D}^4 - 2N_{f_D}^2 + 6m_{\pi}^2}{N_{f_D}^2(N_{f_D}^2 - 1)} \frac{m_{\pi}^2}{f_{\pi}^4} \\
&= \frac{3}{64\pi} \frac{m_{\pi}^2}{f_{\pi}^4} (1 + \mathcal{O}(N_{f_D}^{-2})) \quad (12)
\end{aligned}$$

denoting the two-pion scattering cross section in the limit of vanishing initial velocities.

We emphasize that the thermal width becomes exponentially suppressed at small temperatures, as a result of the suppressed number density of dark pions that can participate in scattering. In the temperature range relevant for freeze-out ($x \approx 20$) the contribution from the dark pion width is therefore negligible unless m_{ρ_D} is extremely close to $2m_{\pi_D}$.

It is instructive to compare our result to the conventional freeze-out of SIMPs through the WZW anomaly. The corresponding Boltzmann equation reads

$$\dot{n}_{\pi_D} + 3Hn_{\pi_D} = \langle \sigma v^2 \rangle_{3\pi_D \rightarrow 2\pi_D} n_{\pi_D}^2 (n_{\pi_D}^{\text{eq}} - n_{\pi_D}), \quad (13)$$

where the thermally averaged cross section is given by

with

$$\kappa_{3\pi_D \rightarrow 2\pi_D} = \frac{N_{f_D}(N_{f_D}^2 - 4)}{(N_{f_D}^2 - 1)^2} = \frac{1}{N_{f_D}} + \mathcal{O}(N_{f_D}^{-3}). \quad (15)$$

The cross sections for the processes $3\pi_D \rightarrow \pi_D \rho_D$ and $3\pi_D \rightarrow 2\pi_D$ can both be written as

$$\langle \sigma v^2 \rangle \equiv \frac{\alpha^{\text{eff}}}{m_{\pi_D}^5}. \quad (16)$$

To understand where the $3\pi_D \rightarrow \pi_D \rho_D$ process dominates over the well-known $3\pi_D \rightarrow 2\pi_D$ SIMP process, it is useful to consider the ratio

$$\begin{aligned}
R &\equiv \frac{\langle \sigma v^2 \rangle_{3\pi_D \rightarrow \pi_D \rho_D}}{\langle \sigma v^2 \rangle_{3\pi_D \rightarrow 2\pi_D}} = \frac{\alpha_{3\pi_D \rightarrow \pi_D \rho_D}^{\text{eff}}}{\alpha_{3\pi_D \rightarrow 2\pi_D}^{\text{eff}}} \\
&\approx (600-2800) \times \frac{1}{N_{c_D}^2 \xi^4} \frac{x^2}{\sqrt{1-y}}, \quad (17)
\end{aligned}$$

where the range for the numerical factor has been obtained by varying y in the range $0.6 < y < 1$ and N_{f_D} between 3 and 6, noting that the process $3\pi_D \rightarrow 2\pi_D$ does not exist for $N_{f_D} = 2$ [8]. We find that the ratio R is much larger than unity for all values that we consider. As expected, R grows rapidly for $y \rightarrow 1$ and also grows with increasing x . This is because the process $3\pi_D \rightarrow 2\pi_D$ proceeds via d wave, whereas the process $3\pi_D \rightarrow \pi_D \rho_D$ proceeds via s wave. We conclude that the latter process will completely dominate dark pion freeze-out for $m_{\rho_D} < 2m_{\pi_D}$. As shown in Supplemental Material [52], also direct annihilations of the dark pions into SM fermions via an off-shell dark ρ meson are negligible as long as the SM coupling of the dark ρ meson is sufficiently small.

IV. RESULTS AND DISCUSSION

It is convenient to express the Boltzmann equation in terms of the dimensionless quantity $Y_{\pi_D} = n_{\pi_D}/s$, where $s = 2\pi^2/45g_{\star}T^3$ is the total entropy density. The Boltzmann equation (7) then takes the form

$$\frac{dY}{dx} = \frac{s^2}{\tilde{H}x} \langle \sigma v^2 \rangle Y(Y_{\text{eq}}^2 - Y^2), \quad (18)$$

where following Ref. [57] we have introduced the modified Hubble rate $\tilde{H} = (\frac{8\pi^3}{90}g)^{1/2} \frac{T^2}{M_{\text{pl}}} [1 + \frac{1}{3} \frac{d(\ln g_{\star})}{d(\ln T)}]^{-1}$, with g (g_{\star}) denoting the energy (entropy) degrees of freedom taken from Ref. [58]. We solve the Boltzmann equation in log-space.

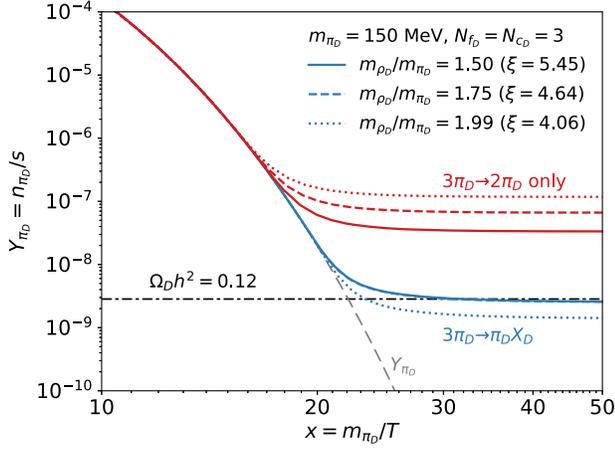


FIG. 2. Solutions of the Boltzmann equation for the (dimensionless) dark pion number density as a function of inverse temperature. For the red lines only the process $3\pi_D \rightarrow 2\pi_D$ is included, whereas the blue lines include $3\pi_D \rightarrow \pi_D X_D$ with $X_D = \pi_D, \rho_D$. The value of Y_{π_D} that corresponds to the observed DM relic abundance is indicated by the dot-dashed black line, while the equilibrium value of Y_{π_D} is represented by the dashed gray line.

In Fig. 2 we show the solution of the Boltzmann equation for $m_{\pi_D} = 100$ MeV, $N_{f_D} = N_{c_D} = 3$, and different values of m_{ρ_D}/m_{π_D} . As expected, the dimensionless dark pion number density Y_{π_D} follows the equilibrium value $Y_{\pi_D}^{\text{eq}} = (N_{f_D}^2 - 1)m_{\pi_D}^3 (x/(2\pi))^{3/2} \exp(-x)$ until it freezes out at $x_f \approx 20$ and becomes constant. For the chosen parameters, the observed DM relic abundance $\Omega_D h^2 = 0.12$ is approximately reproduced. For comparison, we also show the evolution of Y_{π_D} obtained when only the process $3\pi_D \rightarrow 2\pi_D$ is considered [59]. For the same masses, the predicted relic abundance is too large by approximately an order of magnitude.

We can understand this result analytically, by writing Eq. (18) as

$$\frac{dY}{dx} = \frac{\lambda_{3 \rightarrow 2}}{x^5} Y(Y_{\text{eq}}^2 - Y^2), \quad (19)$$

with

$$\lambda_{3 \rightarrow 2} = \frac{2\sqrt{5}}{675} \pi^{5/2} g^{3/2} M_{\text{Pl}} m_{\pi_D}^4 \langle \sigma v^2 \rangle, \quad (20)$$

where $g = g_* = 10.75$ in the temperature range of interest and we can drop the derivative in the expression for \dot{H} .

Typically, $Y_{\text{eq}} \ll Y$ during freeze-out and the corresponding term in the Boltzmann equation can be neglected. Treating $\lambda_{3 \rightarrow 2}$ as a constant during freeze-out by setting $\lambda_{3 \rightarrow 2} \equiv \lambda_{3 \rightarrow 2}(x = x_f)$, Eq. (19) can be approximately solved analytically, which yields the asymptotic solution

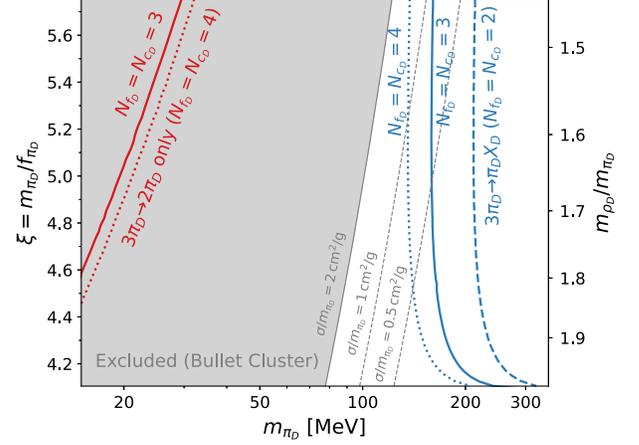


FIG. 3. Combinations of m_{π_D} and $\xi = m_{\pi_D}/f_{\pi_D}$ [or m_{ρ_D}/m_{π_D} via Eq. (6)] that yield $\Omega_D h^2 = 0.12$ in agreement with observations. For the red lines only the process $3\pi_D \rightarrow 2\pi_D$ is included, whereas for the blue lines $3\pi_D \rightarrow \rho_D \pi_D$ is also taken into consideration. Note that the process $3\pi_D \rightarrow 2\pi_D$ does not exist for $N_{f_D} = 2$. The gray shaded region is excluded by the Bullet Cluster bound on the DM self-interaction cross section (evaluated for $N_{f_D} \gg 1$).

$$Y_{\infty} \approx \sqrt{2} \frac{x_f^2}{\sqrt{\lambda_{3 \rightarrow 2}}}. \quad (21)$$

Hence, the DM relic abundance scales with the DM mass and effective coupling [as defined in Eq. (16)] as $\Omega_D h^2 \sim m_{\pi_D}^{3/2} / \sqrt{\alpha_{\text{eff}}}$. Therefore, when $3\pi_D \rightarrow \pi_D \rho_D$ annihilations dominate over $3\pi_D \rightarrow 2\pi_D$, the preferred dark pion mass scale increases as $m_{\pi_D} \sim R^{1/3}$ relative to the usual expectation for SIMPs annihilating via the WZW term. For typical values of ξ , m_{ρ_D}/m_{π_D} , and N_{c_D} this corresponds to a factor of 2–3, and even more for $m_{\rho_D} \rightarrow 2m_{\pi_D}$, see Eq. (17).

We explicitly confirm this expectation in Fig. 3, where we plot the combinations of m_{π_D} and ξ [or, equivalently, m_{ρ_D}/m_{π_D} as related to ξ by Eq. (6)] that yield $\Omega_D h^2 = 0.12$ via a numerical solution of the Boltzmann equation for different choices of N_{f_D} and N_{c_D} . As before, we show for comparison the result when considering $3\pi_D \rightarrow 2\pi_D$ only, which is robustly excluded by the Bullet Cluster constraint on DM self-interactions, given by [33,34]

$$\frac{\sigma_c}{m_{\pi_D}} \lesssim 2 \text{ cm}^2/\text{g}, \quad (22)$$

with σ_c as in Eq. (12). Including the dark ρ mesons in the final state, on the other hand, increases the dark pion mass scale sufficiently to evade these constraints. Because of the interplay between ξ and m_{ρ_D}/m_{π_D} in Eq. (9), we find that the preferred value of m_{π_D} is largely independent of these parameters and can be approximately written as $m_{\pi_D} \approx 330 \text{ MeV}/N_{f_D}^{2/3}$.

In conclusion, we have shown that, for strongly interacting dark sectors with $m_{\rho_D}/m_{\pi_D} < 2$, the dominant process that changes the number density of dark pions in the early Universe is $3\pi_D \rightarrow \pi_D \rho_D$. We emphasize that this process does not depend on any nonperturbative parameters other than m_{π_D} , f_{π_D} , and m_{ρ_D} and—in contrast to the conventionally studied process $3\pi_D \rightarrow 2\pi_D$ —does not rely on the WZW anomaly, i.e., it also exists for theories with only two light flavors. In contrast to the recently proposed Co-SIMP mechanism [60,61], the process that we consider requires no interactions between the dark sector and SM particles beyond those needed for thermalization (which only places a very weak lower bound on the decay width of the dark ρ mesons). As a result, we obtain a theoretically clean and robust prediction for the dark pion mass that reproduces the observed DM relic abundance. We have specifically considered the range $1.45 \leq m_{\rho_D}/m_{\pi_D} < 2$ corresponding to $4 \lesssim m_{\pi_D}/f_{\pi_D} \lesssim 6$, within the validity of chiral perturbation theory. We find typical values of m_{π_D} around 100 MeV reproduce the measured DM relic abundance, with only mild dependence on the other parameters. Crucially, these dark pion masses satisfy the Bullet Cluster constraint on DM self-interactions—unlike the masses favored by freeze-out

via $3\pi_D \rightarrow 2\pi_D$ —and therefore provide an important benchmark scenario for further exploration. We emphasize that the required mass spectrum implies that the dark ρ mesons can only decay into SM particles, leading to exciting signatures at laboratory experiments.

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